Meson, baryon, and glueball masses in the MIT bag model

C. E. Carlson
College of William and Mary, Williamsburg, Virginia 23185

T. H. Hansson
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

C. Peterson
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305 and NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark
(Received 30 August 1982; revised manuscript received 20 December 1982)

We present a consistent and unified study of the spectrum of low-lying mesons, baryons, and glueballs using the MIT bag model, incorporating several improvements in the model. We correct for the center-of-mass motion, use a running coupling $\alpha_s(R)$, include self-energy terms for quarks and gluons confined in a cavity, and get the bag constant from a model of the QCD vacuum. Our fit to the meson and baryon spectrum, including the pion, is good and predictions for the glueball spectrum are given.

I. INTRODUCTION

The aim of this paper is to give a description of the low-lying meson, baryon, and glueball spectrum in the framework of the MIT bag model.[1,2] We differ from earlier works[3-6] on this subject in that we use the same type of approximations and the same parameters for the glueballs as for the mesons and baryons. Also we incorporate several improvements of the model, some new and some partially treated in earlier work.

Our results are given in Sec. IV. A short summary follows here.

We obtain a good description of the low-lying meson and baryon spectrum, including a light pion. Since our model includes several new elements, the parameters are not the same as in earlier bag calculations. We use a running coupling constant, and thus have Λ_{QCD} as one parameter. The bag constant B is computed using a model for the QCD vacuum.[7] No "Casimir" or "zero-point" energy[2] $-Z_0/R$ is included, but instead we have self-energy contributions $(\alpha_s/R)e_i$ for both quarks and gluons. The constants e_i are calculable in cavity perturbation theory, but are in this work kept as free parameters. We also correct for the effects of the center-of-mass (c.m.) motion in a static cavity. These corrections are the main reason for the very light pion.[8] For the ("current") quark masses we take $m_u = m_d = 0$ but keep m_s as a free parameter. The numerical result is shown in Table III.

Proceeding to the glueball sector, all parameters for states of type $(\text{TE})^n$ are fixed from the meson and baryon spectrum. Hence we expect some reliability for mass predictions of these states. The scalar $(\text{TE})^2$ state plays a special role in our scheme since it is assumed to mix strongly with the vacuum. The remaining four $(\text{TE})^2$ and $(\text{TE})^3$ states lie between 1.8 and 2.9 GeV; see Table IV. The other states in Table IV involve (TM) gluons and we have made the simplest assumption about the self-energy parameters and put $e_{\text{TM}} = e_{\text{TE}}$. This gives a relatively light pseudoscalar (TE)(TM) glueball at 0.7–0.9 GeV. In Sec. IV we briefly discuss the relevance of this result for the η-η' mass splitting, and the possible identification of $\iota(1440)$ (Ref. 9) as a glueball.

Before describing the details of our model, we should place our work in context by remarking upon other hadron models.

Have the recent rapid developments in numerical (lattice Monte Carlo) calculations of the hadron spectrum directly from the QCD Lagrangian made bag- and potential-model phenomenology obsolete? For two reasons we believe that the answer is no. First, the calculation of higher excited states, as well as quantities directly related to the shape of the quark and gluon wave functions, such as radiative widths, might be difficult to perform using present numerical techniques. Such problems may be more readily approached in a constituent space-time model such as the bag. Second, the phenomenological hadron models provide a physical picture which we believe has a value regardless of how precise the computer-generated mass estimates become.

One may also ask why we choose the bag-model approach instead of the potential models which are so popular in baryon and charmonium spectroscopy. One reason is that it is more clear how to use the bag model in new situations. We can give the QCD vacuum and glueballs a unified treatment along with mesons and baryons. Potential models have been successful in describing the spectrum of baryons made from light quarks. However, most treatments do so by introducing "constituent masses" and using nonrelativistic kinematics. The bag, using relativistic kinematics for the constituents, deals easily with light quarks and extends straightforwardly to gluons. Also, it is not obvious what potential to use for glueballs, while in the bag model, the bag constant is the same regardless of whether we discuss baryons, mesons, or glueballs.

The paper is organized as follows. Section II summarizes the quantum numbers of the low-lying mesons, baryons, and glueballs. Section III describes our version of the MIT bag model, and Sec. IV presents numerical results for the spectrum and also contains a discussion and conclusions.

II. MESONS, BARYONS, AND GLUEBALLS IN THE BAG

Let us now list the states to be considered and establish our notation. We will use the "static-spherical-cavity approximation" to the MIT bag model.[2] Hadrons are made by combining quark and gluon cavity modes into color-singlet spin eigenstates. We consider mesons and baryons made from quarks in their lowest mode, and glueballs made from TE and TM gluons in their lowest respective modes[3]:

$$\begin{aligned}
&\text{quarks: } {}^1S_{1/2}\,, && && x_q=2.043\,,\\
&\text{gluons: TE}\,, && J^P=1^+\,, && x_E=2.744\,,\\
&\qquad\quad\ \text{TM}\,, && J^P=1^-\,, && x_M=4.493\,.
\end{aligned}$$

The energy of each mode is given by

$$\omega_i=[x_i{}^2+(m_iR)^2]^{1/2}/R$$

where R is the bag radius and m_i the mass of constituent i. The x_q given above is for massless quarks; for massive quarks, x_q is given in Ref. 2.

From these modes, the following low-lying hadron states can be constructed:

$$\begin{aligned}
&\text{mesons} && (q\bar q): && J^{PC}=0^{-+},1^{--}\,;\\
&\text{baryons} && (qqq): && J^{P}=\tfrac{1}{2}^{+},\tfrac{3}{2}^{+}\,;\\
&\text{glueballs} && (\text{TE})^2: && J^{PC}=0^{++},2^{++}\,,\\
& && (\text{TM})^2: && J^{PC}=0^{++},2^{++}\,,\\
& && (\text{TE})(\text{TM}): && J^{PC}=0^{-+},2^{-+}\,,\\
& && (\text{TE})^3: && J^{PC}=0^{++},1^{+-},3^{+-}\,.
\end{aligned}$$

Other multigluon states and states such as $\bar q^2 q^2$ (Ref. 10) or $\bar q q g$ (Ref. 11) will not be considered here.

III. THE IMPROVED BAG MODEL

There have been a number of developments and improvements of the MIT bag model after the original works,[1,2] and we single out for discussion a partial resolution of the center-of-mass motion problem, the perturbative calculations of mass shifts to $O(\alpha_S)$, and the relationship between the QCD vacuum and the bag constant B. We also discuss the stability of glueballs and conclude by summarizing the formulas used in our calculations.

A. The center-of-mass momentum problem

Almost all calculations using the bag model have been done in the "static-spherical-cavity approximation." This means that a spherical bag is frozen in space, and is not an eigenstate of total momentum. A prescription must be given to relate the eigenvalues of the static bag model Hamiltonian H to the hadron masses. This is done in two steps.

First we give a relation between the bag-model Hamiltonian H and the true Hamiltonian $H_{\text{true}}{}^2=P^2+m^2$. We choose

$$H^2=P^2+m^2\,. \tag{1}$$

So for static-bag eigenstates $|E\rangle$,

$$H|E\rangle=E|E\rangle\,, \tag{2}$$

one has

$$E^2=\langle E|P^2|E\rangle+m^2\,. \tag{3}$$

Note that this prescription is not unique. With the choice

$$H=(P^2+m^2)^{1/2}$$

one instead obtains

$$E=\langle E|(P^2+m^2)^{1/2}|E\rangle \tag{4}$$

which is *not* equivalent to Eq. (3) since $|E\rangle$ is not an eigenstate of P^2. This last prescription, which was used in Ref. 8, is somewhat more complicated to implement but gives essentially the same results as Eq. (3).

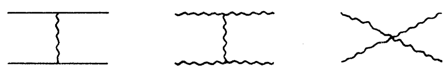

FIG. 1. $O(\alpha_s)$ interaction graphs. Solid lines are quarks and wiggly lines gluons.

The second step is to estimate the matrix elements $\langle E \mid P^2 \mid E \rangle$. For this, one must know the total momentum distribution in the bag. We will take a simple approach and assume that all the momentum is carried by the valence particles. Furthermore, if we neglect all interactions and treat these particles as independent we get the estimate

$$\langle E \mid P^2 \mid E \rangle = \sum_i n_i \left[\frac{x_i}{R} \right]^2 , \tag{5}$$

where n_i is the number of constituents of type i.

Although this prescription only provides a partial resolution of the problem concerning the c.m. energy, it has the virtues of being very simple and not adding any new free parameters. We believe that more refined considerations would at this stage be meaningless because of the overall crudeness of the model.

B. Cavity perturbation theory

Understanding the level splitting in heavy quarkonium systems in terms of potentials generated from one-gluon exchange is one of the great successes of QCD-based phenomenology. The bag model allows us to extend these ideas to hadrons composed of gluons or light quarks or both. Here the nonrelativistic potential description breaks down, but the effect of gluon exchange can still be calculated by evaluating graphs like those in Fig. 1, where the external legs represent cavity wave functions. Tree diagrams like those in Fig. 1 can be calculated, albeit with more labor than for their free-space counterparts.[2,5,12–14]

Techniques are also being developed for handling loop diagrams.[15,16]

In our mass calculations we will include energy shifts due to the lowest-order color interactions. Diagrams 1 give rise to color-spin interactions, while the loop diagrams in Fig. 2 contribute only spin-independent terms. In the notation of Ref. 14 the total $O(\alpha_s)$ energy shift for color-singlet states is given by

FIG. 2. $O(\alpha_s)$ self-energy graphs.

TABLE I. The coefficients a, b, and c in Eq. (7). For the quark case we only quote the $m_q=0$ values. The expressions for $m_q \neq 0$ can be found in Ref. 2.

	a	b	c
$(\mathrm{TE})^2$	0.263	−0.041	0.164
$(\mathrm{TM})^2$	0.247	−0.007	0.028
(TE)(TM)	0.255	−0.017	−0.083
qq, $q\bar{q}$	0.177	0	0

$$\Delta E = \sum_{i \neq j} \Delta E_{ij} + \sum_i \Delta E_i , \tag{6}$$

where

$$\Delta E_{ij} = -\frac{\alpha_s}{R} \langle \mid \vec{\Lambda}_i \cdot \vec{\Lambda}_j (a_{ij} \vec{S}_i \cdot \vec{S}_j + b_{ij} T_{ij} + c_{ij} I_{ij}) + d_{ij} \mathscr{P}_{ij} \mid \rangle , \tag{7}$$

$$\Delta E_i = -\frac{\alpha_s}{R} {\Lambda_i}^2 e_i . \tag{8}$$

Here α_s is the strong coupling constant, $\vec{\Lambda}_i$ and $\vec{S}_i$ are generators of color and spin, respectively, and $\mathscr{P}$ is the projection operator on the color-octet spin-one state. I_{ij} is the unit operator in spin space, and

$$T_{ij} = 2[(\vec{S}_i \cdot \vec{S}_j)^2 - 1] + \vec{S}_i \cdot \vec{S}_j .$$

The values of the constants a, b, and c, for relevant constituents, are listed in Table I, and $d_{(\mathrm{TE})^2} = -0.529$.

Since the calculations of the loop diagrams in Fig. 2 have either not been attempted or not been confirmed,[15] we shall treat the constants e_i, $i = q$,TE,TM, as free parameters.[17]

With massless quarks QCD is a parameter-free theory where the coupling strength is traded for a scale parameter Λ. This happens also in the confined theory since the short-distance properties are not affected by the bag boundary. Thus the effect of many higher-order diagrams, such as those in Fig. 3, will be to change the constant lowest-order α_s to a running $\alpha_s(\Lambda R)$. Because of asymptotic freedom, and the universality of the lowest-order β function, one obtains, for small bag radii,

$$\alpha_s(R) \underset{R \ll \Lambda_{\overline{\mathrm{MS}}}^{-1}}{\approx} \frac{2\pi}{9} \frac{1}{\ln 1/\Lambda R} , \tag{9}$$

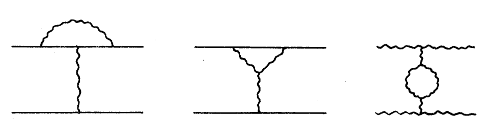

FIG. 3. Examples of $O(\alpha_s{}^2)$ vertex and propagator corrections.

where $\Lambda_{\overline{\rm MS}} \simeq 0.1-0.2$ GeV ($\overline{\rm MS}$ refers to the modified minimal-subtraction scheme). The scale Λ cannot be related to, e.g., $\Lambda_{\overline{\rm MS}}$ before the subtraction and renormalization scheme is specified. In our calculations we keep Λ a free parameter but expect $\Lambda \simeq \Lambda_{\overline{\rm MS}}$. (This is very different from the lattice calculations where $\Lambda_{\overline{\rm MS}} \simeq 30\Lambda_{\rm lattice}$.[18] In the lattice case the short-distance behavior of the theory is changed, while in the bag it is the long-distance behavior that is altered.) For real hadrons,

$$R \approx 5 \text{ GeV}^{-1} \simeq (0.5-1)\Lambda_{\overline{\rm MS}}^{-1}$$

so Eq. (9) cannot be used as it stands. Without a full calculation we do not know the behavior of $\alpha(R)$ for large R so we must resort to a parametrization consistent with Eq. (9). We shall use[7]

$$\alpha_s(R) = \frac{2\pi n}{9} \frac{1}{\ln[1+1/(\Lambda R)^n]} , \tag{10}$$

where n is a positive parameter. There is no strong reason for choosing this particular form (see, e.g., Ref. 8 for another choice) but we have checked that our results are rather independent of the parametrization of $\alpha_s(R)$.

C. The QCD vacuum and the origin of B

In the original version of the bag model, the vacuum energy density, or bag constant, was a free parameter determined from data. Recently, K. Johnson and two of the present authors (T.H.H. and C.P.) proposed a model for the QCD vacuum which allows B to be calculated given $\alpha_s(R)$ and the constant $e_{\rm TE}$ (i.e., the self-energy of the lowest TE gluon mode). The basic idea is that the vacuum is filled by 0^{++} $({\rm TE})^2$ glueballs which form a negative energy condensate. For details we refer to Ref. 7 and here only quote the expression for B:

$$B = \frac{3}{8\pi R_0{}^3}(-m^2)^{1/2} , \tag{11}$$

where $m^2 < 0$ is the mass squared, and R_0 the radii of the condensed (tachyonic) glueballs, as calculated using bag-model wave functions.

D. The QCD vacuum and the stability of glueballs

There is a long-standing question whether or not the bag model allows for spherical glueballs. Let us briefly state the problem. In the original bag model, the gluon fields satisfy not only the linear boundary condition $n_i F^{i\mu} = 0$ ($\vec{n}$ is the normal to the bag surface) which forces all states to be color singlets, but also the quadratic condition

$$B = \tfrac{1}{2}(\vec{E}^2 - \vec{B}^2) + \tfrac{1}{2}\hat{n}\cdot\nabla(\bar{q}q)\,|_{\rm on\ surface} . \tag{12}$$

This equation expresses pressure balance and ensures the stability of the bag surface. For $j=\frac{1}{2}$ quark modes, the pressure is spherically symmetric, and Eq. (12) can be implemented simply by minimizing the energy with respect to R. For gluons the situation is different. One can show that the pressure $\frac{1}{2}(\vec{E}^2 - \vec{B}^2)$ cannot be both spherically symmetric and everywhere >0 for any classical gluon field.[19] We might thus be led to believe that there are no stable spherical glueballs (and even worse, any deformed shape with the same topology is still unstable[19]). This has led to speculations about glue bags with toroidal shape.[20]

However, the above argument is classical, while a quantum-mechanical evaluation of $\langle E^2 - B^2 \rangle$ for $J=0$ states gives a spherically symmetric and positive pressure.[21] Unlike the $j=\frac{1}{2}$ quark modes, however, the individual gluon modes do not exert a uniform pressure.

For $J \neq 0$, we may still suppose that the states are spherical, or nearly so, within the context of the QCD vacuum model referred to in the previous section.[4,7] Since the vacuum is supposed to be filled by *spherical* two-gluon glueballs, the lowest excited states are formed by destroying one such localized state. This stabilizes the spherical shape, and the pressure balance condition should not be imposed. The essence of the argument is that not only the size ($R \sim B^{-1/4}$) but also the shape of hadrons is "built" into the vacuum. In this work we shall assume that spherical glueballs do exist and treat them in the same way as mesons and baryons without imposing Eq. (12).

E. The mass formula

For the convenience of the reader, we will now summarize the formulas used in our calculations. The mass m of a hadron is given by

$$m = \left[E^2 - \sum_i n_i \left[\frac{x_i}{R} \right]^2 \right]^{1/2} , \tag{13}$$

$$E = \sum_i n_i \omega_i + \tfrac{4}{3}\pi R^3 B + \Delta E , \tag{14}$$

where

$$\omega_i = \frac{1}{R}[x_i{}^2 + (Rm_q)^2]^{1/2} , \tag{15}$$

$$B = \frac{3}{8\pi R_0{}^4}\{2x_{\rm TE}{}^2 - [2x_{\rm TE} - 6\alpha_s(R_0)(a_{EE} + 2b_{EE} + e_{\rm TE})]^2\}^{1/2} , \tag{16}$$

$$\Delta E = -\frac{\alpha_s(R)}{R}\text{ const },\tag{17}$$

$$\alpha_s(R) = \frac{2\pi n}{9}\frac{1}{\ln[1+1/(\Lambda R)^n]}\ .\tag{18}$$

The value of R_0 which determines the bag constant is obtained by minimizing the vacuum energy (i.e., maximizing B) for a given Λ and e_{TE}. The resulting value for B is then used to determine the hadron masses.

The (state-dependent) constant in Eq. (17) is a linear combination of the a, b, c, d, and e's with appropriate color and spin factors. (These factors are easily computed and are also listed in Refs. 2 and 14.)

The free parameters are Λ, e_{TE}, e_{TM}, e_q, and m_s. We put $e_s = e_u = e_d = e_q$ and assume that any actual difference can be effectively incorporated into the strange-quark mass m_s.

We do not include any state-independent "Casimir" term of the form $-Z_0/R$. Such a term adds an extra parameter, and is not needed to obtain a good fit to observed meson and baryon masses. It might happen, however, that when the self-energies are calculated *ab initio* the "Casimir" term must be reintroduced.

The constant n is not included among the free parameters. Our attitude is that the scheme is trustworthy only if the mass fits are stable when n varies. Such a stability is explicitly shown in the next section.

Our parametrization can be compared with the one of the early work in Ref. 2. There the free parameters were B, Z_0, α_s, and m_s, so we have traded the first three for Λ, e_{TE}, and e_q. In the original work B, Z_0, and α_s were all free parameters. In our case there is a possibility of computing both e_{TE} and e_q, and, at least in principle, of relating Λ to $\Lambda_{\overline{MS}}$.

IV. HADRON MASSES

We now compute the hadron spectrum. First we deal with the quarkic sector, and fix our parameters by fitting the masses of ρ, p, Δ, and Ω^-. Then we predict the masses of the remaining S-wave mesons and baryons. Finally predictions for the glueball spectrum are made.

TABLE II. Relations between the parameters used in this work and in Ref. 2. The bag constant B is related to e_{TE} and Λ via Eq. (16), and by using a running coupling constant, α_s can be replaced by Λ. The quark self-energy parameter e_q replaces the "Casimir" parameter Z_0.

This work		Ref. 2
e_{TE}	$\longleftrightarrow$	B
Λ	$\longleftrightarrow$	α_s (const)
e_q	$\longleftrightarrow$	Z_0

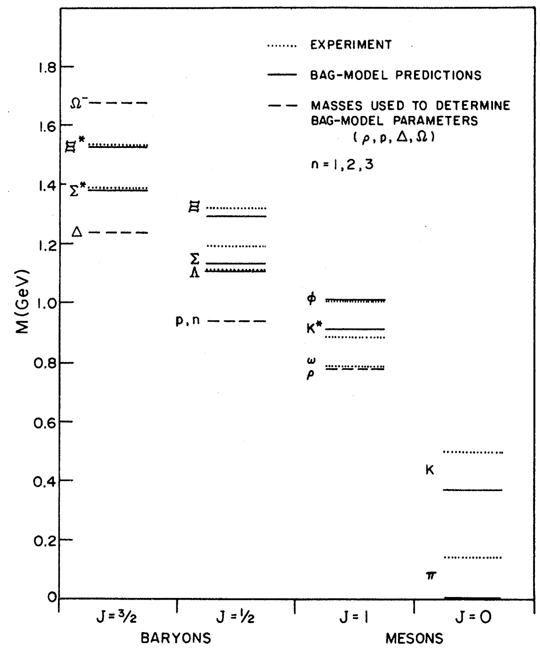

FIG. 4. Meson and baryon masses for $n=2$ with $m_{u,d}=0$. The masses of p, Δ, ρ, and Ω^- were used to determine the parameters, all other masses are predicted.

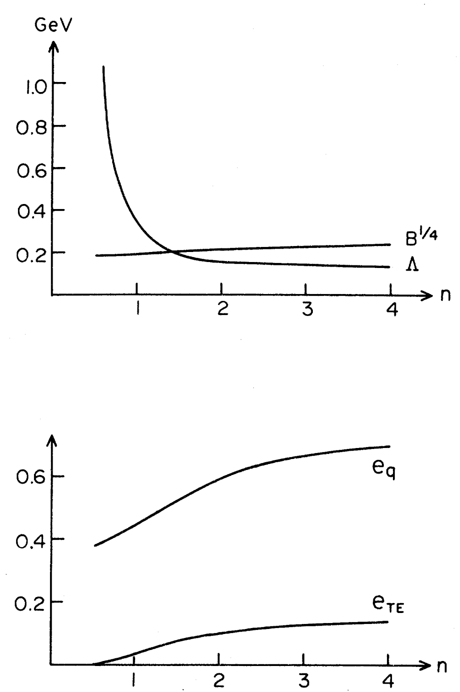

FIG. 5. n dependence of the parameters.

A. The quarkic sector

As discussed above, our approach for the $\bar{q}q$ and qqq states has several differences with earlier calculations[2]:

(1) We minimize $m^2(R)=E^2(R)-\langle P^2(R)\rangle$ rather than $E(R)$.

(2) We use the running coupling constant $\alpha_s(R)$ given by Eq. (18).

(3) We do not include any state-independent term $-Z_0/R$, but instead self-energies of the type $-e_i\Lambda_i^2\alpha_s/R$. These procedures are different even in the quarkic sector since in our scheme mesons and baryons receive different contributions

$$-2\times\tfrac{4}{3}e_q\alpha_s/R$$

and

$$-3\times\tfrac{4}{3}e_q\alpha_s/R\ .$$

(4) The model for the QCD vacuum described in Sec. III C gives the bag constant B as a function of Λ and $e_{\rm TE}$. This means that fitting B to the $(\bar{q}q)$ and (qqq) states in reality involves a determination of $e_{\rm TE}$ since Λ is essentially fixed by the spin splittings.

The points (2)—(4) are illustrated in Table II which shows the connection between our parameters and those of Ref. 2.

We have carried out the calculations for a wide range of n. In the quarkic sector the quality of the fit is very insensitive to n as seen in Fig. 4. In Table III we give the numerical values for $n=2$. The masses of the ρ, p, and Δ are used to obtain the parameters $e_{\rm TE}$, Λ, and e_q. The variation of these parameters with n is shown in Fig. 5. For $n\geq 2$ they are quite stable. Finally, the strange-quark mass m_s is determined from the mass of Ω^-.

A few comments on our results in Table III:

(i) The pion and kaon masses. The pion mass comes out $m_\pi{}^2\lesssim 0$ which signals a small overestimate of the c.m.-motion correction. If one wants, this can easily be remedied by changing Eq. (3) to

$$m^2=E^2-a\langle P^2\rangle \qquad (19)$$

and choosing $\alpha\leq 1$ (e.g., $\alpha=0.95$ for $n=2$) to get $m_\pi{}^2=0$. All strange hadrons come out right except the kaon. Making the above adjustment to get $m_\pi{}^2=0$ pushes m_K up to an appropriate value (e.g., $\alpha=0.95$ and $n=2$ gives $m_K=468$ MeV).

As is well known, a spontaneous breakdown of chiral symmetry results in $m_\pi=0$. This is an exact result. In QCD, chiral symmetry is explicitly broken by a small amount, $m_{u,d}<\Lambda_{\rm QCD}$, so that the pion is an approximate Goldstone boson. Lowest-order chiral perturbation theory gives a relation between m_π and $m_{u,d}$,

$$m_\pi{}^2=m_u\langle\pi\,|\,\bar{u}u\,|\,\pi\rangle+m_d\langle\pi\,|\,\bar{d}d\,|\,\pi\rangle\ . \qquad (20)$$

As noted in Ref. 8 this $m_\pi\sim\sqrt{m_q}$ dependence is expected in the kind of model we study. In fact, expanding E around $m_q=0$,

TABLE III. Meson and baryon masses for $m_{u,d}=0$ and $n=2$ using the parametrization of $\alpha_s(R)$ in Eq. (10). Masses used to fix the parameters are underlined. The pion mass is given as zero; cf. the discussion under (i) in Sec. IV A.

Particle	$m_{\rm exp}$ (GeV)	$m_{\rm bag}$ (GeV)	R (GeV^{-1})	$\alpha_s(R)$
π	0.139	0	2.42	0.74
K	0.495	0.372	2.55	0.77
η	0.549	0.630	2.30	0.70
ρ	<u>0.770</u>	0.770	2.65	0.79
K^*	0.892	0.914	2.66	0.80
ϕ	1.019	1.057	2.70	0.81
p	<u>0.938</u>	0.938	3.21	0.96
Λ	1.116	1.104	3.22	0.97
Σ^+	1.189	1.125	3.21	0.96
Ξ^0	1.321	1.280	3.22	0.97
Δ	<u>1.236</u>	1.236	3.29	0.99
Σ^*	1.385	1.381	3.30	0.99
Ξ^*	1.533	1.525	3.29	0.99
Ω^-	<u>1.672</u>	1.672	3.29	0.99

$\Lambda=0.172$ GeV, $e_{\rm TE}=0.107$, $e_q=0.607$, $m_s=0.288$ GeV;
$B^{1/4}=0.228$ GeV, $R_0=2.73$ GeV^{-1}

TABLE IV. Glueball masses for $n=2$ using the same parameters as in Table III and putting $e_{TM}=e_{TE}$.

State		m_{bag} (GeV)	R (GeV^{-1})	$\alpha_s(R)$
$(TE)^2$	2^{++}	1.88	3.16	0.94
$(TM)^2$	0^{++}	1.58	3.36	1.01
	2^{++}	2.51	3.53	1.07
(TE)(TM)	0^{-+}	0.81	2.91	0.87
	2^{-+}	1.98	3.29	0.99
$(TE)^3$	0^{++}	2.71	3.65	1.11
	1^{+-}	2.36	3.65	1.11
	3^{+-}	2.85	3.67	1.11

$\Lambda=0.172$ GeV, $e_{TM}=e_{TE}=0.107$; $B^{1/4}=0.228$ GeV

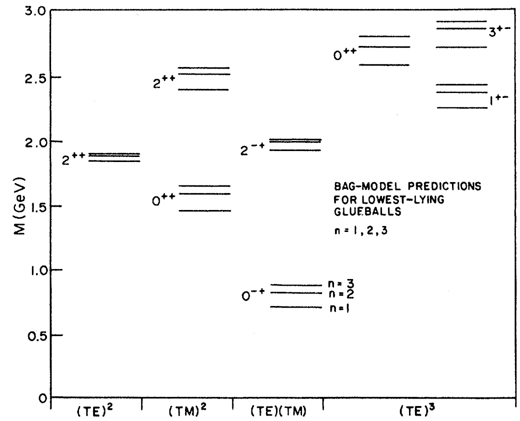

FIG. 6. Lowest-lying glueball masses for $n=1,2,3$ using Λ and e_{TE} as determined from mesons and baryons and assuming $e_{TM}=e_{TE}$.

$$E(m_q)=E(0)+m_q\frac{\partial}{\partial m_q}E(m_q)\,|_{m_q=0}\ , \qquad (21)$$

and using Eq. (3) gives

$$m_\pi{}^2=Am_q+Bm_q{}^2+\cdots\ . \qquad (22)$$

According to Eq. (22) one expects that m_π rises steeply with m_q. This is indeed the case. With $m_q\simeq 70$ MeV one obtains the physical pion mass $m_\pi=140$ MeV.

(ii) The values of R and $\alpha_s(R)$. As is seen from Tables III and IV the hadronic radii R are ~ 3 GeV^{-1}. This is significantly smaller than in earlier works where R typically ~ 5 GeV^{-1}. A lower value for R is phenomenologically favored if one identifies the bag size with the size of hadrons as deduced from scattering experiments.[22,23]

Although, lacking higher-order calculations, nothing can be said with certainty about the convergence at the cavity perturbation expansion, it is appealing that we obtain $\alpha_s(R)\lesssim 1$ rather than $\alpha_s\simeq 2.2$ in earlier calculations.[2] The small α_s is a consequence of R being small, since $\alpha_s(R)/R$ is fixed by the p-Δ mass difference.

(iii) The bag constant. We get $B^{1/4}=200$—220 MeV rather than $B^{1/4}=145$ MeV in Ref. 2. Of course, B itself is not directly observable. However, it is interesting to notice that $B^{1/4}=250$ MeV was favored in a recent bag calculation of the charmonium spectrum using a Born-Oppenheimer-type approximation.[24]

B. The gluonic sector

Now turn to the (gg) and (ggg) states. We limit ourselves to states made from the lowest TE and TM modes, and for (ggg) states consider only $(TE)^3$, since $(TM)(TE)^2$, $(TM)^2(TE)$, and $(TM)^3$ turn out to be too heavy for phenomenological interest.

For $B^{1/4}$, Λ, and e_{TE} the values previously determined were used. The only new parameter needed for the glueball sector is the transverse-magnetic self-energy parameter e_{TM}. The simplest assumption is to put $e_{TM}=e_{TE}$. The predictions with this assumption are shown in Table IV (for $n=2$) and in Fig. 6 (for $n=1,2,3$). As seen from Fig. 5, our predicted glueball masses, as opposed to the baryon and meson masses, do depend on the parameter n. The reason for this is that e_{TE}, by the fit to the known hadron masses, is forced to vary with n as seen in Fig. 5. The n dependence is not too strong, however, so we can make several predictions. The lowest glueball mass is for the 0^{-+} state, in the range 700—900 MeV.[25] This is significantly lower than the experimental 0^{-+} glueball candidate $\iota(1440)$.[9] The next states are the $(TM)^2$ 0^{++} and $(TE)^2$ 2^{++} which populate the mass region where $\theta(1670)$ has been found. Excepting the 0^{-+}, all the states considered here are heavier than previously estimated in the bag-model framework.[3,4,6]

If ι(1440 is a glueball, then our naive predictions are too low. Of course our assumption $e_{TM}=e_{TE}$ could be wrong, but to obtain a (TE)(TM) 0^{-+} state at 1440 MeV would require e_{TM} to have opposite sign from e_{TE} and e_q, and to be significantly larger than e_{TE} (for $n=2$) $|e_{TM}/e_{TE}|\simeq 2.5$ would be needed). Even if this should be the case, and our 0^{-+} state could be identified with $\iota(1440)$, the masses of η and η' would still pose a serious problem. The solution of the U(1) problem will most likely involve a simultaneous understanding of the η, η', and 0^{-+} glueball masses.[26] No simple level mixing scheme (one level always goes down) will do much good, but it is conceivable that a θ-vacuum contribution is present in the pseudoscalar sector as suggested in models based on phenomenological Lagrangians.[27]

In conclusion, we have presented a consistent and unified spectrum calculation for mesons, baryons, and glueballs using the bag model, incorporating several improvements in the model. We have corrected for the center-of-mass motion, used a running coupling $\alpha_s(R)$, included self-energy terms for quarks and gluons confined in a cavity, and obtained the bag constant from a model of the QCD vacuum. The "Casimir" term $-Z_0/R$ has not been used. Our fit to the meson and baryon spectrum, including the pion, is satisfactory and predictions for the glueball spectrum have been given.

ACKNOWLEDGMENTS

One of us (T.H.H.) thanks the theory group at the College of William and Mary for kind hospitality at an early stage of this work. He also thanks Ken Johnson for many fruitful discussions and comments. He was supported in part through funds provided by the U. S. Department of Energy under Contract No. DE-AC02-76ER03069 and by the Swedish Natural Science Research Council under Contract No. F-PD4728-100. C.E.C. was supported in part by the U. S. National Science Foundation under Grant No. NSF PHY-79-08240. C. P. was supported in part by the U. S. Department of Energy under Contract No. DE-AC03-76SF00515.

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[26]It is amusing to see what happens if the $\iota(1440)$ is a glueball with $e_{\rm TM}=e_{\rm TE}$ and the mechanism responsible for bringing the mass up to 1.44 GeV contributes equally to the flavor singlet $q\bar{q}$ state. (This possibility was suggested to us by Ken Johnson.) For $n=2$, we start with two states at 0.00 and 0.63 GeV and do linear mass mixing, neglecting mixing with the glueball. The states get moved to 0.27 and 0.99 GeV (the former

would be brought up by having massive u and d quarks and by c.m. correction adjustments) and the octet-singlet mixing angle is $-28°$. This is to be compared with the experimental values $(-24\pm1)°$ for the linear mixing angle, and 0.55 and 0.96 GeV, respectively, for the physical η and η' masses (Ref. 28).

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