Limits on the composite structure of the τ lepton and quarks from anomalous-magnetic-moment measurements in e^+e^- annihilation

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One of the best tests for compositeness of leptons or quarks is the presence of an anomalous magnetic moment F_2. We determine limits on F_2 for the τ, from $e^+e^- \to \tau^+\tau^-$ at PETRA, of $F_2^\tau \leq 0.02$. We also present limits on the F_2 of the quarks. We suggest lower-energy experiments on the angular distribution to give an independent measurement of F_2^τ. Future experiments near the Z^0 peak are discussed.

There have been many recent papers in which leptons and quarks are considered to be composites of more fundamental constituents. There are some very strong constraints on such models, in particular, the phenomenal agreement[1] between experiment and present theory (electroweak plus hadronic) for the anomalous magnetic moments F_2 of the electron (to a part in 10^{10}) and the muon (to a part in 10^8). The corrections to F_2 for a fermion of mass m from a composite structure are[2] (a) in general $O(m/m_c)$ where m_c is the "constituent mass" and (b) for certain situations $O(mm_f/m_B^2)$ for $m_f << m_B$ where f and B are the constituent fermion and boson (spin-0 or spin-1) particles. As we obtain below, present limits on F_2 for the τ, from total-cross-section measurements of $e^+e^- \to \tau^+\tau^-$ at PETRA,[3] are $F_2^\tau \leq 0.02$. We also obtain similar limits on quark anomalous magnetic moments from PETRA limits[4] on the deviation of R_{had} from QCD. We suggest low-energy experiments on the angular distribution of $e^+e^- \to \tau^+\tau^-$ to give an independent measurement of F_2^τ of comparable or somewhat better accuracy. Future experiments at LEP near the Z^0 peak are discussed for the τ and can be applied similarly to determine quark anomalous magnetic moments. Although the above very strong limits on composite contributions to the electron or muon moments exist, there are models[5] in which the electron family (e, ν_e, u, d) and the muon family (μ, ν_μ, s, c) are elementary, whereas the τ family (τ, ν_τ, b, t) is composite. If a new composite mass scale exists at a few hundred GeV, for example, it could give an anomalous magnetic moment to the τ or b quark of a few percent. Thus it is useful to stress the importance of obtaining separate although much less accurate limits for the F_2's of the τ and the quarks in addition to those for the electron and muon.

The F_2 moment is present at any $|q^2| < \Lambda_c^2$ where Λ_c is the composite mass scale and F_2 may be observed by its effect on the total cross section or in the angular dependence of the produced particles in e^+e^- annihilation. At high q^2 the F_2 contribution grows faster with q^2 than the usual F_1^2 or $(g_V^2+g_A^2)$ contribution and is of the order $(q^2/m^2)F_2^2 \sim q^2/\Lambda_c^2$ with respect to F_1^2. Thus the F_2 contribution for $q^2 << \Lambda_c^2$ is of the same order as the form factor $(1+q^2/\Lambda_c^2)^{-1}$ corrections and it may be detected or bounded simultaneously by comparison with the experimental data corrected for QED and QCD effects. The F_2 contribution gives a different angular dependence than F_1^2 or $(g_A^2+g_V^2)$ and thus provides a complementary test for compositeness to that with the form factor. In this paper we present the details of the effects of F_2 on the total and differential cross sections in $e^+e^- \to \tau^+\tau^-$ or $\bar{q}q$ via photons at low or high q^2 and at the Z^0 peak. We also analyze at low q^2 the directional dependence of the charged particle coming from the two- or three-body decay of the τ.

We begin with the production of $\tau^+\tau^-$ or $q\bar{q}$ of mass m and charge Q in e^+e^- via the electromagnetic current. The unpolarized differential cross section with $\beta=(1-4m^2/q^2)^{1/2}$ is

$$d\sigma/d\cos\theta = Q^2(2\pi\alpha^2/3q^2)\beta \times [G_0(q^2)+P_2(\cos\theta)G_2(q^2)] \quad , \quad (1)$$

where

$$G_0(q^2)=F_1^2(1+2m^2/q^2)+3F_1F_2 + F_2^2(q^2/8m^2+1) \quad , \quad (2)$$

$$G_2(q^2)=\tfrac{1}{2}(1-4m^2/q^2)(F_1^2-F_2^2q^2/4m^2) \quad . \quad (3)$$

At $q^2 >> 4m^2$ or $\beta \to 1$ the total cross section is

$$\sigma = Q^2(4\pi\alpha^2/3q^2)(F_1^2+3F_1F_2+F_2^2q^2/8m^2) \quad . \quad (4)$$

Even though F_2 is expected to be smaller than F_1, the contribution of F_2^2 is enhanced by the factor $q^2/8m^2$ compared to F_1^2.

The preliminary results at PETRA[3] at q^2 up to (37 GeV)^2 show that τ production agrees with the pointlike result $4\pi\alpha^2/3q^2$ to 10% at two standard deviations (2 SD $\triangleq$ 95% C.L.) or 5% at 1 SD. Using Eq. (2) with[6] $F_1=1$ and $q^2=1350$ GeV^2 gives a 2-SD bound $F_2^\tau \leq 0.023$, or at 1 SD $F_2^\tau \leq 0.014 = \frac{1}{70}$. The dominant term containing F_2 here is the $3F_1F_2$ term.

We may also set limits on the anomalous magnetic moments of quarks at high q^2 using the accuracy of the agreement of the PETRA value[4] of $R_{\rm had}$ with that expected from pointlike quarks[7] as compared to that given by Eq. (2) for quarks with anomalous magnetic moments at high q^2. The smallest absolute normalization errors are those of PLUTO[4] and TASSO[4] which are 5% at 1 SD. We use Eq. (4) to compare with the data at $q^2 = (37\ {\rm GeV})^2$ using constituent quark masses $m_u = m_d = 0.3$ GeV, $m_s = 0.45$ GeV, $m_c = 1.8$ GeV, and $m_b = 4.5$ GeV. With a 5% limit on agreement with $R = \frac{11}{3}$ we get the bounds at 1 SD,

$$F_2^u \leq 0.008,\ \ F_2^d \leq 0.017,\ \ F_2^s \leq 0.025\ ,$$
$$F_2^c \leq 0.030,\ \ F_2^b \leq 0.13\ . \tag{5}$$

Use of the current-algebra or bag-model masses of $m_u, m_d \sim 10$ MeV would reduce the limits on F_2^u and F_2^d by a factor of 30 to $F_2^u \leq 0.0003$ and $F_2^d \leq 0.0006$.

The anomalous magnetic moment is included in the electromagnetic-current part of the Z^0 coupling and shares the enhancement factor of ~ 5000 at the Z^0 peak in e^+e^-. The differential cross section in the Z^0 peak is given by

$$d\sigma/d\cos\theta = (G^2 m_Z{}^6/12\pi)[(q^2 - m_Z{}^2)^2 + \Gamma_Z{}^2 m_Z{}^2]^{-1}[(g_V^e)^2 + (g_A^e)^2]$$
$$\times [g_V{}^2 + g_A{}^2 + 3g_V F_2' + F_2'^2 q^2/8m^2 + \tfrac{1}{2}P_2(\cos\theta)(g_V{}^2 + g_A{}^2 - F_2'^2 q^2/4m^2) + 6\cos\theta(g_V + F_2')g_A r_e]\ , \tag{6}$$

where $r_e = g_V^e g_A^e/[(g_V^e)^2 + (g_A^e)^2]$, $F_2' \equiv 2\sin^2\theta_W F_2$, and θ is the angle between the τ^- and e^- directions. For the τ, F_2^τ will only give a small asymmetry but the symmetrical angular dependence at the Z^0 peak is

$$(1 + \cos^2\theta) + 2620\sin^2\theta(F_2^\tau)^2\ .$$

So a 10% limit on the $\sin^2\theta$ coefficient will set a limit $F_2^\tau < 0.006$. For the b quark the angular dependence is

$$(1 + \cos^2\theta) + 284\sin^2\theta\ (F_2^b)^2$$

and a 10% limit would give $F_2^b < 0.02$. The total τ cross section at the Z^0 peak from Eq. (6) is proportional to $(1 + 213\,F_2{}^2)$. A total-τ-production measurement accurate to $1 \pm \sigma$ will give a limit on F_2 of $F_2 < \sqrt{\sigma}/14.6$. For 10% accuracy this gives $F_2^\tau(m_Z{}^2) \leq 0.02$.

Finally, we present here the method of measuring or bounding the τ anomalous moment to an accuracy of the order of 1% at relatively low $E_{\rm c.m.} \leq 10$ GeV $(q^2 = E_{\rm c.m.}{}^2)$ by a high-statistics measurement of the angular distribution of τ decay products. In the preceding high-$E_{\rm c.m.}$ analysis, the τ's direction was essentially that of the charged particle in its decay. At lower $E_{\rm c.m.}$ the τ direction is not precisely known. However, since the nature of the two- and three-body decays of the τ are known, namely, $\tau^- \to \nu_\tau\rho^-$ (22%), $\tau^- \to \nu_\tau\pi^-$ (8%), $\tau^- \to \nu_\tau\bar{\nu}_\mu\mu^-$ (18%), and $\tau^- \to \nu_\tau\bar{\nu}_e e^-$ (17%), we can integrate over the unobserved neutrino directions and predict the angular distribution for the *charged particle* including the effect of the F_2^τ form factor.

For the two-body decays of $\tau^- \to \nu_\tau\pi^-$ and $\tau^- \to \nu_\tau\rho^-$ we denote the final charged particle's mass as m_c and its scaled momentum by $z = P_c/E$ where $E = E_{\rm c.m.}/2$ is the τ's energy and $\beta = (1 - m_\tau{}^2/E^2)^{1/2}$. The angle between the final charged particle's direction and the τ direction is given by

$$\cos\phi = \frac{2E(z^2E^2 + m_c{}^2)^{1/2} - m_\tau{}^2 - m_c{}^2}{2\beta z E^2}\ . \tag{7}$$

The limits on z are given by

$$z_{\max} = [1 + \beta - (1-\beta)m_c{}^2/m_\tau{}^2]/2$$

and

$$z_{\min} = |\beta - 1 + (\beta+1)m_c{}^2/m_\tau{}^2|/2\ .$$

For the two-body decays the differential cross section of the charged π or ρ with angle $\cos\theta_c$ relative to the e^-e^+ beam axis and branching ratio $B(c)$ is given by

$$\frac{d\sigma}{dz\,d\cos\theta_c} = \frac{2\pi\alpha^2}{3q^2}B(c)\frac{1}{(1 - m_c{}^2/m_\tau{}^2)}$$
$$\times (1 + 4m_c{}^2/z^2q^2)^{-1/2}\mathfrak{F}\ , \tag{8}$$

$$\mathfrak{F} = G_0(q^2) + P_2(\cos\phi)P_2(\cos\theta_c)G_2(q^2)\ . \tag{9}$$

A high-statistics study of the angular and z distribution (as contained in $\cos\phi$) can isolate the ratio of the coefficients of the orthogonal angular terms giving a value or limit for F_2^τ.

In the three-body τ decays into μ or e plus two neutrinos, a result similar to the two-body decay is obtained, only now involving an integration over the invariant-mass squared of the two neutrinos $m_{\nu\bar{\nu}}{}^2$ or $y = m_{\nu\bar{\nu}}{}^2/m_\tau{}^2$. Neglecting $m_\mu{}^2/m_\tau{}^2$, we have the angle between the final charged lepton and the τ given by

$$\cos\phi = 1 - (1-\beta)(1-z-y)/2$$

and limits $z_{\max} = (1+\beta)/2$ and $z_{\min} = (1-\beta)/2$. The differential cross section for the charged final lepton

TABLE I. Coefficients $b(E_{c.m.})$ for Eq. (11) and statistical limits on F_2^τ obtainable at various $E_{c.m.}$ from a simplified two-bin analysis of the τ angular distribution using Eq. (12) with $(L_{31}YA)=1$.

$E_{c.m.}$ (GeV)	$b^{e,\mu,\tau}$	b^ρ	$F_{2\tau}^{(\text{joint})}$
5	0.056	0.081	0.014
6	0.11	0.18	0.0074
8	0.20	0.30	0.0053
10	0.27	0.37	0.0049
12	0.32	0.41	0.0051
20	0.41	0.47	0.0068
30	0.45	0.47	0.0096

using $\mathfrak{F}$ in Eq. (9) is

$$\frac{d\sigma}{dz\, d\cos\theta_c}=\frac{4}{3}\frac{\pi\alpha^2}{q^2}B(c) \times \int_0^{1-z/z_{\max}} dy(1-y)(\tfrac{1}{2}+y)\mathfrak{F} \quad . \qquad (10)$$

Again if the angular and z dependence is measured the ratio of the coefficients of the $P_2(\cos\theta_c)$ and $\cos\theta_c$-independent terms in $\mathfrak{F}$ can set a limit on F_2.

In order to estimate the statistics needed and find the best energy for the experiment we have integrated over z (and y in the three-body case) to study just the angular dependence in the form (for $F_2 << F_1 = 1$)

$$d\sigma/d\cos\theta_c \propto [\,1+6F_2/(3-\beta^2) + b(E_{c.m.})P_2(\cos\theta_c)] \quad . \qquad (11)$$

The coefficient $b(E_{c.m.})$ determines how well we can separate the orthogonal angular distributions and set a limit on F_2. By its decrease from the value for the τ itself before decay, $b^\tau=\beta^2/(3-\beta^2)$, we find how much the decay has washed out the angular variation. We find that b^ρ is the largest and $b^\pi \simeq b^\mu = b^e$. Values of the b's are presented in Table I for various total $E_{c.m.}$. Using a simplified two-bin method for estimation [bins with $P_2(\cos\theta_c)$ positive or negative] in isolating an angular distribution using statistical errors only, we find the following limit can be set on F_2,

$$F_2 < \frac{1.2\times10^{-4}E_{c.m.}(1+2m_\tau^2/E_{c.m.}{}^2)}{b(E_{c.m.})(BL_{31}YA)^{1/2}} \quad , \qquad (12)$$

where L_{31} is the luminosity in units of $10^{31}/\text{cm}^2$ sec, Y is effective running time in years, and A is the acceptance. The limits that can be obtained at various energies from the e, μ, τ lumped together with total branching ratio b of 43% (since they have the same b's) and for the ρ are nearly equal. In Table I we show the weighted limit that can be obtained by combining both of these limits. The table shows that a limit $F_2^\tau \leq 0.02$ is obtainable statistically and the favored energy range is $7 \leq E_{c.m.} \leq 20$ GeV.

In conclusion we have shown that including the anomalous-magnetic-moment effects of quarks or leptons is important in determining whether they have a structure. We have used PETRA data on the $e^+e^- \to \tau^+\tau^-$ total cross section to set a 1σ limit on the anomalous magnetic moment of the τ of $F_2^\tau < \frac{1}{70}$, corresponding to a limit on a composite scale of $\Lambda_c > 100$ GeV. We have also set similar limits on the anomalous magnetic moment of the quarks. We have shown how an improved limit can be set by observing the F_2 effect from the quark or lepton electromagnetic-current part of the neutral current in Z^0 production at the next generation of e^+e^- colliders. Finally we have shown the feasibility at present colliding rings of independently setting a limit on anomalous magnetic moments from the angular dependence of the τ's charged decay products or analogously the leading particle in a quark jet.

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[5]See, e.g., the SO_{14} unified gauge model of G. L. Shaw and F. Daghighian, Phys. Rev. D 26, 1798 (1982).
[6]Since the limits on the deviation of the $F_1(q^2)$ form factor from 1 are less than a few percent, we also expect the F_2-form-factor composite contribution to be undiminished from its $q^2=0$ value.
[7]The QCD corrections to $R_{\rm had}$ start with $(R_{\rm had}\,\alpha_s/\pi)$. The value of α_s at these q^2 is obtained from different experiments [P. Duinker, Rev. Mod. Phys. 54, 325 (1982), gives $\alpha_s=0.17\pm0.04$] and the error in α_s only affects $R_{\rm had}$ at the 1% level.