πNN form factor, chiral-symmetry breaking, and three-pion resonances

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The extraction of the πNN form factor $F_{\pi NN}(t)$ from Reggeized one-pion-exchange (OPE) fits to hadronic cross sections is reexamined. The objective is to explain the Goldberger-Treiman discrepancy Δ_π which is determined from the extrapolation of $F_{\pi NN}$ to $t=0$. Until now Reggeized OPE analyses have led to values of Δ_π that disagree with experiment by a factor of 2. It is argued that this failure is likely due to an insensitivity of the OPE amplitudes to heavy-pion (three-pion-resonance) contributions, at small t, when $F_{\pi NN}$ is parametrized by standard forms, e.g., the monopole or the dual model. A new functional form is proposed for $F_{\pi NN}(t)$ which has the virtue of enhancing these contributions at small momentum transfers, thus leading to OPE amplitudes more sensitive to 3π-resonance effects. This form factor is then used to reanalyze differential cross sections for $pp\to n\Delta^{++}$, $pp\to p\Delta^+$, $np\to pn$, $\bar{p}p\to\bar{n}n$, and $\gamma p\to\pi^+ n$, as well as asymmetries for $\gamma p\to\pi^+ n$, at high energies ($E_L\simeq 5$–25 GeV) and $|t|\lesssim 0.3$ (GeV/c)^2. The results show that a self-consistent fit to all these data, with the correct value of Δ_π, is indeed possible.

I. INTRODUCTION

The pionic form factor of the nucleon, $F_{\pi NN}(t)$, is known to play an important role in intermediate-energy physics, in connection with the NN interaction,[1] as well as in high-energy physics through its relation to chiral-SU(2)$\times$SU(2)-symmetry breaking.[2] In a recent series of papers[3–5] a thorough analysis of pion photoproduction[3] and two-body[4] and quasi-two-body[5] hadronic reactions at high energy and small momentum transfers has been carried out in order to extract $F_{\pi NN}(t)$ as well as the value of the Goldberger-Treiman (GT) discrepancy Δ_π. These analyses are based on the Reggeized one-pion-exchange (OPE) approximation to scattering amplitudes,[6] which is known to hold to great accuracy at small t [-0.3 (GeV/c)$^2\lesssim t<0$]. The results of the fits to the differential cross sections for $pp\to n\Delta^{++}$, $pp\to p\Delta^+$, $\gamma p\to\pi^+ n$, $np\to pn$, and $\bar{p}p\to\bar{n}n$, as well as the asymmetries in $\gamma p\to\pi^+ n$, at various energies ($E_L\simeq 5$–25 GeV) and small t indicate that a good parametrization of $F_{\pi NN}(t)$ is that given by the monopole form, i.e.,

$$F_{\pi NN}(t)=\frac{\Lambda^2-\mu_\pi^2}{\Lambda^2-t}\ , \qquad (1)$$

with $\Lambda\simeq 800$–1000 MeV. At the same time, a dynamical model[7] for $F_{\pi NN}(t)$ with sound physical motivation based on ideas from duality and Reggeism, and capable of describing any fully off-mass-shell hadronic vertex,[8] has also been used in these analyses. In the particular case of the πNN vertex this model gives

$$F_{\pi NN}(t)=\Gamma(\beta)\frac{\Gamma(1-\alpha_\pi(t))}{\Gamma(\beta-\alpha_\pi(t))}\ , \qquad (2)$$

where

$$\alpha_\pi(t)=\alpha'(t-\mu_\pi^2)\ , \qquad (3)$$

is the pion Regge trajectory with universal slope $\alpha'=M_\rho^2/2\simeq 0.82$ GeV^{-2}, and β is a free parameter that controls the asymptotic behavior of $F_{\pi NN}(t)$ in the spacelike region, i.e.,

$$F_{\pi NN}(t)\underset{t\to-\infty}{\simeq}(-\alpha' t)^{1-\beta}\ . \qquad (4)$$

In the range of momentum transfers covered by the fits, dictated by the requirement that Reggeized OPE be a good approximation, Eqs. (1) and (2) give essentially identical values of χ^2 if $\beta\simeq 2.5$–3 (in good agreement with theoretical expectations). Unfortunately, due to a breakdown of the Reggeized OPE approximation at higher values of t, it is not practical to extend these analyses beyond $t\simeq -0.3$ (GeV/c)^2 in order to discriminate between Eqs. (1) and (2). In any case, since the pion pole has been removed from $F_{\pi NN}(t)$, both expressions indicate the presence of an important dynamical contribution to the πNN vertex function. In particular, Eq. (2) is the result of summing a number of heavy pions ($J^P=0^-$) or zero-width three-pion resonances at the vertex, with this number being finite or infinite depending on whether β is integer or noninteger. On the other hand, the monopole form factor may be viewed as an effective single-pole approximation to Eq. (2).

These three-pion resonances were first postulated as a result of the impossibility of accounting for the corrections to the GT relation in terms of a πNN form factor built only from ordinary dynamical continua.[9] Later on, they were incorporated into $F_{\pi NN}(t)$ in the framework of the dual model,[7] Eq. (2), and more recently they were shown to be crucial for a correct understanding of the analytic corrections to current-algebra and PCAC (partially conserved axial-vector current) soft-meson theorems in broken $SU(2)\times SU(2)$.[10] In the language of quantum chromodynamics (QCD) these heavy pions correspond to radial excitations of the quark-antiquark bound state.[11] Recently, the first radial excitation of the pion has been found in two independent analyses[12,13] of the reaction $\pi N \to (3\pi)N$, with a mass $\mu_{\pi'} \simeq 1.3$ GeV and a broad width $\Gamma_{\pi'} \simeq 600$ MeV. This experimental value of the mass of π' is in good agreement with the dual-model prediction[7] as well as with quark-model calculations.[11]

In spite of all this progress in our understanding of the dynamics of the πNN vertex function, there still remains the fundamental problem of predicting the correct GT discrepancy from the extrapolated value of $F_{\pi NN}(t)$ to $t=0$. In fact, using the relation

$$F_{\pi NN}(0)=1-\Delta_\pi \ , \tag{5}$$

where

$$\Delta_\pi = 1-\frac{Mg_A(0)}{f_\pi g_{\pi NN}(\mu_\pi{}^2)} \tag{6}$$

is the GT discrepancy, one finds using Eqs. (1) or (2) that $\Delta_\pi = 0.02-0.03$, to be compared with the experimental result[14]

$$\Delta_\pi^{\rm exp} = 0.06 \pm 0.01 \ . \tag{7}$$

At the same time, estimates of $F_{\pi NN}(0)$ based on dispersion relations[2] also fail to reproduce Eq. (7).

In a recent paper,[15] it has been shown that a calculation of the renormalizations induced by chiral-$SU(2)\times SU(2)$-symmetry breaking in all four parameters appearing in Eq. (6) leads to the successful prediction $\Delta_\pi = 0.06 \pm 0.02$. Also, as a result of this new approach one obtains[15] a functional relation between Δ_π and the πN σ commutator $\sigma_{\pi N}$.[16] This provides a strong indication that there should be nothing intrinsically wrong with our current understanding of chiral-symmetry breaking and, in particular, with the representation content of the QCD Lagrangian. However, in view of the impact that a correct prediction of Δ_π has on QCD, it would be highly desirable to succeed in predicting its value from an independent approach, such as that of the πNN vertex function.

In this paper it will be argued that the failure of the approach based on the extraction of $F_{\pi NN}(t)$ from Reggeized OPE analyses is due to the fact that Eqs. (1) or (2) produce OPE amplitudes which are rather insensitive to three-pion-resonance contributions at small t. Since these contributions are expected to be crucial for a correct prediction of Δ_π (Refs. 9 and 10) it should come as no surprise that all previous attempts along this line[3–5] have failed. In order to correct this deficiency a new functional form for $F_{\pi NN}(t)$ will be proposed here which has the virtue of enhancing, at small momentum transfers, the sensitivity of OPE amplitudes to heavy-pion contributions. This form factor will then be used in all the reactions previously analyzed with Eqs. (1) and (2), i.e., Δ production in NN collisions, pion photoproduction, and NN charge-exchange scattering. The results of the new fits to all the differential cross sections and the asymmetries in $\gamma p \to \pi^+ n$ show that it is indeed possible to have $\Delta_\pi \simeq 0.06$ without at the same time degrading the quality of the previous fits done with Eqs. (1) or (2).

II. THE πNN FORM FACTOR AND THREE-PION RESONANCES

Let us consider the matrix element of the divergence of the axial-vector current between nucleons:

$$\begin{aligned}\langle N(P')\,|\,i\partial^\mu A_\mu\,|\,N(P)\rangle &= [2Mg_A(t)+tg_P(t)]\bar{u}(P')\gamma_5 u(P)\\ &= D(t)\bar{u}(P')\gamma_5 u(P) \ ,\end{aligned} \tag{8}$$

where $M=(M_p+M_n)/2$, $t=(P'-P)^2$, and isospin indices have been omitted. The natural framework for the introduction of heavy-pion states is that of extended PCAC (EPCAC), i.e.,

$$\partial^\mu A_\mu = \sum_{n=0}^{N} \mu_{\pi_n}{}^2 f_{\pi_n}\phi_{\pi_n} \ , \tag{9}$$

where the number of heavy pions, $N \geq 1$, need not be specified. The only requirement needed at this stage is that $f_{\pi_n} = 0(\mu_\pi{}^2)$, for $N \geq 1$, which ensures that the heavy pions do not become Goldstone bosons in the $SU(2)\times SU(2)$ limit. An absolutely equivalent way of introducing these $J^P=0^-$ states is by leaving PCAC [$N=0$ in Eq. (9)] unmodified but redefining ϕ_π. Substituting Eq. (9) into Eq. (8) one finds

$$D(t)=\sum_{n=0}^{N} 2f_{\pi_n} g_{\pi_n NN}\frac{\mu_{\pi_n}{}^2}{\mu_{\pi_n}{}^2-t} \ , \tag{10}$$

with the following normalization at $t=0$:

$$D(0)=2Mg_A(0) \ . \tag{11}$$

Since the Reggeized OPE analyses of hadronic reactions at high energy are carried out at small t, i.e., $-0.3\ (\mathrm{GeV}/c)^2 \lesssim t < 0$, one can safely approximate the contribution of the $N \geq 1$ terms in Eq. (10) by a single effective pole term, i.e.,

$$D(t) \simeq 2 f_\pi g_{\pi NN}(\mu_\pi{}^2) \frac{\mu_\pi{}^2}{\mu_\pi{}^2 - t} + c \frac{\Lambda^2}{\Lambda^2 - t} . \quad (12)$$

The coefficient c is determined from the normalization, Eq. (11), and upon using Eq. (6) one finds that Eq. (12) becomes

$$D(t) \simeq 2 f_\pi g_{\pi NN}(\mu_\pi{}^2) \left[\frac{\mu_\pi{}^2}{\mu_\pi{}^2 - t} - \Delta_\pi \frac{\Lambda^2}{\Lambda^2 - t} \right] . \quad (13)$$

The πNN form factor, defined with the pion pole removed, is related to $D(t)$ as follows:

$$D(t) = 2 f_\pi g_{\pi NN}(\mu_\pi{}^2) \mu_\pi{}^2 \frac{F_{\pi NN}(t)}{\mu_\pi{}^2 - t} . \quad (14)$$

Comparing Eqs. (13) and (14) one finally obtains

$$F_{\pi NN}(t) = 1 - \frac{\Delta_\pi}{\mu_\pi{}^2} \Lambda^2 \frac{\mu_\pi{}^2 - t}{\Lambda^2 - t} , \quad (15)$$

where $F_{\pi NN}(0) = 1 - \Delta_\pi$, $F_{\pi NN}(\mu_\pi{}^2) = 1$, and Eq. (15) is to be understood as an approximation for small t.

The correlation between Λ and the number of heavy pions may be established by comparing, e.g., Eqs. (10) and (13). In this case it is straightforward to show that as N increases Λ decreases and in particular that for small t and large N

$$\Lambda^2 \underset{\substack{t \to 0 \\ N \to \infty}}{\simeq} \frac{1}{\alpha' \ln N} , \quad (16)$$

where a linear mass spectrum has been assumed, viz.,

$$\mu_{\pi_n}{}^2 = \mu_\pi{}^2 + n/\alpha' . \quad (17)$$

A particular value of Λ of interest here is

$$\Lambda^2 = \frac{\mu_\pi{}^2}{\Delta_\pi} ,$$

which reduces Eq. (15) to the monopole form, Eq. (1). According to the results of the Reggeized OPE fits mentioned before,[3–5] $\Lambda \simeq 800$–1000 MeV and, therefore, $\Delta_\pi \simeq 0.02$–0.03, i.e., a factor of 2 smaller than the experimental value.

A simple exercise shows that these values of Λ correspond to $N \simeq 1$–3 in Eq. (10). This may be taken as an indication that the Reggeized OPE amplitudes at small t ($t < 0$) are sensitive only to nearby singularities at $t > 0$, with the pion pole giving the main contribution. One would expect that by increasing the range of t covered in the fits the effect of higher radial excitations of the pion would become more important, leading to a decrease in the resulting value of Λ and thus improving the prediction for Δ_π. However, fitting high-energy reactions beyond $t \simeq -0.3\ (\mathrm{GeV}/c)^2$ is not a sensible course of action because the OPE approximation breaks down; one would have to parametrize the scattering amplitudes with additional Regge poles and thus introduce several new free parameters. The power of the Reggeized OPE extractions of $F_{\pi NN}(t)$ rests precisely on the fact that the analyses are essentially one parameter fits with Λ being the only unknown.[17]

One possible way out of this dilemma is to incorporate the effect of high radial excitations of the pion mainly through their contribution at $t=0$ and then let $F_{\pi NN}(t)$ adjust itself in order to fit the data away from $t=0$. Since this strategy requires that Eq. (18) be abandoned it is not obvious that it would still be possible to obtain fits of comparable quality. After reanalyzing from this point of view the high-energy reactions that were studied previously with Eqs. (1) and (2)[3–5] it was found that Eq. (15), with $\Delta_\pi = 0.06$, is indeed capable of producing fits of essentially the same quality as measured by the re-

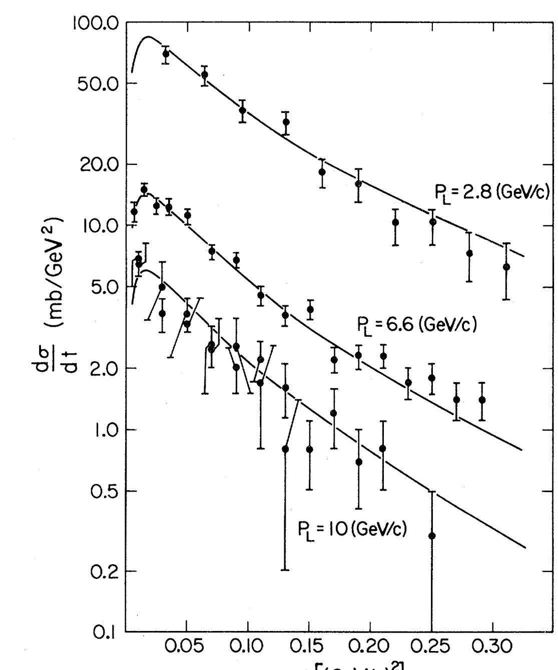

FIG. 1. Experimental data on $pp \to n\Delta^{++}$ at $P_L = 2.8$, 6.6, and 10.0 GeV/c. Solid curves are the results of the best fit with the πNN form factor of Eq. (15) and $\Delta_\pi = 0.06$ (see Table I).

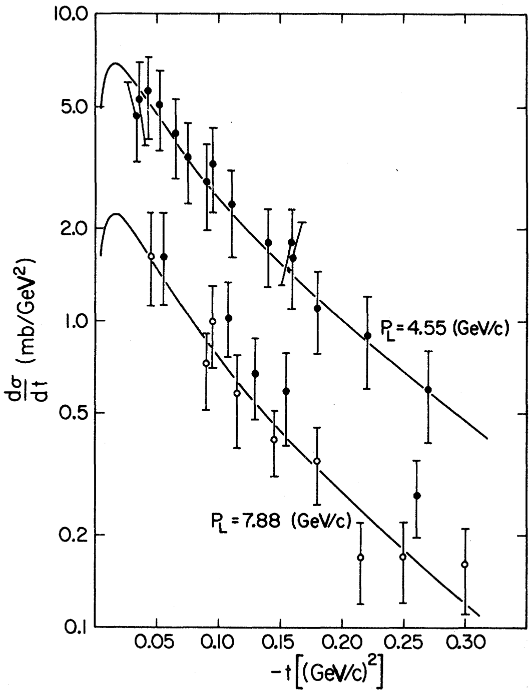

FIG. 2. Experimental data on $pp \rightarrow p\Delta^+$ at $P_L = 4.55$, 6.06, and 7.8 GeV/c. The $P_L = 9.9$ GeV/c data is not shown for clarity but it has been used in the fits. Solid curves are the results of the best fit with the πNN form factor of Eq. (15) and $\Delta_\pi = 0.06$ (see Table I).

sulting values of χ^2. The results of these new fits are discussed in the next section.

III. EXTRACTION OF $F_{\pi NN}(t)$ FROM HIGH-ENERGY REACTIONS

The reactions to be studied here are $pp \rightarrow n\Delta^{++}$, $pp \rightarrow p\Delta^+$, $\gamma p \rightarrow \pi^+ n$, and NN charge-exchange scattering. The formalism and details of the Reggeized OPE analyses used to extract $F_{\pi NN}(t)$ from fits to the differential cross sections and asymmetries of these processes have been already discussed in detail in Refs. 3–5. We then proceed directly to present the results of the new fits using Eq. (15) together with Eq. (7).

A. $pp \rightarrow n\Delta^{++}$ and $pp \rightarrow p\Delta^+$

These two reactions are clearly related by isospin but since there are independent data for them they will be treated separately. The data sets used in the analysis are the following.[18] For $pp \rightarrow n\Delta^{++}$, differential cross sections $d\sigma/dt$ at $P_L = 2.8$, 6.6, and 10.0 GeV/c; for $pp \rightarrow p\Delta^+$, $d\sigma/dt$ at $P_L = 4.55$, 6.06, 7.88, and 9.9 GeV/c. These cross sections were fitted in the interval -0.3 (GeV/c)$^2 \lesssim t < 0$ using Eq. (15) together with Eq. (7) and the Reggeized OPE amplitudes discussed in Ref. 5. It must be pointed out that, unlike pion photoproduction and NN charge exchange, these Δ production reactions do not involve absorption. Therefore, the range Λ in Eq. (15) is the only free parameter in the fits.

The results are shown in Figs. 1 and 2 and in Table I, where the previous result[5] obtained with the monopole, Eq. (1), is indicated for comparison. The monopole prediction is not shown in the figures since it cannot be distinguished on this scale from that of Eq. (15). In Fig. 2 the 6.06 and 7.88 GeV/c curves are practically coincident and although the 9.9 GeV/c data is not shown for simplicity it has been used in the fits.

As expected from the discussion in Sec. II, the influence of high radially excited pion states introduced through Δ_π produces a substantial decrease in the range of the form factor. The important point, though, is that the quality of the fit has not degraded; in fact it has even improved slightly with respect to the monopole fit.

TABLE I. πNN form-factor parameters from best Reggeized OPE fits to hadronic reactions at high energy and $|t| \lesssim 0.3$ (GeV/c)^2.

Reaction	Form factor	Λ (MeV)	$\chi_F{}^2$
$pp \rightarrow n\Delta^{++}$	Monopole, Eq. (1)	788±20	1.3
	Eq. (15)	407± 8	1.0
$pp \rightarrow p\Delta^+$	Monopole, Eq. (1)	807±37	0.7
	Eq. (15)	400±13	0.6
$\gamma p \rightarrow \pi^+ n$	Monopole, Eq. (1)	800 (fixed)	2.4
	Eq. (15)	400 (fixed)	2.5
$np \rightarrow pn$ and	Monopole, Eq. (1)	800 (fixed)	1.2
$\bar{p}p \rightarrow \bar{n}n$	Eq. (15)	400 (fixed)	1.3

B. Charged-pion photoproduction

All the available data[19] above $\nu=3.4$ GeV measured at SLAC, DESY, and CEA has been used, i.e., unpolarized differential cross sections[20] between $\nu=3.4$ and 18 GeV and asymmetries[21] in the range 3.4–16 GeV, with a total of 126 data points. It should be recalled that the existence of two independent sets of experimental data, i.e., unpolarized cross sections and asymmetries, places strong constraints on the parametrization of the $\gamma p \to \pi^+ n$ reaction. In particular, the availability of asymmetry data allows a separation of the natural- and unnatural-parity exchange in the t channel and thus an isolation of OPE.

Unlike the Δ production reactions discussed in A, the process $\gamma p \to \pi^+ n$ involves absorption and, therefore, two additional parameters. It is customary to treat absorptive effects by means of a simple absorption model,[6] in which case the contribution to the relevant amplitudes is of the form Ae^{bt}, with A expected to be very close to unity. In view of this it would be convenient to establish some criterion for comparing the results of the fits to pion photoproduction with those of Δ production. One possibility would be to start with the monopole, Eq. (1), and search for Λ, A, and b in $\gamma p \to \pi^+ n$. Then one could freeze Λ at the value that fits Δ production, always with the monopole, and search only for A and b. A self-consistent analysis should produce only minor changes in the parameters and the χ^2. This procedure could then be repeated for the new form factor, Eq. (15).

Adopting this criterion one finds that the analysis is indeed self-consistent, i.e., A and b change very slightly, typically not more than 1%, while χ^2 varies at most by 6%. The results of the fits are shown in Figs. 3 and 4 and in Table I. Once again, these results show that $F_{\pi NN}(t)$ as given by Eq. (15) together with Eq. (7) fits the data with a statistical quality comparable to that of the monopole.

C. Nucleon-nucleon charge-exchange scattering

The quantity of interest here is the average of the two charge-exchange reactions $np \to pn$ and $\bar{p}p \to \bar{n}n$.

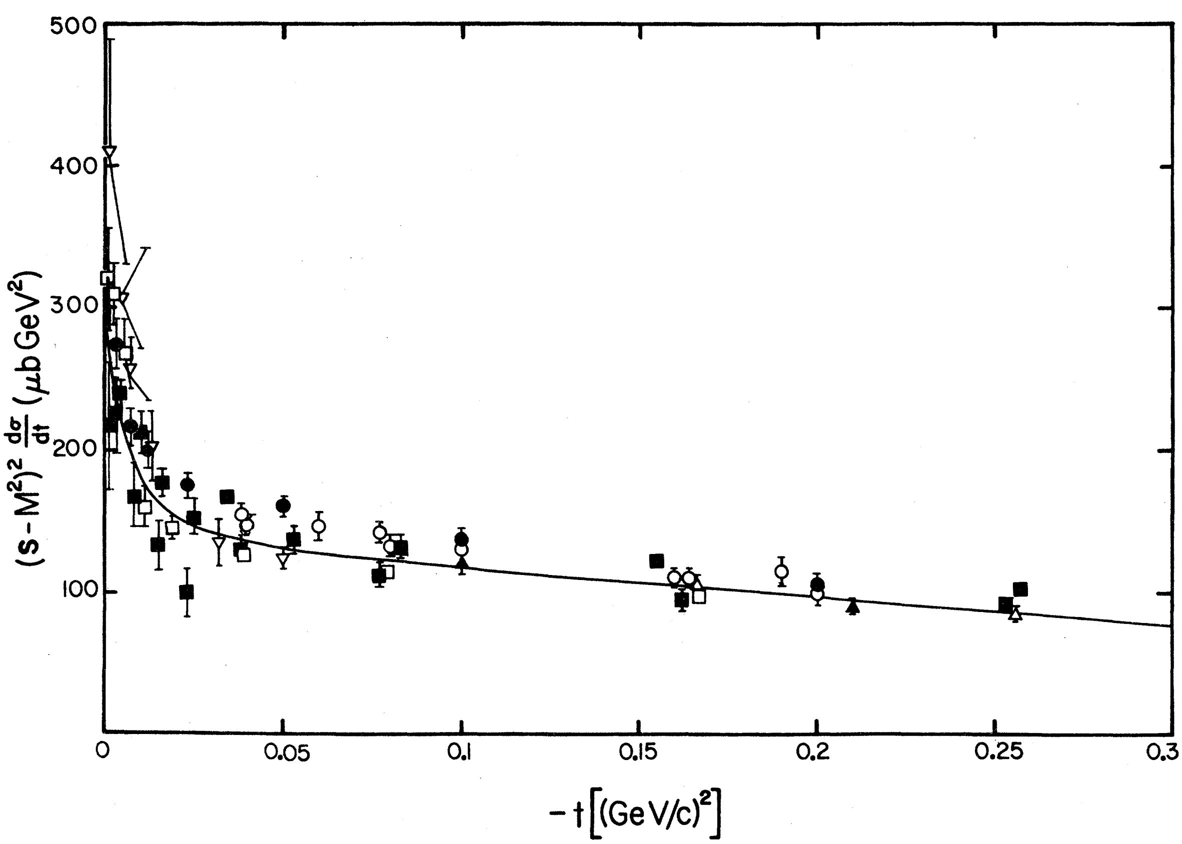

FIG. 3. Unpolarized differential cross sections for $\gamma p \to \pi^+ n$ at various incident photon energies ($\nu=3.4$–18 GeV). Only a part of the data is shown for clarity. The solid curve is the result of the best fit with the πNN form factor of Eq. (15) and $\Delta_\pi=0.06$ (see Table I).

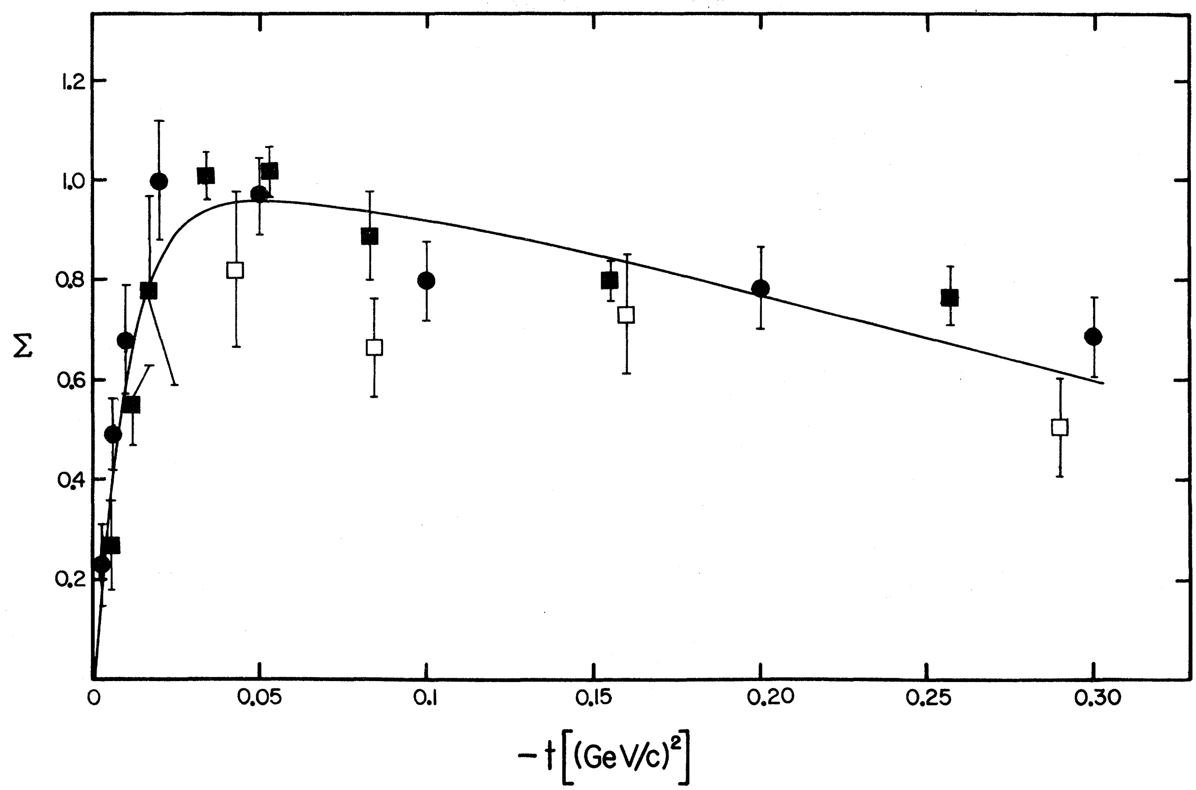

FIG. 4. Asymmetries for $\gamma p \to \pi^+ n$ at various incident photon energies ($\nu=3.4$–16 GeV). Only part of the data is shown for clarity. The solid curve is the result of the best fit with the πNN form factor of Eq. (15) and $\Delta_\pi=0.06$ (see Table I).

The reason for taking the average is that it eliminates completely ρ-A_2 interference[22] and thus simplifies the analysis. The data set used[23] here is at $P_L=5$, 8, and 25 GeV/c, corrected for normalization uncertainties as in Ref. 3. Since NN charge-exchange scattering involves absorption, the same criterion of self-consistency discussed in B has been adopted.

The results indicate that the absorption parameters A and b do not show any appreciable changes if Λ is taken as a free parameter or fixed at the value from Δ production, both for the monopole, Eq. (1), and the new form factor Eq. (15) [together with Eq. (7)]. In fact, A changes at most by 0.6% and b by 5%, well within the one-standard-deviation level, thus ensuring the self-consistency of the whole analysis. The results are shown in Fig. 5 and Table I.

IV. SUMMARY

The problem of accounting for the corrections to the Goldberger-Treiman relation, Δ_π, has received considerable attention through the years.[2] This problem is of a fundamental nature in view of its impact on our current understanding of chiral-symmetry breaking in the framework of QCD. Theoretical predictions based on dispersion relations, πNN form factor extractions from high-energy hadronic reactions, and specific models have led to values of Δ_π a factor of 2 smaller than experiment. Recently, a direct calculation of the renormalizations induced by SU(2)$\times$SU(2) breaking in all four parameters appearing in the GT relation has at last succeeded in accounting for Δ_π.[15] However, a satisfactory and self-consistent solution to this problem would require that Δ_π be correctly predicted by all possible model-independent techniques.

With this in mind, Reggeized OPE fits to high-energy hadronic and electromagnetic reactions have been reexamined in this paper. It has been argued that the failure to predict the correct value of Δ_π from the extracted $F_{\pi NN}$ is likely due to an insensitivity of OPE amplitudes to 3π-resonance effects, at small t, when $F_{\pi NN}(t)$ is parametrized by a monopole or the dual model. In order to correct this deficiency, a new functional form for $F_{\pi NN}(t)$ has been proposed which leads to OPE amplitudes more sensitive to heavy-pion contributions at small momen-

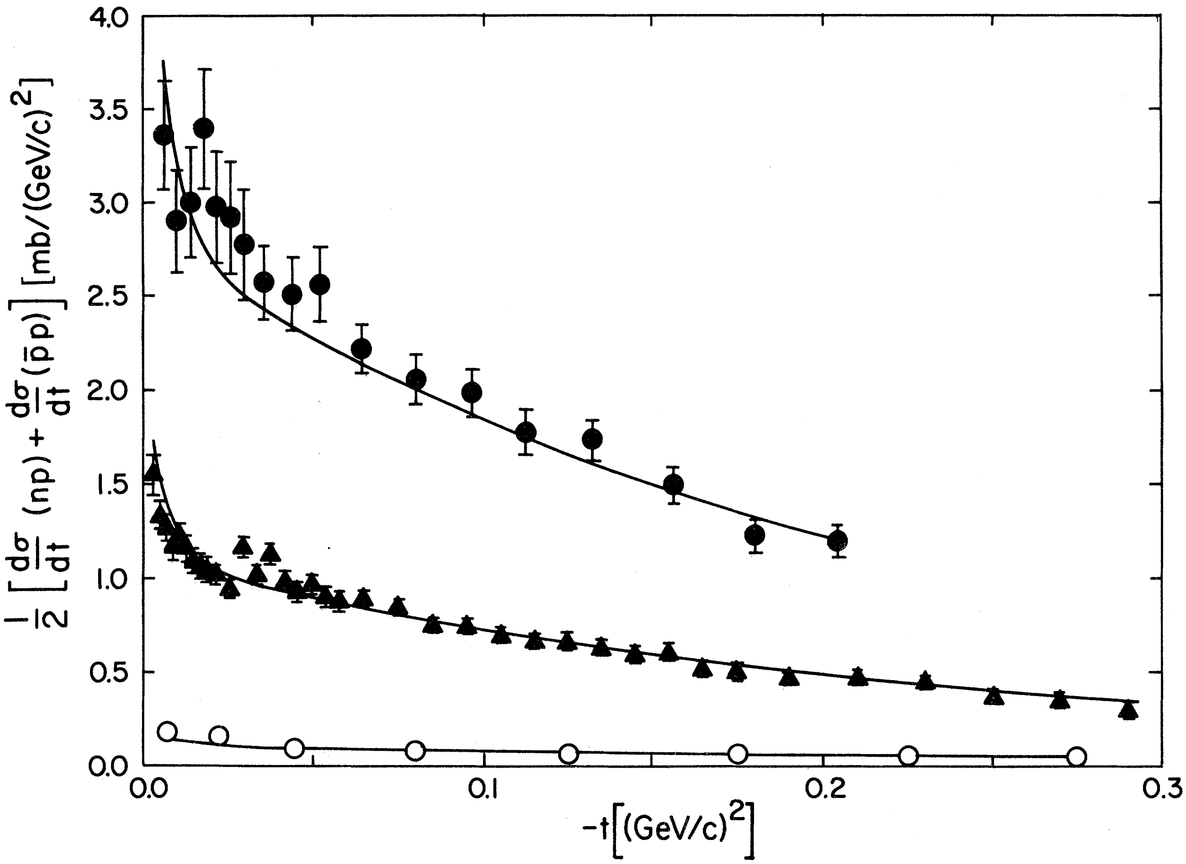

FIG. 5. Average of np and $\bar{p}p$ charge exchange scattering differential cross sections at $P_L=5$, 8, and 25 GeV/c, corrected for normalization effects as in Ref. 3. Solid curves are the results of the best fits with the πNN form factor of Eq. (15) and $\Delta_\pi=0.06$ (see Table I).

tum transfers. This πNN form factor has then been used to reanalyze OPE fits to differential cross sections for $pp\to n\Delta^{++}$, $pp\to p\Delta^{+}$, $\gamma p\to\pi^{+}n$, $np\to pn$, $\bar{p}p\to\bar{n}n$, and asymmetries for $\gamma p\to\pi^{+}n$.

The results show that a self-consistent fit to all these data with the correct value of Δ_π, i.e., $\Delta_\pi=0.06$, is indeed possible. This together with the recent successful prediction of Ref. 15 brings us very close to achieving a complete and correct understanding of the dynamics of the corrections to the Goldberger-Treiman relation. One remaining approach that needs revision in view of the importance of 3π-resonance contributions is that of the dispersion relation calculation of $F_{\pi NN}(t)$. Much more experimental information on the heavy-pion states would be required, though, before this could be attempted.

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