Scaling of the elastic differential cross section in high-energy collisions of hadrons

J. K. Mohapatra
Regional College of Education, Bhubaneswar 751007, India

Jnanadeva Maharana
Institute of Physics, A/105, Saheed Nagar, Bhubaneswar 751007, India
(Received 5 March 1981; revised manuscript received 5 January 1982)

We investigate the existence of scaling of the elastic differential cross section with $tb(s)$ as the scaling variable, where $b(s)$ is the slope of the diffraction peak. High-energy elastic-scattering data over a wide range of energies exhibit scaling with the above scaling variable. We also note that $t(\ln s)^2$ is not a good scaling variable.

I. INTRODUCTION

Auberson, Kinoshita, and Martin[1] were the first to investigate the scaling of the scattering amplitude $F(s,t)$. The following is shown from axiomatic field theory[2]:

(i) $F(s,t)$ is holomorphic in the disk $|t|<t_0$ for any s in the cut s plane, where t_0 is a constant, less than or equal to the t-channel threshold.

(ii) $F(s,t)$ is bounded by Cs^N for $|t|<t_0$ and $s\rightarrow\infty$; N is a finite constant independent of s.

(iii) $F(s,t)$ satisfies unitarity in the s channel.

(iv) $F(s,t)$ satisfies the bound[3]

$$|F(s,t)| \le \left[\frac{4\pi}{t_0}\sigma_{\rm el}\right]^{1/2} s\ln s \times\exp[(|t|/t_0)^{1/2}\ln s]\ . \qquad (1)$$

Using (1) Auberson, Kinoshita, and Martin showed that the crucial relation[1,4] in determining the analytic property of the Pomeranchuk-theorem-violating amplitudes in the high-energy limit is

$$s(\ln s)^2\frac{{\rm Im}F(s,0)}{|F(s,0)|^2}\le C_0 \quad \text{for } s>s_0\ , \qquad (2)$$

where C_0 is a positive constant. Using (1) and (2) they demonstrated that for a sequence of $(s_n)\rightarrow\infty$, $\lim f(s_n.\tau(\ln s_n)^{-2})$ exists and is a nontrivial function of τ, $\tilde{f}(\tau)$, which is entire and of order $\frac{1}{2}$, where

$$f(s,t)=F(s,t)/F(s,0)\ , \qquad (3)$$

$$\tau=t(\ln s)^2\ . \qquad (4)$$

In fact, insofar as (2) is satisfied, the above scaling property applies to any scattering amplitude.

Soon after, assuming that the modulus of the forward symmetric and antisymmetric scattering amplitudes $f_\pm(s,t)$ for large energy are of the type ${\rm const}\times s(\ln s)^{\beta_\pm}$, Cornille and Simao[5] studied the possibility of extending the results of Auberson *et al.*[1] for other forward high-energy behavior of the scattering amplitude. In particular, they assumed (i) $|f_\pm(s,t)|$ is bounded for $t<0$ and (ii) there exists one zero of the type $|t|={\rm const}\times(\ln s)^{-\gamma}$, where γ is such that the maximum number $L_{\max}$ of partial waves which effectively contribute to the scattering amplitudes near $t=0$ is of the type $L_{\max}\simeq{\rm const}\times s^{1/2}(\ln s)^{\gamma/2}$. Then defining

$$\tau=tg(s)\ , \qquad (5)$$

they tried to find $g(s)$ such that, at fixed τ, $\lim_{s_n\rightarrow\infty}f(s_n,\tau g^{-1}(s_n))$ exists and is a nontrivial function of τ. Their analysis showed that $\lim_{s_n\rightarrow\infty}f(s_n,\tau g^{-1}(s_n))=\tilde{f}(\tau)$ exists for $g=(\ln s)^\gamma$. If, in particular,

$$g(s)=(\ln s)^2\ , \qquad (6)$$

then $\tilde{f}(\tau)$ is an entire function of order $\frac{1}{2}$ in the τ plane. Subsequently, considering collisions of particles of arbitrary spins, Cornille and Martin[7] demonstrated that conditions

$$\left.\frac{d}{dt}\ln\frac{d\sigma^A}{d\Omega}(s,t)\right|_{t=0}<{\rm const}\times\frac{{\sigma_T}^2}{\sigma_{\rm el}}\ , \qquad (7)$$

$$C(s)/[b(s)]^2<{\rm const}\ , \qquad (8)$$

where $b(s)$=slope, $C(s)$=curvature, and $d\sigma^A/d\Omega(s,t)$=differential cross section (the superscript A denotes that the cross section is purely absorptive), together with some positivity properties of the scattering amplitude allow one to show the existence of a scaling limit for the differential cross section for an elastic reaction in the nearly forward direction. On the other hand Mahoux[8] proved that the absorptive unpolarized differential cross section for the elastic scattering of particles of arbitrary spin must obey the representation

$$\frac{d\sigma^A}{dt}(s,t) \Big/ \frac{d\sigma^A}{dt}(s,0) = \sum_{l=0}^{\infty} (2l+1) a_l(s) P_l \left[1 + \frac{2t}{s} \right] , \quad (9)$$

where $0 \le a_l \le 1$. Equation (9) is a fundamental consequence of unitarity. The importance[6] of this result for scaling properties was exploited by Auberson and Roy.[9] They deduced bounds on slope and curvature of the diffraction peak and further observed that $\tau = tb(s)$ can be chosen as a scaling variable provided $b(s)$ saturates the unitarity bound qualitatively, i.e.,

$$b(s) \propto (\ln s)^2 \quad (10)$$

and the scaling function is again an entire function of order $\frac{1}{2}$ in the complex τ plane.

Dias de Deus[10] proposed the hypothesis of geometric scaling,[11] namely that

$$T(s,B) = T(B/R(s)) , \quad (11)$$

where T is the elastic amplitude, defined in the impact-parameter B space by

$$T = (e^{2i\delta(B)} - 1)/i \quad (12)$$

and $R(s)$ is a radial-scaling parameter, containing all the s dependence. Equation (11) further implies[12] that

$$\frac{d\sigma}{dt} = R^4 f(R^2 t) , \quad (13)$$

$$\sigma_T \propto R^2 , \quad (14)$$

$$\sigma_{\text{el}} \propto R^2 , \quad (15)$$

$$\sigma_{\text{inel}} \propto R^2 , \quad (16)$$

$$t_0 \propto R^{-2} , \quad (17)$$

$$b(t=0) \propto R^2 , \quad (18)$$

where t_0 is the position of any dip or maximum in $d\sigma/dt$ and b is the usual slope parameter. The first test of geometric scaling i.e., $\sigma_{\text{el}}/\sigma_T$ and $b(t=0)/\sigma_T$ remain constant, showed that geometric scaling seems to hold down to about $P_{\text{lab}} = 100$ GeV/c, below which $\sigma_{\text{el}}/\sigma_T$ starts to rise and b/σ_T starts to fall. Thus geometrical scaling is apparently a property of the dominant diffractive contribution (Pomeron term), since secondary Regge-exchange terms are not expected to have this behavior. It is especially remarkable that the scaling of $d\sigma/dt$ seems to hold[12] even through the dip region, where $d\sigma/dt$ has fallen by six orders of magnitude and should be sensitive to small corrections.

With the scaling phenomena thus qualitatively well established we proceed to give in this paper a simple proof of the existence of a scaling function and then analyze the world data on pp, $p\bar{p}$, $K^{\pm}p$, and $\pi^{\pm}p$ scattering with the new scaling variable proposed by us. The plan of the paper is as follows. Section II contains a summary of relevant previous work as well as our arguments regarding the existence of a scaling function. Analysis of the slope-parameter data and the behavior of diffractive cross section under the new scaling variable are presented in Sec. III. In Sec. IV we analyze some relevant models in the framework of our results and give our concluding remarks.

II. SUMMARY OF PREVIOUS WORK

Defining

$$f(s,t) = \frac{d\sigma^A}{dt}(s,t) \Big/ \frac{d\sigma^A}{dt}(s,0) , \quad (19)$$

several results follow immediately:

(i) $f(s,0) = 1$. (20)

(ii) Due to positivity of a_l of Eq. (9),

$$0 \le f(s,t) \le 1 . \quad (21)$$

(iii) If t_i is a zero of $f(s,t)$, so is t_i^*. This result is due to the fact that a_l are real.

(iv) Auberson and Roy[9] have further shown that for complex values of $|t| < t_0$, $f(s,t)$ is bounded from above as

$$|f(s,t)| \le I_0(\omega(s)[\,|t|/(t_0 - \epsilon)]^{1/2}) , \quad (22)$$

where I_0 is the modified Bessel function of order zero, and

$$\omega(s) \equiv \ln \left[\frac{s^2}{(d\sigma^A/dt)(s,0)} \right] . \quad (23)$$

Note that it follows from the Froissart bound and the Jin-Martin lower bound that

$$(2-\epsilon)\ln s \underset{s\to\infty}{\leq} \omega(s) \underset{s\to\infty}{\leq} 14\ln s \ . \tag{24}$$

Defining the slope and curvature, respectively, as

$$b(s)=\frac{d}{dt}f(s,t)\bigg|_{t=0} , \tag{25}$$

$$C(s)=\frac{d^2}{dt^2}\ln f(s,t)\bigg|_{t=0} , \tag{26}$$

and deriving bounds[14] on the number of zeros, n_γ, of $f(s,t)$ within the disk $|t|<r<t_0-\epsilon=R$, one of us has shown[13] recently the following:

(i) $b(s$ and $C(s)$ are bounded as

$$b(s)\leq\frac{e^2}{4R}[\omega(s)]^2 , \tag{27}$$

$$C(s)\leq\frac{3e^2}{16R^2}[\omega(s)]^4 . \tag{28}$$

(ii) If $b(s)\simeq[\omega(s)]^\alpha$ for s large, and all zeros, t_i of $f(s,t)$ lie in a domain $\text{Im}t_i<\epsilon|t_i|^2$, ϵ being some s-dependent and arbitrarily small positive number if

$$\tfrac{5}{4}<\alpha<2 , \tag{29}$$

then

$$\tau=tb(s) \tag{30}$$

is a scaling variable

Remark: If we choose $\tau=t(\ln s)^\alpha$, then $\tilde{f}(s,\tau)\equiv f(s,\tau(\ln s)^{-\alpha})$ scales, i.e.,

$$\lim_{s\to\infty}\tilde{f}(s,\tau)\to F(\tau) \tag{31}$$

for a sequence $\{s_n\to\infty\}$. The argument is as follows.

Following Cornille and Simao[5] we define $f(s,t)$ as

$$f(s,t)=F(s,t)/F(s,0) , \tag{32}$$

where $F(s,t)$ is the scattering amplitude. Then

$$|f(s,t)|<\text{const} \tag{33}$$

for

$$t\in[-\chi,0] , \tag{34}$$

where χ is finite and $\chi>0$. Using (33), (34), (35), and the bounds on the Legendre polynomials we obtain

$$|f(s,t)|<C_1\exp[\sqrt{|t|}\,C_2(\ln s)^{\alpha/2}] , \tag{35}$$

where

$$|t|\leq\chi \tag{36}$$

and

$$C_i>0 .$$

Using $\tau=t(\ln s)^\alpha$ we then get

$$|\tilde{f}(s,\tau)|<C_i\exp[C_2\sqrt{|\tau|}] \tag{37}$$

for complex τ. Thus the sequence $\tilde{f}_1(s,\tau)$, $\tilde{f}(s_2,\tau),\ldots,\tilde{f}(s_n,\tau)$ with $s_n\to\infty$ constitutes a set of bounded equicontinuous functions and according to the Arzela theorem, there is a subsequence approaching a limit $F(\tau)$. It is not zero since

$$f(s,0)=1 . \tag{38}$$

Unfortunately, at present we are unable to show that the scaling function is nontrivial.

III. ANALYSIS OF SCATTERING DATA

Our arguments for the existence of a scaling function coupled with the sufficient condition of scaling in the form of (29) and (30) under the particular distribution of zeros lead one to analyze the world data on slope parameters and test whether the differential cross section actually does scale or not with $\tau=tb(s)$ as the scaling variable. To this end we parametrized $b(s)$ as

$$b(s)=C_1+C_2(\ln s)^\alpha$$

and made a least-χ^2 fit to the data for $b(s)$ (Ref. 15) of pp, $p\bar{p}$, $K^\pm p$, and $\pi^\pm p$ scattering. The best parameters of our fit are given in Table I.

Table I clearly shows that the slopes for the various processes analyzed favor a value of α which is consistent with the condition (29) and thus satisfies the requirements of scaling. It is important to note that α seems to saturate its lower bound and does not necessarily saturate its unitarity bound as is expected from the earlier works.[1,5,9] Hence as far as the analyzed processes are concerned, $tb(s)$, where $b(s)=C_1+C_2(\ln s)^\alpha$ should be the right scaling variable and if we use $\tau=t(\ln s)^2$, then the data should show deviations from scaling. In Figs. 1 to 12

$$\frac{d\sigma}{dt}(s,t)\bigg/\frac{d\sigma}{dt}(s,0)$$

is plotted vs $\tau=tb(s)$ and also vs $\tau=t(\ln s)^2$ for the processes under analysis and the plots confirm our observations.

TABLE I. The values of the parameters (in GeV units) for the slope of the diffraction peak for various processes. The slope is parametrized as $b(s)=C_1+C_2(\ln s)^\alpha$.

Process	C_1	C_2	α
pp	8.7230	0.3694	1.15 ± 0.3
$\bar{p}p$	12.29	0.035	$1.45^{+0.4}_{-0.63}$
K^+p	3.6	0.5	1.3 ± 0.002
K^-p	5.0378	0.5448	1.16 ± 0.1
π^+p	8.2801	0.0592	1.5 ± 0.07
π^-p	9.02	0.127	$1.3^{+0.015}_{-0.03}$

IV. ANALYSIS OF MODELS AND CONCLUSION

The several models that exist in the field to analyze the experimental data of pp, $p\bar{p}$, $K^\pm p$, and $\pi^\pm p$ scatterings use some assumptions which are reflected in the expressions for scattering amplitudes and differential cross sections obtained from

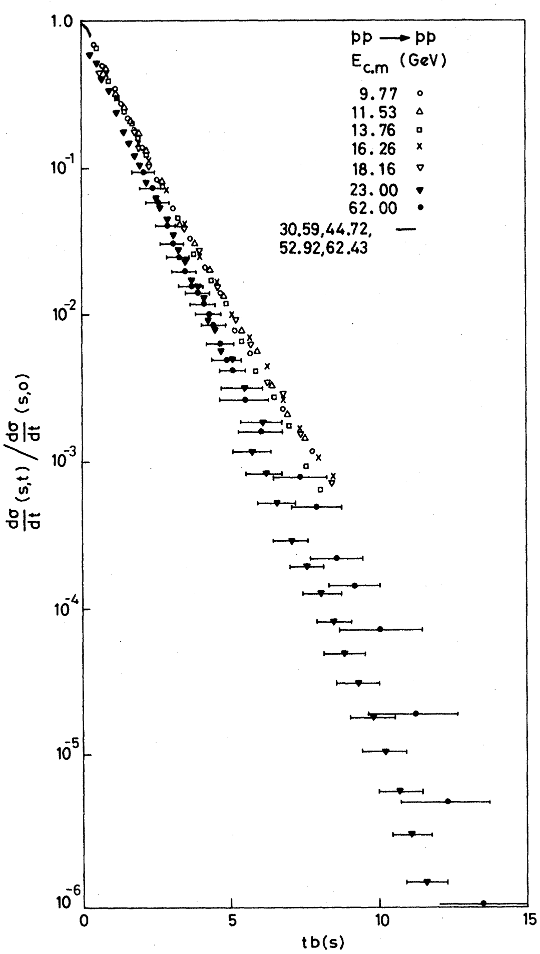

FIG. 1. Plot of $f(s,t)=(d\sigma/dt)(s,t)/(d\sigma/dt)(s,0)$ vs $\tau\equiv tb(s)$ for pp scattering.

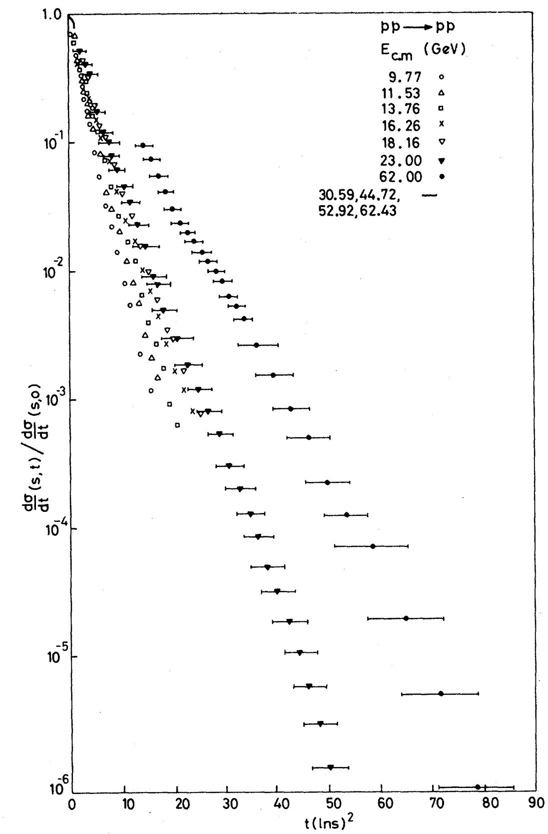

FIG. 2. Plot of $f(s,t)$ vs $t(\ln s)^2$ (t,s in GeV^2 here and in subsequent figures) for pp scattering.

the models. We will refer here to only two such models on pp scattering and analyze them to see what possible conclusions one can arrive at by imposing on them our observations that $\tau=t[C_1+C_2(\ln s)^\alpha]$ is a good scaling variable with α close to 1.25. These conclusions are supposed to demonstrate the validity of the assumptions used in the models.

(i) In a recent model by Schrempp and Schrempp,[16] the concept of hadrons as extended objects (bags) has been used to give a space-time description of hadronic interactions.[17] Introducing the shape of the interaction region as a general variable they have obtained the diffraction amplitude as

$$f\approx 2i\frac{J_i(R_\perp\sqrt{-T}\,)}{R_\perp\sqrt{-t}}\ . \tag{39}$$

This leads to

$$b(s)=R_\perp{}^2/8 \tag{40}$$

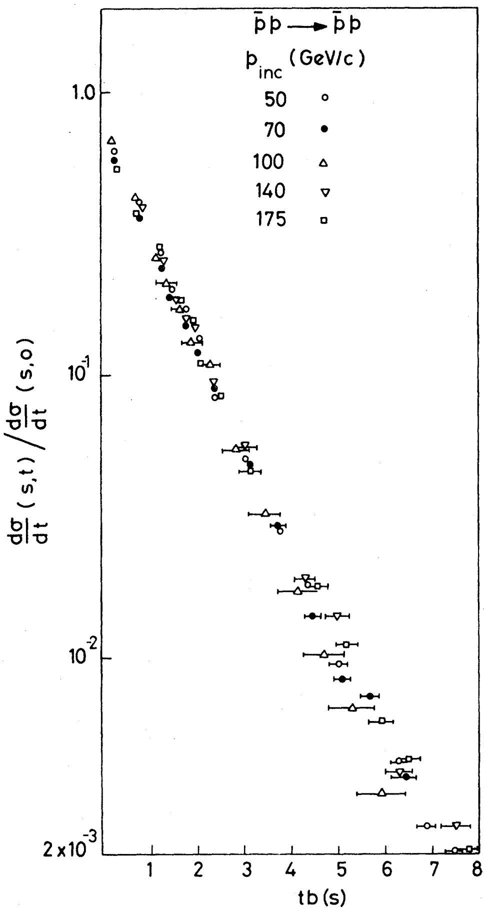

FIG. 3. Plot of $f(s,t)$ vs τ for $\bar{p}p$ scattering.

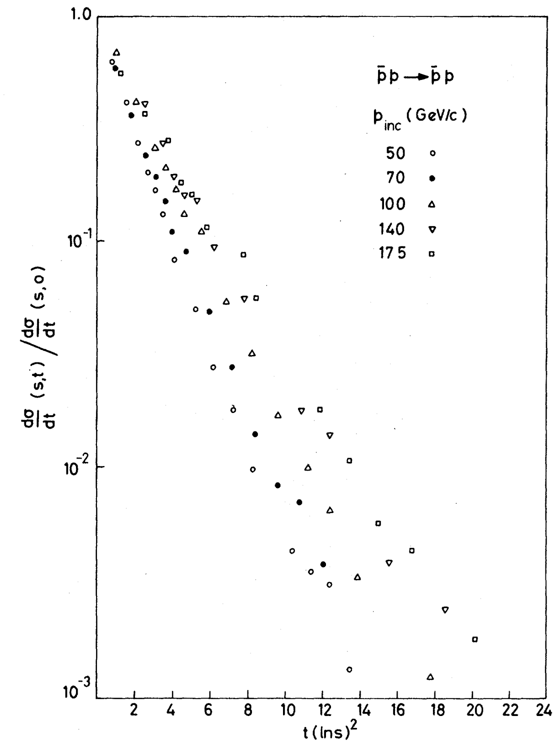

FIG. 4. Plot of $f(s,t)$ vs $t(\ln s)^2$ for $\bar{p}p$ scattering.

and

$$\frac{C(s)}{b^2(s)}=\frac{1}{3} . \tag{41}$$

It was shown in Ref. 13 that

$$\frac{C(s)}{b^2(s)}\simeq O((\ln s)^{2\alpha-4}) , \tag{42}$$

and with $\frac{5}{4}<\alpha\leq 2$ one obtains that (i) $C(s)/b^2(s)\simeq O((\ln s)^{-3/2})$ when $\alpha\simeq\frac{5}{4}$ and (ii) $C(s)/b^2(s)\simeq O(1)$ for $\alpha=2$. Thus $C(s)/b^2(s)\to 0$ in the first case and it is bounded by a constant in the second case. Thus Eq. (41) as obtained from this model is consistent with Eq. (42). However, in this model the s dependence of $R_\perp$ corresponds to $\alpha=2$ and not $\alpha=1.25$. So the $b(s)$ values obtained from this model are expected to show slightly higher values in the near-forward direction as is actually manifested in Fig. 2 of Ref. 7.

(ii) Assuming the scattering amplitude to be purely imaginary in the eikonal-type models[18–22] we represent[19] it as

$$A(s,t)=i\int_0^\infty [1-S(b)]J_0(b\sqrt{-t})b\,db . \tag{43}$$

Assuming further that

$$[1-S(b)]\sim e^{-\lambda b^2}b^n , \tag{44}$$

we obtain the amplitude as[23]

$$A(s,t)=\frac{\Gamma((n+2)/2)}{2[\lambda]^{(n+2)/2}} \times\exp\left[\frac{t}{\lambda}\right]{}_1F_1\left[-\frac{n}{2};1;-\frac{t}{\lambda}\right] . \tag{45}$$

Now using the normalization of Ref. 18, λ is determined to be

$$\lambda=\beta^{2/(n+2)} , \tag{46}$$

where

$$\beta=\frac{2\pi\Gamma((n+2)/2)}{\sigma_T} . \tag{47}$$

Then the slope and the curvature of the diffraction peak are given by

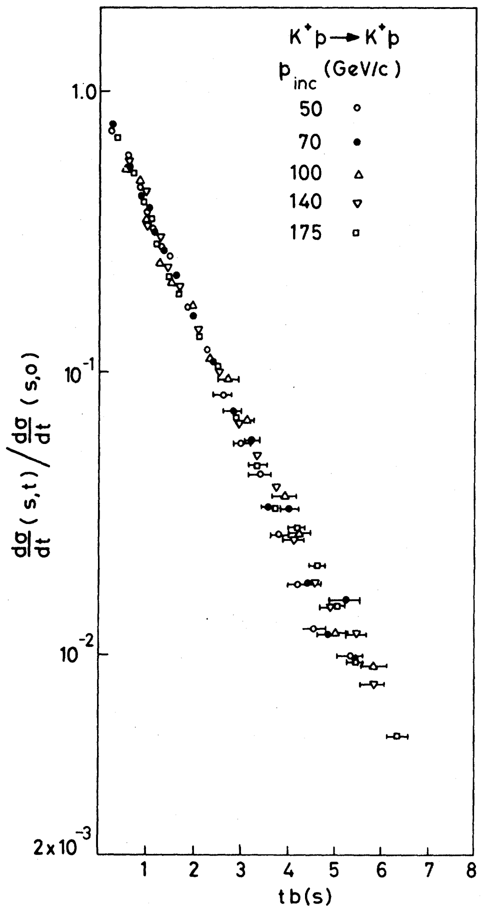

FIG. 5. Plot of $f(s,t)$ vs τ for K^+p scattering.

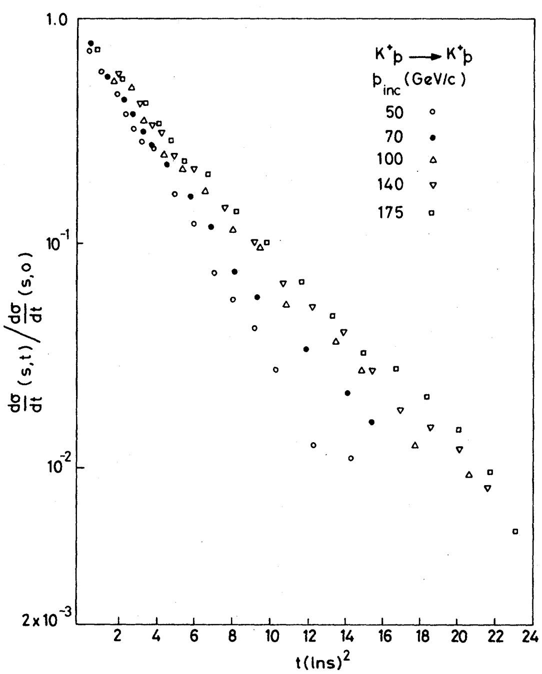

FIG. 6. Plot of $f(s,t)$ vs $t(\ln s)^2$ for K^+p scattering.

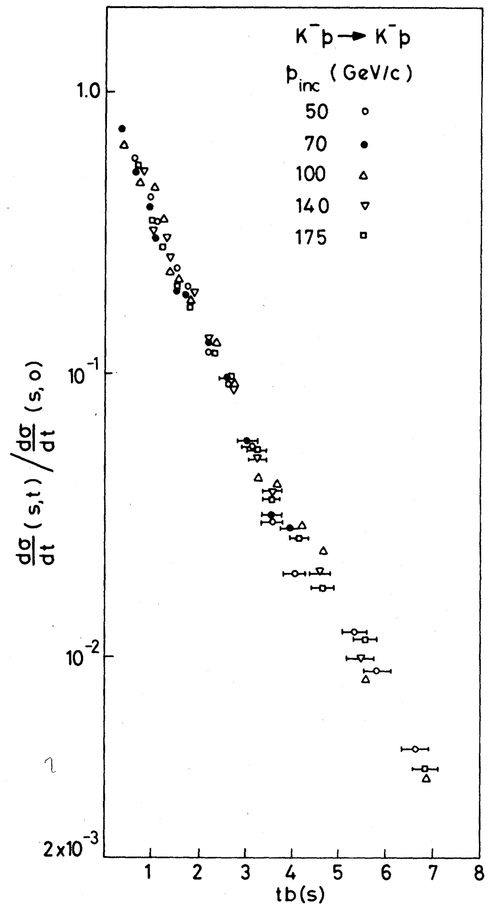

FIG. 7. Plot of $f(s,t)$ vs τ for K^-p scattering.

$$b(s)=\frac{2+n}{2\lambda}\,, \tag{48}$$

$$C(s)=b(s)\,, \tag{49}$$

and

$$\frac{C(s)}{b^2(s)}=\frac{2}{n+2}\left[\frac{2\pi\Gamma((n+2)/2)}{\sigma_T}\right]^{2/(n+2)}. \tag{50}$$

Note that if σ_T increases with energy, as the experimental situation indicates, then the Cornille-Martin criterion for scaling,

$$\frac{C(s)}{b^2(s)}\leq \text{const}\,, \tag{51}$$

is satisfied as long as $n>-2$. This bound is improved to

$$n\geq 0 \tag{52}$$

if σ_T saturates to Froissart bound, otherwise (48)

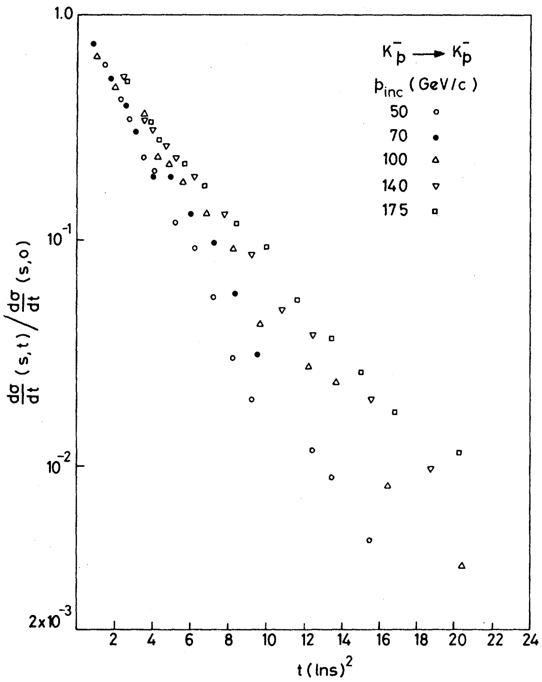

FIG. 8. Plot of $f(s,t)$ vs $(\ln s)^2$ for K^-p scattering.

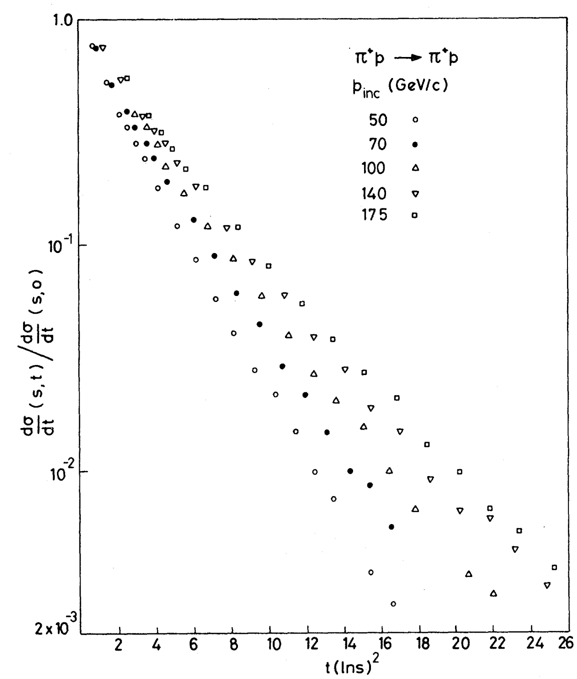

FIG. 10. Plot of $f(s,t)$ vs $t(\ln s)^2$ for π^+p scattering.

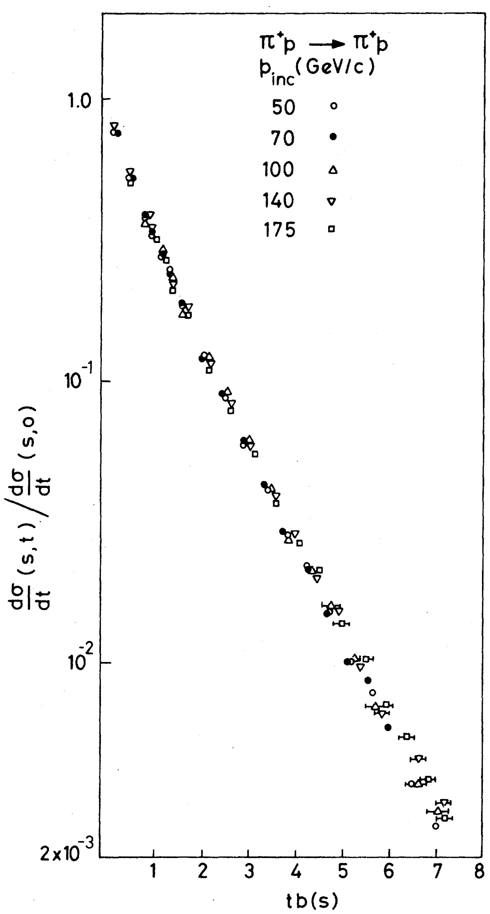

FIG. 9. Plot of $f(s,t)$ vs τ for π^+p scattering.

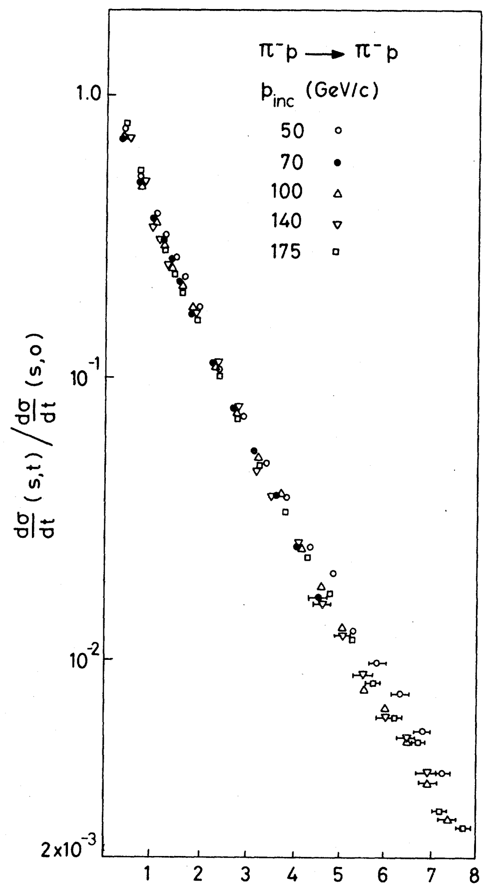

FIG. 11. Plot of $f(s,t)$ vs τ for π^-p scattering.

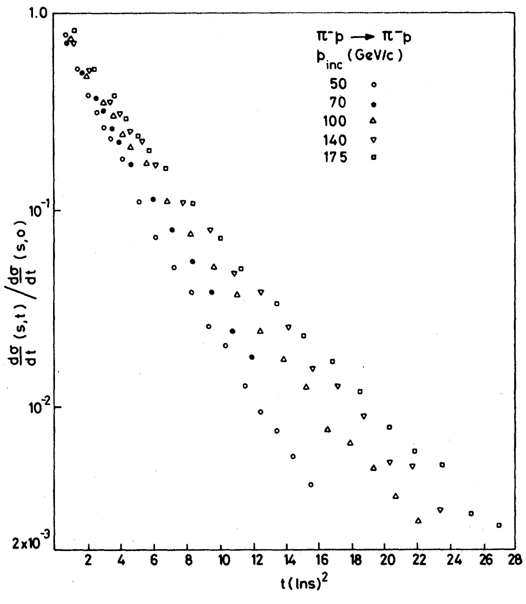

FIG. 12. Plot of $f(s,t)$ vs $t(\ln s)^2$ for $\pi^- p$ scattering.

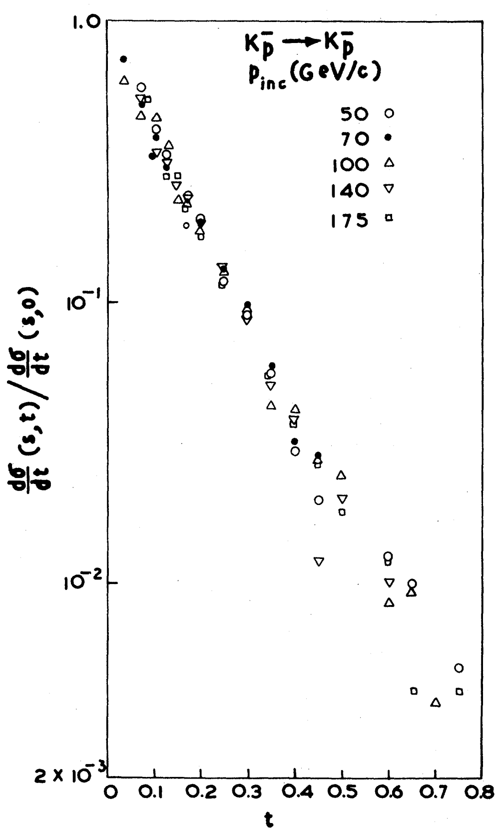

FIG. 13. Plot of $f(s,t)$ vs t for $K^- p$ scattering.

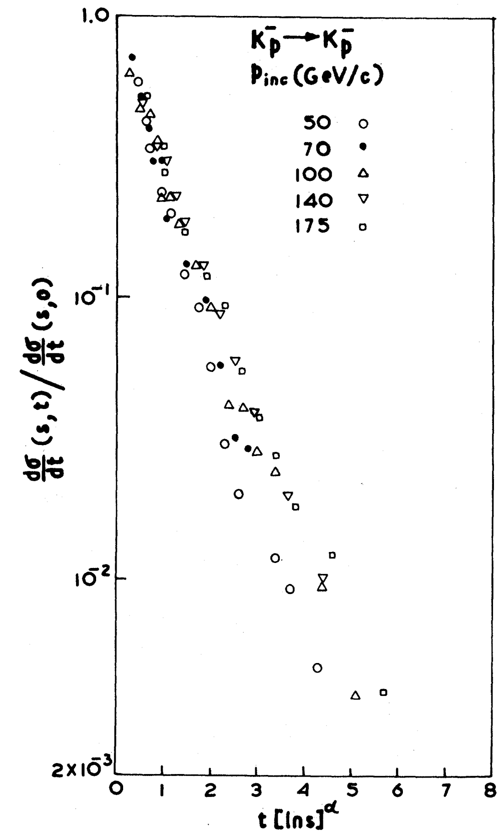

FIG. 14. Plot of $f(s,t)$ vs $t(\ln s)^{\alpha}$ for $K^- p$ scattering.

will violate the unitarity bound. However, if the eikonal-model amplitude has zeros [note that ${}_1F_1(-n/2;l;-t/\lambda)$ is a polynomial in t for even values of n], then one can make the bound (52) more defined by imposing the bounds of (29) and obtain

$$0 \le n \le 1.2\ . \tag{53}$$

Since our analysis shows that the existing experimental data prefer the lower bound of α, n should be close to 1.2 and this is expected to put strong constraints on the structure of $(1-S(b)]$ and therefore on the form of the amplitude.

In conclusion, we note that in recent times there have been various phenomenological attempts[24] to describe the high-energy differential cross section in the diffraction peak in terms of a single variable. On the other hand, using techniques of axiomatic field theory scaling could be established[1,5,9] if σ_T behaves like $(\ln s)^2$. We attempted in this paper to establish a correspondence between the two types of efforts in the sense that following heuristic procedure we tried to show the existence of a scaling function. Then we proposed a scaling variable $\tau = t[C_1 + C_2(\ln s)^\alpha]$ consistent with our observations. We proceeded to make a phenomenological fit to the data for various scattering processes and determine C_1, C_2, and α. We further demonstrated that $t(\ln s)^2$ is not a good scaling variable even at $E_{\text{c.m.}} = 62$ GeV. In fact, it looks obvious from the figures that as higher and higher energies are reached, deviations from scaling will be enhanced if one will use $t(\ln s)^2$ as the scaling variable. However, insofar as our proposed scaling variable is concerned we note that for $s_n \to \infty$,

$$\tau = t[C_1 + C_2(\ln s)^\alpha] \simeq C_2 t(\ln s)^\alpha .$$

On the other hand, at the energies considered by us tC_1 is the dominating term (Table I) compared with the $tC_s(\ln s)^\alpha$ term. Hence one may possibly conjecture that just t may be a good scaling variable at the energies we have considered. We have made illustrative plots for the K^-p process only with t as the scaling variable (Fig. 13) and also with $t(\ln s)^\alpha$ as the scaling variable (Fig. 14). It is heartening to note that the scaling is better for the variable $\tau = t[C_1 + C_2(\ln s)^\alpha]$ as seen in Fig. 7 than in Figs. 13 and 14. But we do agree that the energy regions where the values of $(\ln s)^\alpha$ will be comparable to C_1/C_2 will be the regions where the universal validity of our scaling variable can be tested. However, the figures optimistically demonstrate that $t[C_1 + C_2(\ln s)^\alpha]$ perhaps will be a good scaling variable at those energies also. One of the important conclusions from our fit is that α saturates its lower bound, i.e., 1.25. We then demonstrated that this observation not only can be useful to investigate the validity of models of diffraction scattering insofar as the scaling properties of their amplitudes are concerned, but it also can impose constraints on the form of these amplitudes.

ACKNOWLEDGMENTS

We are extremely thankful to Professor U. Amaldi for supplying us with some of the latest unpublished data on pp scattering. The computational facilities of the Computer Center, Utkal University, is gratefully acknowledged. A part of this work was done while one of us (J.M.) was visiting Theory Division Rutherford Laboratory. He would like to thank Roger Phillips and the members of the theory group for their gracious hospitality.

[1]G. Auberson, T. Kinoshita, and A. Martin, Phys. Rev. D 3, 3185 (1971).

[2]H. Epstein, V. Glaser, and A. Martin, Commun. Math. Phys. 13, 257 (1969).

[3]R. J. Eden and G. D. Kaiser, Nucl. Phys. B28, 253 (1971).

[4]H. Cornille, Lett. Nuovo Cimento 4, 267 (1970).

[5]H. Cornille and F. R. A. Simao, Nuovo Cimento 5A, 138 (1971).

[6]S. M. Roy, Phys. Rep. 5, 125 (1972).

[7]H. Cornille and A. Martin, Nucl. Phys. B115, 163 (1976).

[8]G. Mahoux, Phys. Lett. 65B, 139 (1976).

[9]G. Auberson and S. M. Roy, Nucl. Phys. B117, 322 (1977).

[10]J. Dias de Deus, Nucl. Phys. B59, 231 (1973).

[11]A. J. Buras and J. Dias de Deus, Nucl. Phys. B71, 481 (1974).

[12]V. Barger, J. Luthe, and R. J. N. Phillips, Nucl. Phys. B88, 237 (1975).

[13]J. Maharana, Commun. Math. Phys. 58, 195 (1978).

[14]E. C. Titchmarsh, *The Theory of Functions* (Oxford University Press, London, 1939), p. 171.

[15]N. Kwak *et al.*, Phys. Lett. 58B, 233 (1975); C. W. Akerlof *et al.*, Phys. Rev. D 14, 2864 (1976); D. H. Ayres *et al.*, *ibid.* 11, 3105 (1977); U. Amaldi *et al.*, Phys. Lett. 66B, 390 (1977); J. Dorenbosk, Ph.D. thesis, 1977 (unpublished); A. Schiz *et al.*, Phys. Rev. D 24, 26 (1981).

[16]B. Schrempp and F. Schrempp, in *Deep Scattering and Hadronic Structures,* proceedings of the XII Rencontre de Moriond, edited by J. Trân Thanh Vân (Editions Frontiéres, Dreux, 1977), p. 67; Phys. Lett. 70B, 88 (1977).

[17]J. Koplik and A. H. Mueller, Phys. Rev. D 12, 3638 (1975); F. E. Low, *ibid.* 12, 163 (1975); G. Preparate and N. Craigie, Nucl. Phys. B102, 478 (1976).

[18]T. T. Chou and C. N. Yang, Phys. Rev. Lett. 20, 1213 (1968); Phys. Rev. 170, 1591 (1968); University of Georgia report, (1977) (unpublished).

[19]L. Durand and R. Lipes, Phys. Rev. Lett. 20, 637 (1968).

[20]F. Hayot and V. P. Sukhatme, Phys. Rev. D 10, 2183 (1974).

[21]T. Maehara, T. Yanagita, and M. Yonezawa, Prog. Theor. Phys. 57, 1097 (1977).

[22]A. N. Chao and C. H. Yang, Phys. Rev. D 8, 2063 (1973).

[23]G. N. Watson, *A Treatise on the Bessel Functions* (Cambridge University Press, London 1966), p. 394.
[24]V. Barger, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974,* edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England , 1974); J. Dias deDeus, talk presented at 15th Zakopane Summer School, Poland, 1975 (unpublished).