PHYSICAL REVIEW D VOLUME 27, NUMBER 5 1 MARCH 1983

Quark content of neutral mesons

Jonathan L. Rosner
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
*and The Enrico Fermi Institute and the Department of Physics, University of Chicago, Chicago, Illinois 60637**
(Received 26 July 1982; revised manuscript received 19 November 1982)

Radiative transitions involving neutral mesons with $J^P=0^-$ $[\eta,\eta',\iota(1440)]$ and 2^+ $[f,f',\theta(1640)]$ are used to provide information on the light-quark content of these mesons, and to test predictions of models in which these states are quarkonium-gluonium mixtures. Some suggested observations include (1) $\phi\to\eta'\gamma$, predicted to have a branching ratio of 7×10^{-5} in the standard quarkonium picture of η', (2) $f'\to\gamma\gamma$, predicted to have a partial width $\gtrsim 200$ eV if $f'=s\bar{s}$ but very sensitive to possible destructive interference from small nonstrange-quark admixtures, and (3) $\iota(1440)\to\gamma\gamma$ and $\theta(1640)\to\gamma\gamma$, which will be easily detectable unless these states are almost totally free of quarkonium admixtures.

I. INTRODUCTION

The gauge theory of the strong interactions, quantum chromodynamics (QCD), involves quanta (gluons) which are expected to form bound states with one another.[1] In general these gluonic bound states can be expected to mix with neutral (isoscalar) mesons formed of quark-antiquark pairs, unless forbidden by C and P conservation. In particular, both recent candidates for gluonic bound states,

$$\iota(1440):\ J^{PC}=0^{-+}\ \text{(Ref. 2)}\ , \tag{1}$$

$$\theta(1640):\ J^{PC}=2^{++}\ \text{(Ref. 3)}\ , \tag{2}$$

can have quark-antiquark admixtures which can alter the expected pattern of their decays.

It is our aim here to carry out some simple tests, based on radiative decays, of quark admixtures in neutral mesons. We discuss both 0^- (η,η',ι) and 2^+ (f,f',θ) states. These tests allow one to confront models[4–15] for mixing of quarkonium and gluonium states with experiment, and thus to learn both about the physics underlying the mixing process and about the spectroscopy of the unmixed states. Partial results of such tests have been discussed previously.[16]

We find the following results.

(1) The η is well understood as an SU(3)-flavor octet with a small quarkonium singlet admixture, and not much room for a significant gluonium admixture.

(2) Information on the η' is incomplete without a significant constraint on its $s\bar{s}$ content. We propose a measurement of the rare process $\phi\to\eta'\gamma$ to resolve the issue.[17] A conventional quarkonium picture of η' predicts $B(\phi\to\eta'\gamma)=7\times10^{-5}$.

(3) The rate for $f'\to\gamma\gamma$, predicted to be 200–250 eV if $f'=s\bar{s}$, is very sensitive to small admixtures of nonstrange quarks in f'. The expected sign of this admixture[12,15] is such that it will cause destructive interference. However, a drastic suppression of $\Gamma(f'\to\gamma\gamma)$ is excluded by experiment.[18] This limits the nonstrange-quark content of f'.

(4) The decays $\iota\to\gamma\gamma$ and $\theta\to\gamma\gamma$ are quite likely to proceed with detectable rates in the several-hundred-eV–keV range, and provide useful constraints on the quarkonium content of these mesons (see, e.g., Ref. 6).

We introduce some elementary notation in Sec. II. The pseudoscalar mesons (η,η',ι) are discussed in Sec. III, while the tensor mesons $[f,f',\theta(1640)]$ are treated in Sec. IV. We review one model for mixing[12,13] in Sec. V, and discuss prospective tests. Section VI contains a summary.

II. NOTATION

We shall work in a basis consisting of the states

$$|N\rangle\equiv\frac{1}{\sqrt{2}}\,|u\bar{u}+d\bar{d}\rangle\ , \tag{3}$$

$$|S\rangle\equiv|s\bar{s}\rangle\ , \tag{4}$$

and

$$|G\rangle\equiv|\text{gluonium}\rangle\ . \tag{5}$$

A physical state $|\Psi\rangle$ is assumed to be a linear combination of these:

$$|\Psi\rangle=x\,|N\rangle+y\,|S\rangle+z\,|G\rangle\ , \tag{6}$$

with

 ©1983 The American Physical Society

$$x^2+y^2+z^2=1 \tag{7}$$

and thus

$$x^2+y^2\leq 1\ . \tag{8}$$

A significant gluonic admixture in a state is possible only if $z^2=1-x^2-y^2>0$. We shall neglect everywhere small $c\bar{c}$ admixtures in the states being discussed[10,11]; they would be important only outside the context of questions addressed here. We shall also ignore mixing with radial excitations, though this may be important for a more precise discussion of $\iota(1440)$ and $\theta(1640)$.

III. PSEUDOSCALAR MESONS

The standard unitary-symmetry assignments for an η and η' imply

$$(y/x)_\eta=-\sqrt{2},\quad (y/x)_{\eta'}=1/\sqrt{2} \tag{9}$$

for an octet η and singlet η', or approximately

$$(y/x)_\eta=-1,\quad (y/x)_{\eta'}=1 \tag{10}$$

for an octet-singlet mixing angle of $\sim 10°$.[19] What do radiative decays say about the values of x and y for these mesons? The data we shall use are summarized in Table I.[20–28]

We calibrate $M1$ transition rates between 0^- and 1^- mesons composed of light quarks with the help of $\omega\to\pi\gamma$ and $\rho\to\pi\gamma$.[20,21] An SU(3) calculation implies the ratio $\Gamma(\omega\to\pi^0\gamma)/\Gamma(\rho^-\to\pi^-\gamma)=9\times(\text{phase space})\approx 9.6$. This is in rough accord with experiment. We shall use the nominal value $\Gamma(\omega\to\pi^0\gamma)=790$ keV in what follows, neglecting uncertainties in this quantity. We assume that $0^-\to\gamma\gamma$ partial widths scale as $(m_{0^-})^3$ (Ref. 29) and use the $\pi^0\to\gamma\gamma$ width[22] quoted in Table I to predict $\Gamma(\eta\to\gamma\gamma)$ and $\Gamma(\eta'\to\gamma\gamma)$ in terms of the quark content of η and η'.

We first discuss processes involving η.

(a) $\rho\to\eta\gamma$. An SU(3) calculation predicts

$$\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\left[\frac{m_\rho{}^2-m_\eta{}^2}{m_\omega{}^2-m_\pi{}^2}\frac{m_\omega}{m_\rho}\right]^3 x_\eta{}^2\ , \tag{11}$$

so we find

$$|x_\eta|=0.73\pm0.10\ . \tag{12}$$

(b) $\phi\to\eta\gamma$. We take into account SU(3) breaking to the extent that the strange quark has a smaller magnetic moment than the d quark by a factor m_u/m_s. Then we expect

$$\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{4}{9}\frac{m_u{}^2}{m_s{}^2}\left[\frac{m_\phi{}^2-m_\eta{}^2}{m_\omega{}^2-m_\pi{}^2}\frac{m_\omega}{m_\phi}\right]^3 y_\eta{}^2 \tag{13}$$

and find

$$|y_\eta|=0.74\pm0.06\ . \tag{14}$$

(c) $\eta\to\gamma\gamma$. An SU(3) calculation yields

$$\frac{\Gamma(\eta\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left[\frac{m_\eta}{m_{\pi^0}}\right]^3(5x_\eta+\sqrt{2}y_\eta)^2\ , \tag{15}$$

so that

$$\left|x_\eta+\frac{\sqrt{2}}{5}y_\eta\right|=0.46\pm0.04\ . \tag{16}$$

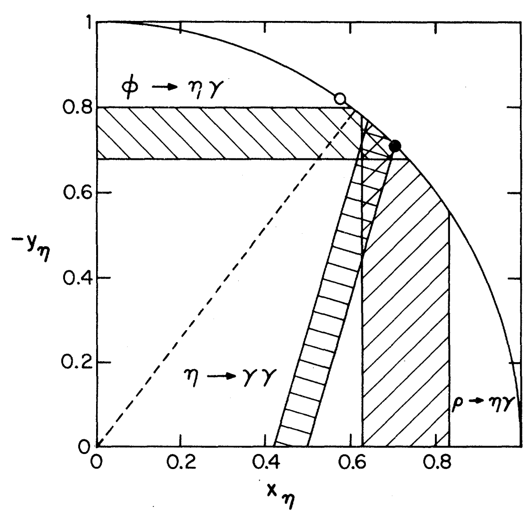

FIG. 1. Constraints on nonstrange- (x_η) and strange- (y_η) quarkonium mixing coefficients in the η, as defined in Eq. (6). The circular boundary denotes the constraint $x_\eta{}^2+y_\eta{}^2\leq 1$. Only the quadrant containing a pure octet η [open circle, with $x_\eta=(\frac{1}{3})^{1/2}$, $y_\eta=-(\frac{2}{3})^{1/3}$] is shown. The mixing solution of Ref. 19, $x_\eta=(\frac{1}{2})^{1/2}=-y_\eta$, is shown as a closed circle. The mixing scheme of Ref. 12, to be discussed in Sec. V, implies a ratio of y_η/x_η shown as the dashed straight line. The vertical, horizontal, and inclined bands are the regions allowed by Eqs. (12), (14), and (16), respectively.

TABLE I. Experimental rates for radiative processes involving pseudoscalar mesons.

Process	Rate	Reference
$\omega\to\pi^0\gamma$	789 ± 92 keV	20
$\rho\to\pi\gamma$	67 ± 7 keV	21
$\pi^0\to\gamma\gamma$	7.92 ± 0.42 eV	22
$\rho\to\eta\gamma$	53 ± 14 keV	23
$\phi\to\eta\gamma$	68 ± 10 keV	23,24
$\eta\to\gamma\gamma$	324 ± 46 eV	25
$\eta'\to\rho\gamma$	84 ± 30 keV	26
$\phi\to\eta'\gamma$	Not quoted	27
$\eta'\to\gamma\gamma$	5.8 ± 2.3 keV	28

The constraints implied on x and y by the above relations are shown in Fig. 1. There exists a consistent solution compatible with the mixing proposed in Ref. 19 (the closed circle on the figure). The allowed gluonic admixture in the η ($z_\eta^2=1-x_\eta^2-y_\eta^2$) is required to be small, $|z_\eta|\lesssim 0.4$, but not necessarily zero.

We now turn to processes involving η'.

(d) $\eta'\to\rho\gamma$. A calculation similar to those performed above yields

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=3\left[\frac{m_{\eta'}{}^2-m_\rho{}^2}{m_\omega{}^2-m_\pi{}^2}\frac{m_\omega}{m_{\eta'}}\right]^3 x_{\eta'}{}^2 \qquad (17)$$

or

$$|x_{\eta'}|=0.63\pm0.12\ . \qquad (18)$$

(e) $\phi\to\eta'\gamma$. Here there are no data, though the large number of ϕ's observed in one experiment[27] suggests that this situation could change. We find

$$\frac{\Gamma(\phi\to\eta'\gamma)}{\Gamma(\phi\to\eta\gamma)}=\left[\frac{m_\phi{}^2-m_{\eta'}{}^2}{m_\phi{}^2-m_\eta{}^2}\right]^3\frac{y_{\eta'}{}^2}{y_\eta{}^2}$$

$$=4.6\times10^{-3}\frac{y_{\eta'}{}^2}{y_\eta{}^2}\ . \qquad (19)$$

Since $B(\phi\to\eta\gamma)=(1.5\pm0.2)\%$,[30] we expect

$$B(\phi\to\eta'\gamma)\approx(7\times10^{-5})\frac{y_{\eta'}{}^2}{y_\eta{}^2}\ . \qquad (20)$$

The mixing of Ref. 19 implies $y_{\eta'}{}^2/y_\eta{}^2=1$. The most distinctive signal for this decay might be

$$\begin{array}{l}\phi\to\eta'\gamma\\ \quad\searrow \eta\pi^+\pi^-\\ \qquad\searrow \pi^+\pi^-\pi^0\\ \qquad\quad\searrow \gamma\gamma,\end{array} \qquad (21)$$

i.e., four charged pions and three photons. Very few other sources of this final state are expected at the ϕ mass.[27]

(f) $\eta'\to\gamma\gamma$. We expect

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left[\frac{m_{\eta'}}{m_{\pi^0}}\right]^3(5x_{\eta'}+\sqrt{2}y_{\eta'})^2 \qquad (22)$$

and find

$$\left|x_{\eta'}+\frac{\sqrt{2}}{5}y_{\eta'}\right|=0.85\pm0.20\ . \qquad (23)$$

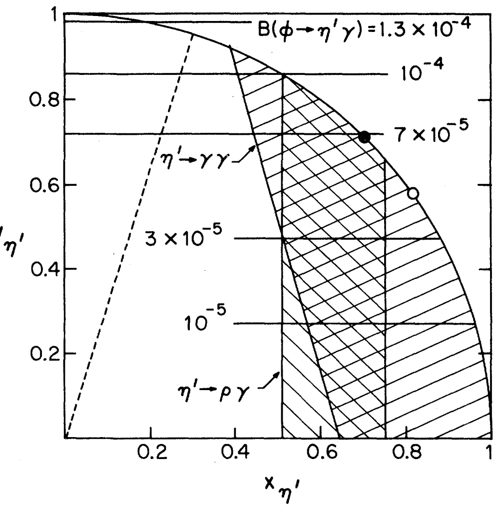

FIG. 2. Constraints on nonstrange- ($x_{\eta'}$) and strange- ($y_{\eta'}$) quarkonium mixing coefficients in the η'. The open circle denotes a pure quarkonium singlet η', $x_{\eta'}=(\frac{2}{3})^{1/2}$, $y_{\eta'}=(\frac{1}{3})^{1/2}$. Closed circle denotes mixing solution of Ref. 19, $x_{\eta'}=(\frac{1}{2})^{1/2}=y_{\eta'}$. Dashed straight line: mixing scheme of Ref. 12 (Sec. V). Vertical and inclined bands are regions allowed by Eqs. (18) and (23), respectively. Horizontal lines are labeled by predicted values of $B(\phi\to\eta'\gamma)$.

The constraints implied by $x_{\eta'}{}^2+y_{\eta'}{}^2\le1$ and by Eqs. (18) and (23) are shown in Fig. 2. The absence of a significant constraint on $y_{\eta'}$ (the strange quarks in η') is keenly felt. With a very small value of $y_{\eta'}$ the 1σ limit in Fig. 2 associated with $\eta'\to\gamma\gamma$ would

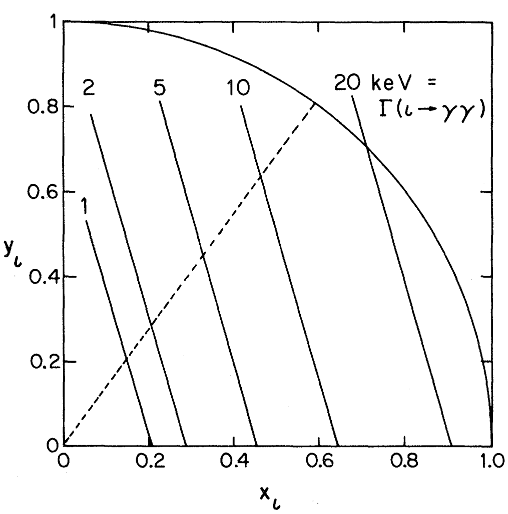

FIG. 3. Constraints on nonstrange- (x_ι) and strange- (y_ι) quarkonium mixing coefficients in the ι. (Radial excitations are ignored here.) Dashed straight line: mixing scheme of Ref. 12 (Sec. V). Inclined lines are labeled by predicted values of $\Gamma(\iota\to\gamma\gamma)$ [calculated as in Eqs. (15) and (22)].

allow a value of $z_{\eta'}{}^2$ exceeding one-half, i.e., more than half the η' could be gluonic.

The horizontal lines in Fig. 2 indicate contours labeled by the predicted values of $B(\phi\to\eta'\gamma)$, ranging from $\gtrsim 10^{-4}$ to zero. Data on $\phi\to\eta'\gamma$ thus will be much help in learning the strange-quark and gluonic content of η'.

There are no observed radiative $0^-\leftrightarrow 1^-$ transitions involving $\iota(1440)$. Nonetheless we can discuss its $\gamma\gamma$ width *as if* it contains admixtures of ground-state quarkonium (ignoring radial excitations).[31] The result is the set of contours shown in Fig. 3. Widths in excess of 1 keV are possible even with tiny quarkonium admixtures ($x_\iota{}^2+y_\iota{}^2\gtrsim 7\%$). A more detailed treatment would, of course, include radial excitations, for which there is some evidence.[32]

IV. TENSOR MESONS

The quarkonium components of f, f', and θ give rise to $\gamma\gamma$ decays which we may relate to those of A_2 (neglecting kinematic factors and radially excited components in θ):

$$\frac{\Gamma(f,f',\theta\to\gamma\gamma)}{\Gamma(A_2\to\gamma\gamma)}=\frac{25}{9}\left[x+\frac{\sqrt{2}}{5}y\right]^2 . \quad (24)$$

The observed widths as quoted in Ref. 30 are[33]

$$\Gamma(f\to\gamma\gamma)=2.95\pm0.40 \text{ keV} , \quad (25)$$

$$\Gamma(A_2\to\gamma\gamma)=0.77\pm0.45 \text{ keV} . \quad (26)$$

An independent analysis[5,15] of mesonic decay modes of f concludes $x>0.9$. We may turn Eqs. (24)–(26) and the bound $x>0.9$ into a prediction

$$0.88\le\Gamma(A_2\to\gamma\gamma)\le 1.12 \text{ keV} . \quad (27)$$

We show in Fig. 4 contours of constant two-photon partial width estimated according to Eq. (24) with the nominal value $\Gamma(A_2\to\gamma\gamma)=1$ keV. The two-photon widths of f, f', and θ are each of great interest.

(a) $f\to\gamma\gamma$. The observed two-photon width (25), when combined with the rather low value (26), supports the conclusion[5] that the gluonic admixture in the f must be rather small. The band corresponding to (25) is shown as the cross-hatched region in Fig. 4.

(b) $f'\to\gamma\gamma$. A new partial width for this process has been quoted[17]:

$$\Gamma(f'\to\gamma\gamma)B(f'\to K\bar{K})=0.15\pm0.03^{+0.03}_{-0.04} \text{ keV} . \quad (28)$$

We may combine Eq. (28) with an estimate[34]

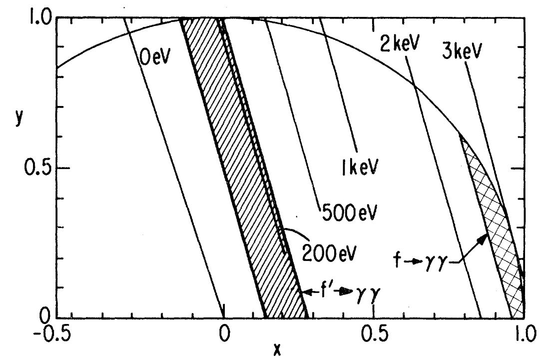

FIG. 4. Constraints on nonstrange- (x) and strange- (y) quarkonium mixing coefficients in f, f', θ. Cross-hatched region: allowed by observed rate for $f\to\gamma\gamma$ [Eq. (25)]. Shaded region: allowed by observed rate for $f'\to\gamma\gamma$ [Eqs. (28)–30)]. Inclined lines are labeled by predicted values of $\gamma\gamma$ widths [Eq. (24)], for $\Gamma(A_2\to\gamma\gamma)=1$ keV.

$$B(f'\to K\bar{K})\ge 0.7 \quad (29)$$

to set the bounds

$$0.14\le\left|x+\frac{\sqrt{2}}{5}y\right|_{f'}\le 0.28 , \quad (30)$$

shown as the shaded band in Fig. 4. (In accord with the expectation that the f' is mainly $s\bar{s}$, we show only the solution containing the point $x\approx 0$, $y\approx 1$.) An unmixed $s\bar{s}$ f' would have a decay width to $\gamma\gamma$ of about 200–250 eV on the basis of Eqs. (24) and (27).

The $f'\to\gamma\gamma$ amplitude is very sensitive to destructive interference from nonstrange quarks. From Fig. 4, one sees that even the rather lax bounds (28) drastically restrict the allowed fraction of nonstrange quarks in the f'.

(c) $\theta\to\gamma\gamma$. For wide ranges of possible mixings (one prediction will be mentioned presently, and another will be discussed in Sec. V), *the partial width* $\Gamma(\theta\to\gamma\gamma)$ *is likely to exceed* $\Gamma(f'\to\gamma\gamma)$. This behavior is seen quite dramatically in Ref. 6, for example.

At present one knows only that[17]

$$\Gamma(\theta\to\gamma\gamma)B(\theta\to K\bar{K})\le 0.5 \text{ keV (95\% C.L.)} . \quad (31)$$

If θ is really a gluonic bound state, mixed with quarkonium as in (say) Ref. 5, it is likely that its branching ratio to $K\bar{K}$ does not exceed several percent, as we shall show below. Thus the bound in (31) is not very powerful.

Another bound which has recently been reported[35] is

$$\Gamma(\theta\to\gamma\gamma)B(\theta\to\rho^0\rho^0)<1.2 \text{ keV (95\% C.L.)} . \quad (32)$$

Even if $\rho\rho$ were the major decay mode of θ, it appears that all one can say from (31) and (32) (for an isoscalar θ) would be

$$\Gamma(\theta\to\gamma\gamma)<4 \text{ keV} . \quad (33)$$

From Fig. 4 we see that *any* state satisfies this bound. (Kinematic corrections have been neglected here.) Thus, present experimental limits on $\Gamma(\theta\to\gamma\gamma)$ do not tell us much about the quarkonium content of this state.

A quarkonium admixture in $\theta(1640)$ could well explain the observed small $\pi^0\pi^0/\eta\eta$ ratio in its decays. Indeed, as noted in Refs. 12 and 13, a gluonic bound state decoupling from $\pi\pi$ by virtue of a small quarkonium admixture was proposed before the discovery of the $\theta(1640)$.[4,5] This quarkonium admixture could be detected via $\theta\to\gamma\gamma$. The model of Ref. 5 predicts

$$\frac{\Gamma(\theta\to\gamma\gamma)}{\Gamma(f\to\gamma\gamma)}\approx(\tfrac{1}{10} \text{ to } \tfrac{1}{4})\times(\text{phase space}) , \quad (34)$$

so $\Gamma(\theta\to\gamma\gamma)$ should be several hundred eV in that model.

The decay $\theta\to K\bar{K}$ should be observable. When the η can be represented as $(u\bar{u}+d\bar{d}-\sqrt{2}s\bar{s})/2$ (the favored mixing cited in Secs. II and III and Ref. 19), one finds[12]

$$\frac{\Gamma(\theta\to K\bar{K})}{\Gamma(\theta\to\eta\eta)}=4\times(\text{phase space})\approx 5.7 , \quad (35)$$

no matter *what* the nonstrange, strange, and gluonic admixtures in the θ. [The corresponding ratios are $4\times(\text{phase space})=12$ for f and 6.4 for f'.] The prediction (35) simply tests SU(3) invariance of couplings.

Preliminary indications, when the present work was first performed, pointed to a substantial $K\bar{K}\eta\eta$ ratio in θ decays. It has now been found[36] that this ratio is indeed large; about 3, but with a range of variation consistent with Eq. (35) as well. A further discussion may be found in Ref. 37.

The model of Ref. 5 predicted $\Gamma(\theta\to K\bar{K})$ of only several MeV, and $\Gamma(\theta\to\eta\eta)\approx 1$ MeV. The total observed width is much larger,[3] $\Gamma_{\text{tot}}(\theta)=220^{+100}_{-70}$ MeV. This also exceeds the prediction of Ref. 5 of $\lesssim 40$ MeV. That prediction was based on an analogy with the $f\to 4\pi$ decay. The $\theta(1640)$ is sufficiently massive that important new channels like $\rho\rho$, not accessible in $f\to 4\pi$, are open. These could contribute to a significantly enhanced total θ decay rate. There is the possibility of a $\rho^0\rho^0$ signal around threshold in $J/\psi\to\gamma\rho^0\rho^0$,[38] but it is too early to tell whether it is due to $\theta(1640)$. Other decay modes of interest include $\rho^+\rho^-(\pi^+\pi^-\pi^0\pi^0)$, $K\bar{K}\pi\pi$ (in several different charge states), and states involving three pseudoscalar mesons such as $\eta\pi\pi$ or $K\bar{K}\pi$ [though these may be suppressed in the limit in which θ is mainly an SU(3) singlet by generalized G-parity arguments; see Ref. 39]. The size of the $\eta\eta$ signal in Ref. 3 is compatible with $\Gamma(\theta\to\eta\eta)\sim 1$ MeV and a total θ width of order 100 MeV if the total branching ratio $B(J/\psi\to\gamma\theta)$ is several percent.[40] This is quite compatible with estimates[41] of the production of 2^+ gluonium states through $J/\psi\to\gamma+$ (2 gluons) if the θ provides a major contribution to the two-gluon spectral function. [It has been estimated that the decays $J/\psi\to\gamma+(2^+ \text{ meson})$ can be evaluated fairly reliably this way.[42]]

To summarize this subsection, we expect processes like $\gamma\gamma\to\theta(1640)\to K\bar{K},4\pi,K\bar{K}\pi\pi$ to occur with observable strength if the $\theta(1640)$ is largely (but not wholly) a gluonic bound state.

V. A SIMPLE MIXING MODEL

Suppose quarkonium states $q_i\bar{q}_i$ (i denotes flavor) mix with other $q_j\bar{q}_j$ with an amplitude α, and with gluonium states β, in a flavor-independent way.[12,13,43,44] Then, in the three-dimensional space (3)—(5) composed of quarkonium states $(u\bar{u}+d\bar{d})/\sqrt{2}$ (mass M_N), $s\bar{s}$ (mass M_S), and gluonium (mass M_G), the physical eigenstates $|\Psi\rangle$ satisfy $\underline{M}\,|\Psi\rangle=M\,|\Psi\rangle$, where

$$\underline{M}=\begin{bmatrix} M_N+2\alpha & \alpha\sqrt{2} & \beta\sqrt{2} \\ \alpha\sqrt{2} & M_S+\alpha & \beta \\ \beta\sqrt{2} & \beta & M_G \end{bmatrix} . \quad (36)$$

If we write Ψ as a linear combination (6) of basis states with coefficients x, y, and z, we find a relation independent of α or β:

$$\frac{y}{x}=(2)^{-1/2}\frac{M-M_N}{M-M_S} . \quad (37)$$

Thus the ratio of nonstrange- to strange-quark content is prescribed in terms of the unmixed quarkonium masses M_N, M_S and the particle mass M.

The possibility has also been raised[44] that the nonstrange- and strange-quarkonium systems have different wave functions at the origin as a result of the different masses of the corresponding quarks. This would lead to a mass matrix of the form

$$\underline{M}'=\begin{bmatrix} M_N+2\alpha & \alpha Z\sqrt{2} & \beta\sqrt{2} \\ \alpha Z\sqrt{2} & M_S+\alpha Z^2 & \beta Z \\ \beta\sqrt{2} & \beta Z & M_G' \end{bmatrix} . \quad (38)$$

One still obtains a simple result for y/x in this case,

$$\frac{y}{x}=(2)^{-1/2}Z\frac{M-M_N}{M-M_S} . \quad (39)$$

This effect may be important for pseudoscalar mesons (a value equivalent to $Z=0.78$ was found in Ref. 44) but will be neglected here.

A. Pseudoscalar mesons

We adopt a model of masses based on the naive nonrelativistic quark picture. Here, only the constituent quark masses m_i and hyperfine splittings contribute to pseudoscalar- and vector-meson masses. Thus

$$M(q_i\bar{q}_j)=m_i+m_j+\frac{\lambda}{m_i m_j}\langle \vec{\sigma}_i\cdot\vec{\sigma}_j\rangle \ , \qquad (40)$$

where $\langle \vec{\sigma}_i\cdot\vec{\sigma}_j\rangle=(-3,1)$ for (pseudoscalar, vector) mesons. This model provides quite a satisfactory description of 0^- and 1^- meson masses with $m_u=310$ MeV, $m_s=483$ MeV.[45] It implies

$$M_N=m_{\pi^0}=135\text{ MeV}\ \ (c\equiv 1)\ , \qquad (41)$$

$$M_S=m_\phi-(m_{K^*}-m_K)^2/(m_\rho-m_\pi)=770\text{ MeV}\ . \qquad (42)$$

We then find, from Eq. (37),

$$(y/x)_\eta=-1.3,\ \ (y/x)_{\eta'}=3.1\ . \qquad (43)$$

These are to be compared with the values (9), (10) for more conventional assignments. The results (43) are shown as the dashed lines on Figs. 1 and 2.

The predictions of the model for the η are not drastically different from the usual estimates. The dashed line in Fig. 1 runs fairly near the region allowed by radiative transition data, corresponding to very little glue in the η.

The mixing scheme of Ref. 12 predicts that most of the quarks in the η' (far more than usually supposed) are strange. Can this be true? Radiative transition data (as noted in Fig. 2) do not appear to permit a preponderance of strange quarks in η'. However, as we have stressed earlier, and has been mentioned by others,[17] $\phi\to\eta'\gamma$ would provide an important constraint. The modifications suggested in Ref. 44 and quoted in Eqs. (38) and (39) lead to some improvement, since they reduce the predicted magnitude of y/x. Even so, the very large value of $(y/x)_{\eta'}$ implied by the mixing scheme of Ref. 12 lies outside the constraints of the data. It could only be correct if the total η' width had been overestimated experimentally.

For the $\iota(1440)$, the mixing model of Ref. 12 implies that the nonstrange- and strange-quarkonium contributions should add constructively to the amplitude. One can see this effect in Fig. 3. Only upper bounds exist for the $\iota\to\gamma\gamma$ partial width: e.g.,[35]

$$\Gamma(\iota\to\gamma\gamma)B(\iota\to\rho^0\rho^0)<1.0\text{ keV}\ . \qquad (44)$$

Since the ι lies below $2m_\rho$, this is not very restrictive. The ι is not seen to decay to $\rho^0\rho^0$ in $J/\psi\to\gamma\rho^0\rho^0$, for example.[38]

B. Tensor mesons

An unmixed nonstrange $J^{PC}=2^{++}$ quarkonium state is the A_2, so we may take[46]

$$M_N=m_{A_2}=1318\pm 5\text{ MeV}\ . \qquad (45)$$

An estimate of the corresponding $s\bar{s}$ mass depends on that of the K^{**}, which we estimate[47] to lie anywhere between 1420 and 1440 MeV. We then find

$$M_S=2m_{K^{**}}-m_{A_2}=1542\pm 21\text{ MeV}\ . \qquad (46)$$

With[30]

$$m_f=1273\pm 5\text{ MeV}\ , \qquad (47)$$

$$m_{f'}=1520\pm 10\text{ MeV}\ , \qquad (48)$$

$$m_\theta=1640\pm 50\text{ MeV}\ , \qquad (49)$$

we then find

$$\left[\frac{y}{x}\right]_f=0.12\pm 0.04\ ,\quad \left[\frac{x}{y}\right]_{f'}=-0.15\pm 0.24\ ,\quad \left[\frac{x}{y}\right]_\theta=0.4\pm 0.3\ . \qquad (50)$$

There is a small but non-negligible strange-quark admixture in the f, and the possibility of a slightly larger nonstrange-quark admixture in the f'. The errors in Eq. (50) reflect extreme limits in Eqs. (45)–(49). The ranges (50) may be expressed as radial sectors in Fig. 4. We see qualitative agreement with constraints derived earlier for f and f', and predict (as promised in Sec. IV) that the nonstrange- and strange-quarkonium components of θ add constructively in $\theta\to\gamma\gamma$.

A more detailed comparison of the model of Ref. 12 with the tensor-meson data is carried out in Ref. 37. There, under the added assumption that the matrix (36) is energy independent, we find that there is no consistent solution whereby one can accommodate all the observed masses and branching ratios.[48] No such assumption, however, was made in arriving at the relations (50).

VI. SUMMARY

We have discussed ways to measure the content of nonstrange and strange quarks in various isoscalar mesons with the help of electromagnetic transitions. We have concentrated on the channels with $J^{PC}=0^{-+}$ and 2^{++}, for which knowledge of this quark content is especially useful when evaluating candidates for gluonic bound states.

We have found a number of results, cited in the Introduction. With the help of measurements of processes such as $\phi\to\eta'\gamma$, and two-photon widths of $\iota(1440)$, f', and $\theta(1640)$, one can begin to sort out the mixing between quarkonium and gluonic bound states that would characterize the 0^{-+} (η,η',ι) and 2^{++} (f, f', θ) systems if these can be treated as three-state problems.

Further constraints on the 2^+ system are provided by such rare processes as $f'\to\pi\pi$, $\theta\to\pi\pi$, and $J/\psi\to\gamma f'$. These constraints have been investigated in Refs. 5–14, 37, and 49. The question of whether the θ is mainly a gluonic bound state, in our opinion, still remains open, and will probably be solved only when a theory of its mixing with quarkonium emerges.

---

*Present address.

[1]Recent reviews, from which further work can be traced, include J. D. Bjorken, in *Proceedings of the European Physical Society International Conference on High Energy Physics, Geneva, 1979,* edited by A. Zichichi (CERN, Geneva, 1980), p. 245, and in *Quantum Chromodynamics,* proceedings of the SLAC Summer Institute on Particle Physics, 1979, edited by Anne Mosher (SLAC, Stanford, 1980), p. 219, and as Report No. SLAC-Pub-2372, 1979 (unpublished); Michael S. Chanowitz, SLAC Summer Institute lectures, 1981 (unpublished); and Lawrence Berkeley Laboratory Report No. LBL-13593 (unpublished); John F. Donoghue, Comments Nucl. Part. Phys. 10, 277 (1982); D. L. Scharre, invited talk at Orbis Scientiae, Coral Gables, Florida, 1982, Report No. SLAC-PUB-2880 (unpublished).

[2]D. L. Scharre *et al.*, Phys. Lett. 97B, 329 (1980); C. Edwards *et al.*, Phys. Rev. Lett. 49, 259 (1982).

[3]C. Edwards *et al.*, Phys. Rev. Lett. 48, 458 (1982).

[4]John F. Donoghue, K. Johnson, and Bing An Li, Phys. Lett. 99B, 416 (1981); John F. Donoghue, Phys. Rev. D 25, 1875 (1982).

[5]Jonathan L. Rosner, Phys. Rev. D 24, 1347 (1981).

[6]Y.-M. Cho, University of Paris Report No. PAR LPTHE 81/19, 1981 (unpublished).

[7]Tadayuki Teshima and S. Oneda, University of Maryland Reports Nos. 82-178 and 83-031, 1982 (unpublished). The first paper contains an extensive list of earlier references to the mixing problem in the 0^- channel.

[8]Nobuyuki Aizawa, Ziro Maki, Isao Umemura, and Katsuji Yamamoto, Prog. Theor. Phys. 63, 962 (1980), and earlier references therein.

[9]V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. 86B, 347 (1979).

[10]H. Fritzsch and J. D. Jackson, Phys. Lett. 66B, 295 (1977).

[11]F. D. Gault and A. B. Rimmer, Z. Phys. C 8, 353 (1981); 9, 274 (E) (1981).

[12]Howard J. Schnitzer, Nucl. Phys. B 207, 131 (1982).

[13]Norman H. Fuchs, Phys. Rev. D 14, 1912 (1976).

[14]S. Minami, Osaka City University Report No. OCU-88, 1982 (unpublished); Lett. Nuovo Cimento 34, 525 (1982).

[15]M. Nikolić, Phys. Rev. D 26, 3141 (1982).

[16]See, in particular, Ref. 7 for $0^-\to\gamma\gamma$ decays, and many of the Refs. 6–10 for other constraints. We do *not* discuss the process $\psi\to 0^-\gamma$ here, however, which is treated in many of these references.

[17]The importance of this decay has also been stressed by Nilendra G. Deshpande and Gad Eilam, Phys. Rev. D 25, 270 (1982). We thank Dr. Eilam for calling attention to this reference.

[18]D. Burke, rapporteur's talk (Report No. SLAC-PUB-2988), and D. Lüke, talk (Report No. DESY 82/073) presented at XVI International Conference on High Energy Physics, Paris, 1982 (unpublished). Slightly different results for $f'\to\gamma\gamma$ are quoted by M. Althoff *et al.*, Report No. DESY 82/071, 1982 (unpublished): $\Gamma(f'\to\gamma\gamma)B(f'\to K\bar{K})=0.11\pm0.02$ (stat.)±0.04 (syst.) keV.

[19]N. Isgur, Phys. Rev. D 13, 122 (1976); 13, 129 (1976); Julian Schwinger, Phys. Rev. 135, B816 (1964).

[20]T. Ohshima, Phys. Rev. D 22, 707 (1980).

[21]D. E. Berg *et al.*, Phys. Rev. Lett. 44, 706 (1980).

[22]A. Browman *et al.*, Phys. Rev. Lett. 33, 1400 (1974).

[23]D. Andrews *et al.*, Phys. Rev. Lett. 38, 198 (1977), as updated in Ref. 20. These authors measure $B(\phi\to\eta\gamma)=(1.35\pm0.29)\%$.

[24]G. Cosme *et al.*, Phys. Lett. 63B, 352 (1976), as updated in Ref. 20. These authors measure $B(\phi\to\eta\gamma)=(1.5\pm0.4)\%$. (Other experiments with somewhat larger error bars are also quoted in Ref. 20.)

[25]A. Browman *et al.*, Phys. Rev. Lett. 32, 1067 (1974).

[26]The value for $\Gamma(\eta'\to\rho\gamma)$ is based on taking $B(\eta'\to\rho\gamma)=0.300\pm0.016$, as quoted by Particle Data Group, Phys. Lett. 111B, 1 (1982), and $\Gamma_{\rm tot}(\eta')=280\pm100$ keV, as measured by D. M. Binnie *et al.*, Phys. Lett. 83B, 141 (1979). An essentially equivalent result for $\Gamma_{\rm tot}(\eta')$ is obtained if one combines $\Gamma(\eta'\to\gamma\gamma)$ and $B(\eta'\to\gamma\gamma)$ as quoted by the Particle Data Group. Ohshima (Ref. 20) quotes $\Gamma(\eta'\to\rho\gamma)=93\pm25$ keV on this basis.

[27]V. Sidorov, in *Proceedings of the 1979 International*

Symposium on Lepton and Photon Interactions at High Energies, Fermilab, edited by T. B. W. Kirk and H. D. I. Abarbanel (Fermi National Accelerator Laboratory, Batavia, Illinois, 1980), p. 490.

[28]G. S. Abrams *et al.*, in *Lecture Notes in Physics, Vol. 134, $\gamma\gamma$ Collisions,* proceedings of the International Workshop, Amiens, France, 1980, edited by G. Cochard and P. Kessler (Springer, Berlin, 1980), p. 47. We are also aware of a new value for $\Gamma(\eta'\to\gamma\gamma)$ from the JADE group, W. Bartel *et al.*, Phys. Lett. 113B, 190 (1982); $\Gamma(\eta'\to\gamma\gamma)=5.0\pm0.5$ (stat.) ±0.9 (syst.) keV.

[29]S. L. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 51, 47 (1969).

[30]Particle Data Group (Ref. 26).

[31]Minami, first work in Ref. 14; M. Chanowitz, Phys. Rev. Lett. 46, 981 (1981); H. J. Lipkin, Phys. Lett. 109B, 326 (1982); S. Ono and O. Péne, *ibid.* 109B, 101 (1982); I. Cohen, N. Isgur, and H. J. Lipkin, Phys. Rev. Lett. 48, 1074 (1982).

[32]N. R. Stanton *et al.*, Phys. Rev. Lett. 42, 346 (1979).

[33]The $\gamma\gamma\to A_2$ width is based on detection of the $\eta\pi$ mode: see C. Edwards *et al.*, Phys. Lett. 110B, 82 (1982).

[34]L. Montanet, in *Proceedings of the 9th International Winter Meeting on Fundamental Physics, Siguenza, Spain, 1981,* edited by A. Ferrando (Instituto de Estudias Nucleares, Madrid, 1981), p. 449.

[35]M. Althoff *et al.*, DESY Report No. 82-062, 1982 (unpublished). It thus appears that the strong $\rho^0\rho^0$ signal observed near threshold in $\gamma\gamma\to\rho^0\rho^0$ by R. Brandelik *et al.*, Phys. Lett. 97B, 448 (1981), is *not* due to θ, but rather to a $J^P=0^+$ state.

[36]M. E. B. Franklin *et al.*, Bull. Am. Phys. Soc. 27, 553 (1982) (abstract HG3), and private communication; M. E. B. Franklin, Thesis, Stanford University, 1982 (unpublished); Elliott Bloom, rapporteur's talk at XVI International Conference on High Energy Physics, Paris, 1982, SLAC Report No. SLAC-PUB-3005, 1982 (unpublished). We thank Gary Feldman for early correspondence regarding the decay $J/\psi\to\gamma K^+K^-$.

[37]Jonathan L. Rosner and San Fu Tuan, University of Chicago Report No. EFI 82/49, 1982 (unpublished).

[38]D. L. Burke *et al.*, Phys. Rev. Lett. 49, 632 (1982).

[39]H. J. Lipkin, Phys. Lett. 109B, 326 (1982); Paul M. Fishbane, David Horn, Gabriel Karl, and Sydney Meshkov, 1981 (unpublished).

[40]Here we are in accord with the analysis of Ref. 14.

[41]A. Billoire, R. Lacaze, A. Morel, and H. Navelet, Phys. Lett. 80B, 381 (1979).

[42]A. Nishimura, Glasgow University report, 1979 (unpublished). Details of such a calculation are given by M. Krammer, Phys. Lett. 74B, 361 (1978).

[43]A. De Rújula, Howard Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[44]Hiroshi Suura and Masaaki Kuroda, Prog. Theor. Phys. 54, 1513 (1975).

[45]See, e.g., Stephen Gasiorowicz and Jonathan L. Rosner, Am. J. Phys. 49, 954 (1981).

[46]See Particle Data Group (Ref. 26). This value seems to be based primarily on the $K\bar{K}$ decay mode of the A_2. The $\rho\pi$ mode gives a value 8 MeV lower and the $\eta\pi$ mode gives one about 16 MeV higher than that in Eq. (45).

[47]The average quoted in the tables of Ref. 30 does not reflect that quoted in the data listings. Our estimate is based on the figure associated with the neutral-$K^*(1430)$ mass quoted in the data listings of Ref. 30.

[48]In this respect we agree with a recent analysis by Paul M. Fishbane, Gabriel Karl, and Sydney Meshkov, UCLA Report No. UCLA/82/TEP/18, 1982 (unpublished).

[49]Fishbane *et al.* (Ref. 48).