Muon polarization in $K_L \to \mu^+\mu^-$

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The longitudinal polarization of muons in the decay $K_L \to \mu^+\mu^-$ is considered in the presence of nonelectroweak interactions.

The decay $K_L \to \mu^+\mu^-$ occurs with a branching ratio $B(K_L \to \mu^+\mu^-)=(9.1\pm1.9)\times10^{-9}$ (Ref. 1). Although it can be accounted for by the usual electroweak interactions, the possibility that new types of interactions are also involved cannot be ruled out. As the precise value of the electroweak contribution is uncertain,[2] it would be difficult, if not impossible, to discern the presence of a new interaction on the basis of the decay rate alone. In this paper we wish to explore what information one could obtain from a measurement of the longitudinal polarization of the muons.

Assuming *CPT* invariance, the K_L state can be expressed in terms of the *CP* eigenstates K_2 and K_1 as

$$K_L = K_2 + \epsilon K_1 \,, \qquad \epsilon \simeq (2\times10^{-3})\exp(i\pi/4) \,, \tag{1}$$

where we have kept only the first-order term in ϵ and used the Wu-Yang phase conventions.[3] We shall consider first the contribution of the $CP=-1$ state K_2 to the muon polarization. The effects of the K_1 component will be discussed at the end of the paper.

The most general matrix element for K_2 decaying into a negative and a positive muon of four-momenta p_- and p_+, respectively, can be written as[4]

$$\mathcal{M}(K_2 \to \mu^+\mu^-) = a_2 \bar{u}(p_-)\gamma_5 v(p_+) + i b_2 \bar{u}(p_-) v(p_+) \,, \tag{2}$$

where a_2 and b_2 are complex numbers.[5] The amplitude a_2 is CP conserving ($\mu^+\mu^-$ in a $^1S_0$ state), whereas the b_2 amplitude is *P*- and *CP*-violating ($\mu^+\mu^-$ in a $^3P_0$ state). It follows that $K_2 \to \mu^+\mu^-$ is a parity-conserving decay if CP invariance holds. In particular, a nonvanishing muon longitudinal polarization in $K_2 \to \mu^+\mu^-$ cannot arise from a CP-invariant effective quark-lepton interaction.

CPT invariance, which we shall assume to hold, requires that b_2 and a_2 be relatively real, apart from "unitarity phases" arising from absorptive parts of the contributing Feynman diagrams.[6] In the following we shall choose the common phase of the amplitudes so that b_2 and a_2 are real in the absence of such phases.

The decay probability in the K_2 rest frame, summed, for example, over the μ^+ variables, is given by

$$dW = \frac{m_K r}{64\pi^2}(|a_2|^2 + r^2|b_2|^2)(1+\mathcal{P}\vec{n}\cdot\vec{\zeta})d\Omega_n \,, \tag{3}$$

where $\vec{n}=\vec{p}_-/|\vec{p}_-|$, $r=(1-4m_\mu^2/m_K^2)^{1/2} \simeq 0.905$, m_K and m_μ are the masses of the kaon and the muon, respectively, and $\vec{\zeta}$ is the polarization vector of μ^- in the rest frame of μ^-. $\mathcal{P}$ is the degree of longitudinal polarization of μ^-,

$$\mathcal{P} = \frac{N_R - N_L}{N_R + N_L} \,,$$

where N_R (N_L) is the number of μ^-'s emerging with positve (negative) helicity. In terms of the decay amplitudes

$$\mathcal{P} = \frac{2r\,\mathrm{Im}(b_2 a_2^*)}{|a_2|^2 + r^2|b_2|^2} = \frac{m_K r^2 \mathrm{Im}(b_2 a_2^*)}{4\pi\Gamma} \,, \tag{4}$$

where Γ is the $K_2 \to \mu^+\mu^-$ decay rate,

$$\Gamma = \frac{m_K r}{8\pi}(|a_2|^2 + |b_2|^2) \,. \tag{5}$$

The longitudinal polarization of positive muons is the same, given by Eq. (4).

We shall write the amplitudes a_2, b_2 as sums of

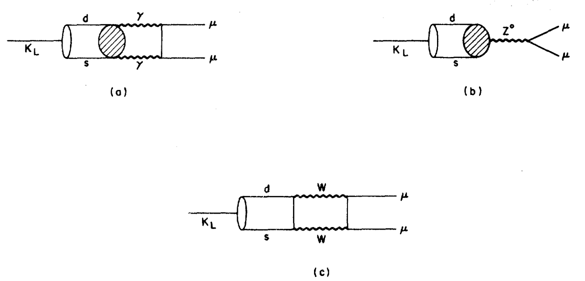

FIG. 1. Electroweak contributions to $K_L \to \mu^+\mu^-$. (a) 2γ-exchange contribution; (b) induced Z^0 contribution; (c) box diagram.

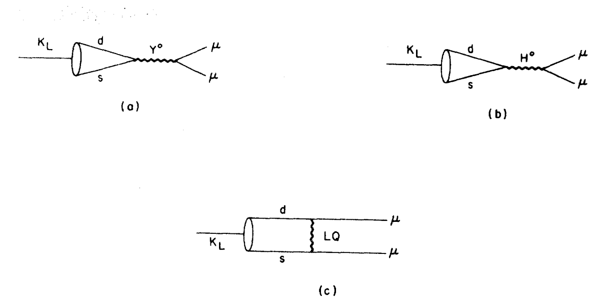

FIG. 2. Possible nonelectroweak contributions to $K_L \to \mu^+\mu^-$. (a) Contribution of a flavor-changing gauge boson; (b) flavor-changing Higgs-boson contribution; (c) leptoquark contribution.

contributions $a_2^{(e)}, b_2^{(e)}$ from the electroweak interactions and of possible nonelectroweak contributions $a_2^{(n)}, b_2^{(n)}$ (Ref. 7),

$$a_2 = a_2^{(e)} + a_2^{(n)} ,$$
$$b_2 = b_2^{(e)} + b_2^{(n)} . \tag{6}$$

Figure 1 shows the lowest-order contributions from the electroweak interactions [in the standard $SU(2)_L \times U(1)$ model]. Possible nonelectroweak contributions are shown in Fig. 2. From the electroweak contributions only the two-photon-exchange diagram (a) has an absorptive part. Ignoring possible contributons from multiboson exchanges (not shown in Fig. 2) involving light bosons, the lowest-order nonelectroweak amplitudes are real. As a result,

$$\mathrm{Re}a_2 = \mathrm{Re}a_2^{(e)} + a_2^{(n)} , \tag{7}$$

$$\mathrm{Im}a_2 = \mathrm{Im}a_2^{(e)} \; [=\mathrm{Im}(a_2^{(e)})_{\gamma\gamma}] , \tag{8}$$

$$\mathrm{Re}b_2 = \mathrm{Re}b_2^{(e)} + b_2^{(n)} , \tag{9}$$

$$\mathrm{Im}b_2 = \mathrm{Im}b_2^{(e)} \; [=\mathrm{Im}(b_2^{(e)})_{\gamma\gamma}] . \tag{10}$$

In the following we shall neglect the amplitude $b_2^{(e)}$. $\mathrm{Im}b_2^{(e)}$ can receive a contribution only from diagram (a) in Fig. 1, dominated presumably by the process $K_2 \to \pi\pi \to \gamma\gamma(CP=+1) \to \mu^+\mu^-$.[8] Thus [since CP violation in $K_2 \to \pi\pi$ is constrained by the experimental limit $|\epsilon'/\epsilon| \le \frac{1}{50}$ (Ref. 9)] one expects $|\mathrm{Im}b_2^{(e)}/\mathrm{Im}a_2^{(e)}| \lesssim 10^{-4}$. On the same grounds one expects $|\mathrm{Re}(b_2^{(e)})_{\gamma\gamma}/\mathrm{Re}a_2^{(e)}| \lesssim 10^{-4}$ for the contribution of diagram (a) to $\mathrm{Re}b_2^{(e)}$. Diagrams (b) and (c) in Fig. 1 do not contribute to $\mathrm{Re}b_2^{(e)}$ in the standard $SU(2)_L \times U(1)$ electroweak model,[10] as they lead to an effective nonderivative four-fermion interaction involving only vector and axial-vector couplings.[8,11] Using Eqs. (7)–(10) with $b_2^{(e)}=0$, the muon polarization can be expressed as

$$\mathscr{P} = -\frac{m_K r^2}{4\pi\Gamma} b_2^{(n)} \mathrm{Im}(a_2^{(e)})_{\gamma\gamma} . \tag{11}$$

The magnitude of $\mathrm{Im}(a_2^{(e)})_{\gamma\gamma}$ can be calculated in a model-independent way using the unitarity relation. Keeping only the dominant two-photon intermediate state one has[12]

$$|\mathrm{Im}(a_2^{(e)})_{\gamma\gamma}| = \frac{1}{4r} \alpha \frac{m_\mu}{m_K} \ln\frac{1+r}{1-r} |\mathrm{Re}F_2| , \tag{12}$$

where F_2 is the $K_2 \to 2\gamma$ decay constant

$$|F_2| \simeq [64\pi\Gamma(K_L \to 2\gamma)/m_K]^{1/2} .$$

The contribution of the absorptive part of the $K_L \to 2\gamma$ amplitude to $\Gamma(K_L \to 2\gamma)$ is expected to be small.[12] Neglecting it and using

$$\Gamma(K_L \to 2\gamma)_{\mathrm{exp}} \simeq 6.2 \times 10^{-12} \text{ eV}$$

[from

$$\Gamma(K_L \to 2\gamma)/\Gamma(K_L \to \text{ all})_{\mathrm{exp}} \simeq 4.9 \times 10^{-4}$$

and

$$\Gamma(K_L \to \text{ all}) \simeq 1.27 \times 10^{-8} \text{ eV}] ,$$

one obtains[13]

$$|\mathrm{Im}(a^{(e)})_{\gamma\gamma}| \simeq 2 \times 10^{-12} . \tag{13}$$

It follows that

$$|\mathscr{P}| \simeq (5.7 \times 10^{11}) |b_2^{(n)}| . \tag{14}$$

The experimental $K_L \to \mu^+\mu^-$ rate,[1]

$$\Gamma(K_L \to \mu^+\mu^-)_{\mathrm{exp}} \simeq 1.16 \times 10^{-16} \text{ eV}$$

implies the bound

$$|b_2^{(n)}| = \frac{1}{r}\left[\frac{8\pi\Gamma}{m_K r} - (\mathrm{Im}a_2^{(e)})^2 - (\mathrm{Re}a_2)^2\right]^{1/2}$$
$$\leq \frac{1}{r}\left[\frac{8\pi\Gamma}{m_K r} - (\mathrm{Im}a_2^{(e)})^2\right]^{1/2}$$
$$\simeq 1.7\times 10^{-12}\ . \quad (15)$$

The muon polarization for the maximum allowed value of $b_2^{(n)}$ (corresponding to $\mathrm{Re}a_2=0$) would be about $|\mathscr{P}|\simeq 0.96$.

The actual value of $|\mathscr{P}|$ depends on the magnitude of $b_2^{(n)}$. To consider the latter, we shall assume that the interaction which would give rise to $b_2^{(n)}$ can be represented by an effective Hamiltonian of the form

$$H_{eff} = \frac{G}{\sqrt{2}}(g_{AA}\bar{s}\gamma_\lambda\gamma_5 d\bar{\mu}\gamma_\lambda\gamma_5\mu + g_{PP}\bar{s}i\gamma_5 d\bar{\mu}i\gamma_5\mu + g_{SP}\bar{s}i\gamma_5 d\bar{\mu}\mu) + \mathrm{H.c.}\ , \quad (16)$$

where g_{AA}, g_{PP}, and g_{SP} are constants, and $Gm_p{}^2\simeq 10^{-5}$. Equation (16) is the most general Hermitian four-fermion coupling not involving derivatives of the fermion fields that can contribute to $K_2\to\mu^+\mu^-$ and $K_1\to\mu^+\mu^-$. It accounts for the important cases of $K_{2,1}\to\mu^+\mu^-$ mediated by the exchange of single gauge bosons, Higgs bosons, or leptoquark bosons. Note that (16) does not contain terms involving $\bar{\mu}\gamma_\lambda\mu$ and $\bar{\mu}\sigma_{\mu\nu}\mu$, as the hadronic part of the $K_{1,2}\to\mu^+\mu^-$ matrix element is proportional to the kaon four-momentum p_λ ($p_\lambda p_\nu$ in the case of the tensor term), and $p^\lambda\bar{\mu}\gamma_\lambda\mu=0$, $p^\lambda p^\nu\bar{\mu}\sigma_{\lambda\nu}\mu=0$.

The contribution of (16) to the $K_2\to\mu^+\mu^-$ amplitudes is given by

$$a_2^{(n)} = i\sqrt{2}Gm_\mu m_K a_A\,\mathrm{Re}g_{AA} + i\frac{Gm_K{}^2}{\sqrt{2}}a_P\,\mathrm{Re}g_{PP}\ , \quad (17)$$

$$b_2^{(n)} = -i\frac{Gm_K{}^2}{\sqrt{2}}a_P\,\mathrm{Re}g_{SP}\ , \quad (18)$$

where a_A and a_P are defined by

$$\langle 0|\bar{s}\gamma_\lambda\gamma_5 d + \bar{d}\gamma_\lambda\gamma_5 s|K_2(p)\rangle = im_K a_A p_\lambda/\sqrt{2m_K}\ , \quad (19)$$

$$\langle 0|\bar{s}i\gamma_5 d + \bar{d}i\gamma_5 s|K_2(p)\rangle = m_K{}^2 a_P/\sqrt{2m_K}\ . \quad (20)$$

The constants a_A is related to the $K^+\to\mu^+\nu_\mu$ decay amplitude as

$$\langle 0|\bar{s}\gamma_\lambda\gamma_5 d + \bar{d}\gamma_\lambda\gamma_5 s|K_2(p)\rangle = \sqrt{2}\langle 0|\bar{s}\gamma_\lambda\gamma_5 u|K^+\rangle \equiv \sqrt{2}f_K p_\lambda/\sqrt{2m_K}\ , \quad (21)$$

while a_P can be expressed in terms of a_A using the relation

$$\partial^\lambda(\bar{s}\gamma_\lambda\gamma_5 d + \bar{d}\gamma_\lambda\gamma_5 s) = (m_s+m_d)(\bar{s}i\gamma_5 d + \bar{d}i\gamma_5 s)\ . \quad (22)$$

One finds[14]

$$a_A\simeq 0.49\ , \quad (23)$$

$$a_P = a_A m_K/(m_s+m_d)\simeq 1.5\ . \quad (24)$$

Consequently,

$$a^{(n)}\simeq(4\times 10^{-7})i\,\mathrm{Re}g_{AA} + (3\times 10^{-6})i\,\mathrm{Re}g_{PP}\ , \quad (25)$$

$$b^{(n)}\simeq(3\times 10^{-6})i\,\mathrm{Re}g_{SP}\ , \quad (26)$$

and

$$|\mathscr{P}|\simeq(1.8\times 10^6)\mathrm{Re}g_{SP}\ . \quad (27)$$

We shall turn to consider now some possible mechanisms which would give rise to an interaction of the form (16), concentrating on the amplitude $b_2^{(n)}$ which enters the expression for the muon polarization.

(1) Flavor-changing gauge-boson exchange [diagram (a) in Fig. 2]. An example is a flavor-changing gauge boson associated with possible horizontal gauge symmetries. The effective quark-lepton interaction corresponding to the exchange of a gauge boson is a linear combination of vector and axial-vector type couplings. These give no contribution to the b amplitude, and consequently $\mathscr{P}=0$ for this mechanism.

(2) Flavor-changing Higgs-boson exchange [diagram (b) in Fig. 2]. In the standard $SU(2)_L\times U(1)$ model with only one Higgs doublet the Higgs-boson—fermion couplings conserve flavor and the Higgs boson couples to scalar rather than pseudoscalar densities. In the standard model with more than one Higgs doublet the Higgs-boson—fermion couplings do not conserve flavor in general, and pseudoscalar-type couplings are in general also present.

Let us consider the contribution of a Hermitian Higgs field ϕ to $\mathscr{P}$ (Ref. 15). The most general coupling of ϕ to (sd) and $(\mu\mu)$ is of the form

$$\mathscr{L} = (f_S'\bar{\mu}\mu + f_P'\bar{\mu}i\gamma_5\mu + f_S''\bar{s}d + f_P''\bar{s}i\gamma_5 d)\phi + \mathrm{H.c.} \quad (28)$$

The constants f_S' and f_P' can be chosen to be real without loss of generality. We shall take also f_S'' and f_P'' to be real, for simplicity.[16] The Lagrangian (28) leads to an effective interaction of the form (16) with

$$g_{PP} = \frac{\sqrt{2}}{G}\frac{f_P' f_P''}{m_H^2} , \tag{29}$$

$$g_{SP} = \frac{\sqrt{2}}{G}\frac{f_S' f_P''}{m_H^2} \tag{30}$$

(and $g_{AA}=0$), where m_H is the mass of the Higgs boson. The bound (15) (which follows from the experimental $K_L \to \mu^+\mu^-$ rate) implies

$$m_H \geq (1.3\times 10^3)[2^{1/4}(f_S' f_P'')^{1/2}/\sqrt{G}\,] . \tag{31}$$

Recall that the equality sign in (31) would correspond to $|\mathscr{P}| \simeq 0.96$.

The interaction (28) leads also to a $\Delta S=2$ nonleptonic term

$$H_{\rm eff}^{|\Delta S=2|} = \frac{1}{2}\frac{(f_P'')^2}{m_H^2}\bar{s}\,i\gamma_5 d\bar{s}\,i\gamma_5 d + \text{H.c.} \tag{32}$$

which gives a contribution

$$(\Delta m_K)_H = \frac{(f_P'')^2}{m_H^2}\langle \bar{K}^0 |\,\bar{s}\,i\gamma_5 d\,\bar{s}\,i\gamma_5 d\,| K^0\rangle \tag{33}$$

to the K_L-K_S mass difference. An estimate of (33) using the vacuum insertion method gives[17]

$$|(\Delta m_K)_H| = \frac{(f_P'')^2}{m_H^2}\left|1-11\left[\frac{m_K}{m_s+m_d}\right]^2\right|\frac{f_K{}^2 m_K}{12}$$
$$\simeq (1.3\times 10^{-1})\frac{(f_P'')^2}{m_H^2}\ \text{GeV}^3 , \tag{34}$$

where $f_K \simeq 1.23\ m_\pi$ (Ref. 18) is the $K^+ \to \mu^+\nu$ decay constant. Barring cancellations among the various contributions to Δm_K we have

$$(\Delta m_K)_H \leq (\Delta m_K)_{\rm exp} \simeq 3.5\times 10^{-15}\ \text{GeV} ,$$

which yields

$$m_H \gtrsim (6\times 10^6)\,|f_P''|\ \text{GeV} . \tag{35}$$

Substituting (35) to (27) yields the bound

$$|\mathscr{P}| \lesssim (6\times 10^{-3})\,|f_S'/f_P''| \tag{36}$$

for the muon polarization.

In the presence of two or more Higgs doublets, the couplings of the Higgs bosons to fermions are undetermined. Special cases include the following[19]

(a) Higgs-boson—fermion couplings are governed roughly by the mass of the fermions they couple to, i.e.,

$$\mathscr{L}_H \simeq 2^{1/4}\sqrt{G}\,(m_1+m_2)\bar{\psi}_1(\alpha_{12}+\beta_{12}\gamma_5)\psi_2\phi + \text{H.c.} \tag{37}$$

(b) Higgs-boson—fermion couplings are proportional to the mass M_F of some heavy fermion in the theory, i.e.,

$$\mathscr{L}_H \simeq 2^{1/4}\sqrt{G}\,M_F\bar{\psi}_1(\kappa_{12}+\rho_{12}\gamma_5)\psi_2 + \text{H.c.} \tag{38}$$

In (37) and (38), α_{12}, β_{12}, κ_{12}, and ρ_{12} are some combinations of mixing angles in the fermion and Higgs sectors.

In case (a) one has

$$f_S' \simeq 2^{5/4}\sqrt{G}\,m_\mu \alpha_{\mu\mu} ,$$
$$f_P'' \simeq 2^{1/4}\sqrt{G}\,(m_s+m_d)\beta_{sd} ,$$
and
$$f_S'/f_P'' \simeq 1.3\alpha_{\mu\mu}/\beta_{sd} . \tag{39}$$

The bounds (31) and (35) are in this case

$$m_H \geq 0.35(\,|\alpha_{\mu\mu}\beta_{sd}|\,)^{1/2}\ \text{TeV}$$
and
$$m_H \gtrsim 3.9\,|\beta_{sd}|\ \text{TeV} ,$$

respectively. The upper limit in (36) would coincide with the maximal $|\mathscr{P}|$ (about 0.96) allowed by the bound (15) for $|\alpha_{\mu\mu}/\beta_{sd}| \simeq 125$.

Taking in case (b) $f_s' = 2^{1/4}\sqrt{G}\,m_\tau \alpha_\tau$ and $f_P'' = 2^{1/4}\sqrt{G}\,m_b \rho_b$,[20] we have

$$f_S'/f_P'' \simeq 0.4\kappa_\tau/\rho_b . \tag{40}$$

The bounds (31) and (35) read now

$$m_H \geq 5.5(\,|\kappa_\tau \rho_b|\,)^{1/2}\ \text{TeV}$$
and
$$m_H \geq 116\,|\rho_b|\ \text{TeV} ,$$

respectively. $\mathscr{P} \simeq 0.96$ would be possible if $|\kappa_\tau/\rho_b| \simeq 440$.

To summarize, if the pattern of Higgs couplings corresponds to cases (a) or (b), the muon polarization could be large (of the order of 1) provided that the ratio of the pertinent combinations of mixing angles in the quark and the leptonic sectors would be of the order of 10^{-2}–10^{-3}.

It should be noted that the constraint (35) from the K_L-K_s mass difference might be weaker due to possible cancellations among the various contributions to Δm_K including those from different Higgs bosons.[21] The uncertainties surrounding an estimate of (33) have to be also kept in mind.

(3) Leptoquark exchange [diagram (c) in Fig. 2]. $K_L \to \mu^+\mu^-$ could also be mediated by leptoquarks (bosons causing quark $\leftrightarrow$ lepton transitions). Lepto-

quarks appear in theories which unify the strong and the flavor interactions. In a class of models due to Pati and Salam[22] they might be sufficiently light to cause observable effects in rare K decays. Relatively light leptoquarks appear also in extended-hypercolor models.[23]

Phenomenological constraints on leptoquarks have been studied recently by Shanker.[24] The strangeness-changing interactions mediated by leptoquarks are not constrained significantly by the K_L-K_S mass difference, since in lowest order leptoquark exchange does not generate a nonleptonic interaction. Leptoquark exchange leads (after a Fierz transformation) to an effective interaction of the form (16). For pseudoscalar-leptoquark exchange (appearing in hypercolor theories) the coupling strength g_{kl} ($kl=AA,PP,SP,\ldots,$) is

$$g_{kl}=\frac{\sqrt{2}}{G}(f^2/M_P{}^2)$$

(M_P = mass of the pseudoscalar leptoquark), with f^2 expected to be of the order of

$$\frac{m_f m_{f'}}{(250\text{ GeV})^2}$$

(m_f, m_f' = fermion masses). For vector-leptoquark exchange

$$g_{kl}\simeq\frac{\sqrt{2}}{G}(g^2/M^2)$$

(M = mass of the vector leptoquark) with $g^2\simeq 0(\alpha)$. Shanker finds[15] that the $K_L\to\mu^+\mu^-$ rate can be reproduced with $M_P\simeq 1$ TeV (assuming $m_f\simeq m_f'=1$ GeV) and $M\simeq 60$ TeV. Constraints from other processes give comparable bounds on M_P and M. It follows that if the leptoquark-fermion couplings violate CP invariance, leptoquark exchange could lead to a large (of the order of 1) muon polarization in $K_2\to\mu^+\mu^-$.

In our discussion so far the K_1 component of K_L has been neglected. When included, the $K_L\to\mu^+\mu^-$ matrix element becomes

$$\mathcal{M}(K_L\to\mu^+\mu^-)=a\bar{u}(p_-)\gamma_5 v(p_+)+ib\bar{u}(p_-)v(p_+) \quad (41)$$

with

$$a=a_2+i\epsilon a_1\ ,\quad b=b_2+i\epsilon b_1\ , \quad (42)$$

where $a_1=a_1^{(e)}+a_1^{(n)}$, $b_1=b_1^{(e)}+b_1^{(n)}$ are the $K_1\to\mu^+\mu^-$ amplitudes. They would be real in the absence of absorptive contributions. Neglecting terms of order ϵ^2, the muon polarization is given by

$$\mathscr{P}=\frac{m_K r^2}{4\pi\Gamma}\text{Im}ba^* \simeq\frac{m_K r^2}{4\pi\Gamma}\text{Im}(b_2a_2^*-i\epsilon^* b_2a_1^*+i\epsilon b_1a_2^*)\ , \quad (43)$$

where the first term has already been considered. The second term, which requires CP violation also in the $K_1\to\mu^+\mu^-$ matrix element, is presumably less important, in view of the factor ϵ. The last term, proportional to $\text{Im}(i\epsilon b_1 a_2^*)$, we know gives a nonvanishing contribution to $\mathscr{P}$: it contributes also in the absence of nonelectroweak interactions and it does not require the presence of a further CP violation, beyond the one involved in the K^0-$\bar{K}^0$ mass matrix. The term $(\text{Im}a_2^{(e)})(\text{Im}b_1^{(e)})\text{Re}\epsilon$ in $\text{Im}(i\epsilon b_1 a_2^*)$, which contains only absorptive amplitudes, can be calculated with relative confidence. Using[25]

$$\text{Im}b_1^{(e)}=7.58\times 10^{-13} \quad (44)$$

and (13), one obtains

$$|\mathscr{P}|_{\text{Im}b_1^e\,\text{Im}a_2^e}\simeq 7.1\times 10^{-4}\ . \quad (45)$$

The remaining electroweak terms in $\text{Im}(i\epsilon b_1 a_2^*)$ should be of comparable magnitude.

We are now ready to summarize our conclusions.

A muon polarization larger than about 10^{-3} would suggest the presence of a nonelectroweak interaction. As gauge-boson exchange gives no contribution to $\mathscr{P}$, the nonelectroweak interaction would presumably originate from Higgs-boson exchange or leptoquark exchange. Values of $|\mathscr{P}|$ near to 1 due to either of these interactions are not ruled out. A large polarization would most likely originate from the $K_2\to\mu^+\mu^-$ amplitudes and thus require the existence of a CP-violating nonelectroweak quark-lepton interaction.

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[13]Note that $\mathrm{Im}(a^{(e)})_{\gamma\gamma}$ given by Eq. (13) includes in addition to the standard electroweak contribution any further contributions that might be involved in $K_2 \to \gamma\gamma$.

[14]P. Herczeg, in *Proceedings of the Kaon Factory Workshop, Vancouver, 1979,* edited by M. K. Craddock (TRIUMF, Vancouver, 1979), p. 20. The value (24) corresponds to the choice $m_s=150$ MeV, $m_d=7.5$ MeV for the current quark masses [cf. S. Weinberg, Trans. N.Y. Acad. Sci. 38, 185 (1977)]. For larger current quark masses [cf. M. D. Scadron, Rep. Prog. Phys. 44, 213 (1981)], a_p would be correspondingly smaller.

[15]Higgs-boson contribution to the $K_L \to \mu^+\mu^-$ rate has been considered by Shanker (Ref. 7).

[16]Note, however, that for real f_P'' (28) does not contribute to $K_1 \to \mu^+\mu^-$. For complex f_P'' (28) generates a $\Delta S=2$ interaction which contributes both to the real and to the imaginary part of the $K^0 \to \bar{K}^0$ amplitude. The constraint on the Higgs-boson mass would be in such a case more severe than the one implied by the K_L-K_S mass difference, unless a small *CP*-violating phase is involved in the couplings.

[17]B. McWilliams and O. Shanker, Phys. Rev. D 22, 2853 (1980). See also O. Shanker, Ref. 7.

[18]R. E. Shrock and S. B. Treiman, Phys. Rev. D 19, 2148 (1979).

[19]See, for example, B. McWilliams and L.-F. Li, Nucl. Phys. B179, 62 (1981).

[20]It is, of course, posssible that other masses are the relevant ones in this case.

[21]A cancellation among the contributions of different Higgs bosons to the *CP*-violating part of the $K^0 \to \bar{K}^0$ amplitude occurs in the model of A. B. Lahanas and C. E. Vayonakis [Phys. Rev. D 19, 2158 (1979)]. However for this to happen some special assumptions have to be made regarding the Higgs-boson—fermion couplings (cf. O. Shanker, Ref. 7).

[22]A recent review of these models is given in J. C. Pati, invited talk at the International Conference on Baryon Nonconservation, Tata Institute of Fundamental Research, Bombay, India, 1982 [University of Maryland Report No. 82-151, 1982 (unpublished)]. Leptoquark masses in Pati-Salam models have been analyzed by T. Goldman, in *Particles and Fields—1981: Testing the Standard Model,* proceedings of the meeting of the Division of Particles and Fields of the APS, Santa Cruz, California, edited by C. A. Heusch and W. T. Kirk (AIP, New York, 1981).

[23]Phenomenological implications of hypercolor schemes and some associated problems of these theories are discussed in J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and P. Sikivie, Nucl. Phys. B182, 529 (1981); S. Dimopoulos and J. Ellis, *ibid.* B182, 505 (1981).

[24]Shanker (Ref. 7).

[25]J. Smith and Z.E.S. Uy, Phys. Rev. D 7, 2738 (1973).