Analytic parametrization of deuteron magnetic form factor

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A modified method of analytic parametrization is used to analyze the existing data on the deuteron magnetic form factor, suggesting an asymptotic behavior of the type $\exp[-0.487(\ln t)^2]\ (\ln t)^2/t^2$ rather than the one implied by the dimensional quark-counting rule. The form factor smoothly extrapolates onto the timelike region, but with an anomalous threshold enhancement. The radius of the magnetic-moment distribution is computed to be 2.698 fm, which is nearly 34% higher than the charge radius. This analysis indicates a zero of the form factor at a rather unreliably large spacelike t value.

The deuteron is the simplest of the definitely known composite systems and its electromagnetic form factor provides an ideal illustration of continuity between nuclear and particle physics at the microscopic level.[1] The successful description of its form factor by analytic methods would also reveal the possibilities of applying the techniques developed for particle physics in nuclear physics.[2] It has been demonstrated recently[2] that a modified form of the N/D method of parametrization, originally developed for the hadronic electromagnetic form factors,[3] works well[4] in the description of the elastic structure factor $A(t)$. But since this structure factor occurs as a combination of the charge, quadrupole, and the magnetic form factor,

$$A(t)=G_c^2(t)+\frac{t^2G_Q^2(t)}{18M^4}-\frac{t}{6M^2}\left[1-\frac{t}{6M^2}\right]G_M^{d\,2}(t)\ , \tag{1}$$

the asymptotic behavior and the extrapolated values[2,4] cannot be ascribed to any one of these three form factors. On the other hand experimental data[5] on the structure factor

$$B(t)=-\frac{t}{3M^2}\left[1-\frac{t}{4M^2}\right]G_M^{d\,2}(t) \tag{2}$$

yield data on the magnetic form factor $G_M^d(t)$ in the spacelike region and these can be analyzed to obtain information on the radius of the magnetic-moment distribution inside the deuteron and values of the form factor in the timelike region on extrapolation, and to suggest a possible nature of asymptotic behavior subject to the well-known limitation that error in the extrapolated quantity increases as one goes away from the data region.[6] In this note we obtain such results by analyzing the data on the magnetic form factor using the type of ansatz specifically developed in Ref. 2 for the deuteron.[4]

We represent the form factor as[2–4]

$$G_M^d(t)/G_M^d(0)=N(t)/D(t)\ , \tag{3}$$

$$N(t)=\exp(-\alpha Z)\sum_m g_m Z^m\ , \tag{4}$$

$$D(t)=\sum_n a_n t^n+h(t)+\frac{2m_\pi^2}{3\pi}\ , \tag{5}$$

where the $h(t)$ has the same definition as in Ref. 2 but with

$$k=\left[\frac{t}{9}-m_\pi^2\right]^{1/2} \tag{6}$$

and $t_R=9m_\pi^2$. With this modification the D function contains the three-pion-cut contribution which is the next-nearest cut, the nearest being the anomalous cut in the t plane. The N function represents the anomalous-cut contribution by conformal mapping and optimized polynomial expansion (OPE).[2,3,7] The representation (3) has the potentiality to satisfy the general type of asymptotic behavior[2,3]

$$G_M^d(t)\underset{t\to\infty}{\sim}\exp[-\alpha(\ln t)^2](\ln t)^{2m}/t^n \tag{7}$$

with $m,n=0,1,2,\ldots$. Considering the deuteron as a six-quark system, the dimensional quark-counting rule (DQCR) yields[1,2] the asymptotic behavior $\sim 1/t^5$. This would correspond to the constraint $\alpha=g_m=0$, for $m\geq 1$ in the N function which is equivalent to retaining only the three-pion-cut[4] contribution in this ansatz.[2,3] With this constraint we first fitted all the available 22 data points[5] by the formula (3). Note that $h(0)=-2m_\pi^2/3\pi$ and we have to use the normalization constraint $g_0=a_0$ for all possible types of fits with formula (3). The values of the parameters, the computed value of the root-mean-square radius of the magnetic-moment distribution defined as $r_d^m=[6G_m^{d'}(0)]^{1/2}$, and the χ^2/DF values are presented in the second column in Table I, and the fit is

TABLE I. Values of parameters, χ^2/DF, and the radius of the magnetic-moment distribution for the two fits described in the text.

Parameters	Curve I $m=0,\ n=5$	Curve II $m=n=2$
α	0	0.487
a_0 (GeV^2)	1.042	1.468
a_1	−13.650	−3.731
a_2 (GeV^{-2})	86.511	7.549
a_3 (GeV^{-4})	0.015	0
a_4 (GeV^{-6})	0.320	0
a_5 (GeV^{-8})	0.0340	0
$g_0=a_0$ (GeV^2)	1.042	1.468
g_1 (GeV^2)	0	−0.046
χ^2/DF	2.160	1.530
r_d^m (fm)	2.28	2.698

shown by the curve I in Fig. 1. Although the fit appears to be good visually, the χ^2/DF value is as large as 2.26. We next removed the constraint $\alpha=g_m=0$ and searched for improved fits allowing α, g's, and a's as free parameters, but with $g_0=a_0$. For the best fit obtained with the parameters reported in the third column of Table I, the χ^2/DF value reduced to 1.53. Increasing the number of parameters did not improve the fit.

Thus, it is found that the OPE approximation of the anomalous cut, when combined with the three-pion-cut contribution by the D function, yields a better fit than the case when only the three-pion-cut contribution is taken, although the two fits do not differ enormously. As noted earlier[2,4] the exponential weight function has a significant role, but in addition the second term in the series (4) also seems to be important. Further the terms proportional to the third and higher powers of t seem to be not required for the fit. These analyses suggest that the asymptotic behavior $\exp[-0.487(\ln t)^2](\ln t)^2/t^2$ is favored, although not strongly, over the $1/t^5$ type of behavior.

Extrapolated values of the form factor in the timelike region are shown in Figs. 2 and 3 for smaller and larger values of t, respectively, but up to and below the threshold of the annihilation process $e^+e^-\rightarrow \bar{d}d$. In Fig. 2 the bump structure in curve II shows the anomalous threshold enhancement, but in curve I the parameters are such that effectively they produce a bump for $t<9m_\pi^2$. The difference in the slopes in the two curves and smoothness of extrapolation from the spacelike to the timelike region is clearly discernible at $t=0$, yielding r_d^m values 2.285 fm for curve I and 2.698 fm for the best fit (curve II). For the best fit the contribution to the mean-square radius is calculated to be nearly 86% (14%) from the anomalous (three-pion) cut. As compared to the deuteron-charge-radius value[2,4] the radius of the magnetic-moment distribution is found to be nearly 34% larger. Extrapolated values of the form factor for large spacelike t values are shown in Fig. 4. Using the definition of the mapping variable Z and the values of g_0 and g_1 for the fit II the possibility of a zero of the form factor is suggested at rather very large spacelike value of $|t|$ ($\sim$650 GeV^2). The possibility of a zero in the proton form factor for $|t|=40-50$ GeV^2 has been recently investigated using a different parametrization of the form-factor data.[8]

The results of our analysis are based upon the data available over a limited range in the spacelike region $0<t\leqslant 1$ GeV^2. Although the value of r_d^m obtained

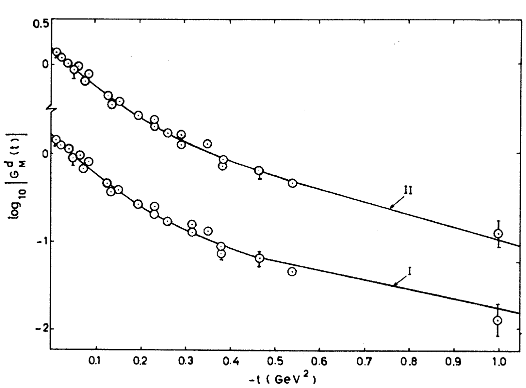

FIG. 1. Fit to the available data on the deuteron magnetic form factor where the curve I (II) corresponds to the parameters given in the second (third) column of Table I.

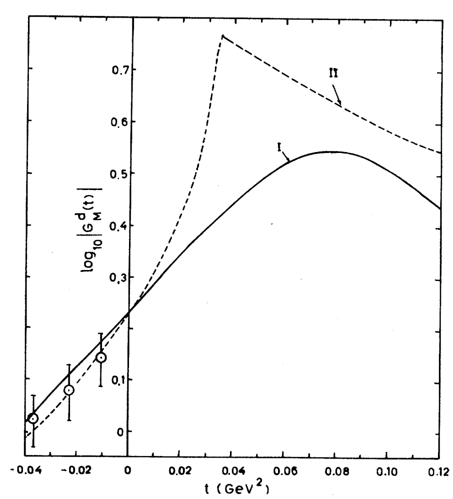

FIG. 2. Extrapolation of the two fits onto the timelike region for small t values.

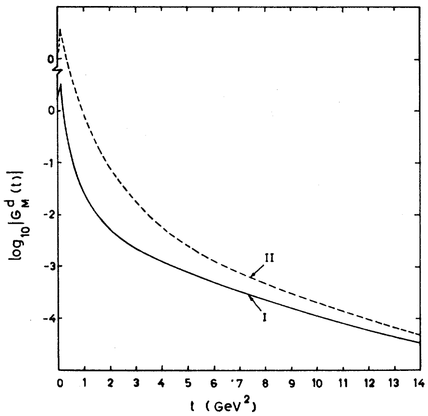

FIG. 3. Same as Fig. 2 but for larger t values.

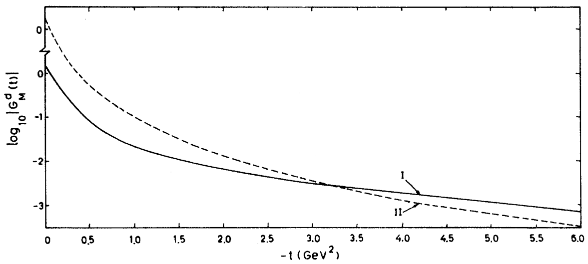

FIG. 4. Extrapolation of the form factor for large spacelike t values.

by our fitting procedure is supposed to be reliable, other results like extrapolated values of form factors for large spacelike and timelike values of t, the asymptotic behavior, and the position of the zero should not be taken as definite, but only suggestive. This is because of the well-known fact that the errors in the extrapolated quantities become larger as one moves farther away from the data region making the extrapolations unstable.

However, it has been demonstrated that such instabilities can be reduced if one uses analyticity and OPE.[7] In the present case only the N function has been approximated by OPE, therefore the stability criterion is not guaranteed.[6,7]

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[4]For the deuteron, besides the anomalous cut which starts at $t_a \simeq 0.033$ GeV^2, the nearest cut is the three-pion cut. But the analyses of Ref. 2 were done by including the two-pion cut, inadvertently. Repeating the analysis of Ref. 2 with formulas (3)–(6) for $A(t)$ and $t_R = 9m_\pi^2$ yields slightly different values of parameters, $\alpha = 0.858$, $a_0 = g_0 = 0.947$, $a_1 = -5.772$, $a_2 = 111.97$, $a_3 = 0.188$. These correspond to the root-mean-square charge-radius value of 2.0 fm and asymptotic behavior of the type $\exp[-0.858(\ln t)^2]/t^3$. Figures 1 and 3 of Ref. 2 are almost the same but in Fig. 2 only the threshold enhancement rises to 9.2 with the other parts of that figure remaining unchanged. Other quantities calculated do not change significantly.

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[6]Two important problems of analytic extrapolation methods, namely, the convergence problem and the stability problem, have been dealt with rigorously by using OPE in Ref. 7. It has been shown that convergence of series can be maximized and the error in the extrapolation reduced by using suitable conformal mapping variables. The error in the extrapolation depends crucially upon the error affecting the data and the number of terms in the convergent series.

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