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How to examine Koba-Nielsen-Olesen scaling for hadron-nucleus collisions

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We present a method to examine Koba-Nielsen-Olesen (KNO) scaling for hadron-nucleus collisions in connection with measurements of the average charged multiplicity and inelastic total cross sections. It is shown that the assumption of a universal KNO scaling curve for the multiplicity distributions turns out to be inconsistent with experimental evidence concerning A dependence of the absorption cross section and the nuclear multiplicity ratio.

Experimental attempts have been made to test whether the multiplicity distributions in hadron-nucleus interactions obey the same Koba-Nielsen-Olesen (KNO) scaling as in hadron-nucleon collisions. KNO scaling is one of the most brilliant phenomenological laws in multiparticle production in h-N (hadron-nucleon) collisions, and it may reflect production mechanisms of hadron dynamics. It is well accepted now that the nucleus enables us to study the produced state before it reaches maturity; the nucleus is part of our laboratory wherein the details of hadron dynamics can be manifested. Therefore, whether universal KNO scaling in h-A (hadron-nucleus) collisions is valid or should be modified is an interesting question.

Analyses made to date[1–18] seem to infer the same KNO scaling curve also in the case of nuclear targets. Those investigations were, however, too incomplete to confirm the scaling hypothesis for various nuclear targets over a wide enery range. Analyses so far were made only of the dispersion of the multiplicity or restricted within one or several nuclei. Universal KNO scaling requires, theoretically, a stringent test: one should check whether all the higher moments of the multiplicity distribution are independent of both energy and mass number, which is difficult.

In this paper we propose a fruitful way to examine the KNO scaling hypothesis for h-A interactions on the basis of the KNO scaling law of h-N interactions. This is a simple way, and we can deal with many data at the same time. It will be shown that the analysis based on this method casts strong doubt on the universal KNO hypothesis.

In order to construct the relation between the charged multiplicity distributions of h-N and h-A interactions, we shall require the inelastic cross section $\sigma_{\rm inel}^{hA}$, mean charged multiplicity $\langle n\rangle_{hA}$, and n-prong cross sections σ_n^{hA} for h-A collisions in contrast with those of h-N interactions. Making use of $\sigma_{\rm inel}^{hN}$ (inelastic cross section for hadron-nucleon interactions), we first put[2]

$$\sigma_{\rm inel}^{hA}=A^{\alpha}\sigma_{\rm inel}^{hN}\ , \tag{1}$$

where we introduce a parameter α, which is given by the model calculations[3] in the usual way. Equation (1) yields the average number of inelastic collisions,

$$\bar{\nu}=\frac{A\sigma_{\rm inel}^{hN}}{\sigma_{\rm inel}^{hA}}=A^{1-\alpha}\ . \tag{2}$$

Subsequently, in accordance with the experimental observations, it is probably correct to write[4–19]

$$R(A)=\frac{\langle n\rangle_{hA}}{\langle n\rangle_{hN}}=1+\beta(\bar{\nu}-1)\ , \tag{3}$$

where β is a dynamical parameter.

Now let us postulate[5] scaling of the multiplicity distributions for h-A collisions,

$$\langle n\rangle_{hA}\frac{\sigma_n^{hA}}{\sigma_{\rm inel}^{hA}}=\psi\left[\frac{n}{\langle n\rangle_{hA}}\right]\ , \tag{4}$$

according to the KNO scaling law of h-N interactions. One can prospect the universal curve ψ for the respective targets, provided ψ does not depend on the nuclear mass number A. With the help of the universal KNO scaling function we will get a relation between those reactions, and hence the choice of the dynamical parameters in Eqs. (1) and (3) by means of a χ^2 fit shall be examined in comparison with the experimental data. Thus we perform an accurate test for the universal KNO scaling hypothesis, i.e., Eq. (4).

By putting the ratio of n-prong cross sections

$$\sigma_n^{hA}/\sigma_n^{hN}=g(n,\langle n\rangle_{hN},A)\ ,$$

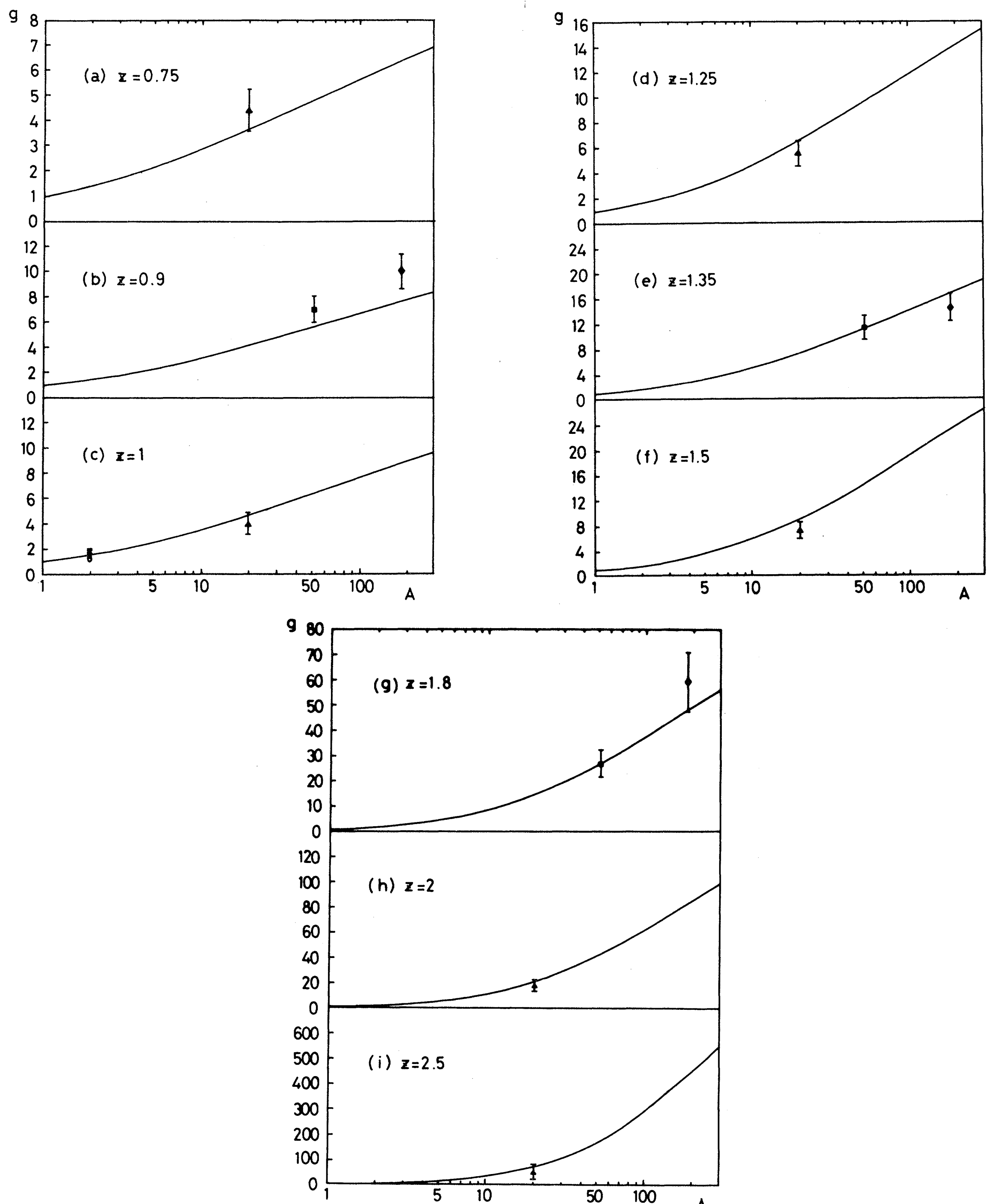

FIG. 1. Plots of $g=\sigma_n^{hA}/\sigma_n^{hN}$ versus A at the fixed value of z. ● (pd 19 GeV)/(pp 19 GeV) (Refs. 10 and 13); ○ (pd 19 GeV)/(pp 102 GeV) (Refs. 10 and 14); × (pd 19 GeV)/(pp 303 GeV) (Refs. 10 and 15); ▲ (π^-Ne 200 GeV/($\pi^- p$205 GeV) (Refs. 11 and 16); ◆ (pW 300 GeV)/(pp 303 GeV) (Refs. 12 and 15); ■ (pCr 300 GeV)/(pp 303 GeV) (Refs. 12 and 15). The solid curve represents the Slattery fit with $\alpha=0.6667$ and $\beta=0.3333$.

we obtain from Eq. (1) that

$$A^{\alpha}P_n^{hA}=gP_n^{hN}\,, \tag{5}$$

where $P_n^{hA}=\sigma_n^{hA}/\sigma_{\rm inel}^{hA}$ and $P_n^{hN}=\sigma_n^{hN}/\sigma_{\rm inel}^{hN}$. It turns out from the universal KNO scaling function and the relations (3) and (5) that we can rewrite $\psi(n/\langle n\rangle_{hA})$ in terms of A and $z=n/\langle n\rangle_{hN}$ as

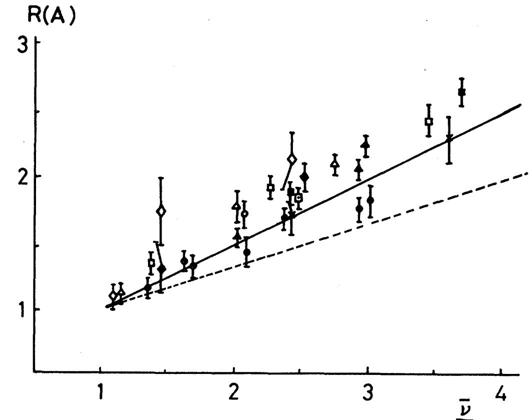

FIG. 2. Plots of $R=\langle n\rangle_{hA}/\langle n\rangle_{hN}$ versus $\bar{\nu}$. Incident π^+ (Ref. 4) △ 50 GeV, ▲ 100 GeV. Incident π^- ○ 60 GeV (Ref. 19), ● 100 GeV (Ref. 18), ⊙ 200 GeV (Ref. 11). Incident K^+ (Ref. 4) ◇ 50 GeV, ◆ 100 GeV. Incident p (Ref. 4) □ 50 GeV, ■ 100 GeV, ⊡ 200 GeV, × 300 GeV (W. Busza in Ref. 1). The solid line (dashed line) represents Eq. (3) with $\beta=\frac{1}{2}$ $(\frac{1}{3})$.

$$\psi\left[\frac{n}{\langle n\rangle_{hA}}\right]=A^{-\alpha}R(A)g(Z,A)\psi(z)\ . \qquad (6)$$

The mth moment is given by

$$M_{hA}^{m}=\frac{\langle n^m\rangle_{hA}}{\langle n\rangle_{hA}^{m}}=\frac{1}{A^{\alpha}R^{m}}\int_0^{\infty}dz\,z^m g(z,A)\psi(z)\ . \qquad (7)$$

We shall investigate the postulate that $\psi(n/\langle n\rangle_{hA})$ shows the same functional form as $\psi(n/\langle n\rangle_{hN})$. Under this assumption, the function $g(z,A)$ in Eq. (6) is determined with two appropriate parameters α and β. Hence we can choose automatically these parameters by comparing the experimental data of $\sigma_n^{hN}/\sigma_n^{hA}$ with $g(z,A)$.

In Figs. 1(a)–1(i), we show comparison of the data with our evaluations of $g(z,A)$. Here we have used Slattery's parametrization[6]

$$\psi(z)=(1.895z+16.85z^3-3.32z^5+0.166z^7)\exp(-3.04z)\ . \qquad (8)$$

Our data fitting with $\alpha=0.667$ and $\beta=0.333$ gives the satisfactory value of $\chi^2/\mathrm{DF}=\frac{12.1}{10}$. The χ^2 value is quite sensitive to the choice of these parameters. Except for the combination of $\alpha\cong 0.67$ and $\beta\cong 0.33$ the results are poor. We will present, for example, the χ^2 fits according to the various parameters obtained from the respective models in the following: we have $\chi^2=45.3$ for $\alpha=\frac{2}{3}$ and $\beta=\frac{1}{2}$, $\chi^2=110$ for $\alpha=\frac{3}{4}$ and $\beta=\frac{1}{2}$, and $\chi^2=307$ for $\alpha=\frac{3}{4}$ and $\beta=\frac{1}{3}$. However, the experimental studies seem to suggest $\alpha=0.7$–0.75 and $\beta=0.5$.[1] Even if we hold an adequate parameter $\beta=0.5$, provided α is changed from 0.67, our analyses yield the inferior χ^2 values; i.e., the fits using $\alpha=0.6$ and 0.75 with $\beta=0.5$ lead to $\chi^2=120$–110, which is all the more unsatisfactory compared with $\chi^2=45.3$ for $\alpha=0.67$ and $\beta=0.5$.

Now let us turn to the investigation of the observed relative multiplicity which suggests $\beta=\frac{1}{2}$,[3,4,7] and accordingly seems to exclude $\beta=\frac{1}{3}$.[8] (See Fig. 2.) Thus, our first assumption that the multiplicity distributions of hadron-hadron and hadron-nucleus collisions fall on the same universal KNO scaling curve leads to inconsistent results concerning observations of $R(A)$. Furthermore, it is probable that $\alpha=\frac{2}{3}$ is slightly small compared with the data of the total inelastic cross sections. It should be noted[9] that we are apt to accept the feigned scaling behavior of $\langle n\rangle_{hA}\sigma_n^{hA}/\sigma_{\mathrm{inel}}^{hA}$ only because of the linear dependence of the dispersion on $\langle n\rangle_{hA}$.

It is astonishing that our assumption that the moments M_{hA}^{m}, which come from the universal KNO scaling function, are independent of both energy and nuclear number yields curious behavior of $R(A)$ and $\sigma_{\mathrm{inel}}^{hA}$. If the parameters $\alpha=0.7$–0.75 and $\beta=0.5$ are extensively suggested in various targets and wide energy regions for further experiments, we must take A dependence into consideration for $\langle n\rangle_{hA}\sigma_n^{hA}/\sigma_{\mathrm{inel}}^{hA}$ in the strict sense.[20]

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