Interplay of glueballs and $q\bar{q}$ mesons: Study of 0^{-+} and 2^{++} mesons

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A general unified treatment of the interplay of the masses, mixing parameters, and couplings among the glueballs and ordinary $q\bar{q}$ mesons is presented, and its implications for the 0^{-+} and 2^{++} mesons are discussed.

The glueballs,[1] if they exist, have to be treated as hadrons along with the $q\bar{q}$ mesons. Even mixings between these two different types of hadrons can take place.[2–5] What then are the constraints on the masses and couplings of the glueballs? For the $q\bar{q}$ mesons we now know pretty well about their approximately ideal structures and the working of the quark-line rule. However, the unusual structure of the 0^{-+} SU(3) nonet and also the fact that the 2^{++} nonet exhibits larger deviation from the ideal structure than the 1^{--} nonet have been puzzles of particle physics. The values of the ω-ϕ and f-f' mixing angles $\theta_{\omega\phi}$ and $\theta_{ff'}$ are about 40° and 26°, respectively, compared with the ideal value $\simeq$35°.

In this paper we argue that a solution of the puzzles may lie in the existence of 0^{-+} and 2^{++} glueballs.[6] We tentatively identify them with $\iota(1440)$ and $\theta(1640)$, respectively. For the 0^{-+} mesons, the solution is usually sought in QCD and SU(4). For example, one can study[4,5] the η' problem by assuming that the effect of gluons on the Adler anomaly is dominated by the $\iota(1440)$. It has also been noted[7] that the contamination of heavy-quark components (such as $c\bar{c}$) in the η and η' is quite possible, as demonstrated by the unusually large hadronic width of η_c. For the 2^{++} mesons, Rosner has tried[3] to explain the observed deviation from the ideal mass prediction $m_{A_2}=m_f$, by introducing f and 2^{++}-glueball mixing. One of the interesting experiments on $\theta(1640)$ is the recent report[6]

$$R(\theta)\equiv\Gamma(\theta\to\eta\eta)/\Gamma(\theta\to\pi\pi)>0.82\pm0.40 \ .$$

If θ is taken to be an SU(3) singlet as usually assumed in perturbative QCD, the value of $R(\theta)$ is predicted to be much smaller. In this paper we show that a unified treatment of the masses, mixing parameters, and couplings of the glueballs as well as the usual $q\bar{q}$ mesons is possible, and that the $\iota(1440)$ and $\theta(1640)$ could play an important role for the puzzles mentioned above.

We work in the theoretical framework[8] in which various current algebras in QCD are regarded as invaluable constraints imposed by confined quarks and gluons upon the world of observable hadrons. By using the (nonperturbative) prescriptions[8] of asymptotic SU(3) symmetry and the $(q\bar{q})$ level realization of asymptotic SU(3) symmetry, the required (asymptotic) realization of the constraint algebras in the world of hadrons produces powerful constraints, providing us with a universal nonet mass-splitting pattern (Schwinger's nonet mass formulas) and also with a derivation of the quark-line rule for the asymptotic single-particle matrix elements of the vector and axial-vector currents and their charges. However, in SU(3) the above two puzzles remained unresolved. Our task is now to study how the theory is modified in the presence of glueballs.

In contrast with our previous work,[5] we do not deal directly with the algebras explicitly involving gluons. Instead, we study the usual chiral SU(3)$\otimes$SU(3) charge algebras ($i=1,\ldots,8$),

$$[V_i,V_j]=if_{ijk}V_k \ , \tag{1a}$$

$$[V_i,A_j]=if_{ijk}A_k \ , \tag{1b}$$

$$[A_i,A_j]=if_{ijk}V_k \ . \tag{1c}$$

We denote the physical nonet t with $t=J^{PC}$ as $(\pi_t,K_t,\eta_t,\eta'_t)$ and the glueball with the *same* t as G_t. In asymptotic SU(3) symmetry,[8] the annihilation and creation operators of "physical" mesons such as $a^t_\eta(\vec{p})$, $a^t_{\eta'}(\vec{p})$, and $a^t_G(\vec{p})$ still transform *linearly* under the SU(3) transformation generated by V_i but only in the limit $\vec{p}\to\infty$. Therefore, at $\vec{p}\to\infty$ these operators can be related linearly to the hypothetical representation operators $a^t_8(\vec{p})$, $a^t_0(\vec{p})$, and $a^t_{G_0}(\vec{p})$ by $[\,|\eta_t\rangle=(a^t_\eta)^\dagger|0\rangle$ and $|(\eta_8)_t\rangle=(a^t_8)^\dagger|0\rangle$, etc.]

$$\begin{aligned} a^t_\eta&=\alpha^t_8a^t_8+\alpha^t_0a^t_0+\alpha^t_Ga^t_{G_0} \ ,\\ a^t_{\eta'}&=\beta^t_8a^t_8+\beta^t_0a^t_0+\beta^t_Ga^t_{G_0} \ ,\\ a^t_G&=\gamma^t_8a^t_8+\gamma^t_0a^t_0+\gamma^t_Ga^t_{G_0} \ , \quad \vec{p}\to\infty \ . \end{aligned} \tag{2}$$

With asymptotic SU(3) symmetry, the realization of

Eq. (1a) implies that the mixing parameters can be expressed in terms of three mixing angles θ_1^t, θ_2^t, and θ_3^t ($c_i^t \equiv \cos\theta_i^t$ and $s_i^t \equiv \sin\theta_i^t$, $i=1,2,3$) as, suppressing the index t, $\alpha_8 = c_1 c_2$, $\alpha_0 = -c_1 s_2 s_3 - s_1 c_3$, $\alpha_G = -c_1 s_2 c_3 + s_1 s_3$, $\beta_8 = s_1 c_2$, $\beta_0 = -s_1 s_2 s_3 + c_1 c_3$, $\beta_G = -s_1 s_2 c_3 - c_1 s_3$, $\gamma_8 = s_2$, $\gamma_0 = c_2 s_3$, and $\gamma_G = c_2 c_3$. θ_1, θ_2, and θ_3 correspond to the 8-1, 8-G_0, and 1-G_0 mixing angles, respectively, when the other two angles are zero. The realization of Eq. (1b) at $\vec{p} \to \infty$ implies that all the single-particle *asymptotic* matrix elements of the axial-vector charge A_i between $(\pi_t, K_t, \eta_t, \eta_t', G_t)$ and the u nonet with $C_t C_u = 1$ (C is the charge-conjugation parity) can be parametrized in general in terms of three independent asymptotic matrix elements

$$I_8^{tu} \equiv \sqrt{3/2} \langle (\eta_8)_t | A_{\pi^+} | \pi_t^-(\vec{p}) \rangle ,$$

$$I_0^{tu} \equiv \sqrt{3}/2 \langle (\eta_0)_t | A_{\pi^+} | \pi_u^-(\vec{p}) \rangle ,$$

and

$$I_{G_0}^{tu} \equiv \sqrt{3}/2 \langle (G_0)_t | A_{\pi^+} | \pi_u^-(\vec{p}) \rangle$$

and the mixing angles. However, as a way to *discriminate* the glueball from the $q\bar{q}$ mesons we propose an aysmptotic assumption,

$$I_{G_0}^{tu} \equiv \langle (G_0)_t | A_{\pi^+} | \pi_u^-(\vec{p}) \rangle = 0 , \quad \vec{p} \to \infty . \tag{3}$$

Equation (3), however, does *not* imply $\langle G_t | A_{\pi^+} | \pi_u^-(\vec{p}) \rangle = 0$ even for $\vec{p} \to \infty$, since the physical G_t could involve the $q\bar{q}$ component via mixing which enables G_t to communicate with the $q\bar{q}$ mesons through A_i. Our assumption Eq. (3) is, therefore, different from the one made in Ref. 3. Intuitively, Eq. (3) is appealing and also leads to an important consequence that the axial-vector matrix elements of the glueball will, in general, be smaller than those of the $q\bar{q}$ mesons with the same J^{PC} and similar masses. The mass splittings of hadrons are constrained[8] by the presence of "exotic" charge commutators involving $\dot{V} \equiv d/dtV$, such as

$$[\dot{V}_{K^0}, V_{K^0}] = 0 \tag{4a}$$

and

$$[\dot{V}_{K^0}, A_{\pi^-}] = 0 . \tag{4b}$$

Asymptotic realization of Eq. (4a) produces a quadratic Gell-Mann—Okubo mass formula as an exact constraint which now involves the glueball ($K_t = m_{K_t}$, etc.),

$$4K_t^2 - \pi_t^2 - 3[\eta_t^2 \alpha_8^{t2} + (\eta_t')^2 (\beta_8^t)^2 + G_t^2 (\gamma_8^t)^2] = 0 . \tag{5}$$

Similar realization of Eq. (4b) using Eq. (3) gives

$$[\eta_t^2 \alpha_8^{t2} + (\eta_t')^2 \beta_8^{t2} + G_t^2 \gamma_8^{t2} - \pi_t^2] I_8^{tu} + [\eta_t^2 \alpha_8^t \alpha_0^t + (\eta_t')^2 \beta_8^t \beta_0^t + G_t^2 \gamma_8^t \gamma_0^t + \delta] \sqrt{2} I_0^{tu} = 0 . \tag{6}$$

Equation (6) is valid *irrespective* of the choice of the u nonet as long as $C_t C_u = 1$. (For the case of $C_t C_u = -1$, we obtain instead a mass relation $K_t^2 - \pi_t^2 = K_u^2 - \pi_u^2$.) In Eq. (6), δ represents the possible *non-negligible* contribution of η_c^t (the mainly $c\bar{c}$ state),

$$\delta \sqrt{2} I_0^{tu} \equiv (\eta_{ct})^2 \sqrt{3/2} R_{\eta_c 8}^t \langle \eta_{ct} | A_{\pi^+} | \pi_u^- \rangle .$$

$R_{\eta_c 8}^t$ denotes the fraction of the octet components in η_{ct}. Although $R_{\eta_c 8}^t$ is certainly small, δ could be non-negligible, if $\langle \eta_{ct} | A_{\pi^+} | \pi_u^- \rangle$ is appreciable (violation of quark-line rule) since $\eta_{ct} \gg \eta_t$ and η_t'. Equation (6) contains the quark-line rule. If we choose $t = u = 1^{--}$, then $\theta_2 = \theta_3 = 0$ (no 1^{--} glueball) and $\delta \simeq 0$ and Eq. (6) reduces to

$$\langle \phi | A_{\pi^+} | \rho^- \rangle / \langle \omega | A_{\pi^+} | \rho^- \rangle$$

$$= -\tan\theta_{\omega\phi} (\omega^2 - \rho^2)/(\phi^2 - \rho^2) ,$$

which demonstrates the interplay

$$\langle \phi | A_{\pi^+} | \rho^- \rangle \simeq 0 \rightleftarrows \omega \simeq \rho .$$

For Eq. (1c), we consider the level realization[8] of asymptotic SU(3) symmetry in the algebra $[A_{\pi^+}, A_{\pi^-}] = 2V_3$, inserting it between the $q\bar{q}$ states $\langle \pi_t^+ |$ and $| \pi_t^+ \rangle$ and $\langle K_t^+ |$ and $| K_t^+ \rangle$ with $\vec{p} \to \infty$. Among the complete set of single-particle intermediate states inserted between the A_π's, we require that the asymptotic SU(3) contents of the algebras be realized *levelwise.* For the $q\bar{q}$ mesons, this usually *reduces* to separate realization by *each* intermediate nonet state u which requires (for $C_t C_u = -1$ realization is *automatic),*

$$(I_8^{tu})^2 = (I_0^{tu})^2 , \quad t \text{ is abitrary and } C_t C_u = 1 . \tag{7}$$

Equation (7) is also compatible with the quark-line rule. $I_8^{tu} = I_0^{tu}$ implies for $t = u = 1^{--}$, for example,

$$\langle \phi | A_{\pi^+} | \rho^- \rangle / \langle \omega | A_{\pi^+} | \rho^- \rangle = -\tan(\theta_{\omega\phi} - \theta_0) ,$$

where θ_0 is the ideal angle. In the presence of a glueball, Eq. (7) implies that G_u appears with η_u and η_u' (and possibly η_{cu}) in the u intermediate state. Even though we assume Eq. (3), $\langle G_u | A_{\pi^+} | \pi_t^- \rangle \neq 0$

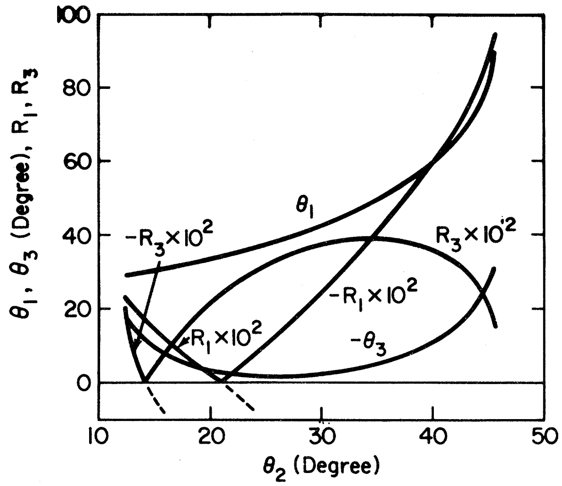

FIG. 1. θ_1, θ_3, R_1, and R_3 vs θ_2 for 2^{++} mesons.

since G_u carries some $q\bar{q}$ component. In this way, we keep track of the $q\bar{q}$ component in G_u leaked from the u nonet. In other words the constraint Eq. (7) remains *intact* in the *presence* of glueballs.

We now discuss numerical results for the 2^{++} and 0^{-+} mesons.

(1) 2^{++} *mesons.* From Eq. (5), by inputting the masses of A_2, K^{**}, f and f', and $G_t=\theta(1640)$, we obtain for two of the f-f'-θ mixing angles $(\theta_1,\theta_2,\theta_3)$, $\theta_1>29°$ and $12°<|\theta_2|<46°$. $\theta_1>0$ is chosen, since f' should mainly be an $s\bar{s}$ state. In Fig. 1 we have shown the case $\theta_2>0$. Since the 2^{++} meson is close to "ideal," we can set $\delta\simeq0$ in Eq. (6). From Eqs. (5), (6), and (7), we determine θ_3 and $I^{tu}\equiv I_8^{tu}/I_0^{tu}$ for $t=2^{++}$ and $u=0^{-+}$ as functions of θ_2. Two solutions are obtained. However, one of them (which gives $|\theta_3|\simeq90°$) is discarded and the other gives $I^{tu}=1$ and a value of $|\theta_3|$ *very close* to zero (see Fig. 1). We define the ratios of *asymptotic* axial-vector matrix elements R_i by

$$R_1\equiv\langle\eta_t|A_{\pi^+}|\pi_u^-\rangle/A,\quad R_2\equiv\langle\eta_t'|A_{\pi^+}|\pi_u^-\rangle/A\ ,$$

$$R_3\equiv\langle G_t|A_{\pi^+}|\pi_u^-\rangle/A,\quad R_4\equiv\langle\eta_t|A_{K^+}|K_u^-\rangle/A\ ,$$

$$R_5\equiv\langle\eta_t'|A_{K^+}|K_u^-\rangle/A,\quad R_6\equiv\langle G_t|A_{K^+}K_u^-\rangle/A\ ,$$

where

$$A\equiv\langle K_t^+|A_{\pi^+}|K_u^0(\vec{p})\rangle,\quad \vec{p}\to\infty\ .$$

The values of $|\theta_3|$ and $|R_i|^2$'s are independent of the signs of θ_1 and θ_2 and their signs are determined uniquely, once the signs of θ_1 and θ_2 are fixed. For $t=2^{++}$, the particle assignments are $\eta_t=f'$, $\eta_t'=f$, and $G_t=\theta(1640)$. In the *absence* of $\theta(1640)$ (i.e., $\theta_2=\theta_3=0$), Eqs. (5) and (7) yield[8] $\theta_1=26.5°$, $R_2=1.40$, $R_4=-0.88$, and $R_5=0.85$. In the *presence* of $\theta(1640)$ we find that, as θ_2 varies from 12° to 46°, R_2 and R_4 do not change appreciably, but R_5 becomes smaller by 6%, if θ_1 *remains* around 30°. Namely, for the values of θ_1 around 30°, the widths of $f\to\pi\pi$ and $f'\to K\bar{K}$ decays do *not* change appreciably from the usual SU(3) values and that of the $f\to K\bar{K}$ decreases by only 12%. However, the values of R_1 and R_3 are more sensitive and they become very close to zero when θ_1 is near 30°. This implies, via PCAC (partial conservation of axial-vector current),[9] that the widths of $f'\to\pi\pi$ and $\theta(1640)\to\pi\pi$ decays become very small, if θ_1 is around 30°. The experimental branching ratio[10] of $f'\to\pi\pi$ is $(1.2\pm0.4)\%$. From $\Gamma(f')=67\pm10$ MeV and $\Gamma(K^{**}\to K\pi)=49.1\pm6.5$ MeV, we find $R_1^{\exp}=0.098\pm0.023$, which is smaller (by factor $\simeq2$) than the predicted value[8] $R_1\simeq0.22$ in the *absence* of $\theta(1640)$. However, with $\theta(1640)$, we find R_1 compatible with $R_1^{\exp}$, if $\theta_1=30.8°-32.4°$ [which gives $\theta_2=16.0°-18.0°$ and $\theta_3=(-7.2°)-(-5.0°)$]. The values of θ_1 then predict $R_3=0.09-0.16$, which is consistent with experiment,[6]

$$R(\theta)\equiv\Gamma(\theta\to\eta\eta)/\Gamma(\theta\to\pi\pi)>0.82\pm0.40\ ,$$

as shown below. If we identify the two-gluon states which connect the $c\bar{c}$ state with light-quark bound states with the glueball, it has to be treated as an SU(3) singlet and

$$R(\theta)=(\text{phase-volume ratio}\simeq0.3)\times\tfrac{1}{3}$$
$$\simeq0.1\ .$$

However, in the present theory we find, with a good approximation $A_\eta\simeq A_8$, $\langle\eta|\simeq\langle\eta_8|$, and $f_\eta\simeq f_\pi$,

$$R(\theta)=\frac{1}{3}\left[\frac{\sqrt{2}c_2s_3-s_2I}{\sqrt{2}c_2s_3+s_2I}\right]^2 \times(\text{phase-volume ratio}\simeq0.3) \simeq0.50-1.84 \qquad (8)$$

for $\theta_1=30.8°-32.4°$. R_6 changes in the same pattern as R_3 and $R_6=(-0.26)-(-0.22)$ for the range of θ_1. We thus find $\Gamma(\theta\to K\bar{K})=4.0-5.2$ MeV, $\Gamma(\theta\to\eta\eta)\sim1.3-1.5$ MeV, and $\Gamma(\theta\to\pi\pi)=0.8-2.7$ MeV. Although we have assumed that $G(2^{++})=\theta(1640)$, no solution exists (without including further corrections) if the mass of $G(2^{++})$ is less than 1.5 GeV.

(2) 0^{-+} *mesons.* We take $t=0^{-+}$ in Eqs. (5) and (6). For Eq. (7), we take $t=2^{++}$ and 0^{++} and $u=0^{-+}$. We can then determine θ_1, θ_3, and R_i, if θ_2 and this time δ in Eq. (6) are given. By inputting the masses of the 0^{-+} nonet and $G(0^{-+})=\iota(1440)$, we first obtain from Eq. (5) $|\theta_1|<10°$ and

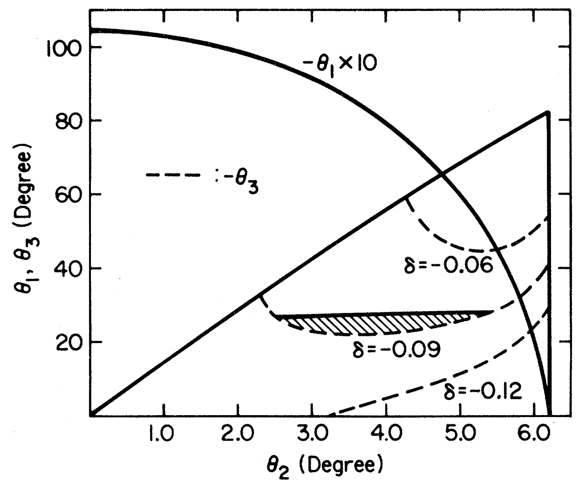

FIG. 2. θ_1 and θ_3 vs θ_2 for 0^{-+} mesons. The solutions are restricted to the wedge-shaped area. The dashed curves describe the θ_2-θ_3 relations for typical values of δ. The shaded area is a region which satisfies $|\delta| < 0.09$ GeV^2 and $|R_3/R_1| < 0.48$.

$|\theta_2| < 6°$. From Eqs. (5)—(7) the solutions are restricted (in the θ_2-θ_3 plane) to the wedge-shaped area in Fig. 2. The dashed curves inside the area show the δ dependence of the θ_2-θ_3 curve. In Fig. 2, θ_1 is negative and $I = +1$. The positive solution of θ_1 is rejected, since it produces too large a value of $|\theta_3|$. If we choose $\theta_2 < 0$, then θ_3 becomes positive without changing the value of $|\theta_3|$. We now make a crude estimate of δ which is now non-negligible. From $\Gamma(\eta_c) = (12.4 \pm 4.1)$ MeV, we estimate[9]

$$|\langle \eta_c | A_{\pi^+} | A_{2^-} \rangle / \langle K^{**+} | A_{\pi^+} | K^0 \rangle| < 0.12$$

if we assume $B(\eta_c \to A_2\pi) < 50\%$. Therefore, if we choose $|R_{\eta_c 8}| < 0.1$, we obtain $|\delta| < 0.09$ GeV^2. On the other hand, from experiments[6] $\Gamma(\iota(1440)) = 55 \pm 25$ MeV and $\Gamma(\delta^- \to \eta\pi^-) = 52 \pm 8$ MeV, we find

$$|\langle \iota | A_{\pi^+} | \delta^- \rangle / \langle \eta | A_{\pi^+} | \delta^- \rangle \equiv |R_3/R_1| < 0.48 .$$

The region where these constraints are *satisfied* is shown in Fig. 2 by the shaded area. It corresponds to $\theta_1 = (-9.5°)-(-4.9°)$, $|\theta_2| = 2.5°-5.4°$, $|\theta_3| = 22°-28°$, and predicts $R_1 = 1.0$, $R_2 = 1.0$, $R_3 = 1.1$, and $R_4 = (-0.21)-(-0.27)$. These should be compared with the case of *no* $\iota(1440)$, i.e., $R_1 = 1.01$, $R_2 = 0.99$, $R_3 = 1.21$, and $R_4 = -0.193$. Therefore, only R_4 becomes larger (by 10 to 40%) with $\iota(1440)$. Thus, while the widths of $A_2 \to \eta\pi$ and $\eta'\pi$ do not change, that of $K^{**} \to K\eta$ becomes larger by 20—100%. With $f_K/f_\pi \simeq 1.17$, the SU(3) value of $\Gamma(K^{**} \to K\eta)$ is 0.54 ± 0.07 MeV, whereas $\Gamma^{\exp}(K^{**} \to K\eta) = 2.5 \pm 2.9$ MeV. Therefore, this change does not give any contradiction. We also find $R_6 \simeq R_3$ and $R_6 = (-0.46)-(-0.58)$ for the values of θ's found. They imply via PCAC that the coupling strengths of ι-κ-K and ι-K^{**}-K are comparable with that of ι-δ-π and their contributions to the $\iota \to K\bar{K}\pi$ decays must be appreciable.

In summary, the observed structures of the 0^{-+} and 2^{++} nonets are better understood, if glueballs exist. The relevant mixing angles are uniquely fixed. The f-f' mixing angle is found to be around 31°—32° and thus becomes closer to the "ideal" value. The $\theta(1640)$ mixes with the $q\bar{q}$ octet rather appreciably ($\theta_2 = 16°-18°$) but less with the singlet $[\theta_3 = (-7°)-(-5°)]$. This is consistent with the experimental ratio of $R(\theta)$. The overall mixing pattern (see *Note added in proof*) is somewhat similar to the one obtained in a particular model discussed in Ref. 3. The $\iota(1440)$ mixes *significantly* with the $q\bar{q}$ singlet ($|\theta_3| = 22°-28°$) but much less with the octet ($|\theta_2| = 3°-5°$). The η-η' mixing angle is in the range $(-10°)-(-5°)$. These values of mixing angles are consistent with the observed widths of η_c and ι. The result is also in agreement with the one[5] obtained using the algebras involving the gluons explicitly.

Note added in proof. The $\bar{q}q$ and glueball content of 2^{++} mesons is given by (choosing a value $\theta_1 = 32.014°$),

$$\begin{pmatrix} f' \\ f \\ \theta \end{pmatrix} = \begin{pmatrix} 0.0529 & -0.9500 & -0.3079 \\ 0.9924 & 0.0843 & -0.0898 \\ 0.1112 & -0.3009 & 0.9472 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \\ \bar{s}s \\ G_0 \end{pmatrix} .$$

One sees that the f turns out to be very *pure* but there exists a somewhat appreciable mixing between the f' and θ. This seems very reasonable, since the masses of f' and θ are close.

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