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Flavor unification in SU(8)

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Flavor unification is proposed in the SU(8) gauge group. This model imbeds the Georgi-Glashow $\mathrm{SU_{GG}}(5)$ as a subgroup. It is necessary to modify Georgi's second postulate for grand unification so that the left-handed fermion representations are complex with respect to $\mathrm{SU}_c(3)\times\mathrm{SU}_W(2)\times U_Y(1)\times\mathrm{U}_J(1)$. There are three families of light quarks, one right-handed isospin-singlet quark with charge $-\frac{1}{3}$, three ordinary lepton families, and one lepton doublet without a right-handed partner.

I. INTRODUCTION

The role of neutral currents in the recent development of the elementary particle physics cannot be overemphasized.[1] It has been pivotal in determining the group structure of currents, which gave strong support for the unified theory of electroweak interaction by the SU(2)×U(1) gauge theories.[2] It should be noted that only the group structure of currents of the SU(2)×U(1) gauge theory has been tested experimentally and no direct proofs for the gauge theory are available so far.

Even though there is no direct confirmation, the non-Abelian gauge theories are believed to be the correct field theories for both strong and electroweak interactions. It is a prevailing hope that one can find a simple gauge group unifying all elementary interactions (strong, weak, and electromagnetic interactions) and explaining fermions in nature, different coupling constants, symmetry-breaking patterns, etc.

It is important to have some criteria to find the correct unifying gauge group among the numerous possibilities which are consistent with presently available experimental data. We believe that further precise experiments on neutral currents will play crucial roles in pinpointing the correct theory. This will provide valuable information on the group structure of the currents at low energy, which, in turn, can allow one to accept or reject many proposed models.

For instance, if the neutral-current structures remain as predicted by the standard model of Kobayashi-Masakawa type,[3] then the minimal unifying theory, the Georgi-Glashow SU(5),[4] would be further supported besides its aesthetical beauty. Frequently suggested left-right symmetry may show some traces at low energy; then the Pati-Salam picture[5] or the SO(10)[6] model would become a good candidate.

If the low-energy currents show more structure, for example, SU(2)×U(1)×U(1), this would have important consequences in model building. It will be especially valuable for flavor unification of all fermion families. For flavor unification Georgi has proposed a few postulates,[7] which led him to SU(11) as the minimal gauge group. He used SU(2)×U(1) as the low-energy structure. With the modified group, say, SU(2)×U(1)×U(1), one can find a smaller group unifying known light fermions.

In this paper we make a systematic search for the smallest possible group to include the three known light sequential leptons (e,μ,τ) with the modified low-energy current structure SU(2)×U(1)×U(1). In Sec. II guidelines for the model construction and details of the minimal model SU(8) are presented with fermion contents. The symmetry-breaking patterns with renormalizations of coupling constants are given in Sec. III. Phenomenological consequences are discussed in Sec. IV.

II. MODEL CONSTRUCTION

For the unification of elementary interactions one should also resolve the old problem of fermion families: Why are there more than one family of quarks and leptons? To assign all the fermions on the irreducible representations of a simple group whose gauge bosons mediate various interactions one needs a suitable set of guidelines. Otherwise there will be too many possibilities to find a reasonable model. In this regard Georgi's postulates are quite useful and instructive.[7] His criteria required that the representation of the left-handed (LH) fermions must be real with respect to the $\mathrm{SU}_C(3)$ but should be complex with respect to the $\mathrm{SU}_C(3)\times\mathrm{SU}_W(2)\times\mathrm{U}_Y(1)$, and no irreducible representation should appear more than once in the representation of the LH fermions.

The first postulate allows the fermion to have masses. This postulate was modified to the reality condition with respect to $\mathrm{SU}_C(3)\times\mathrm{U}_{\mathrm{EM}}(1)$ in order to find restrictions on electromagnetic charge assignment.[8] The second postulate prohibits the fermions from acquiring superheavy masses, which is obviously necessary. This is sometimes called the survival hypothesis,[9] which is modified in various ways.[10] The third postulate is an aesthetic requirement whose modification was used by some authors.[11]

We made a search to find models of flavor unification with a minimally modified survival hypothesis: the LH fermions should be complex with respect to $\mathrm{SU}_C(3)\times\mathrm{SU}_W(2)\times\mathrm{U}_Y(1)\times\mathrm{U}_J(1)$, where J will be determined later. The last U(1) will have experimentally observable neutral-current phenomena which can be tested in the foreseeable future. This U(1) factor allows smaller flavor unifying gauge groups than SU(11).

We list the requirements which a reasonably good theory may satisfy: (1) It is preferable to have asymptotic freedom in the grand unifying group and there must be asymptotic freedom $\mathrm{SU}_C(3)$, (2) the fermion representation must be anomaly free to be renormalizable, (3) no irreducible representation should appear more than once, (4) the LH fermions must be real with respect to $\mathrm{SU}_C(3)\times\mathrm{U}_{\mathrm{EM}}(1)$, (5) the LH fermions should be complex with respect to $\mathrm{SU}_C(3)\times\mathrm{SU}_W(2)\times\mathrm{U}_Y(1)\times\mathrm{U}_J(1)$, and (6) there must be at least three families of leptons (e,μ,τ) and quarks (u,c,t).

The smallest group satisfying the above conditions in SU(N) is SU(8). The fermions are assigned to the representation

$$[\bar{1}]+[\bar{2}]+[3]=\psi_\alpha+\psi_{\alpha\beta}+\psi^{\alpha\beta\gamma}\,, \tag{1}$$

where α,β,γ are SU(8) indices. The only possible electromagnetic charge operator Q is

$$Q=\mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},1,0,0,0,0)\,. \tag{2}$$

This is determined by the reality condition of the fermions with respect to $\mathrm{SU}_C(3)\times\mathrm{U}_{\mathrm{EM}}(1)$.[8]

To see the particle contents it is necessary to decompose the fermion representation of (1) under $\mathrm{SU}_C(3)\times\mathrm{SU}_W(2)\times\mathrm{U}_Y(1)$. There are three $\{3,2,\frac{1}{6}\}$, two $\{3^*,2,-\frac{1}{6}\}$, three $\{3^*,1,-\frac{2}{3}\}$, four $\{3^*,1,\frac{1}{3}\}$, two $\{3,1\frac{2}{3}\}$, and three $\{3,1,-\frac{1}{3}\}$ for the quarks. For the lepton doublets, we have four $\{1,2,-\frac{1}{2}\}$ and three $\{1,2,\frac{1}{2}\}$. For the charged-lepton singlets, we have three $\{1,1,1\}$ and two $\{1,1,-1\}$. For the netural leptons, we have seen $\{1,1,0\}$.

Without extra $\mathrm{U}_J(1)$, we will have only one quark family and one lepton family which can survive down to low-mass particle sectors. With a suitable $\mathrm{U}_J(1)$ we want to keep at least three light families of quarks and leptons. We need a further constraint to require that neutral weak-singlet leptons should be real with respect to $\mathrm{SU}_C(3)\times\mathrm{SU}_W(2)\times\mathrm{U}_Y(1)\times\mathrm{U}_J(1)$. We can then make the neutral-singlet leptons very heavy and the LH neutrinos light by some mechanisms like Witten's.[12]

We find only one possible $\mathrm{U}_J(1)$ whose generator is

$$J=\mathrm{diag}(0,0,0,0,0,-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{2}{3})\,. \tag{3}$$

We then see that all the states except the seven neutral-singlet states are complex under $\mathrm{SU}_C(3)\times\mathrm{SU}_W(2)\times\mathrm{U}_Y(1)\times\mathrm{U}_J(1)$. All the seven neutral-singlet states are self-real and thus can be superheavy. The particle contents are as follows: three families of ordinary-type quarks

$$\{(u,d)_L,(c,s)_L,(t,b)_L,u_R,c_R,t_R,d_R,s_R,b_R\}\,,$$

and one right-hand singlet quark with $Q=-\frac{1}{3}$; three ordinary sequential lepton families,

$$\{(\nu,e)_L,(\nu,\mu)_L,(\nu,\tau)_L,e_R,\mu_R,\tau_R\}\,,$$

and one lepton doublet without a right-hand partner. By "ordinary-type" particles we mean that their weak-interaction vertices are $V-A$ type. There are $(V+A)$-type particles in the representation: two families of quarks, and two families of leptons, and one mixed family of quarks and leptons.

The masses of fermions are the least understood parts of the grand unifying gauge theories. Presently available mass-generating mechanism via Yukawa couplings has not produced acceptable masses so far. In our model we have no particular mechanism to make the $(V+A)$-type particles heavier than $(V-A)$-type particles. The problem is, however, ubiquitous, to any model which has $(V+A)$-type particles.[13] An understanding of the fermion masses is an important area of research which may shed light on the grand unifying theories.

The particle assignments are not fixed because states with the same quantum numbers can mix with each other. For the mass eigenstates one should study mixing processes properly. The only known working mechanism is via Higgs fields, which has not generated reasonable fermion masses. Further studies on dynamical mass-producing mechanisms, may prove to be fruitful.

III. SYMMETRY-BREAKING PATTERN

There are two classes of symmetry-breaking patterns: one with the Georgi-Glashow $\mathrm{SU_{GG}}(5)$ as a subgroup and the other where the weak interaction and the strong interaction are separated from each other at the first stage of the symmetry breaking. We present the first case only because the other variations can be done similarly. The full stages are

$$\begin{aligned}\mathrm{SU}(8) &\to \mathrm{SU_{GG}}(5)\times\mathrm{SU}(3)\times\mathrm{U}(1) \quad \text{at } M_8\\ &\to G_S\times\mathrm{U}_J(1) \quad \text{at } M_G\\ &\to G_S \quad \text{at } M_J\\ &\to G_e \quad \text{at } M_W\ ,\end{aligned} \tag{4}$$

where

$$G_S=\mathrm{SU}_C(3)\times\mathrm{SU}_W(2)\times\mathrm{U}_Y(1)$$

and

$$G_e=\mathrm{SU}_C(3)\times\mathrm{U_{EM}}(1)\ .$$

Since the Georgi-Glashow $\mathrm{SU_{GG}}(5)$ is separated at the first stage, the renormalization of coupling constants of $\mathrm{SU_{GG}}(5)$ and $\mathrm{U}_J(1)$ are independent of each other. The coupling constants at $M_W=100$ GeV are phenomenologically determined quantities. The intermediate mass scales are related[14] by the equations

$$\ln\frac{M_G}{M_W}=\xi=\tfrac{1}{2}\eta\ , \tag{5}$$

where

$$\xi=\frac{6\pi}{11}\left[\sin^2\theta_W/\alpha_{\mathrm{EM}}-\frac{1}{\alpha_c}\right] \tag{6}$$

and

$$\eta=\frac{6\pi}{11}\left(\tfrac{3}{5}\cos^2\theta_W/\alpha_{\mathrm{EM}}-\sin^2\theta_W/\alpha_{\mathrm{EM}}\right)\ . \tag{7}$$

These are well known results, consistent with the Weinberg angle $\sin^2\theta_W \simeq 0.20$. For the evaluation of the $\mathrm{U}_J(1)$ coupling constants we note that J is a generator of SU(3) and it is orthogonal to the $\tilde{\mathrm{U}}(1)$ generator. Therefore, the coupling-constant renormalization is given by

$$\begin{aligned}g_3^{-2}(M_G)&=g_8^{-2}+d(-3+F)\ln M_8/M_G\ ,\\ g_J^{-2}(M_J)&=g_3^{-2}(M_G)+dF\ln M_G/M_J\ ,\end{aligned} \tag{8}$$

where $d=11/24\pi^2$, $F=2$ and Higgs-field contributions are ignored.

From the symmetry-breaking pattern, we see that the interaction strength of $\mathrm{U}_J(1)$ is the weakest at the M_J level, and the relative strength depends upon the mass scales M_8, M_G, and M_J. The M_8 is a completely free parameter, which could have relevance in the early stages of the Universe. The M_G is of order 10^{14} GeV, and has measurable effects, such as proton decay. The present model leaves M_J undetermined, which could be phenomenologically inferred if new neutral-current effects are observed.

It is well known that flavor-changing neutral currents (FCNC's) are strongly suppressed.[15] For the suppression in a grand unification theory it is better to satisfy the Glashow-Weinberg theorem.[16] Our model, however, violates the theorem. To avoid possible conflicts with FCNC data, the M_J must be larger than M_W substantially. Presently available neutral-current data cannot rule out M_J being ten times larger than M_W.[17]

If we assume the mass-generating mechanism via Higgs-field Yukawa couplings, the fermion masses are related to M_J. If M_J is too large, it will then be difficult to understand why fermion masses are light. Hence M_J cannot be either much larger than or too close to M_W.

Further experiments upon neutral currents with more precision and higher energy will be pivotal in determining the low-energy group structure, which will help to find the correct grand unifying group.

IV. DISCUSSION

Decay modes of the proton and its lifetime are similar to those of SU(5). However, these depend upon the symmetry-breaking patterns. If a pattern does not include the $\mathrm{SU_{GG}}(5)$ factor in its intermediate stage, then very different modes are possible.

Recent results from the Cornell Electron Storage Ring[18] show that B-meson decay is in agreement with the Kobayashi-Maskawa standard model,[3] but is in conflict with models without t quarks.[19] These indicate that there are at least three families of quarks. Our model has three ordinary-type quark families and one right-handed singlet quark with charge $-\frac{1}{3}$. The latter does not have weak interactions. Future experiments may reveal the elusive top quark and the singlet quark.

The properties of the τ lepton agree with the sequential lepton picture, in which τ is another lepton with still heavier mass and separately conserved quantum number.[20] The electron-momentum distribution of τ-lepton decay shows that the τ-ν-W vertex is $V-A$ type, as in the electron and muon cases.[21] Our model expects one more lepton doublet of $V-A$ type and three $(V+A)$-type leptons.

If the recently claimed discovery of magnetic monopole is further confirmed,[22] the symmetry-breaking pattern of our model could be in possible conflict with the standard hot-big-bang cosmology.[23] This is common to all grand unifying theories if it has a U(1) factor which is not orthogonal to electromagnetic charge Q at a superheavy mass scale.

Parametrization of neutral currents was studied extensively by Barr and Zee[17] and J. Kim and J. E. Kim.[24] Our model belongs to the simplest case among the many possible low-energy group structures $SU(2)\times U(1)\times G'$.

Other symmetry-breaking patterns which do not have $SU_{GG}(5)$ as an intermediate stage are also interesting because they can have different proton modes. Since these can be studied similarly, further discussions are omitted.

Models based upon SU(9) can be constructed with fermions assigned to the irreducible representations $\psi_{\alpha\beta}+\psi^{\alpha\beta\gamma\delta}$. When the electromagnetic charge operator is

$$Q=\mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},1,0,0,0,0,0,)$$

there are two possibilities for J, namely,

$$J_1=\mathrm{diag}(0,0,0,0,0,-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{2}{3},0)$$

or

$$J_2=\mathrm{diag}(0,0,0,0,0,-\tfrac{1}{3},-\tfrac{2}{3},1,0)\ .$$

For either case the surviving light fermions are exactly the same as those of the SU(8) model.

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