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Pion-to-proton ratio for unaccompanied high-energy cosmic-ray hadrons at mountain altitude using transition-radiation detector

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A transition-radiation (TR) detector, consisting of 24 modules of styrofoam radiators and multiwire proportional chambers, and an ionization calorimeter have been used to measure the pion-to-proton ratio among the unaccompanied cosmic-ray hadrons at a mountain altitude of 730 $\mathrm{g\,cm^{-2}}$. Using the characteristics of the TR detector obtained from calibrations with particle beams at accelerators, the π/p ratio has been determined for cosmic-ray hadrons as $\pi/p = 0.96\pm0.15$ for hadron energy = 400—800 GeV, and $\pi/p = 0.45\pm0.25$ for energy >800 GeV. Monte Carlo simulations of hadron cascades in the atmosphere using the approximate criterion of unaccompaniment suggest that the observed π/p ratio as well as the previously reported neutral-to-charge ratio can be understood by assuming a value of about $\frac{1}{3}$ for the charge exchange in nucleon—air-nucleus inelastic interactions at energies above 400 GeV.

I. INTRODUCTION

Measurements of the composition of high-energy cosmic-ray hadron beams at various levels in the atmosphere provide important information about characteristics of high-energy hadron interactions and primary cosmic rays. These measurements are also needed for proper interpretation of experiments carried out at mountain altitude or sea level for study of hadron interactions. Evidence[1] for increase of proton—air-nucleus interaction cross section with energy and thus the variation of the proton-proton cross section with energy was obtained from the measurements of the unaccompanied hadron flux at various altitudes in the atmosphere as well as from the study of cosmic rays in the atmosphere. This required a knowledge[2] of the pion-to-proton ratio for the hadrons. Similarly a knowledge of the pion-to-proton ratio is also required for interpretation of experiments[3,4] studying the variation of proton—iron-nucleus interaction cross sections at high energies. Since accelerator experiments cannot provide information on the pion-interaction cross section for various nuclei at high energies in the near future, a measurement of the pion content of the hadron beam is necessary for determining the pion-interaction cross section[4,5] at high energies. Many workers have used an indirect approach for determining the pion-to-proton ratio. In this method[3,6] the neutral-to-charged ratio[7] for the hadrons has been used to estimate the pion-to-proton ratio after assuming a value for the ratio of neutrons to protons in the hadron beam. However this latter ratio is rather uncertain due to the lack of knowledge of charge-exchange cross sections at high energies. A direct determination of the π/p ratio at low energies ($\lesssim 100$ GeV) has been carried out using magnet spectrometers[8,9] or Cherenkov counters.[10] The relativistic increase in ionization for pions relative to protons for energies above 100 GeV has also been used for measuring[11–13] the π/p ratio. However, all these methods become insensitive for distinguishing between pions and protons for energies $\gtrsim 300$ GeV. For higher energies, detection of transition radiation, which is emitted by a superrelativistic particle crossing an interface between media with different dielectric properties and which depends sensitively on the Lorentz factor of the particle, is possibly the only method currently available for distinguishing between pions and protons.

We present here the details of an experiment carried out at the Sacramento Ridge Cosmic Ray Laboratory, Sunspot (2900-m altitude, 730 $\mathrm{g\,cm^{-2}}$), New Mexico, during 1973—1976 using a transition-radiation detector (TRD) for determining the pion-to-proton ratio for unaccompanied hadrons of energy above 400 GeV. Preliminary results from this experiment have been reported earlier.[14–16] Since this was the first cosmic-ray experiment attempting to use transition-radiation phenomenon to distinguish between hadrons, the design of the TRD is discussed in some detail in Sec. II along with a brief description of other elements of the experimental system. The expected response of the TRD for cosmic-ray hadrons obtained from Monte Carlo simulations using the response measured in accelerator experiments is discussed in Sec. III. The experimental results obtained from a comparison of observations with expectations are presented in Sec. IV. Results on the pion-to-proton ratio obtained in this experiment are discussed and compared with results from other experiments and calculations in Sec. V. Conclusions derived from this experimental study of composition of unaccompanied hadron flux at mountain altitude are presented in the last section.

II. EXPERIMENTAL SYSTEM

The apparatus used in the experiment is shown in Fig. 1. It consists basically of a calorimeter, which is used to select the hadron and measure its energy, and a transition-radiation detector which is used to measure the transition radiation emitted by the hadron in the radiator modules.

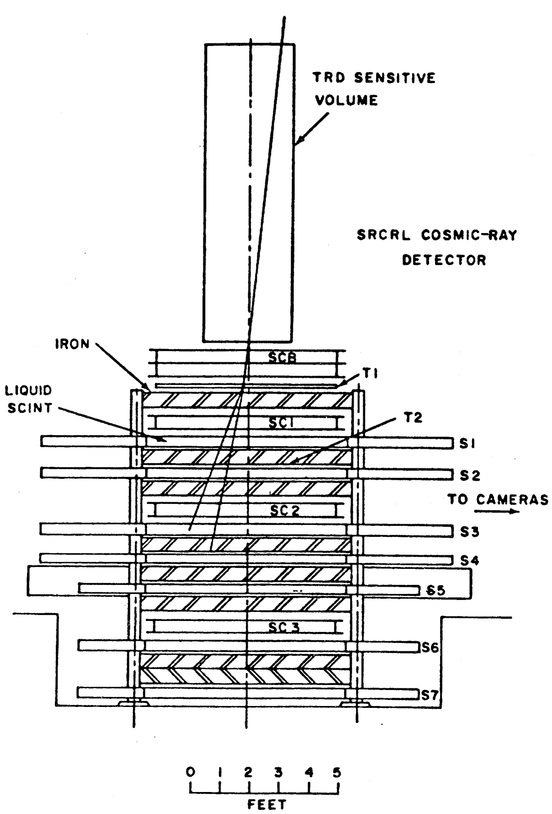

FIG. 1. A schematic diagram of the experimental system.

A. Calorimeter

The calorimeter of area 2.29×2.13 m^2 had seven layers of iron absorber. All layers were 120 g cm^{-2} thick except the second (100 g cm^{-2}) and seventh (240 g cm^{-2}). Under each layer of iron were placed two liquid-scintillator tanks side by side, each with an active area of 2 m$\times$1 m and depth of 7.4 cm (6.5 g cm^{-2}) in the center. The scintillator in each tank was viewed by a total of six photomultipliers (RCA 6655A), three on each of the short sides. Vertically the total absorber thickness was thus 940 g cm^{-2} of iron and 45 g cm^{-2} of liquid scintillator (mineral oil). Two plastic scintillators (T_{1E} and T_{1W}) each of area 1.83×0.91 m^2 and thickness 1.28 cm were placed just above the top iron layer of the calorimeter. Each of these two scintillators is viewed by two photomultipliers (Philips 56 AVP) through Plexiglas light guides placed at either end. In order to make an unambiguous identification of unaccompanied hadrons from hadrons associated with electrons, muons, or low-energy hadrons (charged and neutral), a set of wide-gap chambers (1.83×1.83 m^2 area, 12.7 cm gap width) was placed in the calorimeter. The beam spark chamber (SCB) having a double gap, helped in the selection of hadrons unaccompanied by any charged particle over the area of this chamber. Three spark chambers SC1, SC2, and SC3 located at depths of 120, 340, and 700 g cm^{-2}, respectively, in the calorimeter, helped in rejection of hadrons associated with neutral particles (hadrons) and particles entering from sides of the calorimeter. This set of spark chambers also permitted an accurate ($\delta\theta\sim1°$) determination of the hadron trajectory in space. Incident charged hadrons are selected by requiring a coincidence between a pulse due to a single particle in T_1 (T_{1E} or T_{1W}) and the energy-sum pulse above a preset threshold (100 or 300 GeV) from the calorimeter. The energy-sum pulse (CAL) is obtained by linearly adding the pulses from all the sampling layers of the calorimeter. For each observed hadron event the signal measured for each of the calorimeter tanks is converted to an equivalent number (n_μ) of particles using the calibration obtained with near vertical muons traversing the calorimeter. The conversion of the sum N ($\sum^{14} n_\mu$) of the signal from all the tanks to hadron energy has been carried out using results obtained from Monte Carlo calculations and accelerator calibrations[1] of similar calorimeters. Calculations[1] incorporating the effects due to unsampled energy loss and leakage of energy show that the hadron energy E can be related to N through the simple relation E (GeV)$=0.29N^{1.03}$. The energy resolution is expected[17–19] to improve with increasing hadron energy from about 18% at 100 GeV to about 15% at 1000 GeV.

B. Transition-radiation detector

The phenomenon of transition radiation, first predicted by Ginzburg and Frank[20] has been studied[21] extensively both experimentally and theoretically and has been used for identification of particles in accelerator experiments.[22] Basically, transition radiation is emitted when a charged particle traverses the interface between media of different dielectric properties as the Coulomb field of the particle has to readjust itself. For very high energy (larger Lorentz factor γ) the radiation appears mostly in the higher frequencies (x rays) and propagates close to the direction of the radiating particle within a narrow cone of angle $\theta \sim \gamma^{-1}$. The energy radiated per unit solid angle per unit frequency interval can then be approximated as

$$\frac{d^2W}{\hbar d\omega\, d\Omega}=\frac{\alpha}{\pi^2}\left[\frac{\theta}{\gamma^{-2}+\theta^2+\xi_1{}^2}-\frac{\theta}{\gamma^{-2}+\theta^2+\xi_2{}^2}\right]^2 ,$$

$$\xi=\frac{\omega_p}{\omega} ,$$

where ω is the radiated frequency, ω_p is the plasma frequency of the medium, and α is the fine-structure constant. For a thin foil in a medium, transition radiation (TR) is emitted at both interfaces of the foil with the medium. Since there can be destructive interference between radiation emitted at the two interfaces, the foil thickness should be larger than a characteristic thickness, called formation zone t_f, which depends on the frequency of the radiation and the energy of the particle as

$$t_f=\frac{1}{2c}\left[\frac{1}{\gamma^{-2}+\theta^2+\xi^2}\right]^{-1} .$$

For a simple case of single interface the total energy radiated can be obtained as $E_t=\frac{2}{3}\alpha\omega_p\gamma$. This linear dependence of TR yield on γ is the characteristic which has made TR so very attractive for use in experiments at high energies. However, the energy radiated is very small and it is necessary to use a large number of interfaces to generate a detectable signal. It is important to note that the design considerations for a TRD for use in a cosmic-ray experiment are very different than in an accelerator experiment because the TR energy has to be measured in any detector in the presence of the signal due to the incident hadron in the former case while the hadron can be bent away from the detector in case of the latter. Therefore, the TR yield necessary for a meaningful use in a cosmic-ray experiment has to be comparable to the signal expected from the particle itself requiring the use of a large number of foils to produce TR and thin detectors to minimize signal from ionization. Also the detector area has to be large (for example, 1 m^2 in the present experiment) in a cosmic-ray experiment to observe a statistically significant number of high-energy hadrons, which places limitations on the type of TR radiator (foils of various materials, styrofoam, etc.) that can be used in the experiment.

For a TR radiator with a large number of foils the self-absorption of the TR within the foil stack becomes an important consideration in the design apart from the interference effects expected for radiation emitted at different points in the foil stack. For example, the expected TR yield for a stack of 100 mylar foils (12.5 μm thick each) separated by vacuum gaps (1250 μm thick) is shown in Fig. 2. Note that the interference effects at low energies in a single foil have been suppressed by absorption in the foils. The peak of the TR distribution for particles with γ of 2900 shown in Fig. 2 is at about 10 keV with a significant fraction of radiation at energies above 10 keV. Another important consideration for a good design for a TRD is the type of x-ray detec-

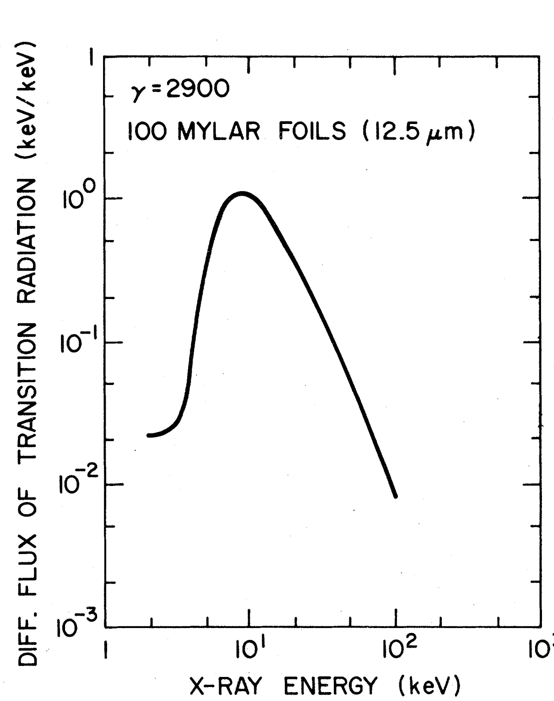

FIG. 2. Expected energy spectrum of transition radiation after a radiator consisting of 100 mylar foils in vacuum due to a charged particle with a Lorentz factor of 2900. Foils of thickness 12.5 μm are assumed to be separated by 1250 μm.

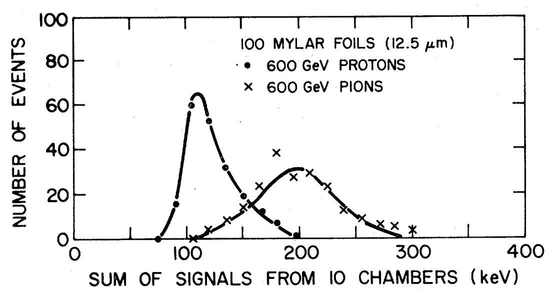

FIG. 3. Expected distributions of signals from a transition-radiation detector having ten radiator-detector modules, for pions and protons of energy 600 GeV. Each radiator-detector module consists of a radiator with 100 mylar foils followed by a 5.1-cm-thick argon-methane-filled proportional chamber.

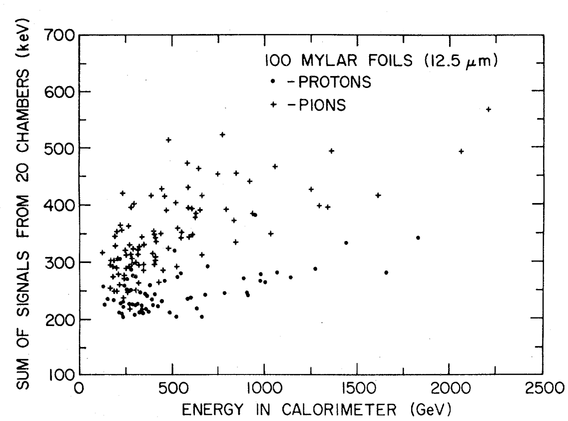

FIG. 4. A scatter plot of the expected signals from a transition-radiation detector having 20 radiator-detector modules, and the energy detected in the calorimeter placed under the TRD for incident pions and protons.

tor to be used. Since solid-state detectors and also inorganic scintillators (for example, NaI) are prohibitively expensive for the large area required in a cosmic-ray experiment, a natural choice is the gas proportional chamber. The thickness of the chamber is an important parameter due to the opposing requirements of a large absorption probability for a TR photon and a smaller ionization (dE/dx) due to the hadron traversing the chamber.

Monte Carlo simulations[14] were carried out for various possible configurations for the radiator as well as the detector, to optimize the design of the TRD for separation of pions and protons of energies above 300 GeV. The number and thickness of mylar foils in the radiator stack, the thickness of the proportional chamber, and the number of radiator-detector modules were varied to find the optimal design. For this purpose it was assumed that the pions and protons are in equal proportion in the beam of unaccompanied hadrons detected by the calorimeter and the energy resolution of the calorimeter was included in the simulation. The energy resolution of the proportional chamber for x-ray photons of various energies, including the effects due to escape of fluorescent photons sometimes, was taken into consideration in the calculations. The ionization energy loss in the proportional chamber due to the hadron was picked from distributions obtained from accelerator experiments for high-energy protons which give values in agreement with calculations of Sternheimer and Peierls[23] for most probable and average energy loss. The distributions of expected signals from a 10 module stack, each module consisting of 100 mylar foils (12.5 μm thick) as a radiator and a 5.1-cm-thick argon-methane-filled proportional chamber as a detector, for monoenergetic protons and pions of 600 GeV are shown in Fig. 3. It is seen that such a TRD can identify pions and protons efficiently. However, in practice, unaccompanied cosmic-ray hadrons have a steep energy spectrum at the mountain altitude with a power-law spectral exponent ~ -3.0 and the energy resolution of the hadron calorimeter is about 15%. This leads to a wider overlap than shown in Fig. 3. A scatter plot of the expected signal from a TRD having 20 modules, of similar type as discussed above, against the energy of the hadron measured by the calorimeter is shown in Fig. 4. For obtaining this plot the energy of the hadron was picked from a $E^{-3}dE$ spectrum and the hadron was assigned to be a pion or a proton with equal probability. It is seen from this figure that the observed signal for pions is considerably larger than for protons. Such scatter plots were obtained from the simulations for various configurations of the TRD. Based on this study a design for the detector consisting of 24 modules was adopted for the experiment. Simulations showed that a module consisting of 100 mylar foils of thickness 12.5 μm with interfoil separation of 1250 μm followed by a 5.1-cm-thick argon-methane-foil multiwire proportional chamber offered an acceptable compromise between factors such as radiation probability, self-absorption in the radiator, x-ray absorption in the detector, dE/dx in the detector due to hadrons, etc. A schematic diagram of the TRD as used in the experiment is shown in Fig. 5. It is to be noted that though the simulations were carried out for mylar foils, Dow styrofoam FR was used as the radiator in the actual

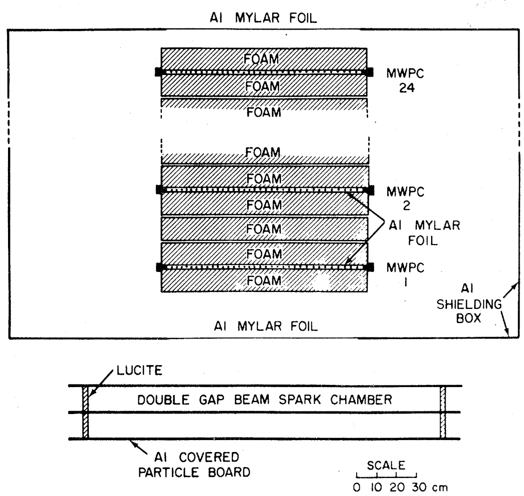

FIG. 5. A schematic diagram of the transition-radiation detector used in the experiment, located above the double-gap beam spark chamber used for identification of unaccompanied hadrons. Dow styrofoam FR was used as the TR radiator.

experiment. This change was necessary for practical considerations, because stretching a very large number of foils over an area of 1 m^2 and holding such a structure steady for a long period of time (~ 3 yr) without distortions presented a formidable problem. Calibrations carried out at CERN and BNL accelerators with electron beams for the Dow styrofoam FR, discussed in some detail in the next section, have shown that a 13.75-cm-thick foam piece is equivalent to 120 mylar foils in radiative power for transition radiation (including self-absorption). As shown in Fig. 5, the 13.75-cm-thick foam piece was composed of three pieces, two with 4.5-cm thickness and the third with 4.75-cm thickness. The thinner pieces also served another important function. Since the cathode planes of the multiwire proportional chambers (MWPC's 1—24) were made of thin 25-μm-thick aluminized mylar foil which bulged outwards due to gas pressure inside, the foam pieces were rigidly clamped on either side of the chambers to minimize the bulge. Proportional chambers, 1 m^2 in area and 5.1 cm thick had 39 anode wires each. These wires, 50-μm diameter stainless steel, were joined together on a printed circuit board outside the chamber. An argon-methane gas mixture (P-10) was flown continuously through the chambers at a slow rate. Gas consumption was kept at a reduced level by connecting six chambers in series which did not produce any undesirable effect on the gains of chambers. Typically the gas flow rate was 60 ml/min through a set of six chambers. A long tube (~ 20 m) connected at the gas output of the last chamber for blowing out the gas into the atmosphere prevented sudden changes in the gas pressure in chambers due to frequent and sudden changes in the atmospheric pressure.

A charge-sensitive preamplifier was used for each chamber and its output was digitized using the LeCroy charge-integrating analog-to-digital converters (ADC's) (LRS 2248). The gate to the ADC's generated by the coincidence ($T_1 \cdot$CAL) was delayed by 500 ns after the pulse maximum. This was necessary to avoid the effects due to the fluctuations in the rise time of the pulse caused by the position and angle of the track of charged particles relative to anode wires. The entire TRD was located inside a rf shield box made of aluminum sheets to minimize the rf noise on the wires and electronics of proportional chambers. However windows on the top and bottom of the rf box permitted the hadrons to enter and leave the stack without interaction in the aluminum sheets. These windows were covered with thin aluminized mylar foils. Further the rf noise due to spark chambers located in the calorimeter was prevented from interfering with data by delaying the trigger to the spark chambers by 1.1 μs relative to the arrival time of the hadron. Extensive tests were carried out to confirm the absence of effects due to spark-chamber noise on the data on pulse amplitudes for various detectors.

III. TRANSITION-RADIATION-DETECTOR CALIBRATION

As discussed in Sec. II, use of styrofoam as the radiator for transition radiation in the present experiment was dictated by practical considerations of difficulties in making large-area stacks of mylar foils. While the radiative power of mylar foils has been measured in experiments,[22] no comparable data were available for this particular type of styrofoam. It was felt necessary to obtain a quantitative estimate of the radiative power for Dow styrofoam FR to be able to compute the expected TR signal for comparison with observed signal for high-energy hadrons in the present experiment. The TR yield from a 4-cm-thick piece of Dow styrofoam FR has been measured by Fabjan[24] in an electron beam of energy 1.3 GeV ($\gamma \sim 2650$) at CERN using a solid-state detector for measurement of energy of TR photons. Since the electron beam was bent away before the detector, the experiment has measured the TR yield directly without contamination due to dE/dx of the particle. This distribution of detected TR energy is shown in Fig. 6. In order to find the equivalent configuration for mylar foils which

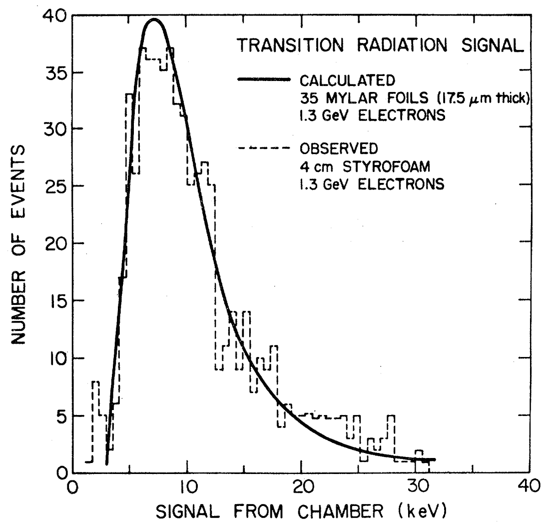

FIG. 6. A comparison of the expected distribution of TR signals from a 35-mylar-foil radiator for electrons of energy 1.3 GeV, with the observed distribution for a 4-cm thick styrofoam radiator. Electrons were bent away from the solid-state x-ray detector used for measuring the TR signal.

would give a similar distribution of detected TR energy, calculations were carried out for various configurations for mylar foils and the expected distribution was compared with experiments. Figure 6 shows such a comparison for the expected distribution from a 35-foil (12.5 μm thick) stack. The interfoil separation in the stack was assumed to be 750 μm in this configuration of mylar foils, but the calculations also showed that the interfoil separation is not a sensitive parameter for the total yield, provided it is much larger than the formation zone in the interfoil medium. For obtaining the configuration which gave a good fit to the observed distribution χ^2 was computed for each expected distribution. These calculations showed that a 4-cm Dow styrofoam FR radiator is equivalent to a stack of 35 mylar foils (each 12.5 μm thick) in terms of TR yield and its distribution.

The calibration described above gave the distribution of TR energy for the styrofoam. But the yield measured in the present experiment also depends on the detected probability for x-ray photons in proportional chambers. This probability is a function of the energy of photons. Also the fluctuations in the ionization (dE/dx) due to the particle itself play a significant role in determination of TR energy. Therefore, in order to calibrate the complete system, a small-size (30 cm$\times$30 cm) TR detector having only three modules, but otherwise identical to the full-size detector used in the cosmic-ray experiment, was constructed. This small TRD was exposed to electrons of various energies (1 , 1.8, and 2.8 GeV) available as test beams at the Brookhaven Alternating Gradient Synchrotron (AGS). The (TR$+dE/dx$) yield was measured for these electron energies for various thicknesses of styrofoam radiator between proportional chambers. A high-statistics measurement of the dE/dx distribution for pions with $\gamma=13$ was also made. A comparison of the calculated (TR$+dE/dx$) yield using radiative power measured in the CERN experiment with these measurements at BNL showed a very good agreement (better than 10%), thus confirming the accuracy of calculations for prediction of TR yield for high-energy hadrons in the cosmic-ray experiment.

IV. EXPERIMENTAL RESULTS

The determination of the pion-to-proton ratio for high-energy cosmic-ray hadrons using a transition-radiation detector depends crucially on the relative amount of energy deposited in proportional chambers due to ionization (dE/dx) and TR photons. Large fluctuations in dE/dx in a single chamber for individual hadrons make it almost impossible to distinguish between dE/dx and the energy deposited by TR photons on an event-by-event basis. However the distribution of detected energy does show the contribution due to TR photons for particles with Lorentz factors large enough to produce detectable TR x rays. In Fig. 7(a) is shown the distribution of energy deposited in a single proportional chamber of the TRD by near-vertical cosmic-ray muons of average energy $\bar{E}_h \sim 4.5$ GeV. Figure 7(b) shows a similar distribution for pions of energy 1.8 GeV observed during calibration runs with the AGS test beam at Brookhaven. These distributions, as expected, are very similar to each other and represent ionization energy loss only. For comparison, the distribution for monoenergetic photons of energy 5.9 keV from a ^{55}Fe source is also shown in Fig. 7(a). The distribution of the energy deposit in a chamber by cosmic-ray unaccompanied hadrons of average energy $\bar{E}_h \sim 120$ GeV is shown in Fig. 7(c). It is seen that this distribution is slightly wider and has a long tail compared to the distributions for low-energy muons and pions shown in Figs. 7(a) and 7(b). While protons of such low energies would not emit practically any TR photons, there is expected to be a small signal due to pions. This contribution due to TR photons emitted by high-energy pions is very clearly seen in the distribution shown in Fig. 7(d) for high-energy hadrons with average energy $\bar{E}_h \sim 600$ GeV.

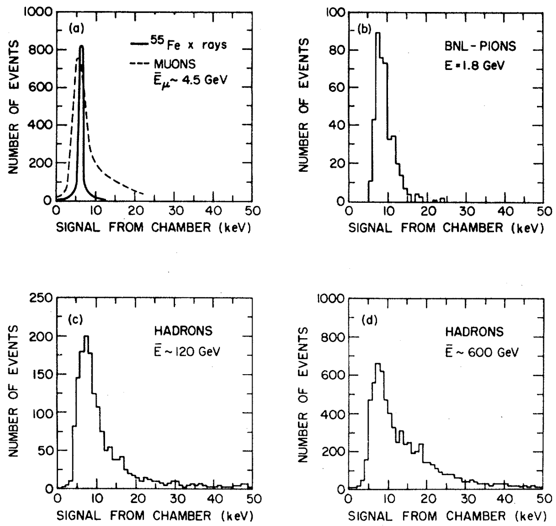

FIG. 7. Observed distributions of signals from a 5.1-cm-thick argon-methane-filled multiwire proportional chamber for traversal due to (a) cosmic-ray muons of average energy of 4.5 GeV, (b) pions of energy 1.8 GeV, (c) cosmic-ray hadrons of average energy of 120 GeV, and (d) cosmic-ray hadrons of average energy of 600 GeV. For comparison the distribution observed due to monoenergetic 5.9-keV x rays is also shown in (a).

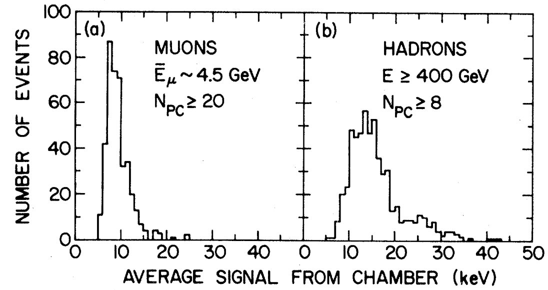

FIG. 8. Observed distributions of the average signals per chamber from multiwire proportional chambers for (a) cosmic-ray muons of average energy 4.5 GeV traversing a string of ≥ 20 chambers, and (b) cosmic-ray hadrons of energy ≥ 400 GeV traversing a string of ≥ 8 chambers.

The contribution of TR photons to the energy deposit observed in proportional chambers can be seen more clearly if the fluctuations in the ionization energy deposited by the charged particle can be reduced by summing the signals from many chambers through which the particle traversed. Figure 8(a) shows the distribution of the average energy deposit per chamber for muons which traversed a string of 20 consecutive proportional chambers of the TRD. Note the reduction in the number of events with signal larger than about 15 keV. Figure 8(b) shows the distribution of the average energy deposit per chamber for high-energy ($E_h > 400$ GeV) hadrons traversing eight or more chambers. It is to be noted that the average energy loss per chamber for high-energy hadrons [Fig. 8(b)] is 16.1 keV which is about 1.7 times larger than the value of 9.3 keV for muons [Fig. 8(a)]. For comparison, the average energy loss per chamber for low-energy hadrons with $\bar{E}_h \sim 120$ GeV is 12.9 keV. While the distribution shown in Fig. 8(b) for high-energy hadrons is broader than for muons, it does not show any double-peak structure which might be ideally expected for a mixed beam of pions and protons. However, the observed increase in the average energy deposit per chamber and the width of the distribution clearly suggest the presence of a significant number of pions among the observed hadrons. In order to obtain a quantitative estimate of the π/p ratio among observed hadrons it is necessary to compare the expected distribution of signals from the TRD, for various values of this ratio, with the observed distribution. Since the distributions for dE/dx as well as TR for particles of various energies have been obtained from accelerator calibrations, as discussed in Sec. III, for the type of radiator and detector used in the present experiment, it is relatively straightforward to simulate the response of the TRD for pions and protons of various energies using Monte Carlo techniques. These simulations have taken into consideration the "string-length" distribution since most of the observed hadrons did not traverse all the proportional chambers of the TRD. This was done by simulating the response from n chambers for incident protons as well as pions corresponding to each event observed to give a signal in a string of n chambers.

A. Single-chamber signal distributions

The expected signal distributions for individual chambers traversed by monoenergetic hadrons are shown in Fig. 9 for three values of Lorentz factors showing the presence of TR energy and its increase with increasing Lorentz factor. The distributions of signal expected in a single proportional chamber of the TRD due to pions and protons of energy ~ 400 GeV (energy spectrum $\sim E^{-3.0}dE$) are compared with the observed distribution in Fig. 10(a). This comparison shows that the unaccompanied hadron beam is neither a pure proton nor a pure pion beam.

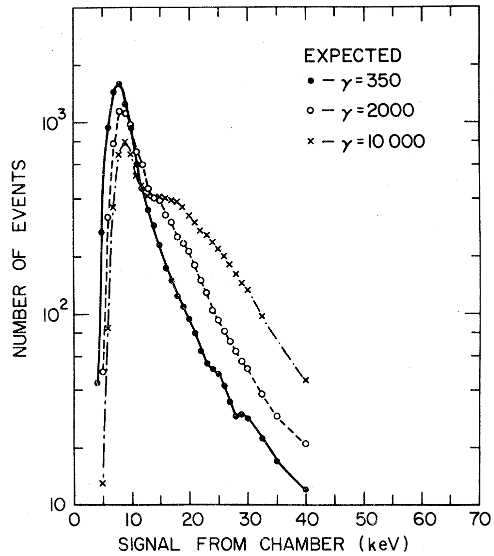

FIG. 9. Expected distribution of signals from multiwire proportional chambers of the TRD used in the experiment for three different values of the Lorentz factors for the particles.

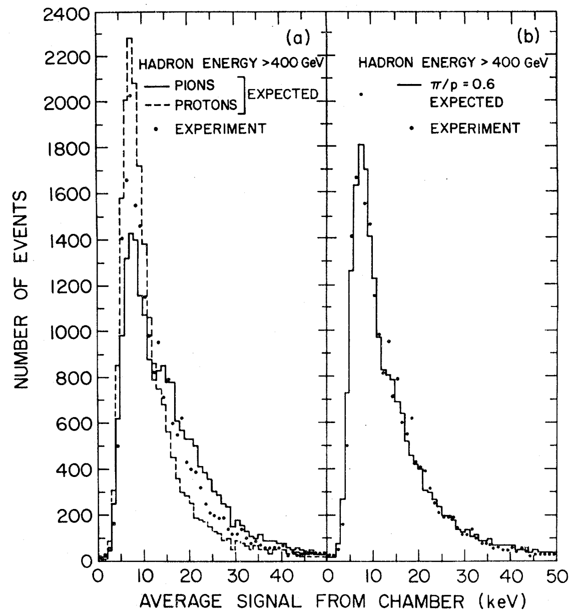

FIG. 10. A comparison of the observed distribution of the signal from a multiwire proportional chamber of the TRD, with the expected distributions for a cosmic-ray hadron beam with energy ≥ 400 GeV consisting of (a) either only pions or only protons, and (b) a mixture of pions and protons in the ratio (π/p) of 0.6.

In order to determine the π/p ratio, the distributions expected for various values of the π/p ratio were computed using the distributions shown in Fig. 10(a) and compared with the observed distribution. Only events giving a signal in ten or more chambers were used for this analysis. The expected distribution for a value of 0.6 for the π/p ratio is shown in Fig. 10(b) along with observations. However this analysis does not use the information from the complete stack of detectors for individual events and is therefore not very sensitive to the π/p ratio.

B. Average chamber signal distribution

The average signal from a string of proportional chambers traversed by a high-energy hadron is expected to have a better sensitivity to the π/p ratio since the effects due to large fluctuations in dE/dx in individual chambers are reduced when the average is taken over ten or more chambers. The distributions of average signal expected for pions and protons traversing a string of ten or more chambers are compared with observations in Figs. 11(a) and 11(b), respectively. The distribution expected for a value of 0.6 for the π/p ratio is compared with the observed distribution in Fig. 11(c). It may be noted that the distributions shown in Figs. 11(a)–11(c) have not taken into account the contribution due to the cosmic-ray background discussed later. The average signals expected for pions and protons [Figs.

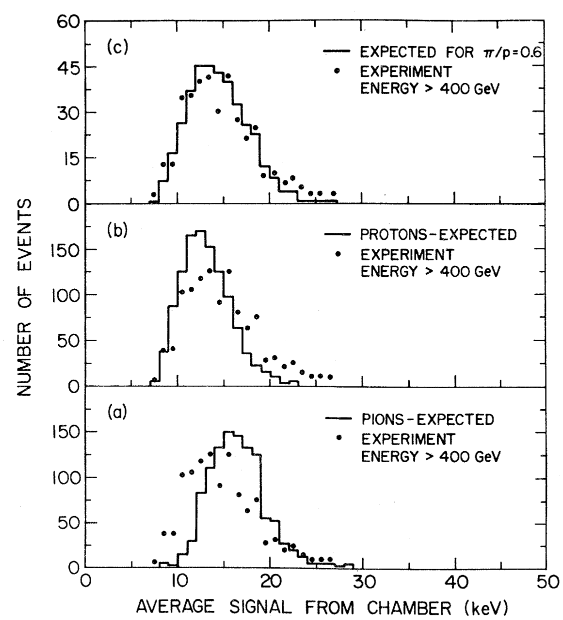

FIG. 11. A comparison of the observed distribution of the average signal from a string of ten or more multiwire proportional chambers of the TRD with the expected distribution for (a) pions, (b) protons, and (c) a mixed beam of pions and protons in the ratio (π/p) of 0.6. The observed distribution in (a) and (b) has been scaled up to normalize with the number of simulated events.

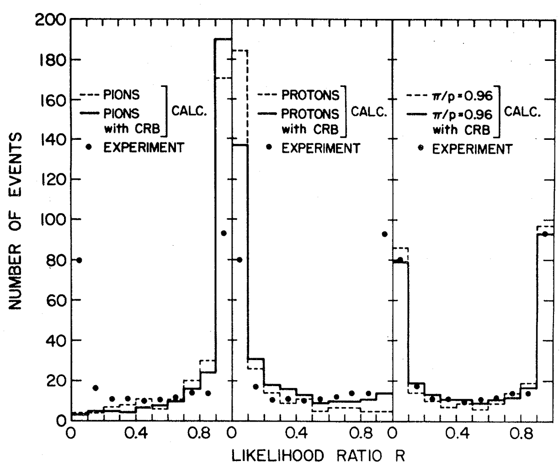

FIG. 12. A comparison of the observed distribution of the likelihood ratio R (see text for definition) for hadrons of energy 400—800 GeV, with the expected distributions for (a) pions, (b) protons, and (c) a mixed beam of pions and protons in the ratio (π/p) of 0.96. The expected distributions with and without inclusion of the signal due to cosmic-ray background (CRB) (δ rays, shower particles, backscattered particles, etc.) are shown separately in each figure.

11(a) and 11(b)] are 16.3 and 12.1 keV, respectively. These values increase to 17.0 and 12.9 keV, respectively, when the contribution due to cosmic-ray background is also considered. For comparison, the experimentally measured value for the average signal is 15.0 keV.

C. Normalized-likelihood-ratio distribution

The methods discussed above for obtaining a value of the π/p ratio, have not used the information on correlations between signals measured in different proportional chambers in individual events. While the fluctuations in dE/dx are nearly independent for various chambers, there are expected to be correlations between signals from various chambers due to detection of TR photons emitted by high-energy hadrons. In order to utilize the information from all chambers in individual events, probabilities $\phi_\pi(x_i)$ and $\phi_p(x_i)$ for observing a signal x_i, for each chamber $(i=k,l)$ in the string of chambers traversed by the high-energy hadron, are computed assuming the incident hadron to be a pion and a proton, respectively. The Lorentz factor γ needed for this computation was obtained from the energy measured by the calorimeter. Then the "normalized" likelihood ratio for a detected hadron to be a pion can be computed as

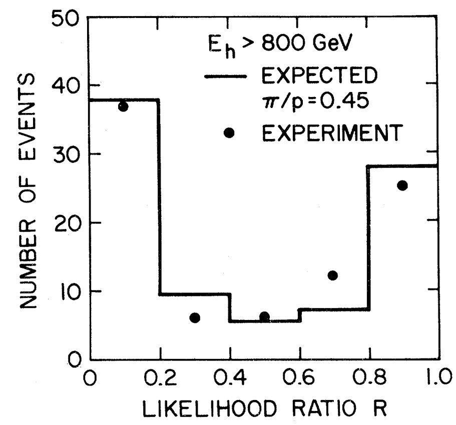

FIG. 13. A comparison of the observed distribution of the likelihood ratio R for hadrons of energy >800 GeV with the expected distribution for a mixed beam of pions and protons in the ratio (π/p) of 0.45.

$$R=\prod_{i=K}^{l}\phi_\pi(x_i)\bigg/\left[\prod_{i=K}^{l}\phi_\pi(x_i)+\prod_{i=K}^{l}\phi_p(x_i)\right].$$

The expected distribution of the ratio R for pions of energies between 400 and 800 GeV ($\bar{E}_h \sim 520$ GeV), with the same traversal configurations as observed hadrons, is shown in Fig. 12(a). A similar distribution expected for protons is shown in Fig. 12(b). The distribution of the ratio R for observed hadrons is shown in Fig. 12(c). Various values of the π/p ratio were tried to find a good fit between the expected and observed distributions. The best fit was obtained for the value 0.96 ± 0.15 for the π/p ratio for hadrons of energy 400—800 GeV. The distribution expected for the likelihood ratio R for this value of the π/p ratio is shown for comparison in Fig. 12(c). A separate but similar analysis was carried out for hadrons of energies >800 GeV ($\bar{E}_h \sim 1300$ GeV) and it was found that a value 0.45 ± 0.25 for the π/p ratio gave the best fit between the expected and the observed distributions of the likelihood ratio R (Fig. 13). For all hadrons of energy above 400 GeV, a π/p ratio of 0.83 ± 0.14 gave the best fit to the observed distribution. Each of the Figs. 11(a)—11(c) show the expected distributions, with and without inclusion of the contribution due to the cosmic-ray background (discussed later). The π/p ratio determined without including the effects of the background is expected to be larger since the background signal simulates the TR signal expected only for pions. It has been seen that the π/p ratio determined from the minimum-χ^2 fit to the observed R distribution is lower by about 20% when the contribution due to the background is included in the ex-

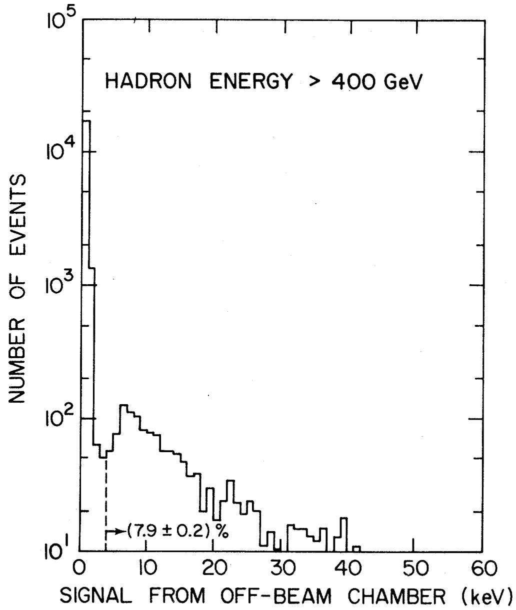

FIG. 14. A distribution of the observed signals in the multiwire proportional chambers missed by the incident hadrons.

pected distribution. For the same value of the π/p ratio the χ^2 is lower by almost a factor of 2 (2.54 versus 5.82) for the expected distribution which includes the contribution due to the cosmic-ray background.

The analysis of experimental data to obtain the π/p ratio, discussed above has taken into consideration some of the effects expected due to interactions of particles in the TRD, e.g., production of energetic knock-on electrons in the radiator material. These effects are included in the distribution of signals observed during accelerator calibrations of the TRD modules which have been used to compute the expected signals for high-energy hadrons. However, there are processes which are specifically relevant for the cosmic-ray experiment and which may have influence on the measurements of the π/p ratio. The background effects are (i) signals due to accompanied charged particles which escaped detection by the beam spark chambers due to their inclined trajectories, and (ii) signals in the proportional chambers due to particles emitted upwards[25] from hadronic and electromagnetic interactions in the calorimeter. The first effect was studied experimentally by looking at the distribution of signals from proportional chambers for high-energy hadrons ($E_h > 400$ GeV) which did not traverse the TRD. This distribution, shown in Fig. 14, is rather similar to that expected due to minimum ionizing particles. These observations show that the probability of a signal in a chamber due to a particle other than the incident hadron, for events classified as an unaccompanied charged hadron on the basis of data from the spark chambers, is about 8%. This additional energy loss in the TRD modules has been taken into consideration while computing the expected signals in the proportional chambers for comparison with observations discussed earlier. It has been observed that this background signal due to accompanied particles does not depend sensitively on the hadron energy. The probability for observing a signal in a chamber due to a particle other than the incident hadron, is about 6% for low-energy hadrons ($E_h \sim 100$ GeV) compared to about 8% for high-energy hadrons ($E_h > 400$ GeV). The signal distribution is very similar at all energies. The effect of backscattered particles on the estimate of the π/p ratio has been studied by selecting events which showed tracks due to such particles in the beam spark chamber. It was noticed that for such events, the signals from only one or two bottommost proportional chambers, which are nearest to the calorimeter, were larger showing the contribution from these particles. Therefore, events with visible backscattered particle tracks in beam spark chambers were included in the data analysis for obtaining the π/p ratio after rejecting signals from three MWPC's closest to the calorimeter. It may be noted that neutral backscattered particles or photons, undetected in the beam spark chamber, are experimentally undistinguishable from accompanied particles which contribute to the signal shown in Fig. 14. Their contribution to the observed signals is therefore included in the expected signals as mentioned above.

It is expected that the larger TR signal would be detected for pions traversing a larger number of TRD modules. Therefore, better discrimination between pions and protons could be expected for hadrons traversing the entire or almost entire TRD. In terms of the normalized likelihood ratio R, a hadron yielding a value less than 0.1 for R is most likely a proton, and a hadron with a value larger than 0.9 for R is most likely a pion. Events with relatively larger fluctuations give a value of R between 0.1 and 0.9. For hadrons traversing a larger number of TRD modules the effects of fluctuations are reduced and a larger number of hadrons can be expected to have values of R either less than 0.1 or larger than 0.9. This expected behavior is clearly seen in Fig. 15, where the variation of the fractional number of hadrons with values of R in the range 0.1—0.9 is plotted as a function of the number of TRD modules used for computation of the value of R.

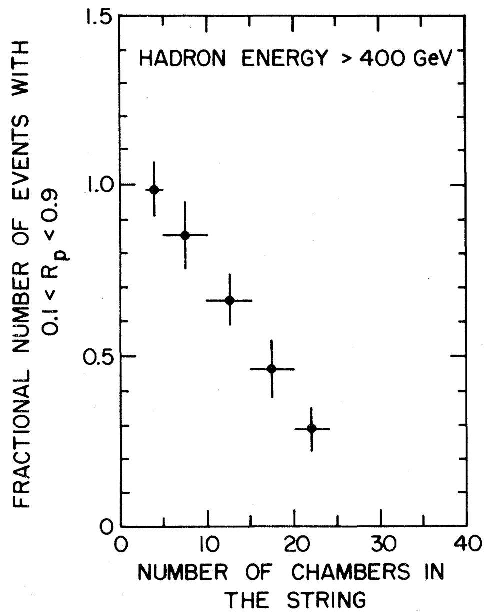

FIG. 15. A plot of the fractional number of events with likelihood ratio R_p in the range 0.1—0.9 against the number of multiwire proportional chambers used for calculating the value of R_p.

However events with number of triggered TRD modules larger than 20 constitute a relatively small fraction of the total number of hadrons detected in the experiment. It has been seen from the present analysis that the estimated π/p ratio obtained using the normalized-likelihood-ratio method, is independent (within errors) of the number of TRD modules for events which have traversed six or more modules. For this reason, the results quoted here [Fig. 12(c)] refer to all events which have traversed ten or more TRD modules.

V. DISCUSSION

The pion-to-proton ratio for unaccompanied hadrons has been measured at mountain altitudes and sea level using a variety of techniques. Conventional techniques, for example, magnet spectrometers, Cherenkov counters, and ionization measurements, have been used for lower-energy hadrons. These techniques lose their sensitivity for distinguishing between pions and protons for hadrons of higher energy ($E_h \gtrsim 300$ GeV). Transition-radiation measurement offers a possible method to distinguish between pions and protons for higher energies and has been used in the present experiment and also by Avakyan *et al.*[5] All the measurements of π/p ratio at mountain altitudes are summarized in Table I. It is seen from this table that the measured π/p ratio for most of the experiments is less than unity. The measurements at higher energies using TRD's are consistent with each other and suggest a decrease in π/p ratio with increasing hadron energy. It is interesting to note that these two experiments have also reported[5,7] their measurements of neutral-to-charged ratio for high-energy unaccompanied hadrons which agree well with each other. These measurements of neutral-to-charged ratio show this ratio to be independent of energy in the energy range ~100—2000 GeV and to have a value of 0.4.

It is of interest to compare these measurements with the expected ratio of pions and protons in this energy range. The measured flux[2] of unaccompanied charged hadrons is much larger than the expected flux of surviving protons at energies ~1000 GeV suggesting that many of the primary protons have suffered interactions in the atmosphere. A large π/p ratio as measured in the present experiment and a large neutral-to-charged (N/C) ratio measured[7] with basically the same detector system, both indicate the presence of many secondary parti-

TABLE I. Pion-to-proton ratio among unaccompanied hadrons at mountain altitude.

Experiment	Altitude ($g\,cm^{-2}$)	Hadron energy (GeV)	Technique	π/p ratio
Lal *et al.* (Ref. 10)	800	20—40	Air Cherenkov counter	0.51±0.09
Denisova *et al.* (Ref. 11)	700	>100	Ionization measurement	0.56±0.16
Bashindjhagyan *et al.* (Ref. 12)	700	>300	Ionization measurement	0.75±0.25
Anoshin *et al.* (Ref. 13)	700	>100	Ionization measurement	1.5 ±0.5
Vardumyan *et al.* (Ref. 9)	800	34—320	Magnet spectrometer	1.4 ±0.25
Avakyan *et al.* (Ref. 5)	700	300—500	Transition radiation	1.34±0.4
		500—1000		0.81±0.18
		1000—2000		0.53±0.24
Ellsworth *et al.*	730	400—800	Transition radiation	0.96±0.15
(present experiment)		800—2000		0.45±0.25

cles among the observed unaccompanied hadrons.[26] Therefore it is necessary to simulate in detail the hadronic cascade in the atmosphere to obtain the expected π/p and N/C ratios for unaccompanied hadrons. It is rather difficult to put the criterion of unaccompaniment into this type of calculation since that requires the calculation of lateral distribution of electrons for every photon created in the atmosphere. In order to simplify the calculation the contribution of all the created photons is summed together and an estimate of the size of the accompanied shower is obtained. Assuming the lateral distribution function of Nishimura-Kamata-Greisen (NKG) type for these small showers, it is easy to see that the shower particle density would be less than about 0.1 m^{-2} for showers of size less than about 1000 particles at 730 $g\,cm^{-2}$ altitude. Since the experiment requires, on the average, an accompaniment density of less than 0.25 m^{-2}, all showers of size less than about 3000 particles would be effectively unaccompanied for the observed hadron. Therefore air showers have been simulated for primary protons of energies larger than 400 GeV and all hadrons arriving at the observational level accompanied by showers of size less than 3000 particles have been accepted as unaccompanied hadrons. However, this criterion can be easily changed to any other threshold for shower density as would be clear from the results of simulation presented below. The primary proton energy spectrum has been assumed to be of the form

$$N(E)dE=KE^{-\gamma}dE,\quad \gamma=-2.65$$

throughout the energy range of interest. It may be noted that the steepening of the proton energy spectrum around 1500 GeV observed in satellite experiments[27] need not be considered here because it has been shown[28] that such a steepening can arise due to backscattered particles. Also, recent direct measurements[29] with nuclear emulsion chambers do not show any discontinuity up to energies $>10^4$ GeV. The presence of other nuclei in the primary cosmic-ray flux is not important for the present purpose because of the relatively small proportion of these nuclei at the same energy per nucleon.

The particle-interaction model assumed in the simulations is the independent-particle-emission model discussed in detail elsewhere[30–32] and assumes Feynman[33] scaling of the inclusive particle cross sections measured at Fermilab and CERN ISR energies. Simulations have been carried out for a narrow band (a factor of 2) of primary energies in order to make it possible to study the dependence of the contribution of various energies to the observed flux. Within this narrow energy band, individual primary proton energies have been picked randomly

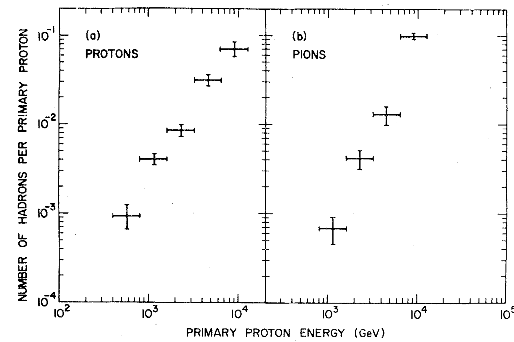

FIG. 16. Expected yield of unaccompanied hadrons of energy ≥ 400 GeV at the observational level of 730 $g\,cm^{-2}$ due to primary protons of various energies, obtained from Monte Carlo simulation of the hadron and electron-photon cascades in the atmosphere: (a) unaccompanied protons, and (b) unaccompanied pions.

using the probability distribution given by the assumed form of the energy spectrum. In Fig. 16(a) is shown the yield per primary proton in terms of the number of nucleons of energy >400 GeV arriving at the observational level for various primary energies. A similar plot of the yield of charged pions per primary protons as a function of primary proton energy is shown in Fig. 16(b). As expected, the yield increases with increasing primary energy. Also, as expected, the yield of pions increases faster than the yield of nucleons and most of the observable pions are produced by primary protons of much larger energies compared to observable nucleons. Integrating the product of the yield and the primary flux from 400 to 1.28×10^4 GeV gives the values of the expected π/p ratio as 0.44.

These simulations have not assumed any charge-exchange probability for proton and pion—air-nucleus interactions. Since the observed π/p ratio is near unity, it is clear that the charge-exchange probability plays an important role in determining the π/p ratio. This is, of course, also expected from the large value of 0.40 for the neutral-to-charged ratio observed[7] for unaccompanied hadrons. The expected values of π/p and N/C ratios, therefore, depend on the charge-exchange probabilities for nucleon and pion—air-nuclei interactions and on the number of interactions in the atmosphere for the observable hadron. Phenomenologically, charge-exchange cross sections correspond to reactions of the type $p+A\rightarrow n+X$, $n+A\rightarrow p+X$, and $\pi^{\pm}+A\rightarrow\pi^0+X$ in the diffractive region of inelastic interactions. In Table II are given the fluxes of different species of particles expected on the basis of different assumptions. The expected π/p and N/C ratios are also

TABLE II. Expected fluxes of unaccompanied protons, neutrons, and pions of energy >400 GeV at 730 $g\,cm^{-2}$ and charge ratios.

Assumptions in calculations	Proton flux ($m^{-2}sr^{-1}s^{-1}$)	Neutron flux ($m^{-2}sr^{-1}s^{-1}$)	Pion flux ($m^{-2}sr^{-1}s^{-1}$)	Charged-hadron flux ($m^{-2}sr^{-1}s^{-1}$)	π/p ratio	N/C ratio
Primary protons only, charge-exchange probability zero	8.6×10^{-4}		3.8×10^{-4}	12.4×10^{-4}	0.44	
Charge-exchange ($n \leftrightarrow p$) probability $=0.33$; average of one interaction per primary proton	5.8×10^{-4}	2.8×10^{-4}	3.8×10^{-4}	9.6×10^{-4}	0.66	0.30
Average of two interactions per primary proton and one interaction per secondary (n,π) particle	4.8×10^{-4}	3.8×10^{-4}	3.8×10^{-4}	8.6×10^{-4}	0.80	0.44
Observations				5.3×10^{-4}	0.83 ± 0.14	0.40 ± 0.04

given in this table. In the first row of Table II are given the expected fluxes of protons and pions assuming the charge-exchange probability to be zero. Under this assumption practically no neutrons are expected in the unaccompanied hadron flux at mountain altitude, ignoring the contribution due to nuclei in the primary-cosmic-ray flux. If it is assumed that most of the hadrons detected at the observational level have interacted, on the average, only once in the atmosphere and that the charge-exchange probability for nucleon interactions ($p \leftrightarrow n$) is 0.33, then the expected flux and charge ratio for unaccompanied hadrons are as given in the second row of Table II. On the other hand, if it is assumed that the primary particles interact, on the average, twice before reaching the observational level and the secondary particles (n,π) interact only once, then the expected flux and charge are as given in the third row of Table II. The calculations, in fact, suggest that for observable unaccompanied hadrons at 730 $g\,cm^{-2}$ altitude, the assumptions made for the third row in Table II are more appropriate. It is interesting to note that the expected charge ratios given in the third row are in good agreement with observations. Further the expected charged hadron flux

$$8.6\times10^{-4}\ m^{-2}sr^{-1}s^{-1}$$

is also in reasonably good agreement with the observed[2] flux

$$\sim 5.3\times10^{-4} m^{-2}sr^{-1}s^{-1} ,$$

considering the basic uncertainty caused by the criterion of unaccompaniment for the calculations. These ratios would not be altered by consideration of the presence of nuclei in the primary flux though the expected unaccompanied-charged hadron flux would be increased by about 10%. However, as discussed by Siohan *et al.*,[2] the absolute flux of unaccompanied charged hadrons is sensitively related to the criterion of unaccompaniment and indeed there are differences of about a factor of 2 between results obtained by different experiments. It may also be noted that the flux and charge ratios would also change slightly if the charge-exchange probability in pion—air-nuclei interactions ($\pi^{\pm} \rightarrow \pi^0$) are also included in the calculations. Kaon production was included in these calculations but for purposes of flux and charge ratios; the charged kaons have been included in the proton flux. However, due to their short lifetimes, kaons constitute less than about 5% of the observable hadrons in the unaccompanied flux at 730 $g\,cm^{-2}$ altitude. It should be pointed out that since observed pions are all secondary particles, the π/p ratio among unaccompanied hadrons is expected to decrease with increasing hadron energy because of the increasing probability of shower accompaniment, as is indeed observed experimentally.

VI. CONCLUSIONS

These first measurements of the pion-to-proton ratio among high-energy (>400 GeV) hadrons in the unaccompanied-charged-hadron flux at mountain altitude have demonstrated the usefulness of the phenomenon of transition radiation for distinguishing between particles of different masses in a cosmic-ray experiment. The observed π/p ratio for hadrons of energy larger than 400 GeV is 0.83 ± 0.14 indicating the presence of a significant number of pions in the unaccompanied-hadron flux. The π/p ratio decreases from 0.96 ± 0.15 for hadrons of energy in the range 400–800 GeV to 0.45 ± 0.25 for hadrons of energy larger than 800 GeV. These observations, combined with our earlier observations of a large and energy-independent ratio of neutral to charged hadrons of 0.40 ± 0.04, require that the phenomenological charge-exchange ($p \leftrightarrow n$) process in nucleon–air-nucleus inelastic interactions plays a significant role in determining the composition of the beam of unaccompanied hadrons at mountain altitude at energies larger than 400 GeV. Our Monte Carlo simulations show that charge-exchange probability of about $\frac{1}{3}$ per interaction gives a good agreement between expected and observed composition of the unaccompanied-hadron flux.

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