Nonperturbative approach to the infrared behavior in physical charged sectors of gauge theories

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Physical charged fields in Abelian gauge theories with unbroken global gauge symmetry are constructed. The Bloch-Nordsieck approximation is then used for the investigation of their mass-shell behavior in both cases of unconfined and confined global charge. In the former case (we consider spinor QED) physical charged fields are found to have vanishing infrared anomalous dimension and to admit the conventional Lehmann-Symanzik-Zimmermann (LSZ) limit. In the latter case (we consider a toy model obtained from QED by replacing the photon with a dipole gluon) it turns out that physical charged fields have zero LSZ limit and the relative Green's functions exhibit an essential singularity in the coupling constant g at $g=0$. Moreover, the perturbative expansion of these Green's functions, although asymptotic, is not Borel summable.

I. INTRODUCTION

The present paper is a continuation of our study[1,2] of the infrared (IR) structure of the physical charged sectors in gauge theories. Here we consider the cases of both unconfined and confined global charge because the comparison is, in our opinion, instructive. Inasmuch as it is founded on the Bloch-Nordsieck (BN) approximation,[3] our approach is nonperturbative. Moreover, it preserves gauge invariance at any stage.

The BN model has been widely considered in the literature.[4] The main reason is that it offers considerable physical insight into the mechanism of soft radiative processes, being therefore an excellent guide in the study of the IR structure of the charged sectors in quantum electrodynamics (QED). As is well known, the importance of this structure stems from the fact that it determines the nature of the asymptotic charged states. However, the existing results on the BN model exclusively concern, to our knowledge, the Green's functions involving the fermion fields ψ and $\bar{\psi}$, local with respect to the electromagnetic field $F_{\mu\nu}$. These Green's functions are gauge dependent and, unfortunately, do not give any information about the properties of the *physical* charged states, i.e., charged states on which Maxwell equations hold. The reason is that, as has been rigorously proven,[5] physical charged states cannot be obtained by applying to the vacuum operators, which are local with respect to $F_{\mu\nu}$.[6]

In particular, it follows from the above discussion that an extrapolation of the IR properties of the fields ψ and $\bar{\psi}$ to the physical charged sectors of the BN model (as well as to the physical charged sectors of QED) *may in principle be misleading*. The present paper shows that such an extrapolation *de facto is misleading*. Indeed, in the case of QED we explicitly construct (Sec. II) physical charged fields, which have (Sec. III) a mass-shell behavior qualitatively different from that of ψ and $\bar{\psi}$ and mild enough to admit the conventional Lehmann-Symanzik-Zimmermann (LSZ) limit.

In the final part of the paper we turn our attention to the study of the IR behavior of physical charged fields in theories with confined global charge. More precisely we are concerned with a specific open problem that is receiving a lot of interest.[7,8] The problem is to find the mass-shell behavior of the quark propagator in quantum chromodynamics (QCD) under the assumption that the full gluon propagator $\mathscr{D}(k^2)$ behaves like

$$\mathscr{D}(k^2)\sim(k^2)^{-2} \quad \text{for } k^2\to 0 . \tag{1.1}$$

Postponing to Eq. (4.7a) below the precise definition of the distribution $(k^2)^{-2}$ in four dimensions, we note that (1.1) provides, in the nonrelativistic limit, a potential[9] linearly rising at large distances.[10] Such a confining potential meets evidence in the spectroscopy of heavy quark-antiquark bound states (charmonium, $b\bar{b}$ bound states, etc.).[11]

In Sec. IV we make a first step towards a solution of the above-mentioned problem. We consider an Abelian gauge model where the gauge field is of dipole type. That is why we refer to this toy model as dipole QED (DQED). We expect that it is a good testing laboratory since, presumably, it shares with QCD [equipped with the assumption (1.1)] the same mechanisms that control the mass-shell behavior of the charged matter fields. The nonperturbative analysis of DQED is done in complete analogy with that performed in Secs. II and III for QED. The output is that the physical charged fields of DQED have, in the BN approximation, a vanishing LSZ limit, in agreement with a widespread belief[7] concerning gauge theories with confinement of the global charge. We also find that the perturbative expansion in the physical charged sectors of DQED, although asymptotic, is not Borel summable.

Section V is devoted to our conclusions.

We work in the metric $(+---)$; the Fourier transform is defined by

$$\hat{f}(p)=\int d^4x\, e^{-ipx}f(x)\ ,$$

and

$$d_4p=(2\pi)^{-4}d^4p\ .$$

II. PHYSICAL CHARGED FIELDS IN SPINOR ELECTRODYNAMICS

In this section we give a concise account of the construction of physical charged fields in the framework of classical spinor electrodynamics.

Our starting point is the *nondegenerate* Lagrangian density[12]

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-B\partial_\mu A^\mu+\frac{1}{2\xi}B^2 +\bar{\psi}(i\partial\!\!\!/-m-gA\!\!\!/)\psi\ , \tag{2.1}$$

where g and ξ are real parameters and

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\ . \tag{2.2}$$

The equations of motion corresponding to (2.1) are

$$\partial^\nu F_{\nu\mu}=j_\mu-\partial_\mu B\ , \tag{2.3}$$

$$j_\mu=g\bar{\psi}\gamma_\mu\psi\ , \tag{2.4}$$

$$\xi\partial^\mu A_\mu=B\ , \tag{2.5}$$

$$(i\overrightarrow{\partial\!\!\!/}-m-gA\!\!\!/)\psi=0\ , \tag{2.6a}$$

$$\bar{\psi}(i\overleftarrow{\partial\!\!\!/}+m+gA\!\!\!/)=0\ . \tag{2.6b}$$

Equations (2.2) and (2.6), respectively, imply

$$\partial^{\nu}\,{}^*F_{\nu\mu}=0 \tag{2.7}$$

with ${}^*F_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, and

$$\partial^\mu j_\mu=0\ . \tag{2.8}$$

The electric charge

$$Q=\int d^3x\, j_0(x) \tag{2.9}$$

is a constant of motion (provided that the current vanishes at spacelike infinity). Furthermore from (2.2), (2.3), and (2.8) one gets

$$\Box B=0\ , \tag{2.10}$$

i.e., the field B, which plays the role of a gauge-fixing Lagrange multiplier in (2.1), remains free. Consequently, the Poisson brackets of B with the fields A_μ, ψ, $\bar{\psi}$, and j_μ are easily derived[1] and have the form

$$\{B(x),A_\mu(y)\}=\partial_\mu D(x-y)\ , \tag{2.11}$$

$$\{B(x),\psi(y)\}=igD(x-y)\psi(y)\ , \tag{2.12a}$$

$$\{B(x),\bar{\psi}(y)\}=-igD(x-y)\bar{\psi}(y)\ , \tag{2.12b}$$

$$\{B(x),j_\mu(y)\}=0\ , \tag{2.13}$$

where $D(z)=(2\pi)^{-1}\epsilon(z^0)\delta(z^2)$ is the well-known Pauli-Jordan commutator function.

According to the theory of dynamical systems with constraints,[13] the *physical* subspace $\mathscr{M}$ of the phase space Γ, corresponding to (2.1), is given by the surface

$$B=0\ . \tag{2.14}$$

In what follows all the equations, which are valid only on $\mathscr{M}$, are called *weak* equations and are denoted by $\approx$. By definition $B\approx 0$. Moreover, owing to (2.3), the Maxwell equation

$$\partial^\nu F_{\nu\mu}\approx j_\mu \tag{2.15}$$

is a weak one.

Following Ref. 14, the *observables* are defined as the quantities whose Poisson brackets with B are *weakly* zero. The set of observables is an algebra under the pointwise multiplication of c-number functions. The observable O is called *local* if

$$\{O(x),O(y)\}=0 \text{ for } (x-y)^2<0;$$

the observables O_1 and O_2 are called *relatively local* if

$$\{O_1(x_1),O_2(x_2)\}=0 \quad \text{for } (x_1-x_2)^2<0\ .$$

The set of all local observables, which are relatively local among one another, forms an algebra *strictly* contained in the algebra of observables. Indeed, as shown a few lines below, the latter involves also nonlocal (electrically charged) fields.

Employing (2.11)–(2.13) one easily verifies that the Hamiltonian H, corresponding to the nondegenerate Lagrangian density (2.1), is an observable. This fact ensures that the time evolution of the system under consideration is consistent with the constraint (2.14), i.e., a point initially on $\mathscr{M}$ is not allowed by time evolution to leave $\mathscr{M}$. As follows from (2.11) and (2.13), other examples of observables are given by $F_{\mu\nu}$, ${}^*F_{\mu\nu}$, and j_μ. Because of (2.11) and (2.12), the fields A_μ, ψ, and $\bar{\psi}$ are not observables.

Let us now consider the fields[15]

$$\Psi(x;f)=\left\{\exp\left[ig\int d^4y\, f_\mu(x-y)\times A^\mu(y)\right]\right\}\psi(x)\;, \tag{2.16a}$$

$$\bar{\Psi}(x;f)=\bar{\psi}(x)\left\{\exp\left[-ig\int d^4y\, f_\mu(x-y)\times A^\mu(y)\right]\right\}. \tag{2.16b}$$

Here f_μ is a tempered distribution satisfying

$$\partial^\mu f_\mu(z)=\delta(z),\quad \bar{f}_\mu(z)=f_\mu(z) \tag{2.17a}$$

and its Fourier transform admits the representation

$$\hat{f}_\mu(p)=i\int d_4q\, F(p,q)q_\mu[c_+(pq+i\epsilon)^{-1}+c_-(pq-i\epsilon)^{-1}]\;, \tag{2.17b}$$

where c_+ and c_- are real parameters and $F(p,q)$ is a tempered distribution. The conditions (2.17a) imply

$$(c_++c_-)\int d_4q\, F(p,q)=1\;, \tag{2.18a}$$

$$\bar{F}(p,q)=F(-p,q)\;. \tag{2.18b}$$

Using (2.11)–(2.13) and the canonical Poisson brackets corresponding to (2.1), one can check that

$$\{B(x),\Psi(y;f)\}=\{B(x),\bar{\Psi}(y,f)\}=0\;, \tag{2.19}$$

$$\{Q,\Psi(x;f)\}=ig\Psi(x;f)\;, \tag{2.20a}$$

$$\{Q,\bar{\Psi}(x;f)\}=-ig\bar{\Psi}(x;f)\;, \tag{2.20b}$$

i.e., $\Psi(x;f)$ and $\bar{\Psi}(x;f)$ are *charged* observables. Due to the convolution in the exponents of (2.16), they are nonlocal with respect to $F_{\mu\nu}$.

From (2.6a) and (2.16a) one gets

$$(i\overrightarrow{\partial}-m)\Psi(x;f)=g\gamma^\mu A_\mu(x;f)\Psi(x;f)\;, \tag{2.21a}$$

where

$$A^\mu(x;f)=\int d^4y[g^{\mu\nu}\delta(x-y)-\partial^\mu f^\nu(x-y)]A_\nu(y)\;. \tag{2.22}$$

Analogously

$$\bar{\Psi}(x;f)(i\overleftarrow{\partial}+m)=-g\bar{\Psi}(x;f)\gamma^\mu A_\mu(x;f)\;. \tag{2.21b}$$

Since

$$\{B(x),A_\mu(y;f)\}=0\;, \tag{2.23}$$

Eqs. (2.21), although equivalent to (2.6), have the advantage of involving observables only. This is true also for the Maxwell equations (2.7) and (2.15) and one has, therefore, a formulation of classical spinor electrodynamics entirely on $\mathscr{M}$. Such a formulation turns out to be very useful in the study of the physical charged sectors of the BN model (as for QED, see Ref. 1) and, in particular, as shown in Sec. III, in the analysis of the IR structure of these sectors. In performing this analysis it will be sufficient to consider the subfamily of (2.17b) with

$$F(p,q)=(2\pi)^4\delta(n-q)\;, \tag{2.24a}$$

where n is an arbitrary four-vector. Because of (2.18a), Eq. (2.24a) implies

$$2c_+=1+c,\quad 2c_-=1-c\;. \tag{2.24b}$$

Equations (2.17b) and (2.24) lead to

$$\hat{f}_\mu(p)=\frac{i}{2}n_\mu[(1+c)(np+i\epsilon)^{-1}+(1-c)(np-i\epsilon)^{-1}]\;, \tag{2.25a}$$

or, in the coordinate representation,

$$f_\mu(z)=\tfrac{1}{2}n_\mu\int_{-\infty}^{\infty}d\alpha[(1+c)\theta(\alpha)-(1-c)\theta(-\alpha)]\delta(z-\alpha n)\;. \tag{2.25b}$$

The fields, obtained from (2.16) and (2.22) by fixing f_μ according to (2.25), have the form

$$\Psi(x;n;c)=\left[\exp\left[\frac{i}{2}g\int_{-\infty}^{\infty}d\alpha[(1+c)\theta(\alpha)-(1-c)\theta(-\alpha)]n^\mu A_\mu(x-\alpha n)\right]\right]\psi(x)\;, \tag{2.26a}$$

$$\bar{\Psi}(x;n;c)=\bar{\psi}(x)\left[\exp\left[-\frac{i}{2}g\int_{-\infty}^{\infty}d\alpha[(1+c)\theta(\alpha)-(1-c)\theta(-\alpha)]n^\mu A_\mu(x-\alpha n)\right]\right]\;, \tag{2.26b}$$

$$A^{\mu}(x;n;c)=A^{\mu}(x)-\tfrac{1}{2}\partial^{\mu}\int_{-\infty}^{\infty}d\alpha[(1+c)\theta(\alpha)-(1-c)\theta(-\alpha)]n^{\nu}A_{\nu}(x-\alpha n)\ . \quad (2.27)$$

[Note that $A_{\mu}(x;n;c)$ identically satisfies $n^{\mu}A_{\mu}(x;n;c)=0$. Moreover, $\Psi(x;n;c)$ and $\overline{\Psi}(x;n;c)$ are special types of string variables.[16] For example, in the case $n^0\neq 0$ one has

$$\Psi(x;n;c)=\left[\exp\left[\frac{ig}{2n^0}\int dy^0[(1+c)\theta(x^0-y^0/n^0)-(1-c)\theta(y^0-x^0/n^0)]\times n^{\nu}A_{\nu}(y^0,\vec{x}-\vec{n}(x^0-y^0/n^0))\right]\right]\psi(x)\ .$$

Therefore, for $n^0>0$ ($n^0<0$) the $\theta(\alpha)$ and $\theta(-\alpha)$ terms in (2.26), respectively, are the retarded (advanced) and the advanced (retarded) parts of the string. Thus a symmetric treatment of particles and antiparticles leads to the choice $c=0$, which, being natural for QED, is adopted in Ref. 1. In the BN approximation of QED we are interested in here, antiparticles are absent. In this case we therefore make the choice

$$c=1\ . \quad (2.28)$$

In what follows we adopt the notation

$$\Psi_r(x;n)=\Psi(x;n;1)\ , \quad (2.29)$$

$$\overline{\Psi}_r(x;n)=\overline{\Psi}(x;n;1)\ ,$$

$$A_r^{\mu}(x;n)=A^{\mu}(x;n;1)\ . \quad (2.30)$$

Furthermore, we define

$$e_r(x)=\int d_4n\, e^{-inx}\left[\int d^4y\,\Psi_r(y;n)e^{iny}\right]\ , \quad (2.31a)$$

$$\bar{e}_r(x)=\int d_4n\, e^{-inx}\left[\int d^4y\,\overline{\Psi}_r(y;n)e^{iny}\right]\ . \quad (2.31b)$$

These fields play a distinguished role, since, as demonstrated in the next section, they have milder IR properties compared to ψ and Ψ_r. From (2.31) one gets, in the momentum representation,

$$\hat{e}_r(p)=\hat{\Psi}_r(p;n)\,|_{n=p}\ ,$$
$$\hat{\bar{e}}_r(p)=\hat{\overline{\Psi}}_r(p;n)\,|_{n=p}\ , \quad (2.32)$$

where

$$\hat{\Psi}_r(p;n)=\int d^4x\, e^{ipx}\Psi_r(x;n)\ ,$$

$$\hat{\overline{\Psi}}_r(p;n)=\int d^4x\, e^{ipx}\overline{\Psi}_r(x;n)\ .$$

We stress that $e_r(x)$ and $\bar{e}_r(x)$ are not string variables. As follows from (2.31), they have an even more complicated structure than the string variables $\Psi_r(x;n)$ and $\overline{\Psi}_r(x;n)$ employed for their definition.

Equations (2.21a), (2.25a), and (2.28) imply

$$(\not p-m)\hat{\Psi}_r(p;n)=g\gamma^{\mu}\int d_4k\, T^{r}_{\mu\nu}(k;n)\times\hat{A}^{\nu}(k)\hat{\Psi}_r(p-k;n)\ , \quad (2.33)$$

with

$$T^{r}_{\mu\nu}(k;n)=g_{\mu\nu}-k_{\mu}n_{\nu}(kn+i\epsilon)^{-1}\ . \quad (2.34)$$

From (2.32) and (2.33) one has

$$(\not p-m)\hat{e}_r(p)=g\gamma^{\mu}\int d_4k\, T^{r}_{\mu\nu}(k;p)\hat{A}^{\nu}(k)\times\hat{\Psi}_r(p-k;p)\ . \quad (2.35)$$

We finish this section with the following remark. The observables H, $F_{\mu\nu}$, j_{μ}, and $A_{\mu}(x;f)$, and $\Psi(x;f)$ and $e_r(x)$ with their Dirac conjugates, are invariant under the local gauge transformations

$$\psi(x)\rightarrow e^{-ig\Lambda(x)}\psi(x),\ \ \bar{\psi}(x)\rightarrow\bar{\psi}(x)e^{ig\Lambda(x)}\ , \quad (2.36a)$$

$$A_{\mu}(x)\rightarrow A_{\mu}(x)+\partial_{\mu}\Lambda(x),\ \ B(x)\rightarrow B(x)\ , \quad (2.36b)$$

where the real function Λ satisfies

$$\Box\Lambda(x)=0\ , \quad (2.37a)$$

$$\Lambda(x)\rightarrow 0\ \text{ for }\ |x|=({x_0}^2+\vec{x}^{\,2})^{1/2}\rightarrow\infty\ . \quad (2.37b)$$

This fact can be checked either directly or by the aid of (2.11)–(2.13), having in mind that the current corresponding to (2.36) has the form

$$J^{\Lambda}_{\mu}(x)=\Lambda(x)\partial_{\mu}B(x)-B(x)\partial_{\mu}\Lambda(x)\ . \quad (2.38)$$

Owing to (2.10) and (2.37a), J^{Λ}_{μ} is conserved and gives rise to the constant of motion

$$Q^{\Lambda}=\int d^3x\, J^{\Lambda}_0(x)\ . \quad (2.39)$$

III. BLOCH-NORDSIECK APPROXIMATION FOR UNCONFINED PHYSICAL CHARGED FIELDS: QED

As is well known (see, e.g., Ref. 17), the BN model is defined by (2.1) with the replacement rule

$$\gamma^{\mu} \to u^{\mu} \ , \tag{3.1}$$

where u is a timelike vector normalized, for simplicity, according to

$$u^2 = 1 \ . \tag{3.2}$$

Consequently, Eqs. (2.33) and (2.35) take the form

$$(up - m)\hat{\Psi}_r(p;n) = g u^{\mu} \int d_4 k \, T^r_{\mu\nu}(k;n)\hat{A}^{\nu}(k) \times \hat{\Psi}_r(p-k;n) \ , \tag{3.3a}$$

$$(up - m)\hat{e}_r(p) = g u^{\mu} \int d_4 k \, T^r_{\mu\nu}(k;p)\hat{A}^{\nu}(k) \times \hat{\Psi}_r(p-k;p) \ . \tag{3.3b}$$

A characteristic feature of the quantized BN model is the vanishing of all diagrams involving at least one fermion loop. This is a consequence of (3.1) and implies that the vacuum polarization vanishes.

We are now interested in the Green's function

$$\mathcal{G}(x-y) = -i\langle T e_r(x)\bar{e}_r(0)\rangle \ . \tag{3.4}$$

In view of (2.32)

$$\hat{\mathcal{G}}(p) = \hat{G}(p;n)\,|_{n=p} \ , \tag{3.5}$$

where

$$G(x-y;n) = -i\langle T\Psi_r(x;n)\bar{\Psi}_r(y;n)\rangle \ . \tag{3.6}$$

In order to find out the mass-shell behavior of (3.5) it will be sufficient to study (3.6) only in a particular range of n, specified below [see Eqs. (3.21) and (3.28)].

By applying the functional integral technique,[18,19] one obtains

$$G(x;n) = \frac{\int [\mathcal{D}A] G(x-y; A^{\mu}_r(x;n)) S_0(A^{\mu}_r(x;n)) \exp\left[i \int d^4 z \, \mathcal{L}_{\rm ph}(z) \right]}{\int [\mathcal{D}A] S_0(A^{\mu}_r(x;n)) \exp\left[i \int d^4 z \, \mathcal{L}_{\rm ph}(z) \right]} \ . \tag{3.7}$$

Here

$$\mathcal{L}_{\rm ph} = -\tfrac{1}{4}(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu})(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}) - B\partial_{\mu}A^{\mu} + \frac{1}{2\xi}B^2 \Big|_{B=\xi\partial_{\mu}A^{\mu}} \ , \tag{3.8}$$

$S_0(A^{\mu}_r(x;n))$ is the determinant of the Dirac operator

$$i u^{\rho}\frac{\partial}{\partial x^{\rho}} - m - g u_{\rho} A^{\rho}_r(x;n) \tag{3.9}$$

for a spinor particle in the external field $A^{\mu}_r(x;n)$, and $G(x-y;A^{\mu}_r(x;n))$ is the corresponding Green's function, i.e.,

$$\left[i u^{\rho}\frac{\partial}{\partial x^{\rho}} - m - g u_{\rho} A^{\rho}_r(x;n) \right] G(x-y; A^{\mu}_r(x;n)) = \delta(x-y) \ . \tag{3.10}$$

The first step in calculating (3.7) is the determination of $G(x-y;A^{\mu}_r(x;n))$. Performing the Fourier transform with respect to $x-y$ in (3.10) one gets

$$\left[up - m + i u^{\rho}\frac{\partial}{\partial x^{\rho}} - g u_{\rho} A^{\rho}_r(x;n) \right] \hat{G}(p; A^{\mu}_r(x;n)) = 1 \ . \tag{3.11}$$

By inserting the representation

$$\hat{G}(p;A^{\mu}_r(x;n)) = -i \int_0^{\infty} d\nu \exp[i\nu(up - m + i0) + iK(\nu; A^{\mu}_r(x;n))] \tag{3.12}$$

into (3.11) and taking into account the identity

$$\lim_{\epsilon\to 0+}\int_0^\infty da\, e^{ia-\epsilon a}=i\ ,$$

Eq. (3.11) is converted into the differential equation

$$\frac{\partial}{\partial v}K(v;A_r^\mu(x;n))=-u^\rho\frac{\partial}{\partial x^\rho}K(v;A_r^\mu(x;n))-gu_\rho A_r^\rho(x;n) \quad (3.13)$$

with the initial condition

$$K(0;A_r^\mu(x;n))=0\ . \quad (3.14)$$

The solution of (3.13), satisfying (3.14), is unique. It reads

$$K(v;A_r^\mu(x;n))=\int d_4k\, V_\rho(-k;x;n)\hat{A}^\rho(k)\ , \quad (3.15)$$

where

$$V_\rho(-k;x;n)=-g\int_0^v d\eta\, u^\sigma T^r_{\sigma\rho}(k;n)\times\exp[ik(x-\eta n)]\ , \quad (3.16)$$

and $T^r_{\sigma\rho}(k;n)$ is given by (2.34).

After $G(x-y;A_r^\mu(x;n))$ has been evaluated, we have to perform the functional integration in (3.7). We first note that, because of the vanishing of the vacuum polarization in the BN model, one has

$$S_0(A_r^\mu(x;n))=1\ . \quad (3.17)$$

Consequently, one is led to a Gaussian integral, which yields

$$\hat{G}(p;n)=-i\int_0^\infty dv\exp[iv(up-m+i0)+I(v;u;n)]\ , \quad (3.18)$$

where

$$I(v;u;n)=ig^2\int_0^v d\eta_1\int_0^{\eta_1}d\eta_2\int d_4k\exp[iku(\eta_1-\eta_2)]u^\lambda T^r_{\lambda\tau}(k;n)u^\rho T^r_{\rho\sigma}(k;n)\times[g^{\tau\sigma}-(1-\xi^{-1})k^\tau k^\sigma(k^2+i\epsilon)^{-1}](k^2+i\epsilon)^{-1}\ . \quad (3.19)$$

By use of the equality

$$T^r_{\mu\nu}(k;n)k^\nu=0\ , \quad (3.20)$$

one realizes that the contribution of the longitudinal part of the photon propagator vanishes. Under the restriction

$$n^2>0\ , \quad (3.21)$$

a straightforward calculation sketched in the Appendix gives, in the limit $\epsilon\to 0$,

$$I(v;u;n)=-d(u;n)\int_0^v d\eta_1\int_0^{\eta_1}d\eta_2(\eta_1-\eta_2)^{-2}\ , \quad (3.22)$$

where

$$d(u;n)=\frac{\alpha}{\pi}\left[2+r(u;n)^{-1}\ln\frac{1-r(u;n)}{1+r(u;n)}\right]\ , \quad (3.23a)$$

$$r(u;n)=[1-u^2n^2/(un)^2]^{1/2}\ , \quad (3.23b)$$

and $\alpha=g^2/4\pi$.

Two remarks are in order:

(1) $\hat{G}(p;n)$ is ξ independent—the first advantage of working with the locally gauge-invariant fields $\Psi_r(x;n)$ and $\overline{\Psi}_r(x;n)$.

(2) $I(v;u;n)$ contains an end-point singularity at $\eta_2=\eta_1$, much as does the corresponding integral in the analogous treatment of the fields ψ and $\overline{\psi}$ [see, e.g., Eq. (8.22) in Ref. 19]. This singularity, which is typical of the BN model, has ultraviolet character, as suggested by (3.19), where $\eta_2=\eta_1$ is the subset of the domain of integration where the oscillation factor

$$\exp[iku(\eta_1-\eta_2)]$$

trivializes. The ultraviolet structure of the BN model is in no way significant for QED, since the BN approximation is reliable only in the IR region. Therefore, adopting the ultraviolet regularization

$$(\eta_1-\eta_2)^{-2}\to\text{P.V.}(\eta_1-\eta_2)^{-2}=\frac{1}{2}\left[\left[\eta_1-\eta_2-\frac{i}{\Lambda}\right]^{-2}+\left[\eta_1-\eta_2+\frac{i}{\Lambda}\right]^{-2}\right]_{\Lambda\to\infty}\ , \quad (3.24)$$

we will be interested exclusively in the mass-shell properties of (3.18).

Performing in (3.22) the integrations in η_1 and η_2 with the prescription (3.24) and neglecting the terms vanishing in the limit $\Lambda \to \infty$, one finds

$$\hat{G}_\Lambda(p;n) = -i \int_0^\infty d\nu \exp\{i\nu(up - m + i0) + d(u;n)[\ln(\nu m) + \ln(\Lambda/m)]\} . \tag{3.25}$$

The mass-shell (ms) behavior

$$p^2 \to m^2, \quad up \to m \tag{3.26}$$

of $\hat{G}_\Lambda(p;n)$ is governed by the terms of $Re I(\nu;u;n)$, which are singular for $\nu \to \infty$. Therefore $\hat{G}_\Lambda(p;n)|_{ms}$ actually is Λ independent and has the form

$$\hat{G}(p;n)|_{ms} = -i \int_0^\infty d\nu\, e^{i\nu(up - m + i0)} (m\nu)^{d(u;n)} . \tag{3.27}$$

The definiteness of (3.27) demands a further restriction on n, namely,

$$d(u;n) > -1 . \tag{3.28}$$

The set of four-vectors n, obeying conditions (3.21) and (3.28), is nonempty. Indeed, for sufficiently small ϵ, the vectors of the form

$$n_\mu = m u_\mu + \epsilon e_\mu, \quad e^2 = -1, \quad eu = 0 \tag{3.29}$$

satisfy them. Finally,

$$\hat{G}(p;n)|_{ms} = \Gamma(1 + d(u;n))(im)^{d(u;n)} \times (up - m + i0)^{-1-d(u;n)} . \tag{3.30}$$

It follows from (3.30) that $\Psi_r(x;n)$ and $\overline{\Psi}_r(x;n)$ have *nontrivial IR anomalous dimension,* much as do the fields $\psi(x)$ and $\overline{\psi}(x)$. We recall that the mass-shell behavior of

$$G(x-y) = -i\langle T\psi(x)\overline{\psi}(y)\rangle \tag{3.31}$$

is given by[17–19]

$$\hat{G}(p) = \Gamma(1 + d(\xi))(im)^{d(\xi)} \times (up - m + i0)^{-1-d(\xi)} \tag{3.32a}$$

with

$$d(\xi) = \frac{\alpha}{2\pi}(3 - \xi^{-1}) . \tag{3.32b}$$

Completely different is the situation with the fields $e_r(x)$ and $\overline{e}_r(x)$. Indeed, on the one hand, Eq. (3.5) and (3.27) imply

$$\hat{\mathscr{G}}(p)|_{ms} = -i \int_0^\infty d\nu\, e^{i\nu(up - m + i0)} (m\nu)^{d(u;p)} ; \tag{3.33}$$

on the other hand, in the regime (3.26), p satisfies (3.21) and (3.28), and according to (3.23),

$$d(u;p) \to 0 \quad \text{for } p^2 \to m^2 \text{ and } up \to m . \tag{3.34}$$

Therefore

$$(up - m)\hat{\mathscr{G}}(p) \to 1 \quad \text{for } p^2 \to m^2 \text{ and } up \to m . \tag{3.35}$$

In this sense the fields $e_r(x)$ and $\overline{e}_r(x)$ have *trivial IR anomalous dimension.* In conclusion they exhibit a qualitatively different mass-shell behavior, compared to ψ and Ψ_r. This nonperturbative result has been verified[1] in QED also independently of the BN approximation by means of the renormalization-group technique.

An intuitive explanation of why, among the fields $\hat{\Psi}_r(p;n)$, only

$$\hat{e}_r(p) = \hat{\Psi}_r(p;p)$$

has a vanishing anomalous IR dimension, can be given in the following way. As can be read off from (3.3a), $\hat{\Psi}_r(p-k;n)$ is coupled to $\hat{A}_\mu(k)$ through the vertex $u^\mu T^r_{\nu\mu}(k;n)$. Since, in the regime (3.26), p_μ becomes proportional to u_μ, the effective IR vertex is of the form $p^\nu T^r_{\nu\mu}(k;n)$. The choice $n = p$ is the only one leading to a vanishing effective IR vertex, due to

$$p^\nu T^r_{\nu\mu}(k;p) = 0 . \tag{3.36}$$

We finally note that the behavior (3.35) holds for any Green's function

$$\langle T\hat{e}_r(p_1) \cdots \hat{e}_r(p_f)\hat{\overline{e}}_r(p_{f+1}) \cdots \hat{\overline{e}}_r(p_{2f}) \cdots \rangle ,$$

which, according to (2.32), is given by

$$\langle T\hat{e}_r(p_1) \cdots \hat{e}_r(p_f)\hat{\overline{e}}_r(p_{f+1}) \cdots \hat{\overline{e}}_r(p_{2f}) \cdots \rangle = \langle T\hat{\Psi}_r(p_1;n_1) \cdots \hat{\Psi}_r(p_f;n_f)\hat{\overline{\Psi}}_r(p_{f+1};n_{f+1}) \cdots \hat{\overline{\Psi}}_r(p_{2f};n_{2f}) \cdots \rangle |_{n_1=p_1,\ldots,n_{2f}=p_{2f}} . \tag{3.37}$$

The proof of this fact follows, with obvious modifications, the proof, given in Ref. 20, of the analogous statement concerning the fields ψ and $\bar{\psi}$. As a consequence, the fields $e_r(x)$ and $\bar{e}_r(x)$ allow for the standard LSZ treatment of the corresponding asymptotic limits $e_r^{\text{ex}}(x), \bar{e}_r^{\text{ex}}(x)$ (ex=in/out), and, we emphasize, the latter are *free* spinor fields.

IV. THE BLOCH-NORDSIECK APPROXIMATION FOR CONFINED PHYSICAL CHARGED FIELDS:DQED

In this section we consider DQED—an Abelian gauge model, formally obtained from QED by replacing the photon field with a dipole gluon field.[21,22] Dipole QED can be given a Lagrangian formulation, which is achieved by the *nondegenerate* Lagrangian density[23]

$$\mathscr{L} = -\tfrac{1}{2}G_{\mu\nu}H^{\mu\nu} - \tfrac{1}{2}M^2 C_\mu C^\mu - C(\partial_\mu B^\mu) - B(\partial_\mu C^\mu) + \frac{1}{\xi}BC + \bar{\psi}(i\partial\!\!\!/ - m - gB\!\!\!/)\psi\ , \tag{4.1a}$$

where

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\ , \quad H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu \tag{4.1b}$$

and M is a constant with a mass dimension. The equation of motion for the field B_μ, following from (4.1), reads

$$\Box^2 B_\mu(x) - (1-\xi^2)\Box\partial_\mu\partial^\nu B_\nu = M^2 j_\mu\ , \tag{4.2}$$

with j_μ given by (2.4). Therefore the *bare* propagator of B_μ behaves like (1.1), and consequently DQED is super-renormalizable[24] and asymptotically free. Since the fermion field ψ is massive, a well-known decoupling argument[25] ensures that the IR behavior of the *full* gluon propagator can be at most as singular as (1.1).[26]

In studying the IR structure of the physical charged fields of DQED, we will apply the manifest locally gauge-invariant formalism developed in Sec. II for QED. This formalism has a straightforward extension to DQED. In analogy with (2.16) one introduces the fields

$$\Phi(x;f) = \left\{\exp\left[ig\int d^4y\, f_\mu(x-y) \times B^\mu(y)\right]\right\}\psi(x)\ , \tag{4.3a}$$

$$\bar{\Phi}(x;f) = \bar{\psi}(x)\left\{\exp\left[-ig\int d^4y\, f_\mu(x-y) \times B^\mu(y)\right]\right\}, \tag{4.3b}$$

where f_μ satisfies (2.17). Let us denote furthermore by $\Phi_r(x;n)$ and $q_r(x)$ the counterparts of $\Psi_r(x;n)$ and $e_r(x)$, respectively. Following, *mutatis mutandis,* the calculation in Sec. III, one obtains for the Fourier transform of

$$G'(x-y;n) = -i\langle T\Phi_r(x;n)\bar{\Phi}_r(y;n)\rangle\ , \tag{4.4}$$

in the BN approximation, the result

$$\hat{G}'(p;n) = -i\int_0^\infty d\nu \exp[i\nu(up - m + i0) + J(\nu;u;n)]\ , \tag{4.5}$$

where

$$J(\nu;u;n) = -ig^2M^2\int_0^\nu d\eta_1 \int_0^{\eta_1} d\eta_2 \int d_4k\{\exp[iku(\eta_1-\eta_2)]\}u^\lambda T^r_{\lambda\tau}(k;n)u^\rho T^r_{\rho\sigma}(k;n) \times[g^{\tau\sigma} - (1-\xi^{-2})k^\tau k^\sigma(k^2+i\epsilon)^{-1}]\hat{E}^c(k;\mu) \tag{4.6}$$

and $\hat{E}^c(k;\mu)$ represents the Fourier transform of the Green's function of the operator $\Box^2$. Among others (see, e.g., Ref. 1), $\hat{E}^c(k;\mu)$ can be given the representation[27]

$$\hat{E}^c(k;\mu) = \underset{\delta\to 0+}{\text{w-lim}}\ \frac{d}{d\delta}[\delta(\mu^2 e)^{-\delta}(-k^2 - i\epsilon)^{-2+\delta}]\ , \tag{4.7a}$$

where w-lim means the weak limit in $\mathscr{S}'(R^4)$. The parameter μ, with dimension of a mass, can take any real value, so that $\hat{E}^c(k;\mu)$ rather represents a one-parameter family of Green's functions. This situation occurs because the distribution $(k^2+i\epsilon)^{-2}$ is well defined only on (test) functions vanishing at $k=0$. $\hat{E}^c(k;\mu)$ stands for the family of all causal extensions of $(k^2+i\epsilon)^{-2}$ that satisfy Lorentz invariance and the normalization condition $(k^2)^2\hat{E}^c(k;\mu) = 1$, following from (4.2) with $j_\mu = 0$. In coordinate space one has

$$E^c(x;l) = i(4\pi)^{-2}\ln\frac{l^2}{-x^2+i\epsilon}\ , \tag{4.7b}$$

$$l^2=4e^{-2\gamma}\mu^{-2}\ , \tag{4.7c}$$

γ being the Euler constant.

Equation (3.20) ensures the ξ independence of $\hat{G}\,'(p;n)$. Performing in (4.6) the integration in k, η_1, and η_2 (see the Appendix) one gets for $n^2>0$, in the limit $\epsilon\to 0$,

$$\hat{G}\,'(p;n)=-i\int_0^\infty dv\exp\left\{iv(up-m+i0)-\frac{\beta}{4\pi}\left[\frac{(un)^2-u^2n^2}{n^2}\right]v^2\right\}, \tag{4.8}$$

with $\beta=g^2M^2/4\pi$. It is remarkable that $\hat{G}\,'(p;n)$ is free both from IR and ultraviolet divergences and is μ independent. Furthermore, because of (3.2) and (3.21), $[(un)^2-u^2n^2]/n^2>0$, which implies that (4.8) is well defined. After the integral in v has been evaluated[28] one is led to the following expression:

$$\hat{G}\,'(p;n)=-i\pi R(u;n)\beta^{-1/2}\{\exp[-\pi(up-m)^2R(u;n)^2\beta^{-1}]\}$$
$$\times\{1+\mathrm{erf}[i\pi^{1/2}(up-m)R(u;n)\beta^{-1/2}]\}\ , \tag{4.9}$$

where

$$R(u;n)=\left[\frac{n^2}{(un)^2-u^2n^2}\right]^{1/2}$$

and

$$\mathrm{erf}(z)=\frac{2}{\sqrt{\pi}}\int_0^z dt\,e^{-t^2}\ .$$

In analyzing the properties of $\hat{G}\,'(p;n)$ we first observe that, due to the exponential appearing in (4.9), it has an *essential singularity* at $\beta=0$.[29] Consequently, the corresponding perturbative expansion in β is not convergent. However, as we are going to show now, it is asymptotic. In order to prove this statement we first derive the explicit form of the just-mentioned perturbative expansion. It is easily obtained by expanding the integrand in the right-hand side of (4.8) in powers of β, formally commuting the infinite sum with the integral and, after that, performing the integration in v. In this way one gets the series

$$(up-m)^{-1}\sum_{k=0}^\infty(2k)!(k!)^{-1}\left[\frac{\beta}{4\pi}(up-m)^{-2}R(u;n)^{-2}\right]^k\ . \tag{4.10}$$

The series (4.10) is obviously divergent. On the other hand, by using the asymptotic expansion formulas[30]

$$x^{-1/2}\exp(-x^{-1})\underset{x\to 0+}{\sim}0\ , \tag{4.11a}$$

$$\mathrm{erf}(x^{-1})\underset{x\to 0+}{\sim}1-\pi^{-1/2}x\exp(-x^{-2})\sum_{k=0}^\infty(-1)^k(2k-1)!!2^{-k}x^{2k} \tag{4.11b}$$

and (4.9), after straightforward manipulations one obtains

$$\hat{G}\,'(p;n)\underset{\beta\to 0+}{\sim}(up-m)^{-1}\sum_{k=0}^\infty(2k)!(k!)^{-1}\left[\frac{\beta}{4\pi}(up-m)^{-2}R(u;n)^{-2}\right]^k, \tag{4.12}$$

which completes the proof.

It is worth stressing that (4.10) is not Borel summable. Indeed the Borel transform[31] of (4.10) reads

$$(up-m)^{-1}\sum_{k=0}^\infty(2k)!(k!)^{-2}\left[\frac{\beta}{4\pi}(up-m)^{-2}R(u;n)^{-2}\right]^k=(up-m)^{-1}\left[1-\frac{\beta}{\pi}(up-m)^{-2}R(u;n)^{-2}\right]^{-1/2}. \tag{4.13}$$

Although convergent for $|\beta|<\pi(up-m)^2\,R(u;n)^2$, it has a cut along the positive real axis with a branch point at $\beta=\pi(up-m)^2\,R(u;n)^2$. Such a singular structure prevents the Borel summability of (4.10).[32] The singular behavior in β of DQED has to be contrasted with the case of QED, where, in the BN approximation, the Green's function $\hat{G}(p;n)$ [see Eq. (3.30)] has a convergent power expansion around $\alpha=0$.

Let us now consider the Green's function

$$\mathcal{G}'(x-y)=-i\langle Tq_r(x)\bar{q}_r(y)\rangle \; . \tag{4.14}$$

Using the relationship

$$\hat{\mathcal{G}}'(p)=\hat{G}'(p;n)\,|_{n=p} \tag{4.15}$$

and (4.9) one gets

$$\hat{\mathcal{G}}'(p)=-i\pi R(u;p)\beta^{-1/2}\{\exp[-\pi(up-m)^2R(u;p)^2\beta^{-1}]\}\{1+\mathrm{erf}[i(up-m)R(u;p)\beta^{-1/2}]\} \; . \tag{4.16}$$

Clearly $\hat{\mathcal{G}}'(p)$ has the same analytic properties in β as $\hat{G}'(p;n)$.

It is instructive at this stage to consider the mass-shell behavior of $\hat{G}'(p;n)$ and $\hat{\mathcal{G}}'(p)$. Equation (4.9) implies that $\hat{G}'(p;n)$ is smooth in the limit (3.26). More precisely

$$(up-m)\hat{G}'(p;n)\to 0 \quad \text{for } p^2\to m^2 \text{ and } up\to m \; . \tag{4.17}$$

From (4.16) it follows that, in the complex p^0 plane, $\hat{\mathcal{G}}'(p)$ has a cut along the positive real axis with a branch point at $p^0=(m+\vec{u}\cdot\vec{p})/u^0$. Nevertheless

$$(up-m)\hat{\mathcal{G}}'(p)\to 0 \quad \text{for } p^2\to m^2 \text{ and } up\to m \; . \tag{4.18}$$

Therefore, in the framework of the LSZ formalism, the fields Φ_r and q_r and their Dirac conjugates have *zero asymptotic limit.* This fact is another substantial difference of DQED compared to QED, where the fields e_r^{ex} and $\bar{e}_r^{\mathrm{ex}}$, which are the analogs of q_r^{ex} and $\bar{q}_r^{\mathrm{ex}}$, are free spinor fields.

V. CONCLUSIONS

The above analysis of Abelian gauge models is an example of the complexity of the relationship between the physical charged fields, which necessarily are nonlocal, and the basic local fields appearing in the Lagrangian of gauge theories. The main result is that the mass-shell behavior of the two sets of fields is, in general, qualitatively different. Therefore, in order to determine the asymptotic particle content of the theory, one is forced to investigate the IR behavior of the physical sectors.

A significant feature of the considered case with confined charge is the singular behavior of the Borel transform of the perturbative series for positive values of the coupling constant. This behavior is similar to the singular structure, signaled by instantons, in non-Abelian gauge theories[33] and implies that the perturbative series is not Borel summable.

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APPENDIX

The integrals $I(v;u;n)$ and $J(v;u;n)$, defined by (3.19) and (4.6), respectively, can be given the following parametrization:

$$I(v;u;n)=ig^2\int_0^v d\eta_1\int_0^{\eta_1} d\eta_2[u^2I_0(\eta_1-\eta_2;u)-2(un)I_1(\eta_1-\eta_2;u;n)+n^2I_2(\eta_1-\eta_2;u;n)] \; , \tag{A1}$$

$$J(v;u;n)=-ig^2M^2\int_0^v d\eta_1\int_0^{\eta_1} d\eta_2[u^2J_0(\eta_1-\eta_2;u)-2(un)J_1(\eta_1-\eta_2;u;n)+n^2J_2(\eta_1-\eta_2;u;n)] \; , \tag{A2}$$

where

$$I_0(\eta;u)=\int d_4k\, e^{iku\eta}(k^2+i\epsilon)^{-1}=i(2\pi)^{-2}[(\eta u)^2-i\epsilon]^{-1} \; , \tag{A3a}$$

$$J_0(\eta;u)=\int d_4k\, e^{iku\eta}\hat{E}^c(k;\mu)=i(4\pi)^{-2}\ln\frac{l^2}{-(\eta u)^2+i\epsilon}, \quad l^2=4e^{-2\gamma}\mu^{-2} \; , \tag{A3b}$$

$$I_1(\eta;u;n)=\int d_4k\, e^{iku\eta}(ku)(k^2+i\epsilon)^{-1}(kn+i\epsilon)^{-1} \; , \tag{A4a}$$

$$J_1(\eta;u;n)=\int d_4k\, e^{iku\eta}(ku)\hat{E}^c(k;\mu)(kn+i\epsilon)^{-1} \tag{A4b}$$

and

$$I_2(\eta;u;n)=-u^\mu\frac{\partial}{\partial n^\mu}I_1(\eta;u;n)\ , \tag{A5a}$$

$$J_2(\eta;u;n)=-u^\mu\frac{\partial}{\partial n^\mu}J_1(\eta;u;n)\ . \tag{A5b}$$

Therefore one is left with the evaluation of $I_1(\eta;u;n)$ and $J_1(\eta;u;n)$. Before performing this calculation, it is useful to notice that, for any four-vector n, condition (3.2) leads to

$$(un)^2\geq u^2n^2\ . \tag{A6a}$$

Furthermore, in the region $n^2>0$ we are interested in,

$$(un)^2\geq u^2n^2>0\ , \tag{A6b}$$

which implies

$$0\leq r(u;n)<1\ , \tag{A7}$$

$r(u;n)$ being given by (3.23b).

In evaluating $I_1(\eta;u;n)$ it is convenient to represent it in the form

$$\begin{aligned}I_1(\eta;u;n)&=-i\frac{d}{d\eta}\int d_4k\,\exp(iku\eta)(k^2+i\epsilon)^{-1}(kn+i\epsilon)^{-1}\\&=-\frac{d}{d\eta}\int_0^\infty dt\int d_4k\{\exp[ik(u\eta+nt)]\}(k^2+i\epsilon)^{-1}\ .\end{aligned} \tag{A8a}$$

Now the integration in k, being a Fourier transform, is easily performed and yields[34]

$$I_1(\eta;u;n)=i(2\pi)^{-2}\frac{d}{d\eta}\int_0^\infty dt[-(u\eta+tn)^2+i\epsilon]^{-1}\ . \tag{A8b}$$

Finally, taking into account (A6), one gets, in the limit $\epsilon\to 0$,

$$I_1(\eta;u;n)=i(2\pi)^{-2}[2\eta^2(un)r(u;n)]^{-1}\ln\frac{1+r(u;n)}{1-r(u;n)}\ . \tag{A9}$$

The calculation of $J_1(\eta;u;n)$ goes along the same lines, but it is slightly more complicated, due to the presence of $\hat{E}^c(k;\mu)$. Using the representation (4.7a) one obtains

$$J_1(\eta;u;n)=-\lim_{\delta\to 0+}\frac{d}{d\delta}\delta(\mu^2e)^{-\delta}\frac{d}{d\eta}\int_0^\infty dt\int d_4k\{\exp[ik(u\eta+nt)]\}(-k^2-i\epsilon)^{-2+\delta}\ . \tag{A10a}$$

Performing the Fourier integral[34] and the derivative with respect to η, one is led to

$$J_1(\eta;u;n)=-2i(4\pi)^{-2}\lim_{\delta\to 0+}\frac{d}{d\delta}\frac{\Gamma(1+\delta)}{\Gamma(2-\delta)}(4/\mu^2e)^\delta\delta\int_0^\infty dt[(un)t+u^2\eta][-(u\eta+nt)^2+i\epsilon]^{-1-\delta}\ , \tag{A10b}$$

which gives

$$J_1(\eta;u;n)=i(4\pi)^{-2}\frac{(un)}{n^2}\left[\ln\frac{l^2}{-u^2\eta^2+i\epsilon}+r(u;n)\ln\frac{1+r(u;n)}{1-r(u;n)}\right]\ . \tag{A11}$$

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