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Quark-tadpole transitions and the $\Delta I=\frac{1}{2}$ rule in K decays

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We show that the Nambu-Goldstone structure of the tightly bound $\bar{q}q$ pion and kaon allows the weak s-d quark tadpole to contribute to the $\Delta S=1$ nonleptonic weak decay amplitudes. This tadpole, bound to the kaon in a quark loop, accounts for the $\Delta I=\frac{1}{2}$ rule and the observed magnitude of the $\Delta I=\frac{1}{2}$ $K\to 2\pi$ amplitude. With a simple vacuum-saturation model of the $\Delta I=\frac{3}{2}$ $K\to 2\pi$ amplitude the observed relative sign of the $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ amplitudes is also correctly obtained.

I. INTRODUCTION

Pions and kaons play a special role in hadron physics because they are the "almost Nambu-Goldstone bosons" associated with the spontaneous breakdown of chiral symmetry. We suggest that it is this special property peculiar to kaons which is to be exploited in the context of the QCD (quantum chromodynamics) quark model to understand their nonleptonic decays. In particular we propose that the important part of the $\Delta I=\frac{1}{2}$ weak Hamiltonian is the s-d quark tadpole generated by a single $W^{\pm}$ exchange,[1,2] and that it is precisely the Nambu-Goldstone nature of the kaon which prevents this tadpole from being transformed away by higher-order QCD or QFD (quantum-flavor-dynamic) interactions. Coupling this tadpole to a QCD quark loop which binds the $s\bar{d}$ quark-antiquark pair into the pseudoscalar $\bar{K}^0$, we find that the experimental $\Delta I=\frac{1}{2}$ $K_{2\pi}$ amplitude is approximately reproduced.

In Sec. II we extract the relevant $K_{2\pi}$ $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ amplitudes from experiment and the usual PCAC (partial conservation of axial-vector current) analysis.[3] Then in Sec. III we present two quantitative lemmas which suggest that the $\Delta I=\frac{1}{2}$ s-d quark tadpole Hamiltonian cannot be transformed away by QCD or QFD renormalization procedures. We evaluate the matrix element $\langle 0|\mathscr{H}_{\rm tad}|\bar{K}^0\rangle$ in Sec. IV, linking the s-d tadpole to the $\bar{K}^0$ bound state by a quark loop. We find that this matrix element is consistent with the magnitude of the measured $\Delta I=\frac{1}{2}$ $K_{2\pi}$ amplitudes. Then in Sec. V we compare the relative sign of the $\Delta I=\frac{1}{2}$ quark tadpole amplitude and the $\Delta I=\frac{3}{2}$ amplitude obtained by vacuum saturation. Even this sign is consistent with experiment and confirms the validity of the s-d quark tadpole.

Finally in Sec. VI we summarize our results and draw our conclusions. We emphasize that the dynamical mechanism producing the $\Delta I=\frac{1}{2}$ rule is *not universal*, but that kaon decays and hyperon decays each have their own *natural* dynamical processes which generate a $\Delta I=\frac{1}{2}$ rule. In the kaon case this mechanism is the Nambu-Goldstone nature of the pseudoscalar mesons as relativistic $\bar{q}q$ tightly bound states, whereas in the $B\to B'\pi$ hyperon case the analogous mechanism is the nonrelativistic qqq loosely-bound-state nature of the baryons, and the resulting SU(6) symmetry.[4–6]

II. PHENOMENOLOGY OF TWO-BODY KAON DECAYS

We begin by extracting the $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ $K_{2\pi}$ amplitudes, which we call $a_{1/2}$ and $a_{3/2}$. An isospin decomposition of the final 2π state and the inclusion of final-state interaction leads to the usual expression[3]

$$iM(K_S^0\to\pi^+\pi^-)=a_{1/2}e^{i\delta_0}+\tfrac{2}{3}a_{3/2}e^{i\delta_2}\ , \qquad (1a)$$

$$iM(K_S^0\to\pi^0\pi^0)=a_{1/2}e^{i\delta_0}-\tfrac{4}{3}a_{3/2}e^{i\delta_2}\ , \qquad (1b)$$

$$iM(K^+\to\pi^+\pi^0)=a_{3/2}e^{i\delta_2}\ . \qquad (1c)$$

The intrinsic weak amplitudes $a_{1/2}$ and $a_{3/2}$ are relatively real. The data on the partial decay rates[7] allows us to deduce from Eqs. (1) that

$$a_{3/2}/a_{1/2}=0.047\pm0.002\ ,\qquad \Delta=\delta_0-\delta_2=(57\pm2)^\circ\ . \qquad (2)$$

Alternatively we may input the value[8] $\Delta=(47\pm7)^\circ$

to obtain from (1a) and (1b)

$$a_{3/2}/a_{1/2}=0.038\pm0.005 \quad (3a)$$

or from (1a) and (1c)

$$|a_{3/2}/a_{1/2}|=0.047\pm0.002\ . \quad (3b)$$

The agreement between Eqs. (2) and (3) is just outside one standard deviation. A weighted average is

$$a_{3/2}/a_{1/2}=0.046\pm0.006 \quad (4)$$

which we will use as the "experimental" value of this ratio.

We then obtain the individual amplitudes from the observed $K_{2\pi}$ decay rates:

$$a_{1/2}=e^{i\phi}(3.84\pm0.01)\times10^{-7}\ \text{GeV}\ , \quad (5a)$$

$$a_{3/2}=e^{i\phi}(1.83\pm0.23)\times10^{-8}\ \text{GeV}\ . \quad (5b)$$

In Eq. (5) the overall phase ϕ is unknown, but from (4) $a_{3/2}/a_{1/2}>0$.

Next we review the current-algebra—PCAC analysis which will enable us to convert (5) to a more useful form for comparison with calculations. We make our computations in a four-quark model, rather than a six-quark model. The b and t quarks are so heavy that they can be decoupled from the calculation in the Appelquist-Carazzone sense,[9] leaving us with a four-quark theory in which the flavor-mixing angles and quark masses are to be regarded as effective parameters applicable only to the four-quark effective theory.[10] The weak left-handed current is then of the standard four-quark Cabibbo—Glashow-Iliopoulos-Maiani (GIM) form[11] (neglecting CP-violating effects):

$$j_\mu=\bar{u}\gamma_\mu^L(d\cos\theta_1+s\sin\theta_1)+\bar{c}\gamma_\mu^L(s\cos\theta_2-d\sin\theta_2)\ , \quad (6)$$

where $\theta_1\cong\theta_2\simeq\theta_C=13.2°$. As we shall see, (6) and the usual model of weak interactions lead to tadpole and current $\times$ current terms in $\mathscr{H}_w$, the effective weak Hamiltonian density. $\mathscr{H}_w$ may be written in terms of tadpole and current $\times$ current parts

$$\mathscr{H}_w=\mathscr{H}_{\text{tad}}+\mathscr{H}_{JJ}\ , \quad (7)$$

or in terms of $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ components,

$$\mathscr{H}_w=\mathscr{H}_{1/2}+\mathscr{H}_{3/2}\ . \quad (8)$$

$\mathscr{H}_{3/2}$ receives contributions only from $\mathscr{H}_{JJ}$, but $\mathscr{H}_{1/2}$ contains both $\mathscr{H}_{JJ}$ and $\mathscr{H}_{\text{tad}}$ pieces. Notwithstanding this we have the familiar current-algebra relation

$$\langle 0|[Q_5^3,[Q_5^3,\mathscr{H}_{1/2}]]|K\rangle=\tfrac{1}{4}\langle 0|\mathscr{H}_{1/2}|K\rangle\ , \quad (9)$$

which, applied to the $K_{2\pi}$ amplitude

$$M_{1/2}^{ij}=-\langle\pi^i\pi^j|\mathscr{H}_{1/2}|K\rangle\ , \quad (10)$$

gives the current-commutator term

$$M_{\text{cc},1/2}^{ij}=\frac{i}{f_\pi}\langle\pi^j|[Q_5^i,\mathscr{H}_{1/2}]|K\rangle=\frac{\delta^{ij}+i\epsilon^{ijk}\tau^k}{4f_\pi^2}\langle 0|\mathscr{H}_{1/2}|K\rangle\ . \quad (11)$$

Here $f_\pi\cong93$ MeV.

The $K\to2\pi$ process contains a rapidly varying pole illustrated by Fig. 1,[12] so the on-shell $\Delta I=\frac{1}{2}$ amplitude is given by

$$M_{\text{on shell}}^{(\frac{1}{2})}=M_{P(\text{on shell})}^{(\frac{1}{2})}-M_{P(\text{soft})}^{(\frac{1}{2})}+M_{\text{cc}}^{(\frac{1}{2})}\ , \quad (12)$$

where M_p is the pole term corresponding to Fig. 1. When evaluating $M_{P(\text{soft})}$ the four-momentum $k=q_i+q_j$ is conserved so that when $q_i\to0$, $k^2=q_j^2=m_k^2$. The $i\epsilon^{ijk}\tau^k$ term in (11) is then canceled by the rapidly varying pole term and we finally obtain

$$M_{1/2}^{ij}=\frac{\delta^{ij}}{2f_\pi^2}\left[1-\frac{m_\pi^2}{m_K^2}\right]\langle 0|\mathscr{H}_{1/2}|K\rangle\ . \quad (13)$$

We note that (13) satisfies the Gell-Mann—Cabibbo SU(3) null theorem[13] in that $M_{1/2}^{ij}$ vanishes in the SU(3) limit $m_\pi^2/m_K^2\to1$. Moreover $M_{1/2}^{ij}$ given in (13) is a factor of 2 larger than the amplitude found in the earlier current-algebra analyses[14] which averaged the results obtained by taking first one and then the other pion soft. This averaging procedure was adopted to eliminate "by hand" a term analogous to the $i\epsilon^{ijk}\tau^k$ term in (11) which depends on the order in which the pions are taken soft. It is no longer necessary when the rapidly varying pole term is taken into account.

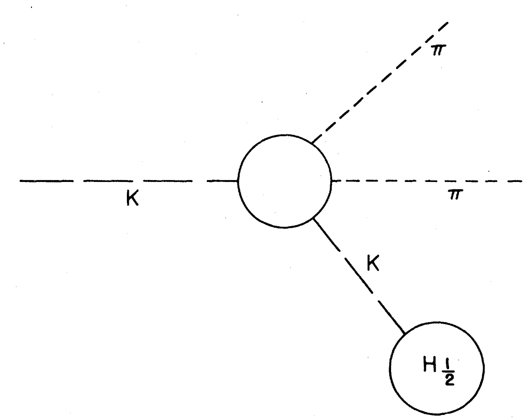

FIG. 1. The rapidly varying pole term in $K\to2\pi$ decay. The open circle represents the strong K-π scattering amplitude.

In Eq. (13) we specialize to the amplitudes for $\bar{K}^0 \to \pi^+\pi^-$ and $\bar{K}^0 \to \pi^0\pi^0$ (recall $\sqrt{2}\bar{K}_0 = K_L - K_S$)

$$M^{\bar{K}^0}_{+-,1/2} = M^{\bar{K}^0}_{00,1/2} = \frac{1}{2f_\pi^2}\left[1-\frac{m_\pi^2}{m_K^2}\right]\langle 0|\mathscr{H}_{1/2}|\bar{K}^0\rangle$$

$$= ia_{1/2} \qquad (14)$$

which gives from (5),

$$-i\langle 0|\mathscr{H}_{1/2}|\bar{K}^0\rangle = e^{i\phi}(5.10\pm0.02)\times 10^{-9}\ \text{GeV}^3 . \qquad (15)$$

As a further check on the magnitude of $\langle 0|\mathscr{H}_{1/2}|\bar{K}^0\rangle$ we may use a pion pole model of $K_L \to \gamma\gamma$, noting that η and η' pole contributions approximately cancel each other,[3] to obtain

$$|\langle \pi^0|\mathscr{H}_w|K_L\rangle| \cong (m_K^2 - m_\pi^2)\,|F_{K_L\gamma\gamma}/F_{\pi\gamma\gamma}|$$
$$\cong (3.0\pm0.2)\times 10^{-8}\ \text{GeV}^2 .$$

Current algebra and PCAC then give

$$|\langle 0|\mathscr{H}_w|\bar{K}^0\rangle| = \sqrt{2}f_\pi\,|\langle \pi^0|\mathscr{H}_w|K_L\rangle|$$
$$\cong (4.0\pm0.3)\times 10^{-9}\ \text{GeV}^3$$

in reasonable agreement with the earlier value (15), which we will use as it is the more accurate and is pure $\Delta I = \frac{1}{2}$.

III. THE NONDISAPPEARING TADPOLE

To leading order in the weak interaction, the one-loop contribution to the s-d quark tadpole transition is illustrated in Fig. 2. The left-handed nature of the Cabibbo-GIM current (6) ensures that there is no mass term in the tadpole. All that remains is a left-handed kinetic term proportional to $\not{p}_L = \not{p}(1-i\gamma_5)$, which may be expressed as[1,2]

$$\mathscr{H}_{\text{tad}} = b(p^2)(\bar{d}\not{p}_L s - \bar{s}\not{p}_L d) , \qquad (16a)$$

$$= b(p^2)(\tilde{S}^7 - \tilde{P}^7) , \qquad (16b)$$

where $\tilde{S}^i = i\bar{q}\lambda^i\not{p}q$ and $\tilde{P}^i = -\bar{q}\lambda^i\not{p}\gamma_5 q$ with $\gamma_5^2 = -1$. It is the opinion of many workers in this field that a series of papers by Weinberg[15] (which in turn rely on a result of Feinberg, Kabir, and Weinberg[16]) imply that self-energy tadpoles of the form given in Eq. (16) can always be transformed away. We agree that this is so for transitions involving quarks loosely bound in baryons or heavy mesons, but we do not believe that the Weinberg "theorem" applies to transitions such as $\langle 0|\mathscr{H}_{\text{tad}}|\bar{K}^0\rangle$ involving quarks tightly bound in light Nambu-Goldstone pseudoscalar mesons. We base the nondisappearance of the tadpole on two observations relating to QCD and QFD renormalizations.

Lemma 1: The immunity of the tadpole $\langle 0|\mathscr{H}_{\text{tad}}|\bar{K}^0\rangle$ to QCD renormalizations. For concreteness we consider the quark tadpole graph embedded in the matrix element $\langle 0|\mathscr{H}_{\text{tad}}|\bar{K}^0\rangle$. In quark-graph language $\langle 0|\mathscr{H}_{\text{tad}}|\bar{K}^0\rangle$ is represented in Fig. 3.

The vertex labeled γ_5 represents the pseudoscalar binding of quark-antiquark pairs in the 0^{-+} mesons. Figure 3 is completely analogous to the quark-graph representation of the matrix element $\langle 0|A_\mu|\pi\rangle = if_\pi q_\mu$ illustrated in Fig. 4. In Figs. 3 and 4 the heavy dots on the quark lines indicate that the quark lines are dressed by gluon self-energy loops but that gluon exchanges between the quark lines are not permitted. The reason for this prohibition is that such gluon exchanges are already included in the Bethe-Salpeter graphs which bind the $\bar{q}q$ pairs into the Nambu-Goldstone pseudoscalar K and π mesons as $q^2 \to 0$. This was shown by Nambu and Jona-Lasinio[17] in a four-fermion-interaction model, and their argument identifying the Bethe-Salpeter binding equation and the self-energy dressing equation was extended to chirally invariant non-Abelian gauge theories including QCD by Delbourgo and Scadron.[18] The graph of Fig. 4 has been analyzed to calculate the slope[3,19] $f'_\pi(q^2=0)$ and magnitude[20,21] $f_\pi(q^2=0)$ of f_π. Similar graphs involving dressed quarks with gluon exchanges between quark lines prohibited give an adequate account of the π and K charge radii,[22] and also reproduce[3] the Adler-Bell-Jackiw $\pi^0 \to 2\gamma$ anomaly.[23] In all such cases it suffices to use a constant πqq coupling of γ_5 type with $g_{\pi qq}$ given by the quark-level Goldberger-Treiman relation assuming $f_\pi > 0$,

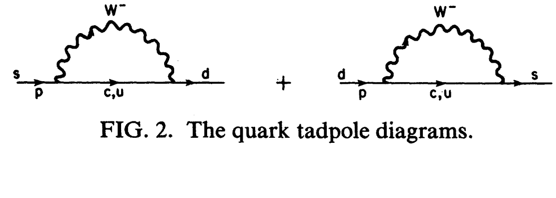

FIG. 2. The quark tadpole diagrams.

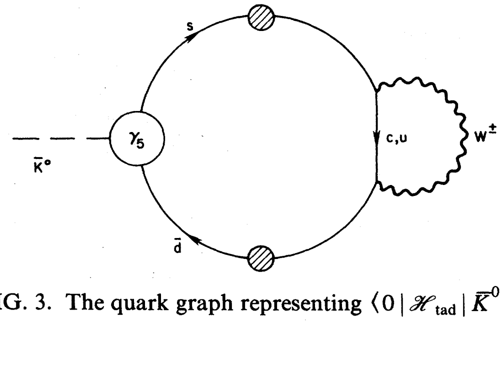

FIG. 3. The quark graph representing $\langle 0|\mathscr{H}_{\text{tad}}|\bar{K}^0\rangle$.

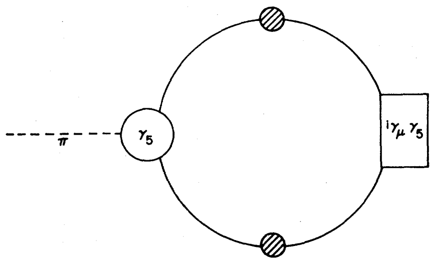

FIG. 4. The quark-graph representation of $\langle 0 | A_\mu | \pi \rangle$.

$$g_{\pi qq} = \frac{\hat{m}_{\text{con}}}{f_\pi} = 3.6 \ , \tag{17}$$

where $\hat{m}_{\text{con}}$ is the constituent mass of the nonstrange quarks (of order 340 MeV).

Now we return to Fig. 3, emphasizing the following:

(i) Gluon exchanges of the self-energy type are already included in the dressed quark lines with constituent quark masses.

(ii) Gluon exchanges between the quarks lines are already included in the Kqq vertex function because of the Nambu-Goldstone nature of the strongly bound kaon. Thus these are no further gluon exchanges allowed in evaluating Fig. 3, and no QCD counterterms can enter to renormalize the tadpole away.

We can reinforce this condition by looking at the complete $K \to 2\pi$ quark tadpole graph of Fig. 5(a). The absence of gluon exchanges between the quarks in the tadpole leg prevents us from deforming Fig. 5(a) into Fig. 5(b). However, Fig. 5(b) corresponds to the process $K \to \rho \to 2\pi$. (The net parity violation of the $K \to 2\pi$ process allows only scalar and vector intermediate particles, but the coupling to scalar particles is expected to be weak and the scalar pole is not rapidly varying.) This amplitude vanishes, so it is no surprise that our arguments do not grant immunity to the s-d tadpole in Fig. 5(b).

Lemma 2: The immunity of $\langle 0 | \mathscr{H}_{\text{tad}} | \bar{K}^0 \rangle$ *to QFD flavor transformations.* Now that we know QCD renormalizations leave the tadpole (16) intact, we must decide whether it will survive QFD flavor renormalizations.

We begin by reviewing the argument of Feinberg, Kabir, and Weinberg[16] on which the general opinion that tadpoles can be transformed away is based. One starts with a general quadratic Lagrangian density of the form

$$\mathscr{L}_0 = \bar{\psi}[A_1 + \gamma_5 A_2 + (B_1 + i\gamma_5 B_2)\not{p}]\psi \ , \tag{18}$$

where ψ are vectors in flavor space and spinors in Dirac space, and A_i, B_i are Hermitian matrices in

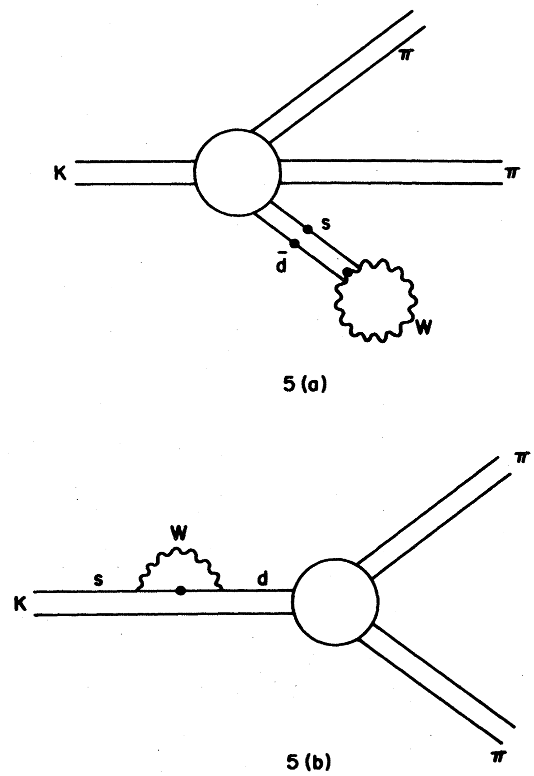

FIG. 5. Tadpole contributions to $K \to 2\pi$. (a) The rapidly varying tadpole graphs. (b) The nonrapidly varying tadpole contribution to $K \to \rho \to \pi\pi$.

flavor space. The Lagrangian $\mathscr{L}_0$ may arise to one-loop order in the QFD interactions—in fact (16) represents an off-diagonal term in the matrices B_1 and B_2. It is convenient to rewrite (18) in a representation in which

$$i\gamma_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and

$$\gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

so that

$$\mathscr{L}_0 = \psi^+[\mathscr{A} + \mathscr{B}(p_0 - i\gamma_5\sigma_i p_i)]\psi \ , \tag{19}$$

where

$$\mathscr{A} = \begin{bmatrix} A_1 & -iA_2 \\ iA_2 & -A_1 \end{bmatrix}$$

and

$$\mathscr{B} = \begin{bmatrix} B_1 & -B_2 \\ -B_2 & B_1 \end{bmatrix} \tag{20}$$

are Hermitian matrices. We now make a transformation of the fields ψ,

$$\psi \rightarrow \psi' = \mathscr{S}^{-1}\psi \ , \tag{21}$$

where $\mathscr{S}$ is a nonsingular (but in general *nonunitary*) matrix which can be shown to be of the form

$$\mathscr{S} = S_1 + i\gamma_5 S_2 = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_1 \end{bmatrix} . \tag{22}$$

Under this transformation (19) is converted to

$$\mathscr{L}'_0 = \psi'^{\dagger}[\mathscr{A}' + \mathscr{B}'(p_0 - i\gamma_5\sigma_i p_i)]\psi \ , \tag{23}$$

where $\mathscr{A}' = \mathscr{S}^{+}\mathscr{A}\mathscr{S}$ and $\mathscr{B}' = \mathscr{S}^{+}\mathscr{B}\mathscr{S}$. In the language of matrix theory $\mathscr{A}'$ and $\mathscr{B}'$ are Hermitian congruent (or conjunct) to $\mathscr{A}$ and $\mathscr{B}$. Now we rely on standard results in matrix theory which assure us that if $\mathscr{B}$ is positive definite then a matrix $\mathscr{S}$ can be found such that

$$\mathscr{B}' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and $\mathscr{A}'$ is diagonal,[24] of the form[25]

$$\mathscr{A}' = \mathrm{diag}(m_1, m_2, \ldots, m_n, -m_1, -m_2, \ldots, -m_n) \ . \tag{24}$$

A further calculation is necessary to demonstrate that $\mathscr{S}$ is of the form (22). Thus (23) can be written in the diagonal (nontadpole) form

$$\mathscr{L}'_0 = \sum_{i=1}^{n} \bar{\psi}_i(\not p + m_i)\psi_i \tag{25}$$

representing n free fermions of mass $m_i, \ldots, m_n$.[26]

Thus far we have given an *incomplete* statement of the Feinber-Kabir-Weinberg result, but this is the form which entered the folklore through Weinberg's 1973 papers.[15] Feinberg, Kabir, and Weinberg[16] completed their *theorem by demonstrating that $\mathscr{L}_0$ [Eq. (18)] and $\mathscr{L}'_0$ [Eq. (23)] give identical results for S-matrix elements involving* on-shell ψ_i states. The Green's functions generated by the Lagrangians $\mathscr{L}_0$ and $\mathscr{L}'_0$ are *not* in general identical because the transformation (21) is a general nonsingular transformation (i.e., it involves a wave-function renormalization as well as a rotation in flavor space) and not a unitary transformation. In fact, Ref. 16 provides an explicit example of a Green's function involving a transition between fermions which does not vanish in general but does vanish on the fermion mass shell.

Now we can see why the QFD transformation (21) applied to Fig. 5(a) or equivalently $\langle 0 | \mathscr{H}_{\rm tad} | \bar{K}^0 \rangle$ will not transform the matrix element to zero, even though $\mathscr{H}_{\rm tad}$ corresponds to off-diagonal B_1 and B_2 terms in Eq. (18). The reason is that *the quark lines in Figs. 3 and 5(a) are well off mass shell,* because of the Nambu-Goldstone nature of the kaon. This follows from $m_K = m_s + \hat{m} - E_B \simeq 0$, whence the binding energy E_B is of the order of the constituent quark masses forcing the quarks far off mass shell. That we can conjunctively transform (18) into (25) is irrelevant for the calculation of Fig. 3; (25) and (18) give identical results only on the quark mass shell. However, (18) represents the physically relevant theory, and it is useful to transform it to (25) only for the process involving quarks on their mass shell. We note that the quarks in the vector mesons and baryons are essentially on their (constituent) mass shell, so that the Feinberg-Kabir-Weinberg theorem does apply in that case and, e.g., $\langle \rho | \mathscr{H}_{\rm tad} | K \rangle$ and $\langle N | \mathscr{H}_{\rm tad} | B \rangle$ do vanish. On the other hand in the kaon the quarks are well off their mass shell and $\langle 0 | \mathscr{H}_{\rm tad} | K \rangle$ survives.

We conclude this section emphasizing that, contrary to the general opinion, the *s-d* quark tadpole of Fig. 2 cannot be transformed away when it is embedded in a quark loop which generates a Nambu-Goldstone boson. We therefore go on to compute $\langle 0 | \mathscr{H}_{\rm tad} | K \rangle$ from the loop of Fig. 4.

IV. THE QCD QUARK-LOOP CALCULATION OF $\langle 0 | \mathscr{H}_{\rm tad} | \bar{K}^0 \rangle$

We cannot transform the *s-d* quark tadpole of Fig. 2 away, so we calculate the magnitude of $\langle 0 | \mathscr{H}_{\rm tad} | \bar{K}^0 \rangle$ from the loop diagram of Fig. 3, to compare it to the experimental value (15). Using the form of the tadpole in (16), Fig. 3 gives

$$\langle 0 | \mathscr{H}_{\rm tad} | \bar{K}^0 \rangle = \frac{i(-3)(\sqrt{2}g_{Kqq})}{(2\pi)^4} \int d^4p \frac{b(p^2)\mathrm{Tr}[(\not p + \frac{1}{2}\not q + m_s)\gamma_5(\not p - \frac{1}{2}\not q + m_d)\not p(1-i\gamma_5)]}{[(p+\frac{1}{2}q)^2 - m_s{}^2][(p-\frac{1}{2}q)^2 - m_d{}^2]} \ . \tag{26}$$

The factor (-3) in Eq. (26) is the color factor of 3 for the loop, with a factor (-1) from the Feyman rules for fermion loops. In (26) the pseudoscalar quark-boson coupling constant g_{Kqq} is given its Goldberger-Treiman value with $f_K \simeq 1.2 f_\pi > 0$,

$$g_{Kqq} = \tfrac{1}{2}(m_s + m_d)_{\rm con}/f_K \simeq 3.8 \ , \tag{27}$$

where $(m_s)_{\rm con} = 510$ MeV and $(m_d)_{\rm con} = 340$ MeV are the strange and down constituent quark masses. Just as in the calculation associated with Fig. 4, relating f_π to the nonstrange-quark mass,[20,21] we take (27) as a constant, remove it from the integral and note in passing the near equality of g_{Kqq} [Eq. (27)] and $g_{\pi qq}$ [Eq. (17)]. We lose no generality in setting $q=0$ in Eq. (26). In that case the quark trace is proportional to $m_s - m_d = [m_s(p^2) - m_d(p^2)]_{\rm curr}$, as the dynamically generated part of the quark mass is flavor independent, but the current quark masses are not. In the asymptotically free deep-Euclidean limit of QCD we may write[27]

$$[m_s(p^2)-m_d(p^2)]_{\rm curr}=[m_s(M)-m_d(M)]_{\rm curr}\left[\frac{\ln(M^2/\Lambda^2)}{\ln(p^2/\Lambda^2)}\right]^d , \tag{28}$$

where $d=12(33-2n_f)^{-1}=0.48$, $\Lambda \cong 150$ MeV is the QCD renormalization invariant for four quark flavors,[28] and M is the fixed renormalization point which we take as $M \cong 2$ GeV for processes involving s and d quarks. Equation (26) then becomes

$$\langle 0 | \mathscr{H}_{\rm tad} | \bar{K}^0(0)\rangle = \frac{3\sqrt{2}g_{Kqq}}{4\pi^4}[m_s(M)-m_d(M)]_{\rm curr}\left[\ln\frac{M^2}{\Lambda^2}\right]^d \int_{|p|=\mu_0}^{|p|=M} d^4p \frac{b(p^2)p^2[\ln(p^2/\Lambda^2)]^{-d}}{[p^2-m_s^2(p^2)][p^2-m_d^2(p^2)]} . \tag{29}$$

The lower limit on this integral is chosen to be 1 GeV, where the QCD coupling and $m_{\rm curr}$ are expected to freeze out.[29]

In the chiral limit the right-hand side of (29) goes to zero as the current quark masses go to zero. This is consistent with the vanishing of the left-hand side as $\bar{K}^0$ becomes soft ($q \to 0$) because the tadpole algebra

$$[Q_5^i, \tilde{S}^j] = if^{ijk}\tilde{P}^k, \quad [Q_5^i, \tilde{P}^j] = if^{ijk}\tilde{S}^k \tag{30}$$

forces $\langle 0 | \tilde{P}_7 | \bar{K}^0(0)\rangle$ to zero. Thus we see that $\langle 0 | \mathscr{H}_{\rm tad} | \bar{K}^0(0)\rangle$ is a chiral-symmetry-breaking quantity and as such can be M dependent, just like $m_{\rm curr}(M)$. Unfortunately the integral in (29) is sensitively dependent on M.

To proceed further we must evaluate the tadpole strength parameter $b(p^2)$ in (16). To calculate Fig. 3 we work in the 't Hooft–Feynman gauge, in which the W-boson propagator is $(-g_{\mu\nu})(k^2-m_w^2)^{-1}$, and the Higgs contributions to the s-d tadpole vanish if we regard the current quark masses as generated by the Higgs vacuum expectation values. (Since in this case diagonalization of the quark mass matrix by a flavor-space rotation also diagonalizes the Higgs-boson–quark couplings in flavor space and the Higgs loop cannot then generate an s-d transition.) Then the four-quark current (6) gives

$$p_\mu b(p^2) = -\frac{iG_W m_W}{\sqrt{2}\,4\pi^2}\sin\theta_C\cos\theta_C \int d^4k \frac{[m_c^2(k^2)-m_u^2(k^2)](p-k)_\mu}{(k^2-m_W^2)[(p-k)^2-m_c^2][(p-k)^2-m_u^2]} . \tag{31}$$

Note that, as in the many examples of W-loop integrals considered by Gaillard and Lee,[30] the GIM mechanism ensures that the divergence in the individual contributions from the intermediate quarks is canceled when the terms are added up. When (31) is inserted into the quark loop integral (29) the ultraviolet cutoff ensures that $p^2 \ll m_W^2$ in (31), so that we may replace $b(p^2)$ by the constant

$$b \equiv b(0) \cong 0\frac{G_W \sin\theta_C\cos\theta_C}{\sqrt{2}\,4\pi^2} m_{c,\rm curr}{}^2(M')\left[\ln\frac{M'^2}{\Lambda^2}\right]^{2d} I_1(m_W) , \tag{32}$$

where

$$I_1(m_W) = \int_{\mu_0{}^2}^{\infty} \frac{x\,dx}{(x+1)^3\left[\ln\dfrac{m_W{}^2x}{\Lambda^2}\right]^{2d}} . \tag{33}$$

The integrand in (33) vanishes as $x \to 0$, so that $I_1(m_W)$ depends only very weakly on the lower cutoff μ_0, which is chosen as 1 GeV as before. The integral is also convergent at the upper limit and depends weakly on m_W. Explicitly, for $d \cong \frac{1}{2}$ we evaluate (33) numerically and find

$$I_1(m_W = 80 \text{ GeV}) = 0.038 , \tag{34a}$$

$$I_1(m_W = 250 \text{ GeV}) = 0.031 . \tag{34b}$$

Using (33), the tadpole matrix element (29) becomes

$$-i\langle 0|\mathscr{H}_{\rm tad}|\bar{K}^0\rangle \cong \frac{3}{4\pi^2}\sqrt{2}g_{Kqq}\frac{G_W}{4\pi^2\sqrt{2}}\sin\theta_C\cos\theta_C\, m_{c,\rm curr}{}^2(M')\left[\ln\frac{M'^2}{\Lambda^2}\right]^{2d}[m_s(M)-m_d(M)]_{\rm curr}I_1(m_W)I_2(M) \tag{35}$$

with a new integral which must be evaluated numerically:

$$I_2(M)=\int_{\mu_0^2}^{M^2}dy\left[\frac{\ln(M^2/\Lambda^2)}{\ln(y/\Lambda^2)}\right]^d$$

$$\cong 3.2\ {\rm GeV}^2 \text{ at } M=2\ {\rm GeV}\ . \tag{36}$$

[Note that $I_2(M)\sim M^2$ for M in this region.] The renormalization point M' should be chosen as being suitable for processes involving c quarks. We choose $M'=3$ GeV, near the Ψ mass, where[27]

$$m_c(M'=2m_c)=m_{c,\rm con}\cong m_{c,\rm curr}\simeq 1.5\ {\rm GeV}\ . \tag{37}$$

To complete the evaluation of (35) we need $[m_s(M)-m_d(M)]_{\rm curr}$. In the strong PCAC scheme of perturbative chiral-symmetry breaking[31] one usually takes $(m_s-m_d)_{\rm curr}^{\rm SPCAC}\simeq 140$ MeV, while in the neutral PCAC scheme[3,32] one has $(m_s-m_d)_{\rm curr}^{\rm NPCAC}\cong 240$ MeV. Thus we have for the tadpole matrix element

$$-i\langle 0|\mathscr{H}_{\rm tad}|\bar{K}^0\rangle \cong \begin{cases}4\times 10^{-9}\ {\rm GeV}^3, & \text{SPCAC}\ ,\\ 7\times 10^{-9}\ {\rm GeV}^3, & \text{NPCAC}\ .\end{cases} \tag{38}$$

The sensitivity of our final result to the choice of M precludes using (38) and the experimental value (15) to distinguish the two schemes of chiral-symmetry breaking. However, the fact that the two values in (38) span the experimental value of $|\langle 0|\mathscr{H}_{1/2}|\bar{K}^0\rangle|$ is a strong indication to us that the tadpole Hamiltonian $\mathscr{H}_{\rm tad}$ of Fig. 2 is indeed the dominant contribution to the $\Delta I=\frac{1}{2}$ weak Hamiltonian, and provides the explanation of the $\Delta I=\frac{1}{2}$ rule in $K\to 2\pi$ decays.

V. THE SIGN OF $a_{1/2}/a_{3/2}$

As a further indication that the s-d tadpole dominates the $K_{2\pi}$ decays we calculate $a_{1/2}/a_{3/2}$ using (35) for $a_{1/2}$ and a simple vacuum saturation argument for $a_{3/2}$. First of all we note from Eqs. (14) and (35) that the tadpole amplitude gives

$$a_{1/2}=\frac{3}{2f_\pi{}^2}\left[1-\frac{m_\pi{}^2}{m_K{}^2}\right]\frac{(m_s+m_d)_{\rm con}}{2f_K}\frac{G_W}{(2\pi)^4}\sin\theta_C\cos\theta_C\, m_{c,\rm curr}{}^2\left[\ln\frac{M'^2}{\Lambda^2}\right]^{2d}(m_s-m_d)_{\rm curr}I_1I_2 \tag{39}$$

which for θ_C, $G_W\propto g_W{}^2/m_W{}^2$, and f_K taken positive is clearly positive:

$$a_{1/2}>0\ . \tag{40}$$

We evaluate $a_{3/2}$ by vacuum saturation of the $K^+\to\pi^+\pi^0$ amplitude, using Eqs. (5c) and (14):

$$a_{3/2}=-i\langle\pi^+\pi^0|\mathscr{H}_{JJ}|K^+\rangle$$

$$=-i\frac{G_W}{2\sqrt{2}}\sin\theta_C\cos\theta_C\langle\pi^0|J_\mu^{4-i5}|K^+\rangle\langle\pi^+|J^{1+i2,\mu}|0\rangle$$

$$\cong\frac{G_W}{2\sqrt{2}}\sin\theta_C\cos\theta_C f_\pi(m_K{}^2-m_\pi{}^2) \tag{41}$$

$$\cong 1.9\times 10^{-8}\ {\rm GeV}>0\ . \tag{42}$$

In this evaluation we have ignored QCD corrections[33] and perhaps should not take the remarkable agreement of (42) with the observed magnitude of $a_{3/2}$ in Eq. (5b) too seriously. But we feel confident of the calculated sign of $a_{3/2}$, and with the prediction

$$a_{1/2}/a_{3/2}>0 \tag{43}$$

which is in agreement with experiments (2)—(5).

VI. CONCLUSION

We have shown that the Nambu-Goldstone strong binding nature of the kaon plays a critical role in determining its weak decays, in that the s-d quark tadpole cannot be transformed away by QCD or QFD renormalization. The tadpole matrix element $\langle 0|\mathscr{H}_{\rm tad}|\bar{K}^0\rangle$ depends on the QCD renormalization point M used in its evaluation, and on the current-quark-mass difference $m_s - m_d$. However, for $M \sim 2$ GeV the strong PCAC and neutral PCAC values of $m_s - m_d$ lead to values of $\langle 0 | \mathscr{H}_{\text{tad}} | \bar{K}^0 \rangle$ which span the experimental value of $\langle 0 | \mathscr{H}_{1/2} | \bar{K}^0 \rangle$, suggesting that the s-d tadpole provides a natural explanation of the $\Delta I = \frac{1}{2}$ rule in $K_{2\pi}$ decays. Even the relative sign of $a_{1/2}$ as found from the quark tadpole and $a_{3/2}$ obtained from the $\mathscr{H}_{JJ}$ vacuum saturation supports this conclusion. Since $K \to 3\pi$ decays are quantitatively related to $K \to 2\pi$ decays in the current-algebra—PCAC program, the s-d tadpole provides an explanation of these decays as well. This includes the positive $\Delta I = \frac{1}{2} / \Delta I = \frac{3}{2}$ enhancement of (21) as both magnitude and relative sign can also be extracted from $K_{3\pi}$ decays.[3]

We do not claim that the same diagrams explain the $\Delta I = \frac{1}{2}$ rule in K and hyperon decays. In our opinion the $\Delta I = \frac{1}{2}$ rule has its origin in the QCD dynamics of the decaying particle. In the case of K mesons the dynamics is the strong binding of $\bar{q}q$ into a Nambu-Goldstone boson, and we have argued that this leads, via the s-d tadpole, to a quantitative understanding of the $\Delta I = \frac{1}{2}$ rule.

In the hyperon decays the QCD dynamics are those of loosely bound constituent quarks which give rise to the observed SU(6) symmetry. A satisfactory explanation of the $\Delta I = \frac{1}{2}$ rule for hyperon decays already exists.[3] In particular the current-algebra—PCAC program reduces the 14 hyperon amplitudes to only three fitted parameters: the large value of the $(\Delta I = \frac{1}{2})/(\Delta I = \frac{3}{2})$ ratio, the scale of $\langle B' | \mathscr{H}_{JJ}(\Delta I = \frac{1}{2}) | B \rangle$, and the weak octet d/f ratio.[3,34]

The *nonrelativistic SU(6) constituent quark model* then explains these last three parameters:

(i) Fierz reshuffling and the hyperon product SU(6) wave functions manifests the $\Delta I = \frac{1}{2}$ rule.[4]

(ii) The scale $\langle B' | \mathscr{H}_{JJ}(\Delta I = \frac{1}{2}) | B \rangle$ is set by $W^{\pm}$ scattering graphs.[5,6]

(iii) The ratio of $d/f \cong -1$ also follows from the $W^{\pm}$ scattering graphs.[6]

Thus the understanding of the $\Delta I = \frac{1}{2}$ rule for strange-particle decays does not follow from a universal mechanism, but from a careful appreciation of the QCD dynamics which is qualitatively different for kaons and hyperons, but nonetheless in each case leads to the observed $\Delta I = \frac{1}{2}$ enhancement.

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[25]That $\mathscr{A}'$ has this particular form does not follow from the general theorem, but from the observation that for $\mathscr{A}$ and $\mathscr{B}$ given by (20), if $\det(\mathscr{A}-\lambda\mathscr{B})=0$ then $\det(\mathscr{A}+\lambda\mathscr{B})=0$, so that both $\pm\lambda$ are characteristic values of the matrix pencil $\mathscr{A}+\lambda\mathscr{B}$.

[26]We remark in passing that if we regard (18) as produced by perturbation from a theory with a bare Lagrangian of the form (25) then we can write $\mathscr{L}_0=\mathrm{diag}(\mu_1,\mu_2,\ldots,\mu_n,-\mu_1,-\mu_2,\ldots,-\mu_n)+\Phi(\alpha_1,\ldots,\alpha_k)$ where $\Phi(\alpha_1,\ldots,\alpha_k)$ is a continuous matrix function of the coupling constants $\alpha_1,\ldots,\alpha_k$ which vanishes as $(\alpha_1,\ldots,\alpha_k)\to(0,\ldots,0)$. It then follows from Ref. 24 (p. 309) that if Φ is nonsingular for all values of $(\alpha_1,\ldots,\alpha_k)$ on a continuous curve joining the origin in coupling-constant space $(0,\ldots,0)$ to the physical point $(\alpha_1^{(0)},\ldots,\alpha_k^{(0)})$ then the set $m_1,\ldots,m_n$ contains the same number of zeros as $\mu_1,\ldots,\mu_n$; the perturbation cannot introduce additional zero-mass particles under these circumstances. The relevant theorem is "if under a continuous change of the elements the rank of an Hermitian matrix is unchanged then its signature is unchanged"—the discussion of p. 309 applying to quadratic forms (symmetric matrices) applies *mutatis mutandis* to the Hermitian case.

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