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Massive, degenerate neutrinos and cosmology

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Modifications to the standard big-bang neutrino cosmology resulting from a nonzero chemical potential are discussed. Cosmological neutrino mass limits as a function of μ/T are derived. Implications for galaxy formation in a universe with degenerate neutrinos are emphasized.

I. INTRODUCTION

Great interest in neutrino cosmology has been stimulated by recent experiments[1,2] and by theoretical work in grand unification[3] suggesting that neutrinos may be massive. If neutrino masses are greater than a few eV, neutrinos may be the dominant matter in the universe and certainly play an important role in galaxy formation.[4] The maximum value of the neutrino Jeans mass determines the scale on which structure first forms in a neutrino-dominated universe,[5] and neutrinos may supply the dark matter in the universe.

It is usually assumed that neutrinos are nondegenerate, i.e., the neutrino chemical potential, μ, is much less than the neutrino temperature, T. Although the ratio μ/T (or equivalently the lepton number $L \equiv n_L/n_\gamma$) is likely to be small (recall that the baryon number of the universe is $\lesssim 10^{-9}$, a large value for μ/T has not been ruled out by observations. A large value of μ/T (corresponding to $L \gtrsim 1$) would have dramatic cosmological effects.

In this paper, we examine the effects of a nonzero chemical potential on various aspects of galaxy formation. We obtain constraints on μ/T and the lepton number L. We consider stable neutrinos with masses up to $\sim$ 200 eV; stable neutrinos more massive than this contribute too much mass density unless they are more massive than $\sim$2 GeV (Ref. 6). In this case, their number density relative to photons must be less than $\sim 10^{-9}$, so that L and μ/T are less than $\sim 10^{-9}$.

In the remainder of the Introduction we briefly review neutrino statistics, neutrino decoupling, and the relationship of chemical potential to lepton number. We find that the temperature at which neutrinos drop out of chemical equilibrium is a function of the chemical potential. A neutrino species with $\mu/T \gtrsim 15$ decouples very early and does not share in the entropy release when various massive particle species annihilate (e.g., $\mu^+\mu^-$ pairs). If $\mu/T \gtrsim 15$, then the present temperature of such a species is $T_\nu \simeq T_\gamma/3$, rather than the familiar $T_\nu \simeq (\frac{4}{11})^{1/3} T_\gamma$. Details are given in the Appendix.

In Sec. II, by requiring $\Omega < 2$, we derive an upper limit on the mass of a neutrino species as a function of μ/T. In particular, if the experiment of Lubimov *et al.*[2] is correct and $m_{\nu_e} \simeq 30$ eV, we find that $L_e \lesssim 1.5$. In Sec. III, we discuss various limits on μ/T. In previous work[7,8] the weak limit $\mu/T \lesssim 66$ was obtained for a massless neutrino species by requiring that $\Omega \lesssim 2$. Taking into account the correct neutrino temperatures for high μ/T, the limit becomes even weaker: $\mu/T \lesssim 141$. However, if we require that the matter density exceed the neutrino radiation density at a red-shift $1+z=100$, then the stronger limit $\mu/T \lesssim 44$ follows. (Matter domination by at least this time is necessary so that perturbations undergo sufficient growth for galaxy formation.[9])

The maximum value of the Jeans mass in a neutrino-dominated universe sets the scale for "pancake" formation in the Zeldovich picture of galaxy formation. In Sec. IV, we calculate this maximum Jeans mass for a neutrino species with nonzero chemical potential. For large-scale structure to survive in a neutrino-dominated universe, the maximum Jeans mass should not exceed supercluster sizes $\lesssim 10^{17} M_\odot$; this requirement can only be satisfied for $\mu/T \leq 5$.

A. Neutrino Statistics

Since neutrinos are spin-$\frac{1}{2}$ particles they obey Fermi-Dirac statistics. At very high temperatures ($T \gg 1$ MeV), neutrinos are in thermal equilibrium with the rest of the universe. Since we are concerned only with neutrinos of masses much less than 1 MeV, they are also relativistic at this time, with number densities given by

$$n(\nu_i) = \frac{1}{2\pi^2} \int dp\, p^2 / \{\exp[(p-\mu)/T_\nu] + 1\} \ . \tag{1.1}$$

Throughout we take $\hbar = k_B = c = 1$, and assume that $\mu > 0$. For $\mu < 0$, one only needs to interchange "neutrino" with "antineutrino" everywhere. The neutrinos drop out of chemical equilibrium once the weak-interaction rates for their production ($e^+e^- \to \nu\bar{\nu}$, etc.) can no longer keep up with the expansion of the universe. Their number density can be described by the same form after decoupling, with a red-shifted neutrino temperature T_ν since the momenta simply red-shift with the scale factor. The ratio μ/T_ν (hereafter simply μ/T without the subscript ν) remains a constant from decoupling until today.

The temperature T_D at which a neutrino species decouples depends on its chemical potential. Highly degenerate neutrinos with $\mu/T \geq 15$ have decoupling temperatures exponentially related to μ/T; they decouple very early ($T \gg 100$ MeV), and therefore do not participate in the heating due to the annihilation of $\mu^+\mu^-$ pairs and other massive species. T_ν and T_γ are related by $T_\nu/T_\gamma \simeq (3.9/g_{*D})^{1/3}$, where g_{*D} is the number of relativistic degrees of freedom in equilibrium at neutrino decoupling ($\equiv \sum g_B + \frac{7}{8}\sum g_F$). The decoupling temperature is related to μ/T by

$$T_D \simeq 10 \text{ MeV } (\mu/T)^{-2/3} \exp(\mu/3T) \tag{1.2}$$

(see the Appendix for details). For $\mu/T \gtrsim 15$ we shall take $g_{*D} \simeq 110$, which is the number of relativistic degrees of freedom in the minimal $SU(3)\times SU(2)\times U(1)$ model at temperatures above ~ 100 GeV, and so $T_\nu/T_\gamma \simeq \frac{1}{3}$.

B. Lepton Number

A separate lepton number is associated with each leptonic species: e, μ, τ, and perhaps others. For example, for the electron

$$L_e = \frac{n_{e^-} - n_{e^+}}{n_\gamma} + \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_\gamma} \ , \tag{1.3}$$

where the photon number density n_γ has been inserted in the denominator to scale out the expansion of the universe. Since the first term is known to be small by charge neutrality ($\lesssim 10^{-9}$), the neutrino densities in the second term make the dominant contribution to the lepton number. Using the Fermi-Dirac form of the number densities given in Eq. (1.1), it is simple to show that the lepton number is related to μ/T by

$$L_i = 6.9\times 10^{-2} (T_\nu/T_\gamma)^3 [\pi^2(\mu_i/T) + (\mu_i/T)^3] \ . \tag{1.4}$$

The baryon number of the universe is known to be small,

$$B = \frac{n_b - n_{\bar{b}}}{n_\gamma} \leq 10^{-9}$$

(Ref. 10). Although the usual grand unified theories (GUT's) scenario for baryogenesis results in B comparable to L (Ref. 11), one can have models with baryogenesis and yet a big L (Ref. 12). Observations do not rule out a very large lepton number, $L \gg 1$. The standard results of big-bang nucleosynthesis are changed significantly if $L \gtrsim 1$ (Ref. 13). A large lepton number prevents high-temperature restoration of spontaneously broken symmetries,[14] eliminating the possibility of an inflationary universe, possibly resolving the monopole problem since monopoles are only produced thermally, and invalidating previously obtained cosmological bounds on Higgs masses.[15] Higher neutrino densities might also enable the detection of the cosmic neutrino background.

II. LIMITS ON NEUTRINO MASSES AND μ/T

Various observations limit Ω to be less than 2 (Ref. 7), where Ω is the ratio of the density of the universe ρ to the closure density ρ_c. The value of the closure density today is $\rho_c = 1.88\times10^{-29}\ h_0^{2}$ $\mathrm{g\,cm^{-3}} = 8.1\times10^{-11}\ h_0^{2}$ $\mathrm{eV^4}$, where the Hubble parameter $H_0 = 100\ h_0\ \mathrm{km\,sec^{-1}\,Mpc^{-1}}$. We shall require the energy density of each neutrino species to satisfy $\rho(\nu)+\rho(\bar{\nu})\le 2\rho_c$. To calculate the present energy density of a massive, degenerate neutrino species, we use the momentum distribution at neutrino decoupling (when the neutrinos were still relativistic and in thermal equilibrium) and red-shift the momenta to their values today. We treat ν and $\bar{\nu}$ separately because for $\mu/T \gtrsim 1$ their distributions are very different, namely $\langle E_\nu\rangle \simeq \frac{3}{4}\mu$, while $\langle E_{\bar{\nu}}\rangle \simeq 3T$. [For $\mu/T \gg 1$, this distinction becomes somewhat irrelevant because there are so few antineutrinos, $n_{\bar{\nu}}/n_\nu \simeq \exp(-\mu/T)$.] Throughout, quantities evaluated at the present epoch will carry the subscript 0, whereas those evaluated at decoupling will carry the subscript D. Thus we have

$$2\rho_c \ge \rho_0(\nu)+\rho_0(\bar{\nu}) = \langle E_\nu\rangle_0 (n_\nu)_0 + \langle E_{\bar{\nu}}\rangle_0 (n_{\bar{\nu}})_0$$
$$= [\langle p_\nu\rangle_D^{2}(T_{\nu0}/T_D)^2 + m_\nu^{2}]^{1/2}(n_\nu)_0 + [\langle p_{\bar{\nu}}\rangle_D^{2}(T_{\nu0}/T_D)^2 + m_\nu^{2}]^{1/2}(n_{\bar{\nu}})_0 \,. \qquad (2.1)$$

In the $\mu=0$ limit this becomes

$$[3\zeta(3)/2\pi^2] m_\nu T_{\nu0}^{3} \le 1.6\times10^{-10} h_0^{2}\ \mathrm{eV}^4$$

or

$$m_\nu \le 190\ \mathrm{eV}\ h_0^{2}(2.7\ \mathrm{K}/T_{\gamma0})^3 \,,$$

where $T_{\gamma0}$ is the photon temperature today. In the range $1 \ll \mu/T < 15$ we obtain

$$m_\nu < 2.1\times10^{3}\ \mathrm{eV}\ h_0^{2}(\mu/T)^{-3}(2.7\ \mathrm{K}/T_{\gamma0})^3 \,, \qquad (2.2)$$

while for $\mu/T \gtrsim 15$

$$m_\nu < 2.0\times10^{4}\ \mathrm{eV}\ h_0^{2}(\mu/T)^{-3}(2.7\ \mathrm{K}/T_{\gamma0})^3 \,. \qquad (2.3)$$

Larger values of μ and L require smaller neutrino masses. Figure 1 shows a graph of the upper limit to the neutrino mass for different values of μ/T and L, where μ/T and L are related by Eq. (1.4). Lubimov *et al.* find evidence for an electron neutrino mass: $m_\nu \gtrsim 14$ eV (99% confidence level). From Fig. 1, we see that this implies a lepton number $L_e \lesssim 3.5$ and $\mu/T \lesssim 5$. Using $m_\nu \simeq 30$ eV, we find that $L_e \lesssim 1.5$.

III. LIMITS ON μ FOR MASSLESS NEUTRINOS IMPOSED BY GALAXY FORMATION

An upper limit to μ/T has been previously obtained for massless neutrinos by requiring that $\Omega_\nu \lesssim 2$ (Refs. 7 and 8):

$$\rho_{\nu0} = \frac{1}{2\pi^2}\int p^3 dp/\{\exp[(p-\mu)/T_{\nu0}]+1\}$$
$$\simeq \frac{\mu_0^{4}}{8\pi^2} \le 2\rho_c \quad (\mu/T \gg 1) \,. \qquad (3.1)$$

In the previous work the present photon and neutrino temperatures were assumed to be related by $T_{\nu0} = T_{\gamma0}(\frac{4}{11})^{1/3}$ and thus the bound became $(\mu/T) \lesssim 66(2.7\ \mathrm{K}/T_{\gamma0})h_0^{1/2}$. However, for such large values of μ/T, as explained in the Appendix, neutrinos decoupled very early so that

$$T_{\nu0} \simeq T_{\gamma0}(3.9/g_{*D})^{1/3}$$
$$\simeq T_{\gamma0}/3 \simeq 0.9\ K(T_{\gamma0}/2.7\ \mathrm{K})$$

(for $g_{*D} = 110$). Whereas the upper limit on μ remains the same, dividing by the correct neutrino temperature, we obtain the less stringent limit

$$\frac{\mu}{T_\nu} \le 141 \left[\frac{2.7\ \mathrm{K}}{T_{\gamma0}}\right] h_0^{1/2} \,. \qquad (3.2)$$

Note that this results in the upper limit to L_i of $L_i \lesssim 7\times10^{3}\ (2.7\ \mathrm{K}/T_{\gamma0})^3 h_0^{3/2}$.

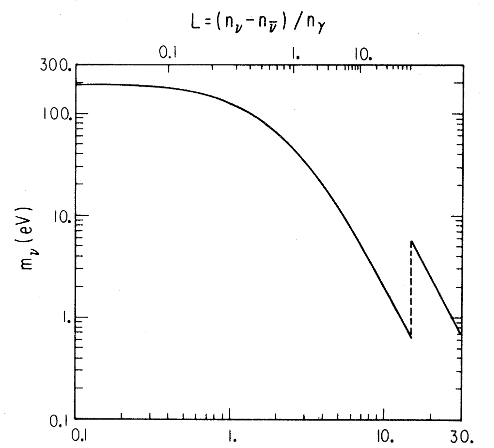

FIG. 1. Upper limit to the neutrino mass as a function of chemical potential μ/T and lepton number L derived by requiring $\Omega_\nu \lesssim 2$. For $\mu/T \gtrsim 15$ neutrinos decouple very early and T_ν/T_γ is about 0.3 rather than 0.7. Since these neutrinos are only about a tenth as abundant as nondegenerate neutrinos, at $\mu/T \simeq 15$ the mass limit abruptly becomes less restrictive. Because lepton number L is related to μ/T by a factor of $(T_\nu/T_\gamma)^3$ [cf. Eq. (1.4)], their functional relationship changes abruptly at $\mu/T \simeq 15$; for this reason $L = 90$ is off scale.

We will now derive a more restrictive limit on μ/T for massless neutrinos by observing that galaxy formation requires matter domination by at least $1+z=100$. Not until the energy density in the universe is dominated by the species (e.g., baryons, massive neutrinos) responsible for the present structure can the density perturbations that lead to galaxy formation start to grow.[16] Perturbations grow as the scale factor, and must reach nonlinearity ($\delta\rho/\rho=1$) to collapse and form stable, self-bound structures. Matter domination by $1+z=100$ allows a growth factor of only about 100 up to the present epoch; in order to reach nonlinearity within this growth period the initial perturbations would have to be very large, so large that they are likely to be in conflict with the isotropy measurements of the microwave background.[9] Matter domination by $1+z=100$ is a very conservative assumption.

Massless, degenerate neutrinos contribute to the radiation density

$$\rho_\nu(T)=\rho_{\nu 0}(1+z)^4=\frac{{\mu_0}^4}{8\pi^2}(1+z)^4 , \tag{3.3}$$

where their energy density today is evaluated in the high μ/T regime, since we are interested in an upper limit to μ/T. Similarly the matter energy density is

$$\rho_m(T)=\rho_{mo}(1+z)^3=\rho_c\Omega_m(1+z)^3 , \tag{3.4}$$

where the density today is given by $\rho_{mo}=\Omega_m\rho_c$. Matter domination by $1+z=100$ requires $\rho_m(T) \geq \rho_\nu(T)$, or

$$\rho_c\Omega_m(1+z)^3 \geq \frac{{\mu_0}^4}{8\pi^2}(1+z)^4 . \tag{3.5}$$

Since the chemical potentials of different neutrino species are not necessarily the same, we require each species independently to satisfy this inequality. We obtain the upper limit

$$\mu_0 \leq 2.8\times 10^{-3}{\Omega_m}^{1/4}\ \mathrm{eV}\ {h_0}^{1/2}\left[\frac{100}{1+z}\right]^{1/4} , \tag{3.6a}$$

$$\frac{\mu}{T} \leq 44{h_0}^{1/2}(\Omega_m/2)^{1/4}\left[\frac{2.7\ \mathrm{K}}{T_{\gamma 0}}\right]\left[\frac{100}{1+z}\right]^{1/4} . \tag{3.6b}$$

If the chemical potentials for all the species are the same the limit becomes more restrictive by a factor of $3^{1/4}$: the 44 in Eq. (3.6b) becomes 33. Matter domination by $1+z=100$ probably does not allow enough growth of the perturbations for galaxy formation to proceed. A larger value of $1+z$ in Eq. (3.6) leads to a more stringent upper limit to μ/T.

IV. NEUTRINO JEANS MASS

A. $\mu=0$ (Ref. 5)

The neutrino Jeans mass is (up to factors of ~ 1) the rest mass of neutrinos inside a sphere whose radius is given by the Jeans length λ_J. The Jeans length is the scale on which radiation pressure forces are just able to balance the gravitational forces. It is also the largest distance a sound wave of speed v_s can cross in a collapse time scale: $\lambda_J \simeq v_s t \simeq v_s/(G\rho)^{1/2}$, where ρ is the total energy density of the universe. (For a collisionless fluid the velocity dispersion plays the role of the pressure.) Scales bigger than the Jeans mass ($M > M_{J\nu}$) are gravitationally unstable, whereas smaller scales are stable and just oscillate as sound (pressure) waves.

While the neutrinos are relativistic the universe is radiation dominated with sound speed $v_s \simeq c/3^{1/2}$. Up to factors of order unity the neutrino Jeans mass is just the rest mass of neutrinos within the horizon, and varies with T as $M_{J\nu} \propto T^{-3}$. When the neutrinos go nonrelativistic, at a temperature where the average neutrino momentum is equal to the neutrino mass $\langle p_\nu \rangle = m_\nu$, they also start to dominate the energy density and the universe begins the matter-dominated era. Thereafter $v_s \simeq \langle p_\nu \rangle / m_\nu \propto T$, and $M_{J\nu} \propto T^{3/2}$. The neutrino Jeans mass reaches its maximum value at about the time the neutrinos become nonrelativistic. The maximum value has been calculated to be[5]

$$\begin{aligned} M_{\nu M} &\simeq 1.8{m_{\mathrm{Pl}}}^3/{m_\nu}^2 \\ &\simeq 3\times 10^{18} M_\odot/(m_\nu/\mathrm{eV})^2 , \end{aligned} \tag{4.1}$$

where $m_{\mathrm{Pl}}=G^{-1/2}\simeq 1.2\times 10^{19}$ GeV is the Planck mass. Perturbations on scales smaller than this are strongly damped by free-streaming of the neutrinos (Landau damping). Only perturbations on larger scales survive, and thus large structures (e.g., superclusters) must form first and then fragment to smaller ones (galaxies).

B. $\mu \neq 0$

(i) Neutrinos. In the case of a nonzero chemical potential, the maximum neutrino Jeans mass is again proportional to the horizon mass at the time when the neutrinos go nonrelativistic. For $\mu/T \gtrsim 1$, ν and $\bar\nu$ have different distributions, and we will calculate a maximum Jeans mass for ν's and $\bar\nu$'s separately. Since the antineutrinos have smaller momenta ($\langle p_{\bar\nu} \rangle \simeq 3T$) than the neutrinos ($\langle p_\nu \rangle \simeq 3\mu/4$), they become nonrelativistic first. In addition, their number density is smaller; both effects lead to a smaller antineutrino Jeans mass. However, for

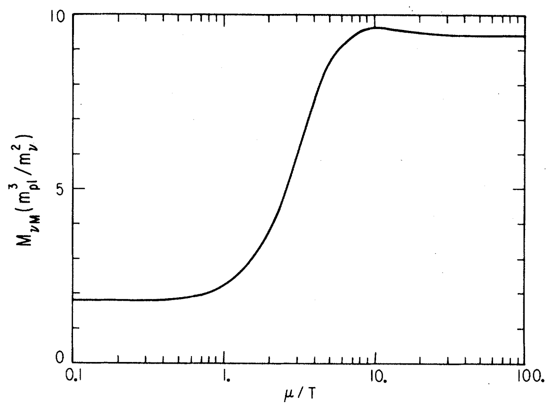

FIG. 2. Maximum neutrino Jeans mass as a function of μ/T.

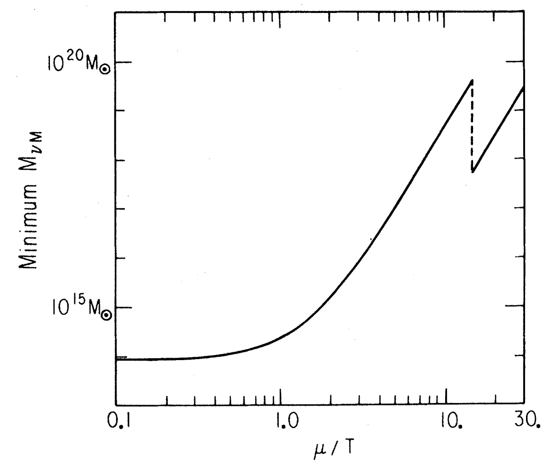

FIG. 3. Minimum allowed value of the maximum neutrino Jeans mass as a function of μ/T. Since the maximum Jeans mass $M_{\nu M} \propto m_\nu^{-2}$, $M_{\nu M}$ attains its minimum for the maximum allowed value of m_ν. The discontinuity for $\mu/T \simeq 15$ results from the discontinuity in the maximum allowed value of m_ν.

$\mu/T \gg 1$ antineutrinos are far outnumbered by neutrinos, $n_{\bar\nu}/n_\nu \simeq \exp(-\mu/T)$, and the neutrino Jeans mass is the relevant scale for galaxy formation. It is given by

$$M_{\nu M} \propto (m_\nu n_\nu t^3)\Big|_{\langle p_\nu\rangle \simeq m_\nu} . \tag{4.2}$$

The age of the universe, t, is determined by the total energy density ρ of the universe: photons, two massless, nondegenerate neutrino species, and one massive, degenerate neutrino species contribute to give $t \sim H^{-1} = (8\pi G\rho/3)^{-1/2}$.

To find the temperature at which the neutrinos (or antineutrinos) go nonrelativistic, we compute the average momentum at decoupling, $\langle p_\nu\rangle_D = \rho_{\nu D}/n_{\nu D}$, and red-shift it to $T_{\rm NR}$ when $m_\nu = \langle p_\nu\rangle_D (T_{\rm NR}/T_D)$. Since the mass we calculate [cf. Eq. (4.2)] is only proportional to the maximum value of the Jeans mass, we normalize our results to those calculated for the $\mu=0$ case.[5] In Fig. 2, we show the maximum Jeans mass for degenerate neutrinos as a function of μ/T. For large μ/T it approaches

$$M_{\nu M} \simeq 9.4 m_{\rm Pl}^3/m_\nu^2 , \tag{4.3}$$

which is more than a factor of 5 larger than the results for $\mu=0$.

In Sec. II, we found an upper limit to the neutrino mass as a function of μ/T. Using this value, and the maximum of the neutrino Jeans mass computed above, we obtain the *minimum* allowed value of the maximum neutrino Jeans mass as a function of μ/T, which is shown in Fig. 3. Perturbations are strongly damped on scales less than the maximum neutrino Jeans mass. To ensure the formation of su-

TABLE I. The ratio of $\rho_{\bar\nu}$ to ρ_ν, maximum ν and $\bar\nu$ Jeans masses, and the minimum-allowed $M_{\nu M}$ and $M_{\bar\nu M}$ for $\mu/T = 1-10$.

μ/T	$\rho_{\bar\nu}/\rho_\nu$	$M_{\nu M}\Big/\left[\frac{m_{\rm Pl}^3}{m_\nu^2}\right]$	$M_{\bar\nu M}\Big/\left[\frac{m_{\rm Pl}^3}{m_\nu^2}\right]$	Minimum $M_{\nu M}/M_\odot$	Minimum $M_{\bar\nu M}/M_\odot$
1	0.15	2.2	0.29	2.4×10^{14}	3.2×10^{13}
2	2.3×10^{-2}	3.7	7.7×10^{-2}	1.5×10^{15}	3.1×10^{13}
3	3.9×10^{-3}	5.8	1.7×10^{-2}	8.2×10^{15}	2.4×10^{13}
4	7.1×10^{-4}	7.5	3.2×10^{-3}	3.4×10^{16}	1.5×10^{13}
5	1.4×10^{-4}	8.6	5.9×10^{-4}	9.7×10^{16}	6.8×10^{12}
6	2.9×10^{-5}	9.1	1.1×10^{-4}	2.7×10^{17}	3.2×10^{12}
7	6.4×10^{-6}	9.4	2.1×10^{-5}	6.0×10^{17}	1.4×10^{12}
8	1.5×10^{-6}	9.5	4.1×10^{-6}	1.3×10^{18}	5.5×10^{11}
9	3.6×10^{-7}	9.6	8.5×10^{-7}	2.4×10^{18}	2.1×10^{11}
10	9.1×10^{-8}	9.6	1.8×10^{-7}	4.4×10^{18}	8.2×10^{10}

perclusters (and hence the subsequent formation of smaller scale structure), we must require that $M_{\nu M} \lesssim M_{sc} \lesssim 10^{17} M_{\odot}$. In a universe dominated by massive, degenerate neutrinos, this implies that μ/T be less than 5, or equivalently that $L \lesssim 4.5$.

(ii) Antineutrinos. In the same way we calculated the maximum Jeans mass for ν's, we have calculated it for the antineutrinos. In Table I, we have tabulated $M_{\nu M}, M_{\bar{\nu} M}$, the minimum allowed values for $M_{\nu M}$ and $M_{\bar{\nu} M}$, and $\rho_{\bar{\nu}}/\rho_{\nu}$ for values of μ/T in the range from 1 to 10. For $\mu/T \simeq 7$ the minimum allowed value for $M_{\bar{\nu} M}$ is $\sim 10^{12} M_{\odot}$; however, since $\rho_{\nu}/\rho_{\bar{\nu}} \gg 1$, growth of perturbations on this interesting scale is very slow since the neutrinos are smooth on this scale. At best the antineutrinos could act as "seeds" for later development of structure on small scales.

V. SUMMARY

Nonzero neutrino chemical potentials (or, equivalently, nonzero lepton numbers) have significant consequences for cosmology. For $\mu/T \gtrsim 15$ neutrinos decouple at a temperature much greater than $\sim$ 100 MeV. This leads to a present neutrino temperature of $\sim T_{\gamma 0}/3$, rather than the familiar result $T_{\nu 0} \simeq (\frac{4}{11})^{1/3} T_{\gamma 0}$ which holds only for $\mu/T \lesssim 15$.

We have derived the μ/T-dependent upper limit to neutrino masses which follows from requiring $\Omega_{\nu} \lesssim 2$. If the mass of the electron neutrino is ~ 30 eV, then L_e must be less than ~ 1. For massless neutrinos, it follows from $\Omega_{\nu} \lesssim 2$ that μ/T must be less than 141 ($L_i \lesssim 7 \times 10^3$). Our result is less stringent than previously derived bounds,[7,8] since we have taken into account that for $\mu/T \gtrsim 15$, $T_{\nu 0} \simeq T_{\gamma 0}/3$. By requiring that the universe be matter-dominated by at least a red-shift of $1+z=100$ for galaxy formation to proceed in the usual way, we have obtained the more restrictive limit $\mu/T \lesssim 44$.

In a neutrino-dominated universe the maximum value of the neutrino Jeans mass sets the scale for the formation of the largest scale structures. We have calculated the maximum Jeans mass for both the neutrinos and the antineutrinos in a universe dominated by massive, degenerate neutrinos. Survival of large-scale structure requires a maximum Jeans mass smaller than a supercluster mass ($\lesssim 10^{17} M_{\odot}$); in a universe dominated by degenerate neutrinos this requires μ/T to be less than ~ 5.

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APPENDIX: NEUTRINO DECOUPLING

Neutrinos drop out of chemical equilibrium when the rate for the weak processes, which can change their number (e.g., $e^+ + e^- \rightarrow \nu_i + \bar{\nu}_i$), can no longer keep up with the expansion of the universe: $\Gamma \lesssim H$. The expansion rate, H, is given by

$$H^2 \equiv (\dot{R}/R)^2 = 8\pi\rho/3 m_{\rm Pl}{}^2 , \tag{A1}$$

where ρ is the total energy density. For $\mu \ll T$, the rate for the weak processes is

$$\Gamma \simeq \begin{cases} 0.6 G_F{}^2 T^5 & (\nu_e) , \\ 0.1 G_F{}^2 T^5 & (\nu_\mu, \nu_\tau) , \end{cases} \tag{A2}$$

where $G_F \simeq 10^{-5}$ GeV^{-2} is the Fermi constant and $\sin^2\theta_w \simeq 0.23$ was used. Note the rate for $e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$ is slightly faster since there are both charged- and neutral-current processes which contribute to the total rate. The energy density $\rho = g_*(\pi^2/30)T^4$, where g_* $(\equiv \sum g_B + \frac{7}{8}\sum g_F \simeq \frac{43}{4}$, for $T \simeq$ few MeV) is the total number of relativistic degrees of freedom. Equating (A1) and (A2), we obtain the standard result that electron neutrinos decouple at $T_D \simeq 2$ MeV, while μ and τ neutrinos decouple at $T_D \simeq 3.5$ MeV. After the neutrinos decouple, the e^+e^- pairs annihilate, heating the photons but not the neutrinos. From entropy conservation, one obtains the standard result that $T_\nu = (4/11)^{1/3} T_\gamma$, for $T_\gamma \ll m_e$.

When μ is much greater than T the rate for $\nu\bar{\nu}$ pair creation differs significantly from expression (A2). This is because the neutrinos are highly degenerate, and all the momentum states up to $E_\nu \simeq O(\mu)$ are filled. Thus neutrinos must be created with energy greater than $O(\mu)$. For $\mu \gg T$ we calculate the rate for $e^+ + e^- \rightarrow \nu + \bar{\nu}$ to be

$$\Gamma \simeq \frac{C_i G_F{}^2}{72\pi^3 \zeta(3)} \mu^4 T \exp(-\mu/T) , \tag{A3}$$

where $\zeta(3) \simeq 1.20206$, $C_i = 5$ for $i = e$, and $C_i = 1$ for $i = \mu, \tau$. In this case the energy density is dominated by the degenerate species and $\rho \simeq \mu^4/8\pi^2$. Equating (A1) and (A3), we find that

$$T_D \simeq \begin{bmatrix} 6 & e \\ 10 & \mu,\tau \end{bmatrix} \text{MeV} \; (\mu/T)^{-2/3} \exp(\mu/3T) . \tag{A4}$$

Note that for $\mu/T \gtrsim 15$, $T_D \gtrsim 100$ MeV, and thus that neutrino species would not share in the heating due to the annihilation of the $\mu^+\mu^-$ pairs, and possibly other massive species. Using entropy conservation, it is easy to calculate that $T_\nu = (3.9/g_{*D})^{1/3} T_\gamma$ for $T_\gamma \ll m_e$, where g_{*D} is the number of relativistic degrees of freedom in thermal equilibrium at $T = T_D$.

Since T_D increases exponentially, we will take $g_{*D} \simeq 110$ for $\mu \gtrsim 15$; this is the total number of relativistic degrees of freedom for $T \gtrsim 100$ GeV in the minimal $SU(3) \times SU(2) \times U(1)$ model. Hence, we find that $T_\nu \simeq T_\gamma/3$ for $T_\gamma \ll m_e$.

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