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Analysis of preon models with a small number of flavors. II

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We extend our analysis of four- and five-flavor preon models to six-flavor preon models; only a single color-triplet preon and three color-singlet preons are allowed by the trace constraints and the composite spectrum. Assuming that the precolor group is either SU(N) or SO(N) we find three models of each kind which satisfy the anomaly- and asymptotic-freedom constraints along with possessing a reasonable composite spectrum. All these models may be grand unified at some large mass scale far above the precolor scale $\Lambda_{\rm PC}$.

I. INTRODUCTION

In a recent paper,[1] hereafter referred to as I, we performed a search for a "successful" preon model with a small number of flavors. The idea that quarks and leptons may be composite objects[2] has grown in popularity recently with the apparent growth of the number of fermion families, and the possibility that Higgs bosons are also composite has also been explored within hypercolor models.[3] Although there is some theoretical support for these preon ideas, experimentally quarks and leptons appear quite pointlike. Accelerator experiments[4] set a lower limit of $\simeq$150 GeV on the scale of any composite substructure, $\Lambda_{\rm PC}$ (PC for precolor) while limits from flavor-changing reactions such as $\mu\rightarrow e\gamma$ (Ref. 5) and the anomalous magnetic moment of the electron (Ref. 6), g_e-2, indicate $\Lambda_{\rm PC}$ may be as large as 100–1000 TeV. This is beyond the region which can be explored directly by any planned or existing accelerator and so preon ideas must be tested indirectly.

The idea of our earlier work, I, was to find the minimum number of preon flavors which are consistent with a set of reasonable criteria:

(i) Preons are spin-$\frac{1}{2}$ particles which obey Fermi-Dirac statistics; three-preon bound states are quarks or leptons in analogy with baryons in QCD with $\mathrm{SU}(3)_C$.

(ii) Preons transform as $\underline{1}$, $\underline{3}$, or $\overline{\underline{3}}$ under $\mathrm{SU}(3)_C$.

(iii) The precolor gauge interactions of preons are asymptotically free and are responsible for preon binding into precolor-singlet composites. The precolor gauge group $G_{\rm PC}$ is either SU(N) or SO(N).

(iv) There must exist a normal generation of quarks and leptons among the composites.

(v) Both $G_{\rm PC}$ and the preflavor group $G_{\rm PF}$ must be anomaly[7] free; 't Hooft[8] has shown this as a necessary condition for forming massless composite states from massless preons. Quarks and leptons are essentially massless on the scale of $\Lambda_{\rm PC}$; their small masses are produced via electroweak interactions.

(vi) It should be possible to unify $G_{\rm PC}$ with the usual strong and electroweak interactions at *some* mass scale.

The results of our previous analysis strongly indicates that four- and five-flavor preon models are not consistent with these constraints; we also found that the only possibly allowed six-flavor preon model is one with a single color triplet and three color singlets of preons. Models with two color triplets or one triplet and one antitriplet cannot be anomaly free and reproduce the correct composite charges simultaneously. We will use the following notation remembering that the flavor groups we will analyze are left-right symmetric $G_F=G_L\times G_R$; $G_{L(R)}$ must contain at least $\mathrm{SU}(3)_{L(R)}\times\mathrm{SU}(2)_{L(R)}$ as a subgroup in order to reproduce the ordinary strong and electroweak gauge groups. Under SU(3)$\times$SU(2) we have

$$\begin{aligned} &A\sim(\underline{3},\underline{1})\ ,\\ &(B,C)\sim(\underline{1},\underline{2})\ ,\\ &D\sim(\underline{1},\underline{1})\ , \end{aligned} \tag{1.1}$$

so that color and weak isospin are incorporated at the preon level.

There have been many attempts[9] to find reasonable composite models, with few successes; therefore, we try to impose as few constraints as possible.

In order to study the asymptotic-freedom and unification constraints we are imposing on six-flavor preon models we must examine the β-function coefficients[10] of the color and precolor groups above the precolor scale $\Lambda_{\rm PC}$. Since $\alpha_{\rm PC}>\alpha_C$ at $\Lambda_{\rm PC}$ (since

TABLE I. Representations, dimensions, anomaly contribution, and β-function contribution for SU(N).

Representation	Dimension	T	K
n	N	$\frac{1}{2}$	1
$\bar{n}$	N	$\frac{1}{2}$	-1
2nd-rank A	$N(N-1)/2$	$(N-2)/2$	$N-4$
2nd-rank S	$N(N+1)/2$	$(N+2)/2$	$N+4$
3rd-rank A	$N(N-1)(N-2)/6$	$(N-2)(N-3)/4$	$(N-3)(N-6)/2$
3rd-rank S	$N(N+1)(N+2)/6$	$(N+2)(N+3)/4$	$(N+3)(N+6)/2$
Adjoint	N^2-1	N	0

precolor is strong at $\Lambda_{\rm PC}$) a minimal condition for unification is that $\beta_{\rm PC}>\beta_C$ so that $\alpha_{\rm PC}\simeq\alpha_C$ *somewhere*. This is a minimal condition and the only one we will impose in what follows. In general, we may write

$$\beta=\tfrac{11}{3}C_2(G)-\tfrac{2}{3}S_2(F)\ , \tag{1.2}$$

where $C_2(G)$ [$=N$ for SU(N) and $2(N-2)$ for SO(N)] represents the gauge-boson contribution to the β function and $S_2(F)$ depends on the number and representations of chiral fermions present in the theory. Remember, above $\Lambda_{\rm PC}$, it is the preons which control the fermion contributions to the β function. The QCD β function above $\Lambda_{\rm PC}$ is given by

$$\beta_C=11-\tfrac{2}{3}d_{\rm PC}\ , \tag{1.3}$$

where $d_{\rm PC}$ is the dimension of the precolor representation of A. Note $d_{\rm PC}\le 16$ for SU(3)$_C$ to be asymptotically free.

The fermion contribution, $S_2(F)$, can be written as

$$-\tfrac{2}{3}S_2(F)=-\tfrac{2}{3}\times 2(3T_1+2T_2+T_3)\ , \tag{1.4}$$

where T_1 is the contribution of A, T_2 that of B or C, and T_3 that of D. This will be used in what follows.

II. ANALYSIS OF CONSTRAINTS

The first constraints we will impose on our six-preon models are the following:

(a) The precolor group $G_{\rm PC}$ must be asymptotically free (AF).

(b) Both $G_{\rm PC}$ and the gauged part of the preflavor group $G_{\rm PF}$ must be anomaly free.

(c) Unification of $G_{\rm PC}$ with the usual electroweak and strong interactions should be possible. The actual constraint imposed here is that $\beta_{\rm PC}>\beta_C>0$ above $\Lambda_{\rm PC}$ ($\simeq$100–1000 TeV).

(d) At least one ordinary composite generation must exist. Let us now turn to individual cases.

A. SU(N)

For SU(N) we have

$$\beta_{\rm PC}=\tfrac{11}{3}N-\tfrac{4}{3}(3T_1+2T_2+T_3) \tag{2.1}$$

and β_C given by (1.3). Table I shows the values of T_i for various SU(N) representations.

(i) In this case we take $A\sim R$ under SU(N) and B, C, and D as $\bar{R}$ so that anomalies cancel automatically and $T_1=T_2=T_3=T$. We assume that either $R^3\sim\underline{1}$ or $R^2\bar{R}\sim\underline{1}_{\rm PC}$. The only possibility here is that R is the second-rank antisymmetric representation of SU(N) (which we denote by $\underline{A}$) since for larger representations $G_{\rm PC}$ will not be AF. We find

$$\beta_{\rm PC}=-\tfrac{1}{3}N+8,\ \ \beta_C=\frac{33-N(N-1)}{3}\ . \tag{2.2}$$

These lead to the constraints that $3\le N\le 7$. We find that the only N for which A satisfies $\underline{A}^2\bar{\underline{A}}$ or $\underline{A}^3\sim 1_{\rm PC}$ is $N=6$ where $\underline{A}$ is the 15-dimensional representation of SU(6). Here $G_F={\rm SU}(6)_L\times{\rm SU}(6)_R\times U(1)_V$.

(ii) We now assume that we have at least two different representations of $G_{\rm PC}$ and try to make $3T_1+2T_2+T_3$ as small as possible. The first choice is to choose $A,B,C\sim\underline{n}$ and $D\sim\underline{A}$ (or $\bar{\underline{n}}$ and $\bar{\underline{A}}$, respectively). Any larger representations will not satisfy the anomaly constraints. In this case $T_1=T_2=\frac{1}{2}$, $T_3=(N-1)/2$ so that the constraint $\beta_{\rm PC}>\beta_C>0$ implies that $4\le N\le 16$. We now must turn to the anomaly cancellation problem; let K_1, K_2, and K_3 represent the contributions to the anomaly by A, B or C, and D, respectively. For anomaly cancellation we must have $3K_1+2K_2+K_3=0$; Table I shows the contributions to the anomaly of various SU(N) representations. There are three subcases:

(α) $K_1=K_2=1$, $K_3=-(N-4)\Longrightarrow N=9$ only. Here we have $A,B,C\sim\underline{n}$, $D\sim\bar{\underline{A}}$; the flavor group is

TABLE II. Precolor singlets for SU(N) preon models as discussed in the text.

	i	iiα	iiβ	iiiα	iiiβ	ivα	vα	vβ
				Color triplets				
ABB	×					×		
$AB\bar{B}$								
$A\bar{B}\bar{B}$								
ABC	×					×		
$AB\bar{C}$								
$A\bar{B}C$								
$A\bar{B}\bar{C}$								
ABD	×	×		×		×		
$AB\bar{D}$					×			
$A\bar{B}D$			×					
$A\bar{B}\bar{D}$								
ACC	×					×		
$AC\bar{C}$								
$A\bar{C}\bar{C}$								
ACD	×	×		×		×		
$AC\bar{D}$					×			
$A\bar{C}D$			×					
$A\bar{C}\bar{D}$								
ADD	×					×		
$AD\bar{D}$								
$A\bar{D}\bar{D}$								
$AA\bar{A}$								
$\bar{A}\bar{A}B$								×
$\bar{A}\bar{A}\bar{B}$	×			×	×		×	
$\bar{A}\bar{A}C$								×
$\bar{A}\bar{A}\bar{C}$	×			×	×		×	
$\bar{A}\bar{A}D$								×
$\bar{A}\bar{A}\bar{D}$	×	×	×				×	

$$G_{\rm PF}={\rm SU}(5)_L\times{\rm SU}(5)_R\times{\rm U}(1)_L\times{\rm U}(1)_R\times{\rm U}(1)_V\ . \tag{2.3}$$

(β) $K_1=-K_2=1$, $K_3=-(N-4)\Longrightarrow N=5$ only. We find then that $A\sim\underline{n}$, $(B,C)\sim\underline{\bar{n}}$, and $D\sim\underline{\bar{A}}$; the flavor group is the same as in (α) above.

(γ) $K_1=-K_2=-1$, $K_3=-(N-4)\Longrightarrow N=3$ only. This does not satisfy the asymptotic-freedom bound obtained above and so this case is ruled out.

(iii) Now we take $T_1=T_3=\frac{1}{2}$, $T_2=\frac{1}{2}(N-2)$ such that we again find $4\le N\le 16$ from the constraint $\beta_{\rm PC}>\beta_C>0$. There are again three cases that have to be considered for anomaly cancellation:

(α) $K_1=K_3=1$, $K_2=-(N-4)\Longrightarrow N=6$ only; this gives the representations $A,D\sim\underline{n}$, $(B,C)\sim\underline{\bar{A}}$. The flavor group is

$$G_{\rm PF}={\rm SU}(4)_L\times{\rm SU}(4)_R\times{\rm SU}(2)_L\times{\rm SU}(2)_R\times{\rm U}(1)_V\ . \tag{2.4}$$

(β) $K_1=-K_3=1$, $K_2=-(N-4)\Longrightarrow N=5$ only; we find $A\sim\underline{n}$, $D\sim\underline{\bar{n}}$, $(B,C)\sim\underline{\bar{A}}$ with the same $G_{\rm PF}$ as in (a) above.

(γ) $K_1=-K_3=-1$, $K_2=-(N-4)\Longrightarrow N=3$ only; this is inconsistent with our asymptotic freedom constraints and so this model is ruled out.

(iv) The next case we consider has $T_1=(N-2)/2$, $T_2=T_3=\frac{1}{2}$; we find that N=4, 5, and 6 are the only values allowed by the constraint $\beta_{\rm PC}>\beta_C>0$. There are again three cases:

(α) $K_2=K_3=1$, $K_1=-(N-4)\Longrightarrow N=5$ only; we then have $A\sim\underline{\bar{A}}$, $(B,C)\sim\underline{n}$, and $D\sim\underline{n}$. The flavor group in this case is

$$G_{\rm PF}={\rm SU}(3)_L\times{\rm SU}(3)_R\times{\rm SU}(3)'_L\times{\rm SU}(3)'_R\times{\rm U}(1)_V\ . \tag{2.5}$$

(β) $K_3=-K_2=+1$, $K_1=-(N-4)$. No solution exists.

(γ) $K_3=-K_2=-1$, $K_1=-(N-4)$. No solution

TABLE II. (*Continued.*)

	i	iiα	iiβ	iiiα	iiiβ	ivα	vα	vβ
				Color singlets				
AAA								
BBB								
$BB\bar{B}$	×							
BBC								
$BB\bar{C}$	×							
$B\bar{B}\bar{C}$	×							
BBD		×						
$BB\bar{D}$	×		×					
$B\bar{B}\bar{D}$	×							
BCC								
$BC\bar{C}$	×							
$B\bar{C}\bar{C}$	×							
BCD		×						
$BC\bar{D}$	×		×					
$B\bar{C}D$	×							
$B\bar{C}\bar{D}$	×							
BDD				×				
$BD\bar{D}$	×							
$B\bar{D}\bar{D}$	×				×			
CCC								
$CC\bar{C}$	×							
CCD		×						
$CC\bar{D}$	×		×					
$C\bar{C}D$	×							
CDD				×				
$CD\bar{D}$	×							
$C\bar{D}\bar{D}$	×				×			
DDD								
$A\bar{A}B$	×							
$A\bar{A}C$	×							
$A\bar{A}D$	×							

exists.

(v) The last case we consider is $T_1=\frac{1}{2}$, $T_2=T_3=(N-2)/2$. There are many subcases here but only two are successful:

(α) $A\sim\underline{n}$, $(B,C),D\sim\bar{\underline{A}}$ with $N=5$ only; $G_{\rm PF}$ is the same in this case as in (2.5) above.

(β) $A\sim\underline{n}$, $(B,C)\sim\bar{\underline{A}}$, $D\sim\underline{A}$ with $N=7$ only; $G_{\rm PF}$ is again the same as in (2.5) above.

To summarize the SU(N) situation, we have several models which are anomaly free and satisfy the constraint $\beta_{\rm PC}>\beta_C>0$: SU(5) for models iiβ, iiiβ, ivα, and vα, SU(6) for models i and iiiα, SU(7) for model vβ, and SU(9) for model iiα. We now must turn to the spectrum of composites produced by these various models; these are shown in Table II where the singlets under $G_{\rm PC}$ are indicated. We see immediately that five of these eight models do not have the correct particle spectrum; ivα, vα, and vβ have no color singlets while in iiα and iiβ color singlets form isotriplets only. The only models surviving are now i, iiiα, and iiiβ so that $N=5$ or 6.

We will return to these models below.

TABLE III. Representations, dimensions, and β-function contributions for SO(N).

Representation	Dimension	T
Vector	N	2
2nd-rank A	$N(N-1)/2$	$2(N-2)$
2nd-rank S	$N(N+1)/2-1$	$2(N+2)$
Spinorial (N even)	$2^{N/2-1}$	$2^{N/2-3}$
Spinorial (N odd)	$2^{(N-1)/2}$	$2(N-3)/2$

B. SO(N)

For SO(N), $\beta_{\rm PC}$ is given by

$$\beta_{\rm PC}=\tfrac{22}{3}(N-2)-\tfrac{4}{3}(3T_1+2T_2+T_3)\,, \qquad (2.6)$$

while β_c is still given by (1.3). The advantage of

TABLE IV. Precolor singlets for SO(*N*) preon models as discussed in the text.

	1	2	3	4	5	6	7	8
				Color triplets				
ABB			×		×			×
$AB\bar{B}$			×		×			×
$A\bar{B}\bar{B}$			×		×			×
ABC			×		×			×
$AB\bar{C}$			×		×			×
$A\bar{B}C$			×		×			×
$A\bar{B}\bar{C}$			×		×			×
ABD	×	×	×					×
$AB\bar{D}$	×	×	×					×
$A\bar{B}D$	×	×	×					×
$A\bar{B}\bar{D}$	×	×	×					×
ACC			×		×			×
$AC\bar{C}$			×		×			×
$A\bar{C}\bar{C}$			×		×			×
ACD	×	×	×					×
$AC\bar{D}$	×	×	×					×
$A\bar{C}D$	×	×	×					×
$A\bar{C}\bar{D}$	×	×	×					×
ADD			×	×				×
$AD\bar{D}$			×	×				×
$A\bar{D}\bar{D}$			×	×				×
$AA\bar{A}$								
$\bar{A}\bar{A}B$		×				×	×	
$\bar{A}\bar{A}\bar{B}$		×				×	×	
$\bar{A}\bar{A}C$		×				×	×	
$\bar{A}\bar{A}\bar{C}$		×				×	×	
$\bar{A}\bar{A}D$	×					×	×	
$\bar{A}\bar{A}\bar{D}$	×					×	×	

SO(*N*) as precolor groups is that all SO(*N*) representations are automatically free; this however leads to a much more complex composite spectrum in general. In this case, we will find that the preons may be in the vector, spinorial, and second-tank tensor (both symmetric and antisymmetric) representations without violating the constraint $\beta_{PC} > \beta_C > 0$. Table III shows a list of relevant SO(*N*) representations and their corresponding values of *T*. The basic procedure is the same as in the case of SU(*N*) except that we need not consider any anomaly constraint conditions since no anomalies exist within SO(*N*) groups. $\underline{A}$ and $\underline{S}$ will label the 2nd-rank antisymmetric and symmetric tensor representations, respectively. We will simply list our results below.

1. $A,B,C \sim \underline{n}$, $D \sim \underline{A}$ $(4 \leq N \leq 16)$ or $D \sim \underline{S}$ $(7 \leq N \leq 16)$. The flavor group is

$$G_F = \mathrm{SU}(5)_L \times \mathrm{SU}(5)_R \times \mathrm{U}(1)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_V \; . \tag{2.7}$$

2. $A,D \sim \underline{n}$, $(B,C) \sim \underline{A}$ $(10 \leq N \leq 16)$; no solution exists if $(B,C) \sim \underline{S}$. The flavor group is

$$G_F = \mathrm{SU}(4)_L \times \mathrm{SU}(4)_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_V \; . \tag{2.8}$$

3. $B,C,D \sim \underline{n}$, $A \sim \underline{A}$ or $\underline{S}$. No solution exists.

4. $A,B,C \sim \underline{n}$, $D \sim \underline{\mathrm{SP}}$ (the spinorial representation). If N is even, $6 \leq N \leq 16$; if N is odd, $7 \leq N \leq 15$. G_F is given by (2.7).

5. $A,D \sim \underline{n}$, $(B,C) \sim \underline{\mathrm{SP}}$. For even N, $6 \leq N \leq 16$, while for odd N we have $7 \leq N \leq 15$ only. G_F is given by (2.8).

(6) $B,C,D \sim \underline{n}$, $A \sim \underline{\mathrm{SP}}$. For even N, $6 \leq N \leq 14$; for N odd, $5 \leq N \leq 11$. The flavor group is

$$G_F = \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{SU}(3)'_L \times \mathrm{SU}(3)'_R \times \mathrm{U}(1)_V \; . \tag{2.9}$$

7. $A \sim \underline{n}$, $B,C,D \sim \underline{A}$ $(3 \leq N \leq 16)$ or $\underline{S}$ $(10 \leq N \leq 16)$. G_F is given by (2.9).

8. $A \sim \underline{n}$, $B,C,D \sim \underline{\mathrm{SP}}$. For even N, $6 \leq N \leq 16$, while for N odd we get $7 \leq N \leq 15$. G_F is given by (2.9).

We now must examine the composite-fermion spectrum in each of these cases.

TABLE IV. (*Continued.*)

	1	2	3	4	5	6	7	8
				Color Singlets				
AAA								
BBB								
$BB\bar{B}$								
BBC								
$BB\bar{C}$								
$B\bar{B}\bar{C}$								
BBD	×				×			
$BB\bar{D}$	×				×			
$B\bar{B}\bar{D}$	×				×			
BCC								
$BC\bar{C}$								
$B\bar{C}\bar{C}$								
BCD	×				×			
$BC\bar{D}$	×				×			
$B\bar{C}D$	×				×			
$B\bar{C}\bar{D}$	×				×			
BDD		×		×				
$BD\bar{D}$		×		×				
$B\bar{D}\bar{D}$		×		×				
CCC								
$CC\bar{C}$								
CCD	×				×			
$CC\bar{D}$	×				×			
$C\bar{C}D$	×				×			
CDD		×		×				
$CD\bar{D}$		×		×				
$C\bar{D}\bar{D}$		×		×				
DDD								
$A\bar{A}B$		×				×	×	
$A\bar{A}C$		×				×	×	
$\bar{A}AD$	×					×	×	

Table IV shows a listing of the precolor singlets in each of the eight models discussed above; note that models 1 and 5 are ruled out since these models contain only weak isotriplets of color singlets. Models 3 and 8 contain no color singlets and model 4 has isosinglet color triplets; these models are all ruled out leaving only 2, 6, and 7 as viable—there are only three models based on SO(N) consistent with the AF, unification, and anomaly constraints with reasonable composite spectra.

III. THE COMPOSITE SPECTRUM

We now turn to an examination of the composite fermion spectrum of each of the three SU(N) and three SO(N) which passed the initial constraints. We turn first to the SU(N) models iiiα and iiiβ which have rather simple composite spectra.

Model iiiα. In this model we have

$$\begin{bmatrix} \nu \\ e \end{bmatrix} = \begin{bmatrix} CDD \\ BDD \end{bmatrix} \tag{3.1}$$

and either

$$\text{A:} \quad \begin{bmatrix} u \\ d \end{bmatrix} = \begin{bmatrix} ACD \\ ABC \end{bmatrix} \tag{3.2}$$

or

$$\text{B:} \quad \begin{bmatrix} u \\ d \end{bmatrix} = \begin{bmatrix} \bar{A}\bar{A}\bar{B} \\ \bar{A}\bar{A}\bar{C} \end{bmatrix} . \tag{3.2$'$}$$

In either case, the other color-triplet isodoublet is exotic; in case A we have

$$\begin{bmatrix} \bar{A}\bar{A}\bar{B} \\ \bar{A}\bar{A}\bar{C} \end{bmatrix}, \text{ with } Q = \begin{bmatrix} -\frac{1}{3} \\ -\frac{4}{3} \end{bmatrix} \text{ and } B-L = -\tfrac{5}{3} , \tag{3.3}$$

while in case B we find

$$\begin{bmatrix} ACD \\ ABD \end{bmatrix}, \text{ with } Q=\begin{bmatrix} \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix} \text{ and } B-L=-\tfrac{1}{6}\ . \tag{3.3$'$}$$

We find

$$Q_A=\tfrac{1}{6},\ \ Q_B=Q_C-1=0,\ \ Q_D=-\tfrac{1}{2}$$

and

$$(B-L)_A=\tfrac{1}{3}\ ,$$

$$(B-L)_B=(B-L)_C=-(B-L)_D=1$$

in model A; in model B we have

$$Q_A=-\tfrac{1}{3}Q_D=\tfrac{1}{24},\ \ Q_B=Q_c-1=-\tfrac{3}{4}\ ,$$

and

$$(B-L)_A=-\tfrac{1}{3}(B-L)_D=\tfrac{1}{12}\ ,$$

with

$$(B-L)_B=(B-L)_C=-\tfrac{1}{2}\ .$$

Model iiiβ. In this case we find

$$\begin{bmatrix} \nu \\ e \end{bmatrix}=\begin{bmatrix} \bar{B}\bar{D}\bar{D} \\ \bar{C}\bar{D}\bar{D} \end{bmatrix} \tag{3.4}$$

and

$$\begin{bmatrix} u \\ d \end{bmatrix}=\begin{bmatrix} AC\bar{D} \\ AB\bar{D} \end{bmatrix} \text{or} \begin{bmatrix} \bar{A}\bar{A}C \\ \bar{A}\bar{A}B \end{bmatrix}\ . \tag{3.5}$$

Either choice leads to the usual charge assignment for both color-triplet isodoublets:

$$Q_A=\tfrac{1}{3}Q_D=\tfrac{1}{6},\ \ Q_C=Q_B+1=1$$

and

$$(B-L)_A=\tfrac{1}{3}(B-L)_D=\tfrac{1}{3}\ ,$$

$$(B-L)_B=(B-L)_C=1\ .$$

Model i. This model has a very complex composite spectrum:

Triplets:

$$\begin{bmatrix} ACC \\ ABC \\ ABB \end{bmatrix},\quad \begin{bmatrix} ACD \\ ABD \end{bmatrix},\quad \begin{bmatrix} \bar{A}\bar{A}\bar{B} \\ \bar{A}\bar{A}\bar{C} \end{bmatrix},\ \ \bar{A}\bar{A}\bar{D},\ \ ADD\ ;$$

Singlets:

$$\begin{bmatrix} C\bar{D}\bar{D} \\ B\bar{D}\bar{D} \end{bmatrix},\quad \begin{bmatrix} CDD \\ BDD \end{bmatrix},\quad \begin{bmatrix} A\bar{A}C \\ A\bar{A}B \end{bmatrix},\quad \begin{bmatrix} CC\bar{D} \\ CB\bar{D} \\ BB\bar{D} \end{bmatrix},$$

$$\begin{bmatrix} C\bar{B}D \\ B\bar{B}D, C\bar{D}D \\ \bar{C}BD \end{bmatrix},\quad \begin{bmatrix} & CC\bar{B} & \\ B\bar{B}C & & C\bar{C}C \\ B\bar{B}B & & C\bar{C}B \\ & BB\bar{C} & \end{bmatrix}. \tag{3.6}$$

There are two isodoublet color triplets, two isosinglet color triplets, and an isotriplet color triplet. The lepton spectrum contains four isodoublets, two isotriplets, two singlets, and a quartet. There are at least four possible charge and $B-L$ assignments in this model; a sample solution is

$$Q_A=\tfrac{1}{6},\ \ Q_D=\tfrac{3}{2},\ \ Q_C=Q_B+1=0\ ,$$

$$(B-L)_A=\tfrac{1}{3},\ \ (B-L)_D=3\ ,$$

$$(B-L)_B=(B-L)_C=-1\ .$$

In the case of SO(N), models 6 and 7 have identical composite spectra

$$\begin{bmatrix} \nu \\ e \end{bmatrix}=\begin{bmatrix} A\bar{A}B \\ A\bar{A}C \end{bmatrix},\ \ A\bar{A}D\ ,$$
$$\begin{bmatrix} u \\ d \end{bmatrix}=\begin{bmatrix} \bar{A}\bar{A}B \\ \bar{A}\bar{A}C \end{bmatrix} \text{ or } \begin{bmatrix} \bar{A}\bar{A}\bar{C} \\ \bar{A}\bar{A}\bar{B} \end{bmatrix},\ \ \bar{A}\bar{A}D,\ \ \bar{A}\bar{A}\bar{D}\ , \tag{3.7}$$

so that $Q_B=Q_C+1=0$ and $(B-L)_B=(B-L)_C=-1$ from the lepton sector alone; note Q_D and $(B-L)_D$ are arbitrary at this point since D does not make up any isodoublet members. We find $Q_A=\frac{1}{2}(B-L)_A=-\frac{1}{3}$, or $Q_A=-\frac{1}{2}(B-L)_A=\frac{1}{6}$ depending upon which doublet corresponds to (u,d); the isosinglet properties depend on the nature of D. In one case we also get an isodoublet color triplet with

$$Q=\begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \end{bmatrix} \text{ and } B-L=-\tfrac{1}{3} \tag{3.8}$$

or we have

$$Q=\begin{bmatrix} -\frac{1}{3} \\ -\frac{4}{3} \end{bmatrix} \text{ and } B-L=-\tfrac{5}{3} \tag{3.8$'$}$$

(3.8′) is similar to (3.3).

The last model we will consider is model 2 which has a very lengthy spectrum of weak isodoublets:

Triplets:

$$\begin{bmatrix} ADB \\ ADC \end{bmatrix}, \begin{bmatrix} A\bar{D}B \\ A\bar{D}C \end{bmatrix}, \begin{bmatrix} AD\bar{C} \\ AD\bar{B} \end{bmatrix},$$
$$\begin{bmatrix} A\bar{D}\bar{C} \\ A\bar{D}\bar{B} \end{bmatrix}, \begin{bmatrix} \bar{A}\bar{A}B \\ \bar{A}\bar{A}C \end{bmatrix}, \begin{bmatrix} \bar{A}\bar{A}\bar{C} \\ \bar{A}\bar{A}\bar{B} \end{bmatrix}. \tag{3.9}$$

Singlets:

$$\begin{bmatrix} BDD \\ CDD \end{bmatrix}, \begin{bmatrix} BD\bar{D} \\ CD\bar{D} \end{bmatrix}, \begin{bmatrix} B\bar{D}\bar{D} \\ C\bar{D}\bar{D} \end{bmatrix}, \begin{bmatrix} A\bar{A}B \\ A\bar{A}C \end{bmatrix}.$$

We can choose the Q and $(B-L)$ values in many ways at this point; an interesting choice is

$$Q_D=(B-L)_D=0,\ \ Q_B=Q_C+1=0\ ,$$
$$(B-L)_B=(B-L)_C=-1\ ,$$

which yields four generations of ordinary lepton doublets. We also obtain two ordinary quark doublets:

$$\begin{bmatrix} ADB \\ ADC \end{bmatrix}, \begin{bmatrix} A\bar{D}B \\ A\bar{D}C \end{bmatrix} \sim \begin{bmatrix} u \\ d \end{bmatrix}, \tag{3.10}$$

as well as four exotic doublets

$$\begin{bmatrix} AC\bar{D} \\ AB\bar{D} \end{bmatrix}, \begin{bmatrix} A\bar{C}\bar{D} \\ A\bar{B}\bar{D} \end{bmatrix} \sim Q=\begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \end{bmatrix}_{B-L=7/3},$$
$$\begin{bmatrix} \bar{A}\bar{A}B \\ \bar{A}\bar{A}C \end{bmatrix} \sim Q=\begin{bmatrix} -\frac{4}{3} \\ -\frac{7}{3} \end{bmatrix}_{B-L=-11/3}, \tag{3.11}$$
$$\begin{bmatrix} \bar{A}\bar{A}\bar{C} \\ \bar{A}\bar{A}\bar{B} \end{bmatrix} \sim Q=\begin{bmatrix} -\frac{1}{3} \\ -\frac{4}{3} \end{bmatrix}_{B-L=-5/3},$$

where $Q_A=\frac{2}{3}$ and $(B-L)_A=\frac{4}{3}$ has been chosen.

Can we apply further constraints to reduce these six models further? In our previous analysis our flavor group was just $SU(5)_L\times SU(5)_R$ and so the charge operator Q had to be traceless; if we gauge the left-right-symmetric[11] subgroup of G_F then $B-L$ must also be a sum of generators and traceless as well. This condition was also used in our previous analysis. Can we apply such considerations here?

The flavor groups we have to consider are

$$\begin{aligned} &G_F=SU(4)_L\times SU(4)_R\times SU(2)_L\times SU(2)_R \\ &\qquad \times U(1)_V,\ \text{for iii}\alpha \text{ and } iii\beta\ , \\ &G_F=SU(6)_L\times SU(6)_R\times U(1)_V,\ \text{for i}\ , \\ &G_F=SU(5)_L\times SU(5)_R\times U(1)_L \\ &\qquad \times U(1)_R\times U(1)_V,\ \text{for 1}\ , \\ &G_F=SU(3)_L\times SU(3)_R\times SU(3)'_L \\ &\qquad \times SU(3)'_R\times U(1)_V,\ \text{for 6 and 7}\ . \end{aligned} \tag{3.12}$$

For model i, if we leave $U(1)_V$ as a global symmetry, both Q and $B-L$ are generators of $SU(6)_L\times SU(6)_R$ and, hence, must be traceless. This has already been incorporated into the assignments for this model; we imagine the embedding

$$SU(6)_i\rightarrow SU(3)_i\times SU(2)_i\times U(1)_i\times U(1)'_i \tag{3.13}$$

and hence obtain the left-right-symmetric model[12] as the relevant, natural electroweak group. In this case $\mathrm{Tr}Q=\mathrm{Tr}(B-L)$ imposes no strict conditions and the model survives intact.

For model 1 we can gauge either the entire G_F [except for $U(1)_V$] or just the $SU(5)_L\times SU(5)_R$ subgroup; if we choose the latter then D is a singlet under $SU(5)_L\times SU(5)_R$ and has no Q or $B-L$ value. Then we must satisfy

$$\begin{aligned} &3Q_A+Q_B+Q_C=0\ , \\ &3(B-L)_A+2(B-L)_B=0\ . \end{aligned} \tag{3.14}$$

We must have the values of $Q_B, Q_C, (B-L)_B=(B-L)_C$ given above in order to make leptons and thus Q_A and $(B-L)_A$ are totally determined by the trace conditions:

$$Q_A=\tfrac{1}{3},\ \ (B-L)_A=\tfrac{2}{3} \tag{3.15}$$

and none of the color-triplet isodoublets can correspond to (u,d). Thus, if this model is not to be dropped we must also gauge the "extra" $U(1)_L\times U(1)_R$ factor since we cannot embed Q and $B-L$ successfully within $SU(5)_L\times SU(5)_R$. Therefore $Q_D=\frac{1}{2}(B-L)_D\neq 0$. We cannot, however, constrain Q_D and $(B-L)_D$ within this G_F alone since the SU(5) and U(1) factors are unrelated; only an embedding in a larger group could accomplish this task.

A similar situation occurs for the flavor group of models iiiα and iiiβ; we imagine the embedding

$$SU(4)_i\rightarrow SU(3)_i\times U(1)_i\ , \tag{3.16}$$

such that

$$\begin{aligned} SU(4)_L\times SU(4)_R &\rightarrow SU(4)_C \\ &\rightarrow SU(3)_C\times U(1)_{B-L}\ . \end{aligned} \tag{3.17}$$

Only the embedding into a larger group[12] can fully constrain the charges; a particular choice has been taken above.

For models 6 and 7 we have the same situation and we cannot make further restrictions.

IV. CONCLUSIONS

We have analyzed six-flavor preon models based on precolor SU(N) or SO(N) and have found three models, based in either group, which pass all of the constraints. The preon content consists of a single color-triplet weak isosinglet, a color-singlet weak isodoublet, and an extra singlet. The models found are free of anomalies, are asymptotically free, have a reasonable composite spectrum and may lead to a unification of precolor with the other interactions—electroweak and strong.

In our earlier work we found that four- and five-flavor preon models are ruled out and so were two kinds of six-preon models. The simplest possible models are thus of the six-flavor variety unless further constraints are imposed which could eliminate them as well.

All of the models we have found contain the $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ subgroup within G_{PF} and, hence are naturally left-right symmetric; these models also contain exotic composites in either the color-triplet or color-singlet sectors or both.

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