Strong gravity: An approach to its source

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We study the source of strong gravity. In the first part we find analytic expressions for a hadron source, in which an electric charge and a scalar field are assigned to it, thus obtaining an extended Reissner-Nordström-scalar-field solution to strong gravity. We then show the correspondence of a massive charged scalar-field wave equation with a generalized Gross-Pitaevskii equation for superfluidity. This allows us to relate the parton-parton average (gluonlike) potential strength U with the overall (confining) strong-interaction strength κ. Hadron structure arises, in our model, unlike QCD, from the interplay of two different strong-coupling constants.

I. INTRODUCTION

Some progress has been achieved in the theory of strong gravity as formulated in its original f-g form[1]:

(a) Short-range solutions were obtained in its asymptotic limit. These solutions were shown to resemble Yukawa's potentials for the nuclear force.[2,3]

(b) Exact solutions for the potentials were obtained. These solutions where shown to be of long-range nature, and were called type I,[4] in order to distinguish from the solutions mentioned in (a), which were called type-II solutions.

A new start was made in a recent work,[5] in which simpler field equations were solved:

$$R^{f}_{\mu\nu}-\tfrac{1}{2}f_{\mu\nu}R^{f}-\Lambda_f f_{\mu\nu}=-\frac{8\pi}{c^4}G_f T^{H}_{\mu\nu}\ , \qquad (1)$$

where the Riemann tensor and scalar are written in terms of the tensor field for strong gravity $f_{\mu\nu}$; both the cosmological constant Λ_f and the coupling constant G_f have values given by strong-interaction physics to be discussed below. In other words, a geometrical background was assumed for strongly interacting matter, and an attempt was made to understand the energy-momentum tensor for hadronic matter:

$$T^{H}_{\alpha\beta}=\rho u_\alpha u_\beta+(p/c^2)(u_\alpha u_\beta-f_{\alpha\beta})\ . \qquad (2)$$

This classical source was studied in connection with type-II solutions, in the limit of vanishing f-meson mass. Two options are presented to us: Either we try to construct more realistic sources in a semiclassical approach by bringing in gradually quantum features of the hadronic source, therefore generalizing the well-known Reissner-Nordström solution,[6] or alternatively we may study the question of whether hadronic matter could show some correlations of the type which occurs in superfluidity, and hence a source such as Eq. (2) may be a reasonable starting point for strong gravity. These two questions shall be touched upon in this work. Clearly, the alternative to our approach is to proceed with a full quantization of strong gravity. This, however, is beyond the scope of the present work, which in both options mentioned above studies the semiclassical problem of a hadronic source in a classical background of Riemannian geometry.[7]

The remaining part of this paper proceeds as follows. In Sec. II, we take the hadronic energy-momentum tensor as

$$T^{H}_{\alpha\beta}=T^{Y}_{\alpha\beta}+T^{\rm EM}_{\alpha\beta}\ . \qquad (3)$$

We shall consider the hadron in first approximation as a point particle placed at the origin of the polar system of coordinates producing spherical symmetry, somewhat like the Born-Oppenheimer approximation to the atomic nucleus. We assign the hadron an electromagnetic (EM) and a massive scalar Yukawa (Y) field. We complete this section with a discussion of the scalar as well as the Maxwell field. In Sec. II, we present an exact first integration of the generalized Reissner-Nordström-scalar-field differential equations; we conclude this section with a discussion of a related work.[8] Then, in Sec. IV, we let the scalar field have a self-interaction and find the generalization of the Hartree liquid model for superfluidity,[9–11] coupled to (strong) curved space. Finally, in Sec. V we discuss our results with special emphasis on the need for strong-gravity theory to incorporate into the study of the source the beautiful results from lepton-hadron and hadron-hadron scattering, which in terms of the quark-parton model have given us remarkable insight into the hadron structure.[12,13]

II. THE SOURCE OF STRONG GRAVITY

A. The scalar field

We study the massive scalar field ϕ, which will be associated with the Yukawa field. First we suppose that ϕ and its complex conjugate ϕ^* satisfy the field equations

$$(\partial^\mu\partial_\mu+m^2)\phi=(\partial^\mu\partial_\mu+m^2)\phi^*=0 . \tag{4}$$

In this section, as well as in Sec. III, we have chosen units so that $\hbar=c=1$, with metric signature $(+---)$.

The minimal coupling of ϕ to the strong-gravity field is obtained as

$$(\nabla^\mu\nabla_\mu+m^2)\phi=(\nabla^\mu\nabla_\mu+m^2)\phi^*=0 , \tag{5}$$

where we have replaced ordinary derivatives ∂^μ by covariant derivatives ∇^μ. We may evaluate the corresponding flat-space energy-momentum tensor,

$$T^Y_{\alpha\beta}=\frac{1}{4\pi}[\partial_\alpha\phi^*\partial_\beta\phi+\partial_\alpha\phi\partial_\beta\phi^* -\eta_{\alpha\beta}(\partial^\mu\phi^*\partial_\mu\phi-m^2\phi^*\phi)] , \tag{6}$$

where $\eta_{\alpha\beta}$ denotes the Minkowski metric. The corresponding strong-gravity energy-momentum tensor is obtained by replacing the Minkowski metric by the strong-gravity metric

$$T^Y_{\alpha\beta}=\frac{1}{4\pi}[\partial_\kappa\phi^*\partial_\beta\phi+\partial_\alpha\phi\partial_\beta\phi^* -f_{\mu\nu}(\partial^\mu\phi^*\partial_\mu\phi-m^2\phi^*\phi)] . \tag{7}$$

We restrict ourselves to a simple source which has spherical symmetry due to a point hadron at the origin of a polar system of coordinates, such that

$$\frac{\partial\phi}{\partial\Phi}=\frac{\partial\phi}{\partial\theta}=\frac{\partial\phi}{\partial x^0} , \tag{8}$$

where the temporal coordinate $x^0=ct$. Further, with this hypothesis, the metric assumes the form

$$f_{\mu\nu}=\mathrm{diag}(e^\nu,-e^\lambda,-r^2,-r^2\sin^2\theta) . \tag{9}$$

The nonvanishing elements of $T^Y_{\alpha\beta}$ may be calculated from Eq. (7):

$$T^Y_{00}=\frac{1}{4\pi}(e^{\nu-\lambda}\partial_r\phi^*\partial_r\phi+m^2e^\nu\phi^*\phi) , \tag{10}$$

$$T^Y_{11}=\frac{1}{4\pi}(\partial_r\phi^*\partial_r\phi-m^2e^\lambda\phi^*\phi) , \tag{11}$$

$$T^Y_{22}=\frac{-1}{4\pi}(r^2e^{-\lambda}\partial_r\phi^*\partial_r\phi+m^2r^2\phi^*\phi) , \tag{12}$$

$$T^Y_{33}=\frac{-1}{4\pi}(r^2e^{-\lambda}\partial_r\phi^*\partial_r\phi+m^2r^2\phi^*\phi)\sin^2\theta . \tag{13}$$

B. The electromagnetic field

We next assign a charge e to the point hadron. In order to make this paper self-contained, although the form of our equations is similar to the Reissner-Nordström equations, we replace the strong-gravity field by that of ordinary gravity. The coupling of Maxwell equations is obtained through the covariant derivatives to give

$$\nabla_\nu F^{\mu\nu}=\frac{4\pi}{c}j^\mu , \tag{14}$$

where $F^{\mu\nu}$ is the electromagnetic tensor field and j^μ denotes the current density. Further,

$$\nabla_\mu F_{\alpha\beta}+\nabla_\alpha F_{\beta\mu}+\nabla_\beta F_{\mu\nu}=0 . \tag{15}$$

Finally, the current density satisfies the conservation law

$$\nabla_\mu j^\mu=0 . \tag{16}$$

The corresponding $T^{\mathrm{EM}}_{\mu\nu}$ in curved space is

$$T^{\mathrm{EM}}_{\mu\nu}=-\frac{1}{4\pi}(F_\mu{}^\alpha F_{\nu\alpha}-\tfrac{1}{4}f_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}) . \tag{17}$$

We have raised indices with the $f_{\mu\nu}$ metric: $F_\mu{}^\alpha=f^{\alpha\nu}F_{\mu\nu}$. Since the point hadron is placed at the origin of the coordinate system, the electromagnetic field will then correspond to an electrostatic field in the x_1 direction, with radial symmetry:

$$F_{\mu\nu}=E(r)\begin{bmatrix}0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} . \tag{18}$$

We are now in a position to solve this strong Reissner-Nordström system of equations with a null current density to obtain

$$E(r)=e^{(\nu+\lambda)/2}\frac{\epsilon}{r^2} , \tag{19}$$

where ϵ is an integration constant, which in the classical case is identified with the electric charge, using the Minkowski boundary conditions. We may then obtain $T^{\mathrm{EM}}_{\mu\nu}$ from the above equations:

$$T_{\mu\nu}^{\rm EM}=\frac{1}{8\pi}\frac{\epsilon^2}{r^4}{\rm diag}(e^{\nu},-e^{\lambda},-r^2,-r^2\sin^2\theta)\ . \tag{20}$$

C. Derivation of the hadronic energy-momentum tensor

From what has been said above, in subsections A and B, we may add Eqs. (7) and (20) in order to reconstruct our semiclassical model of the hadronic source. This, in turn, leads us to the problem of solving the strong Reissner-Nordström-scalar-field differential equations. This will be the topic of the following section.

III. EXACT ANALYTIC EXPRESSIONS FOR THE STRONG-GRAVITY POTENTIALS

Under the assumption of spherical symmetry, the nontrivial field equations (1) are as follows:

$$-e^{\nu-\lambda}\left[\frac{\lambda'}{r}-\frac{1}{r^2}\right]-\frac{e^{\nu}}{r^2}+\Lambda_f e^{\nu}=-\frac{A}{r^4}e^{\nu}-2B(e^{\nu-\lambda}\partial_r\phi^*\partial_r\phi+m^2e^{\nu}\phi^*\phi)\ , \tag{21}$$

$$-\left[\frac{\nu'}{r}+\frac{1}{r^2}\right]+\frac{e^{\lambda}}{r^2}-\Lambda_f e^{\lambda}=\frac{A}{r^4}e^{\lambda}-2B\left[\frac{d}{dr}\phi^*\frac{d}{dr}\phi-m^2e^{\lambda}\phi^*\phi\right]\ , \tag{22}$$

$$-r^2e^{-\lambda}\left[\frac{\nu''}{2}-\frac{\lambda'\nu'}{4}+\frac{\nu'^2}{4}+\frac{\nu'-\lambda'}{2r}\right]-\Lambda_f r^2=-\frac{A}{r^2}+2B\left[r^2e^{-\lambda}\frac{d}{dr}\phi^*\frac{d}{dr}\phi+r^2m^2\phi^*\phi\right]\ , \tag{23}$$

where A and B are defined by

$$A=G_f\epsilon^2/c^4,\quad B=G_f/c^4\ . \tag{24}$$

The fourth equation has been omitted, as usual, since it gives no information beyond that of Eq. (23). Multiplying Eq. (21) by $-e^{-(\nu-\lambda)}$, and (22) by (-1), adding and integrating the resulting equation, we obtain

$$\nu+\lambda=4B\int r\,dr\frac{d}{dr}\phi^*\frac{d}{dr}\phi+\ln\Omega\ , \tag{25}$$

where $\ln\Omega$ is the integration constant. We next multiply Eq. (21) by $-re^{(-\nu+2\lambda)}$ (both sides), and make the substitution $F=e^{\lambda}$, to obtain

$$F'+\left[\frac{1}{r}-\Lambda_f r-\frac{A}{r^3}-2Bm^2r\phi^*\phi\right]F^2-\left[\frac{1}{r}+2Br\frac{d}{dr}\phi^*\frac{d}{dr}\phi\right]F=0\ . \tag{26}$$

It is convenient to define the expressions in the parentheses of Eq. (26) as $q(r)$ and $p(r)$, respectively; for, in this case we may readily identify Eq. (26) as the well-known Ricatti equation

$$F'+q(r)F^2-p(r)F=0\ , \tag{27}$$

whose solution is[14]

$$[F(r)]^{-1}=\left[h+\int dr\,q\exp\left[\int dr\,p\right]\right]\times\exp\left[-\int p\,dr\right]\ , \tag{28}$$

where h is an integration constant. In terms of the original functions, we find

$$e^{-\lambda}=\left\{h+\int dr\left[\frac{1}{r}-r\Lambda_f-\frac{A}{r^3}-2Bm^2r\phi^*\phi\right]\exp\left[\int dr\left[\frac{1}{r}+2Br\frac{d}{dr}\phi^*\frac{d}{dr}\phi\right]\right]\right\}\times\exp\left[-\int dr\left[\frac{1}{r}+2Br\frac{d}{dr}\phi^*\frac{d}{dr}\phi\right]\right]\ . \tag{29}$$

The integrations in the exponentials of the above equation may be performed partially to yield

$$e^{-\lambda}=\left[\frac{h}{r}+\frac{1}{r}\int dr\left[1-r^2\Lambda_f-\frac{A}{r^2}-2Bm^2r^2\phi^*\phi\right]\exp\left[2B\int r\,dr\frac{d}{dr}\phi^*\frac{d}{dr}\phi\right]\right]$$

$$\times\exp\left[-2B\int r\,dr\frac{d}{dr}\phi^*\frac{d}{dr}\phi\right], \tag{30}$$

which may be considerably simplified by defining the integral

$$I(r)=2B\int r\,dr\frac{d}{dr}\phi^*\frac{d}{dr}\phi\ , \tag{31}$$

so that

$$e^{\lambda}=\left[\frac{h}{r}+\frac{1}{r}\int dr\left[1-r^2\Lambda_f-\frac{A}{r^2}-2Bm^2r^2\phi^*\phi\right]e^{I(r)}\right]^{-1}e^{I(r)}\ . \tag{32}$$

On the other hand, in Eq. (25) we may solve for λ, which, in turn, gives from Eq. (32)

$$e^{\nu}=\left[\frac{h}{r}+\frac{1}{r}\int dr\left[1-r^2\Lambda_f-\frac{A}{r^2}-2Bm^2r^2\phi^*\phi\right]e^{I}\right]e^{I}\ . \tag{33}$$

In writing Eq. (33) we have redefined the time coordinate $x^0=ct$, rather as $\bar{x}^0=(\Omega)^{-1/2}x^0$, in order to incorporate the overall factor Ω, thus retaining the simpler expression given in Eq. (33).

As in the original case of Schwarzschild, we must be certain to have a mathematically consistent solution, since Eq. (23) has not been used so far. In Ref. 15, it has been verified that our expressions for e^{λ} and e^{ν} satisfy Eq. (23); it is a straightforward, although very tedious, calculation.

We remark that Raut and Sinha[8] solve approximately the field equations with a source, the exact form of which has been given in Eq. (7), and were unable to obtain exact first integrals, since their metric was asymptotically non-Minkowskian, unlike the work of this section. Yet, their work together with ours illustrates the possibilities of strong gravity, since they succeeded in deducing the strong-interaction coupling constant $g^2/\hbar c$, to a reasonable approximation. We believe that our analytic solutions give a foundation to more accurate work along these lines.

On the other hand, we also believe that progress will not occur by ever-increasing mathematical accuracy in the analytic study of hadron sources, which yield eventually exact type-II solutions to the strong-gravity potentials. Rather, we should accept the overwhelming experimental evidence,[12,13] and attempt to incorporate sea quarks in the source, together with a small set of valence quarks. We now turn our attention to this problem.

IV. A SUPERFLUID HADRON SOURCE

One application of the approach of the previous two sections is in the problem of hadron structure, in which the quantum liquid aspects are also taken into account.[16–24]

Let us consider the charged scalar field, but take the electromagnetic field as being decoupled initially, for simplicity. The action for the scalar field may then be written as[7]

$$S=\int d^4x\,\mathscr{L}\ , \tag{34}$$

where the Lagrangian density has been written in units $\hbar=c=1$, and with metric signature $(-+++)$

$$\mathscr{L}=-\frac{1}{4\pi}\sqrt{-f}\,(f^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi+\xi R^f|\phi|^2+m^2|\phi|^2)\ , \tag{35}$$

where ξ is an arbitrary real number, and m denotes the mass. From the expression for the energy-momentum tensor,

$$T^{\mu\nu}(x)=\frac{2}{\sqrt{-f}}\frac{\delta S}{\delta f_{\mu\nu}}\ , \tag{36}$$

we find for the charged scalar field, after the variation, that

$$T^H_{\alpha\beta}=\partial_\alpha\phi^*\partial_\beta\phi+\partial_\alpha\phi\partial_\beta\phi^* -f_{\alpha\beta}(\partial^\mu\phi^*\partial_\mu\phi-m^2\phi^*\phi) +2\xi(f_{\alpha\beta}\nabla_\mu\nabla^\mu-G^f_{\alpha\beta}-\nabla_\alpha\nabla_\beta)\phi^*\phi\ , \tag{37}$$

where ∇_α denotes the covariant derivative. The field equation is

$$-f^{\mu\nu}\nabla_\mu\nabla_\nu\phi+(m^2+\xi R^f)\phi=0 , \tag{38}$$

which allows us to write the trace of the energy-momentum tensor

$$T^\mu{}_\mu=12(\xi-\tfrac{1}{6})\partial^\alpha\phi^*\partial_\alpha\phi-12(\xi-\tfrac{1}{3})m^2\phi^*\phi -12\xi(\xi-\tfrac{1}{6})R^f\phi^*\phi . \tag{39}$$

We study the choice of parameters $m\neq0$, $\xi=\frac{1}{6}$. This leads to the trace anomaly

$$T^\mu{}_\nu=2m^2\phi^*\phi . \tag{40}$$

In this case, from the field equation (1), with zero cosmological constant (for simplicity), we find that $R^f=\kappa T$, where $\kappa=(8\pi/c^4)G_f$ and, therefore, that

$$R^f=2\kappa m^2\phi^*\phi , \tag{41}$$

which, in turn, gives the following scalar field equation:

$$\Box\phi+m^2\phi=-\tfrac{1}{3}\kappa m^2|\phi|^2\phi , \tag{42}$$

where $-f^{\mu\nu}\nabla_\mu\nabla_\nu$ denotes the d'Alembertian. Equation (42) is the generalization into curved space of the Hartree liquid model of superfluidity,[9–11] which in the nonrelativistic case may be written as

$$i\partial_t\phi=-\frac{1}{2m^2}\partial_r{}^2\phi+\lambda\phi +|\phi|^2\int U(\vec{r}-\vec{r}\,')\phi(\vec{r}\,')d\vec{r}\,' . \tag{43}$$

In order to explain the notation, and for simplicity, we suppose that we are dealing with a parton system for the source of strong gravity, in which there is an averaged parton-parton peaked interaction

$$U(\vec{r}-\vec{r}\,')=U\delta^{(3)}(\vec{r}-\vec{r}\,') . \tag{44}$$

Hence, we have the simpler Gross-Pitaevskii equation

$$\left[\frac{1}{i}\partial_t-\frac{1}{2m}\partial_r{}^2\right]\phi+\lambda\phi=-U|\phi|^2\phi , \tag{45}$$

where λ denotes the chemical potential, U denotes the strength of the average interaction, and ϕ denotes the condensate macrowave function. We remark that in the context of superfluidity of He II, this equation (42) has been used in the context of (weak) gravity.[25] We should emphasize that the mathematical difference between the differential equations (42) and (45) is due to the fact that the nonrelativistic expression for the energy in the Schrödinger picture was used in driving Eq. (45), whereas the special-relativistic result for the energy in the Schrödinger picture was used in Eq. (42), and then the Minkowski metric is replaced by the strong-gravity metric. From a comparison of equivalent terms in Eqs. (42) and (45) we obtain $m^2\sim\lambda$ and $\kappa m^2/3\sim U$. In other works, we have related the average condensate interaction strength U with the confining strong-gravity interaction κ, through

$$U=\kappa\lambda/6 . \tag{46}$$

We notice that an important feature of Eq. (46) is that we have eliminated the m^2 parameter, which would be difficult to interpret in the superfluid hadron.

From the previous work in the field of hadronic quantum liquids already referred to, it seems natural to identify the superfluid with strongly correlated Cooper[26] or more accurately, BCS quark-antiquark pairs[27] in the sea. We shall consider this question in more detail in the last section.

V. DISCUSSION AND CONCLUSION

We have shown in Sec. III how exact first integrals may be obtained for the strong-interaction potentials, in the case in which the infinitely heavy hadron source (Born-Oppenheimer approximation) produces spherical symmmetry. We then went on to consider a second, more promising interpretation of the scalar field. In this second interpretation we provided self-interactions for the scalar field. By replacement of ordinary derivatives with covariant derivatives, we ensured coupling to the gravitational field. In this approach the electric charge, which earlier had been supplied through a generalized Reissner-Nordström mechanism, would now be carried by a few valence quarks: In the context of spherical symmetry, test particles (our valence quarks) of fermionic nature can form bound states,[28,29] through the strong effect of the gravitational field. We envisage hadron structure as a hadronic source in a state of superfluidity (identified with the sea quarks), curving space-time by means of our strong-gravity field equations; this, in turn, has the effect of confining a few test particles (the valence quarks), which then are responsible for carrying the quantum numbers of the hadron (for instance, electric charge, baryon number, strangeness, charm, etc.).

Two further points should be stressed:

(a) We have taken the initiative of masking all the gluon effects by means of the average Hartree-Fock potentials $V_{kk'}$ of the underlying pairing theory, which produces the bosonlike stages ϕ in the simpler Hartree liquid formalism adopted here [cf. Eq. (42)]. Phonons play a similar role in the theory of superconductivity.[27]

(b) By letting the self-interaction scalar field play the role of the missing sea, we are able to incorporate processes in which the sea plays a dominant role, as in the case of the Drell-Yan process[30]

$$pp \to \mu^+\mu^- + \text{anything}\ , \tag{47}$$

where a large contribution to the cross section comes from annihilation of a valence quark with an antiquark in the sea.

We may conclude by saying that 12 years after the theory of strong gravity was formulated, high-energy physics has made an enormous jump. At that time the parton model was also being born.[31] We now have a fairly complete insight into the quark probability distributions $u(x)$, $\bar{u}(x)$, $d(x)$, etc., for finding quarks and antiquarks with fractional momentum x inside the proton.[12] Today, the most pressing need in the theory of strong gravity is to make a beginning in *understanding its source.* In this work we have made a modest approach in making some progress along this particular line of development.

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