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Upper limits to fermion masses in the Glashow-Weinberg-Salam model

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Upper limits on the mass m_f of a fermion in the Glashow-Weinberg-Salam model were previously found by requiring that our vacuum be an absolute minimum of the one-loop Higgs potential; for a Higgs-boson mass $m_H \lesssim 150$ GeV, it was found that $m_f \lesssim 100$ GeV. We note that including the effect of running couplings in the one-loop term enlarges the range of allowed masses based on this constraint. Requiring only that the universe arrive in our vacuum and stay there for over 10^{10} yr further increases this bound to $m_f \lesssim 200$ GeV.

In the Glashow-Weinberg-Salam model[1] the fermion masses are arbitrary and could be quite heavy. Some phenomenological upper limits to fermion masses, based on the ρ parameter[2] or on the K_L-K_S mass difference,[3] exist but these limits are either very weak or very sensitive to unknown matrix elements. In this paper we consider theoretical upper bounds to fermion masses. For simplicity, we assume (for the moment) that the top quark is the only undiscovered heavy fermion.

Three years ago, it was noted[4-6] that there is a theoretical upper bound to the top-quark mass m_t, obtained by considering one-loop corrections to the Higgs potential. The one-loop potential is well known[4-7]; writing it in terms of its minimum σ (we choose units so that $\sigma = 1$) and the Higgs-boson mass $m_H{}^2 = d^2V/d\phi^2$ (at $\phi = \sigma$), yields

$$V = \tfrac{1}{8} m_H{}^2 [2\Xi\phi^4 \ln\phi^2 - (3\Xi - 1)\phi^4 + (4\Xi - 2)\phi^2] \ ,$$

where $\phi^2 = \phi^\dagger\phi$, $\Xi \equiv (6M_W{}^4 + 3M_Z{}^4 + M_H{}^4 - 12m_t{}^4)/16\pi^2 m_H{}^2$. Here, $M_H{}^2\phi^2 = -\mu^2 + 3\lambda\phi^2$, where $-\frac{1}{2}\mu^2$ $(\frac{1}{4}\lambda)$ is the quadratic (quartic) coefficient in V. This potential is plotted for various values of Ξ in Fig. 1(a). It is easy to see that if m_t is sufficiently large, then $\Xi < 0$ and the potential is unbounded.

In Ref. 4, it was argued that the minimum at σ must be an absolute minimum, thus Ξ must be positive. This leads to an upper limit on m_t given by the $\Xi = 0$ line of Fig. 1(b). For large values of ϕ, the potential becomes negative at a value of $\phi = \phi_1 = \exp(-\frac{1}{4}\Xi)$; it was noted in Ref. 5 that if this value is outside the region of validity of perturbation theory, then one cannot say the potential is unbounded. About a year later, the two-loop potential was calculated[8] (without heavy fermions) and it can be shown that perturbation theory is accurate up to the Planck scale; $\phi_1 \gtrsim M_{\rm Pl}$ corresponds to $\Xi > -0.006$. In Ref. 6, it was pointed out that the $SU_2 \times U_1$ potential is only valid up to the unification scale; $\phi_1 \gtrsim 10^{15}$ GeV gives $\Xi > -0.008$. As can be seen from Fig. 1(b), all of these limits give similar upper bounds to m_t.

In this paper, we note two effects which significantly weaken these limits. First, for large values of ϕ,

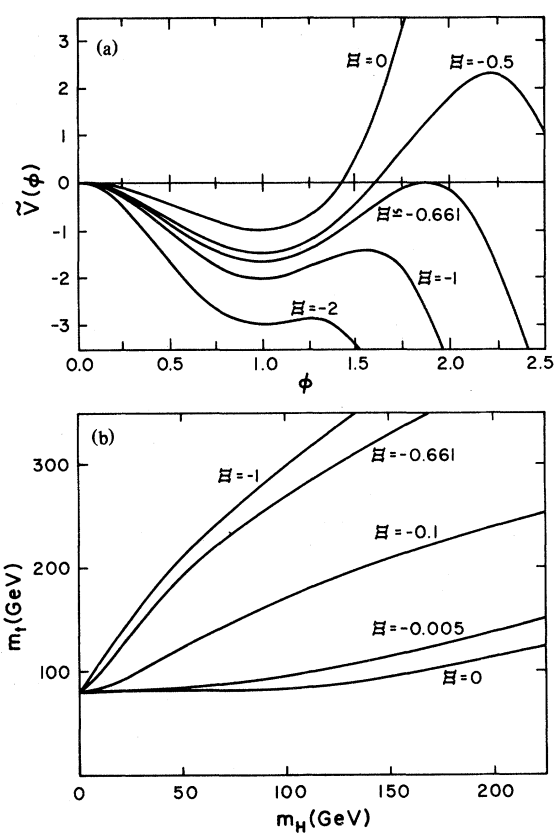

FIG. 1. (a) The one-loop effective potential V (in units in which $\sigma = 1$) for various values of Ξ. (b) The value of Ξ for given values of m_H and m_t. Here, $V = \tilde{V} m_H{}^2/8$.

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the scale dependence of the coupling constants in the one-loop term becomes crucial (this is the leading two-loop effect). This extends the region of masses for which our vacuum is absolutely stable, but is somewhat sensitive to uncalculated two-loop effects. Second, there is no phenomenological requirement that we currently live in a stable vacuum, only that the universe arrive in our current vacuum and stay there for over 10^{10} years. This weakens the upper limits even further and is quite insensitive to two-loop effects.

It has been shown[8] that the major contribution of the two-loop terms in the potential can be included by using running couplings in the one-loop term [this then includes the $\alpha^2(\ln\phi^2)^2$, but not the $\alpha^2\ln\phi^2$ or α^2 terms]. The β function for the Yukawa coupling g_t is

$$(9g_t^3/2-32\pi\alpha_s g_t-9\pi\alpha_w g_t-17\pi\alpha' g_t/3)(16\pi^2)^{-1}\,.$$

If $g_t(\sigma)$ is small enough ($m_t<250$ GeV), then g_t decreases with scale, and this has the effect of *reversing* the sign of Ξ at large scales, bounding the potential. Some subtlety arises in running the $M_H{}^4$ term; we have chosen a scale-invariant regularization scheme in which μ^2 is invariant. The sensitivity of our results to the scheme used, as well as to the uncalculated two-loop terms, will be discussed below.

In Fig. 2(a), we have plotted the potential including this scale dependence for $m_H=10$ GeV, $m_t=125$ GeV. We see that at large values of ϕ, the sign of Ξ changes, bounding the potential (in this case, ours is still an unstable vacuum). There is another effect of this scale dependence; it expands the region of masses for which our vacuum is absolutely stable. This is because the potential may not become negative until a very large value of ϕ, Ξ may change sign between σ and this value, and thus the potential would never turn over. The lower solid curve in Fig. 3 gives the upper limit to the region of absolute stability; one can see that the bounds are already much weaker than those of Refs. 4–6.

There is, however, no phenomenological requirement that we live in an absolute vacuum. A given potential is acceptable if (a) during, say, the grand unified phase transition, the universe goes into the correct $SU_3\times SU_2\times U_1$ vacuum, (b) it stays there until the electroweak transition, (c) during the electroweak transition, the universe goes into the $SU_3\times U_1$ vacuum, and (d) it stays there for at least 10^{10} yr.

To consider requirements (a)–(c), it is necessary to consider the potential at finite temperature. Using the standard finite-temperature formalism[9] we have plotted the full potential for $m_H=50$ GeV, $\Xi=-1$ in Fig. 2(b) as a function of temperature. Here the symmetric vacuum vanishes at $T\sim0.36$; the transition occurs very near this temperature. At the time of the transition, we see that a large barrier separates the asymmetric vacuum from the "unbounded" region; this generally occurs—if the top quark is heavy enough to make the $T=0$ barrier small, then it is so heavy that the transition temperature is large enough that the barrier is large during the transition.

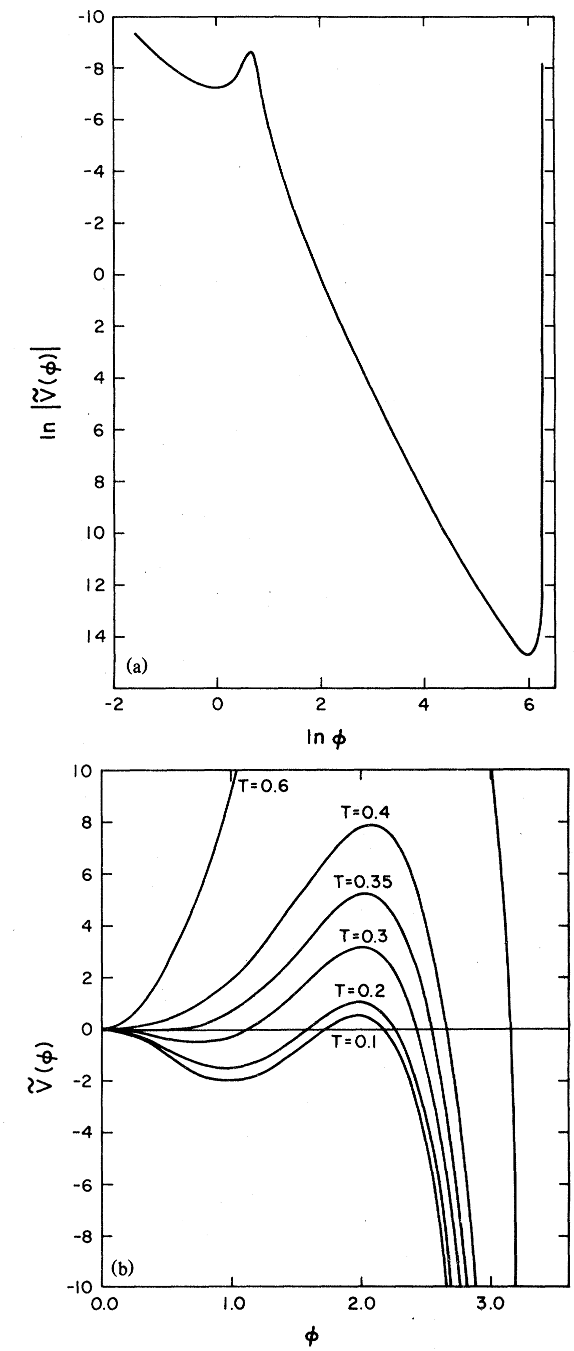

FIG. 2. (a) The effective potential V (for $m_t=125$ GeV and $m_H=10$ GeV) when the scale dependence of couplings is included. (b) The temperature-dependent potential for $m_H=50$ GeV, $\Xi=-1$.

As we will see, requirement (d) will give the most stringent bound on the top-quark mass. First, con-

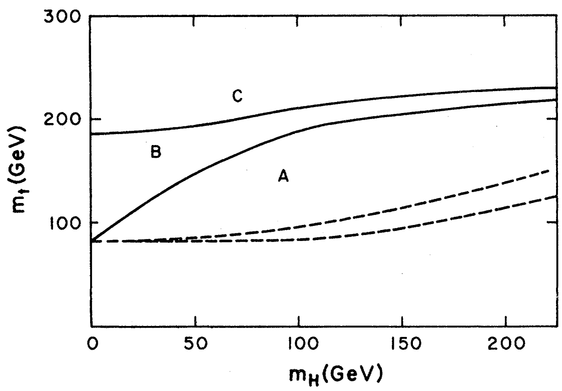

FIG. 3. Upper bound on M_t. The lower (upper) dashed curve is the previous limit of Ref. 4 (5). Below the lower full curve, the present vacuum is absolutely stable (region A). In region B our vacuum is unstable, but with a lifetime $\tau > 10^{10}$ yr. In region C, $\tau < 10^{10}$ yr; thus region C is disallowed.

sider (a): From Fig. 2(b) one can see that at temperatures well above the electroweak scale, there will be a large barrier separating the $SU_3 \times SU_2 \times U_1$ vacuum from the unbounded region (generally larger than T^4). The size of the barrier separating the SU_5, say, and the $SU_3 \times SU_2 \times U_1$ vacua depends on unmeasurable parameters of the combined 24-plet and 5-plet potential. It is certainly plausible that one can choose these parameters such that this barrier vanishes at a temperature sufficiently high that the barrier separating the "unbounded" region is still large; the universe will then go into the correct vacuum. A detailed analysis was done for a hierarchy of the form $O_3 \rightarrow O_2 \rightarrow$nothing, including one-loop corrections at finite temperature for the combined potential; one can show that a wide range of parameters (which preserve the hierarchy) exists for which the universe goes into the correct vacuum. Since a typical grand unified theory has many more parameters (many unmeasurable), no significant bound on m_t can be found from (a). It is easy to show that for masses which satisfy (d), requirement (b) is satisfied (since the barrier is much larger and the time scale much shorter).

If the barrier is small at the time of the electroweak transition, one might worry about the Higgs field "rolling down the hill" and over the barrier. Since the field loses energy at a rate comparable to its lifetime, and the oscillations about σ are at a rate comparable to its mass, it will not lose much energy "rolling down the hill." However, as seen from Fig. 2(b), the barrier has a maximum *above* the $SU_3 \times SU_2 \times U_1$ vacuum for virtually all values[10] of the Higgs-boson and fermion masses; thus energy conservation precludes this possibility, and (c) is satisfied.

The strongest bound comes from requirement (d). As shown in Ref. 11, the nucleation rate per unit volume, f, is (in units where $\sigma = 1$) $\sim \exp(-A)$, where A is the least-action bubble solution (see Ref. 12 for details). The fraction of space filled with new phase at time t is[12] $1-\exp(-ft^4)$. Since, in our units, the age of the universe is e^{101}, the fraction of space filled with "unbounded" region today is $1-\exp[-\exp(404-A)]$. Since A itself is extremely sensitive to the top-quark mass,[13] the uncertainties associated with the precise expansion rate, bubble overlap, the prefactor in the nucleation rate, etc., are utterly negligible in determining our upper bound. Requiring that the fraction of new phase be negligible is the same as requiring that $A \geq 404$. For a given potential, the action can be calculated using standard techniques[12] and this requirement becomes an upper bound on m_t. Our results are plotted in Fig. 3—the upper solid curve corresponds to the $A = 404$ line and is an upper bound to m_t. Our upper bound varies from 188 to 215 GeV as m_H varies from 5 to 200 GeV. Note that for very small m_H, the barrier (for $m_t \sim 190$ GeV) is extremely small ($\Xi \sim -60$) but quantum fluctuations are small as well. Note also that for large m_H, the limits both approach the Pendleton-Ross[14] fixed point (the point at which the β function for g_t vanishes).

There are two principal sources of uncertainty in these results; the scheme dependence of the "running" of the $M_H{}^2$ term and the uncalculated two-loop effects. Strictly speaking, the Higgs-boson-loop contribution to V is $(-\mu^2+3\lambda\phi^2)^2\ln(-\mu^2+3\lambda\phi^2)$. For small values of λ (we always have $\phi > 1$), this may become complex, indicating a breakdown of the renormalization-group approach. Fortunately, this term is small for small values of m_H. Since it is generally positive (since $\phi >> \mu$ in the relevant region) it will only lower our limits (positive terms stabilize our vacuum), and thus the results in Fig. 3 were obtained by multiplying the term by $\theta(-\mu^2+3\lambda\phi^2)$. If, instead, we take the real part of this term, we find that our limits are changed by $< 5\%$ for $m_H < 175$ GeV. Since we cannot prove that the Higgs-boson one-loop contribution is positive, our limits may not be reliable for larger Higgs-boson masses. One might expect that the limit regarding absolute stability in Fig. 3 would be very sensitive to uncalculated two-loop effects, since the leading two-loop terms have such a large effect. This is, in fact, the case; changing α_s by changing Λ_{QCD} from 100 to 300 MeV will change the limits by 10 GeV for $50 < m_H < 200$ GeV, for example. Thus the lower solid curve in Fig. 3 is uncertain by 10–20 GeV. This is not our upper bound, however, and the upper bound bordering region C turns out to be very insensitive to two-loop effects; dropping the running of the couplings entirely changes the limit by $\lesssim 5\%$. Thus, for $m_H \lesssim 175$

GeV, we expect our upper bound to be uncertain by at most 10 GeV.

Extending our results to more complicated models is simple. If there are additional quarks, one replaces m_t^4 by $\sum_q m_q^4$; the limits on fourth-generation masses are more severe. If there are additional Higgs scalars, the abscissa in Fig. 3 refers to one of the neutral scalars and the ordinate is replaced by $(m_t^4 - \frac{1}{12} \sum m_{\text{scalars}}^4)^{1/4}$. In supersymmetric theories, the gauge fermions and Higgs fermions must be included as well as the partners to the top quark; the proliferation of parameters makes any attempt to find limits meaningless.

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