

Scaling violations in inclusive e^+e^- annihilation spectraC. Peterson,* D. Schlatter, I. Schmitt,[†] and P. M. Zerwas[‡]*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305*

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The origin of the observed scaling violations in inclusive e^+e^- annihilation is investigated. Perturbative jet evolution is not necessarily the only reason for scale breaking in the hadron spectra at present energies. Remnants of finite-transverse-momentum and mass effects are still important in nonperturbative, cascade-type, jet formation in the ~ 10 GeV range. Heavy-quark fragmentation has a strong impact on hadronic inclusive spectra. A simple parametrization for the heavy-quark fragmentation function is given which describes well recently measured charmed-meson spectra. Taking these effects into account, good agreement with the observed scaling violations is obtained in cascade-type jet models with hard-gluon bremsstrahlung.

I. INTRODUCTION

The inclusive hadron spectra in e^+e^- annihilation show significant scale-breaking effects.^{1,2} The purpose of this paper is to investigate the origin of these effects. We ask whether these are necessarily due to perturbative QCD or if there could be substantial remnants of nonperturbative effects contributing as well in the energy range between $\simeq 6$ GeV and $\simeq 30$ GeV.

The observation of three-jet events in e^+e^- annihilation has been taken as evidence for gluon radiation off quarks expected in quantum chromodynamics.³ Distributing the available energy among more than two quanta on the scale of fm softens the final hadron spectra. Since the bremsstrahlung probability rises with energy, the spectra should show scale-breaking effects. The magnitude of scale breaking which one expects from perturbative QCD calculations depends on the value of the mass cutoff at which nonperturbative effects set in.

If the perturbative evolution of quark and gluon partons proceeds undisturbed by confinement effects down to masses as low as $\simeq m_p$,^{4,5} this is the only source of scale breaking. Such a scheme might be problematic as the multiplicities of gluons and quarks radiated at distances of the order of 1 fm appear to be too small to avoid large color separation.

Alternatively one might assume the jets to be a nonperturbative phenomena at present energies, with all their properties fixed by confinement forces.⁶ There is no perturbative evolution in such a jet. Gluon bremsstrahlung is significant only if the transverse momentum of the bremsstrahlung quantum is large enough to pass unaffected through the field of confinement forces and to create a jet on its own.⁷ Here, perturbative QCD is considered to be

meaningful only on the scale of fm. Nevertheless, scale-breaking effects in inclusive particle spectra are not necessarily small in, e.g., a picture of nonperturbative jets,⁸ where hadrons are produced in the flux field created by the outgoing quarks. Finite masses and transverse-momentum effects inevitably give rise to an energy dependence of the spectra. For a typical transverse-mass scale of $m_{\perp} \sim \frac{1}{2}$ GeV, scale-breaking effects of the order $\langle n \rangle_{\text{jet}}(m_{\perp}/E_{\text{jet}})^a$ with $a \sim 1-2$ are expected. This energy dependence of the spectra is of the same magnitude as predictions of leading-logarithmic perturbative QCD.

In e^+e^- annihilation, the production of heavy quarks c and b with large mass scales poses an additional complication for the understanding of inclusive spectra. A quantum-mechanical argument will lead us to a simple parametrization for the fragmentation function of a heavy quark Q to, e.g., a heavy meson ($Q\bar{q}$). The resulting spectrum has the expected kinematical feature that the inertia of the heavy quark Q is retained by the ($Q\bar{q}$) meson.⁹

The paper is organized as follows. In Sec. II we study the influence of finite masses and finite transverse momenta on scale breaking in a nonperturbative cascade model.¹⁰ We also investigate scale breaking due to the emission of a third quantum, a hard gluon, in the Born approximation. Section III is devoted to perturbative QCD predictions for the energy dependence of hadron distributions in quark jets. The jet evolution is derived from Altarelli-Parisi-type equations¹¹ in first- and second-order¹² QCD, and the result is confronted with the Born approximation of Sec. II. In Sec. IV heavy-quark production is discussed with a simple conjecture for the fragmentation function. In Sec. V we compare the above models with recently obtained single-particle spectra in e^+e^- annihilation. A short outlook sum-

marizes some attempts to get more insight into non-perturbative and perturbative jet structures.¹³

II. SCALING VIOLATIONS FROM NONPERTURBATIVE PARTON FRAGMENTATION

The creation of light-quark mesons with exponential falloff in transverse momentum during the fragmentation process suggests a nonperturbative treatment of jet formation. Such a picture has been sketched in the form of an inside-outside cascade by Bjorken.⁶ A parton moving out of the perturbative interaction region on the scale of fm builds up a color flux tube. When the length of the flux tube grows larger than ~ 1 fm, spontaneous creation of quark-antiquark pairs occurs and the string breaks, leaving us with mesons and yet another quark as the new color source. Repetition of this mechanism results in a jet of hadrons with limited transverse momentum.

Based on such a dynamical picture of jet formation, algorithms have been developed for Feynman-Field-type cascades with *ad hoc* built-in (approximate) scale invariance and limited transverse momenta of the hadrons. Nonzero transverse momenta and finite hadron masses in the jet inevitably induce scale-breaking effects. The correct incorporation of these kinematical parameters in partonic jet cascades is a first, necessary step in the understanding of any scale breaking. This way we hope to get an estimate of the possible size of nonperturbative scale-breaking effects. Calculating their precise form in QCD requires solving this theory at large distances, which has not been achieved yet.

For an average multiplicity $\langle n \rangle_{\text{jet}}$ in a jet we expect the scaling violation to be $\sim [\langle n \rangle_{\text{jet}} (m_{\perp}/E_{\text{jet}})^a]$, $m_{\perp} \sim \frac{1}{2}$ GeV being the average transverse meson mass in a jet and $a \sim 1-2$. For 3 GeV jet energy with a multiplicity of primary hadrons in a jet of $\langle n \rangle_{\text{jet}} \simeq 4$, the influence of these kinematic factors is huge, $\sim 40\%$. Although the scale breaking disappears as $\propto (\log E_{\text{jet}})/E_{\text{jet}}^a$ with rising energy, the effect is still $\sim 10\%$ at $E_{\text{jet}} = 15$ GeV with $\langle n \rangle_{\text{jet}} \simeq 8$. Changes of order 2 are therefore expected in the (scaled) energy spectra of mesons in quark jets when the e^+e^- energy is raised from 6.5 to 30 GeV. These expectations are substantiated by an elaborate Monte Carlo study of π mesons in light- u, d, s -quark jets. The Lund model¹⁰ has been used for this purpose. Figure 1 (dashed line) presents the results of this study for various energy ratios.

It is clear that these scale-breaking effects are not artifacts of cascade Monte Carlos. This can nicely be demonstrated in longitudinal phase-space models in which particles are produced with limited trans-

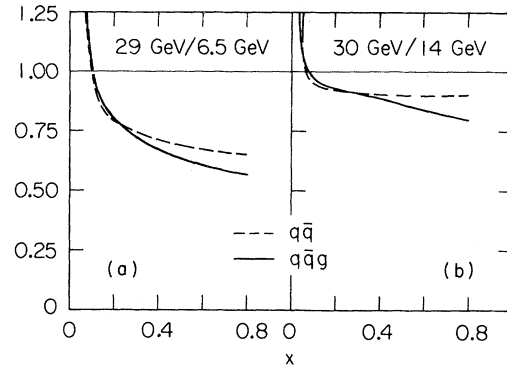


FIG. 1. (a) Ratio of $(1/\sigma)d\sigma/dx$ at 29 and 6.5 GeV for $q=u, d, s$ from a Monte Carlo simulation of $q\bar{q}$ events (dashed line) and $q\bar{q}g$ events (full line, $\Lambda = 200$ MeV). (b) Same as (a) but for 30 GeV/14 GeV.

verse momentum along the jet axis, and probabilities given otherwise just by phase space.¹⁴

In addition to these scale-breaking effects of purely nonperturbative nature we expect additional effects due to perturbative gluon bremsstrahlung in QCD. As the nonperturbative section of QCD is not solved, we have to rely on conjectures for the merger of perturbative and nonperturbative effects. In a first attempt to construct a phenomenological model of gluon bremsstrahlung combined with hadronization of quark and gluon jets, a cutoff of $\gtrsim 4$ GeV for the invariant mass of quark and gluon pairs is introduced.^{7,13} Gluons emitted below this mass submerge in the nonperturbative $q\bar{q}$ two-jet formation. The magnitude of this cutoff coincides with the limit of e^+e^- energy below which two-jet structures cannot be resolved any more. A sudden change from the nonperturbative QCD regime is suggestive as it occurs in lattice gauge theories and bag-type models as well. (The magnitude of the cutoff mass need not be the same in different dynamical problems though.) The solid lines in Fig. 1 describe first-order gluon bremsstrahlung added to the cascade jet evolution of light quarks, with a cutoff parameter $M(qg) \simeq 4$ GeV. At low energies the scale-breaking effects due to bremsstrahlung are small as compared with nonperturbative effects. At large energies, however, all scale-breaking effects are due to gluon bremsstrahlung as finite p_{\perp} and mass effects disappear asymptotically. We have checked this explicitly in the Lund Monte Carlo for $x > 0.1$ by comparing hadron spectra at 30 and 90 GeV.

III. PERTURBATIVE QCD JETS

Assuming that QCD perturbation theory is valid down to cutoff parameters of order m_{ρ} , all energy dependence of the inclusive cross section for light

mesons originated from gluon radiation off quarks and gluon splitting into gg and $q\bar{q}$ pairs.

The change of the meson spectrum with rising en-

ergy can then be described by evolution equations closely related to the well-known Altarelli-Parisi equations^{11,15}:

$$\frac{\partial}{\partial \ln Q^2/\Lambda^2} D_q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} [D_q(y, Q^2) P_{qq}(x/y) + D_g(y, Q^2) P_{gq}(x/y)], \quad (1a)$$

$$\frac{\partial}{\partial \ln Q^2/\Lambda^2} D_g(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[\sum D_q(y, Q^2) P_{qg}(x/y) + D_g(y, Q^2) P_{gg}(x/y) \right]. \quad (1b)$$

D_q is the quark fragmentation function into mesons (for simplicity all are assumed to be π 's), D_g is the gluon fragmentation function. For D_g the same shape was taken as for D_q at the initial value Q_0 .¹⁶ We have fixed $Q_0 = 6.5$ GeV where data are available. The value of Λ was chosen to be 200 MeV. Figure 2 shows the evolution of the meson spectrum for the same energies as considered in the previous section.

Before discussing these results in greater detail we add some comments on second-order corrections. All higher-order QCD corrections are plagued by infrared problems. The positive divergences due to soft-gluon radiation are canceled by negatively infinite vertex corrections. The result is finite once the parameters of jet resolution are properly chosen. In a space-time picture this cancellation involves infinite distances. However, confinement in QCD introduces a cutoff preventing partons from traveling freely to distances of many fm. Thus we do expect corrections to perturbatively calculated cross sections to ~ 1 GeV even without considering any detailed hadronization model. To check the internal consistency of the perturbative calculation, we leave

these provisos aside for the moment, and use the second-order corrections to fragmentation functions from Ref. 12. The solution of the integrodifferential equations is found numerically by adapting a method developed in Ref. 17. The ratio of scaling violations of second order to first order is bigger in e^+e^- than in deep-inelastic scattering. In the evolution from 6.5 to 29 GeV, however, the higher-order corrections remain small compared to the leading-logarithmic corrections; they are of order 10%, rising slowly with x (see Fig. 2, dotted line). These higher-order corrections increase scaling violations slightly.

The results obtained in the cascade model (full line in Fig. 2) and the perturbative jet evolution (dashed and dotted lines in Fig. 2) are remarkably close to one another at present energies. This is no accident. Both are branching processes and both are characterized by energy parameters of similar size, the cascade by $\langle p_\perp \rangle \simeq 400$ MeV and π, ρ masses and the perturbative parton shower by $\Lambda = 200$ MeV. A large lever arm in energy, starting at a high Q_0 , can separate these effects.

IV. HEAVY-QUARK FRAGMENTATION

Whereas light-quark fragmentation into mesons is characterized by a behavior $z^{-1}(1-z)^2$, quite a dif-

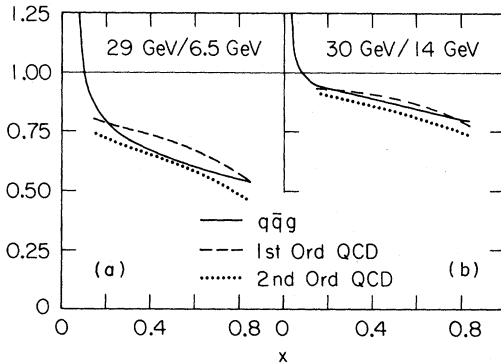


FIG. 2. (a) Ratio of $(1/\sigma)d\sigma/dx$ at 29 and 6.5 GeV for $q = u, d, s$. Full line is a Monte Carlo simulation of $q\bar{q}g$ events. The dashed line is the first-order QCD evolution with $\Lambda = 200$ MeV, the dotted line is the second-order result. (b) Same as (a) but for 30 GeV/14 GeV.

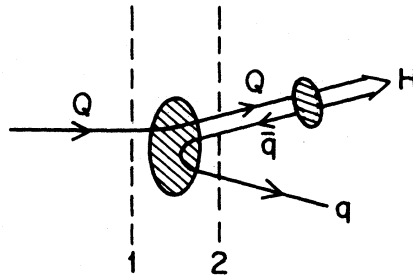


FIG. 3. The fragmentation of a heavy quark Q into a meson $H(Q\bar{q})$. Dashed lines are time slices used in the derivation of Eq. (3).

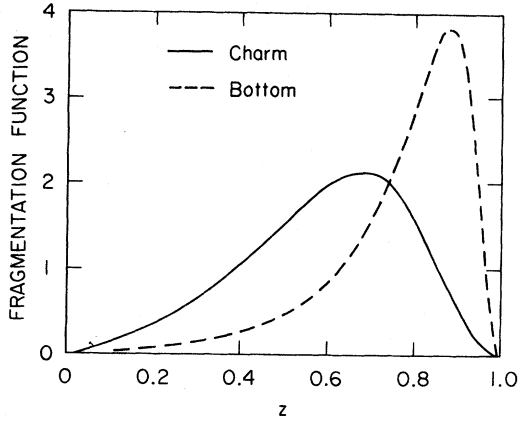


FIG. 4. Fragmentation functions $D_c(z)$ and $D_b(z)$ from Eq. (4) using $\epsilon_c=0.15$ and $\epsilon_b=0.15(m_c/m_b)^2=0.016$.

ferent shape is expected for the fragmentation of heavy quarks $Q=c,b,\dots$ into hadrons containing Q . This follows from simple kinematical considerations as first pointed out by Bjorken and Suzuki.⁹ Attaching a light antiquark \bar{q} to a heavy quark Q (or a diquark qq for baryon production) decelerates the heavy quark in the fragmentation process only slightly. Thus Q and $(Q\bar{q})$ or (Qqq) should carry almost the same energy. This kinematical effect is expected to dominate over more suitable dynamical details.

In this section we derive a simple form for the heavy-quark fragmentation functions by adopting the standard quantum-mechanical parton-model recipe to estimate transition amplitudes, recently dis-

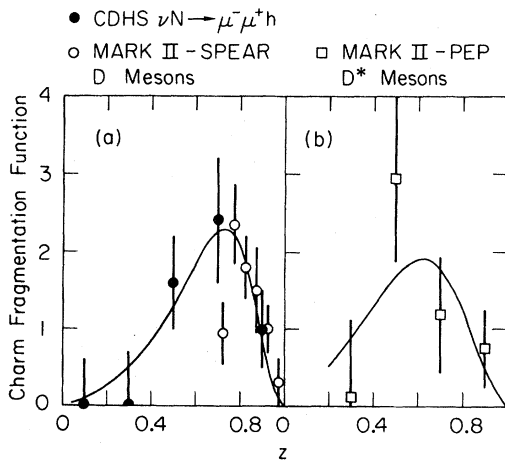


FIG. 5. (a) Charm fragmentation function at lower energies from the CDHS $\nu N \rightarrow \mu^- \mu^+ h$ neutrino data (Ref. 20), and the MARK II-SPEAR D data (Ref. 19) compared with the prediction of Eq. (4) with $\epsilon=0.1$. (b) $D_c(z)$ from the MARK II-PEP D^* data (Ref. 21) at 29 GeV together with the parametrization of Eq. (4) with $\epsilon=0.2$.

cussed in Ref. 18. The gross features of the amplitude for a fast moving heavy quark Q fragmentation into a hadron $H=(Q\bar{q})$ and light quark q (Fig. 3) are determined by the value of the energy transfer $\Delta E=E_H+E_q-E_Q$ in the breakup process,

$$\text{amplitude } (Q \rightarrow H+q) \propto \Delta E^{-1}. \quad (2)$$

Expanding the energies about the (transverse) particle masses ($m_H \simeq m_Q$ for simplicity),

$$\begin{aligned} \Delta E &= (m_Q^2 + z^2 P^2)^{1/2} + (m_q^2 + (1-z)^2 P^2)^{1/2} \\ &\quad - (m_Q^2 + P^2)^{1/2} \\ &\propto 1 - (1/z) - (\epsilon_Q/1-z) \end{aligned} \quad (3)$$

and taking a factor z^{-1} for longitudinal phase space, we suggest the following ansatz for the fragmentation function of heavy quarks Q

$$D_Q^H(z) = \frac{N}{z[1 - (1/z) - \epsilon_Q/(1-z)]^2}. \quad (4)$$

The normalization N is fixed by summing over all hadrons containing Q ,

$$\sum \int dz D_Q^H(z) = 1.$$

According to the above derivation, the parameter ϵ_Q is $\sim m_q^2/m_Q^2$, the ratio of the effective light- and heavy-quark masses. We expect the parameter m_q to be of the order of the nonperturbative strong-interaction scale $\sim (\frac{1}{2} \text{ to } 1)m_\rho$ which gives $\epsilon_Q \sim (\frac{1}{8} \text{ to } \frac{1}{2})/m_Q^2$. The fragmentation function peaks at $z \simeq 1 - 2\epsilon_Q$ with a width $\sim \epsilon_Q$. This function is illustrated in Fig. 4 for $Q=c$ and b with $\epsilon_c=0.15$ and $\epsilon_b=(m_c/m_b)^2\epsilon_c$.

A few comments ought to be added. The second step in Eq. (3) is strictly valid in the $P \rightarrow \infty$ limit. For finite energies light-cone variables are supposedly more appropriate. The fragmentation function is then defined by Eq. (4) above the minimum value of the light-cone variable where we cutoff the function sharply. Although this account of threshold effects is very crude it follows the pattern of bremsstrahlung models. This can be shown by calculating $e^+e^- \rightarrow \gamma^* \rightarrow Q + H(Q\bar{q}) + q$ for a pseudoscalar meson H with effective γ_5 -type coupling to quarks and the transverse momentum cutoff by hand. In practice we can use Eq. (4) for the energy distribution, cut off at m_H/E . [Even for a beam energy $E=3$ GeV this is surprisingly close to the energy distribution which is obtained from postulating (4) for the light-cone variable of charmed particles.]

We will discuss here three measurements of the charm fragmentation function. At low energies the direct measurement of D production in e^+e^- annihilation at SPEAR by the MARK II group¹⁹ and

an indirect measurement by analyzing opposite-sign dimuon production in νN interactions by the CERN-Dortmund-Heidelberg-Saclay (CDHS) collaboration.²⁰ The data of the second experiment indicate that charmed mesons are produced predominantly at large values of $z = E_H/E$, in good agreement with our simple ansatz [Fig. 5(a)]. The SPEAR data, which have a cutoff at about $z=0.7$, show that threshold effects indeed extend only very little beyond the cutoff. At higher energies the MARK II group has measured the D^* spectrum at PEP at 29 GeV.²¹ These data show the same general trend as the CDHS data. The two curves correspond to ϵ parameters of 0.1 and 0.2, the range which we anticipated for charm fragmentation. In regard to the small mass difference between D and D^* we expect the same shape for the D^* fragmentation function as for D . At $Q=30$ GeV, D^* 's are either produced through fragmenting c quarks in

$$e^+e^- \rightarrow c\bar{c} \rightarrow D^{*+}, D^{*0} \quad (5)$$

or in b cascade decays,

$$e^+e^- \rightarrow b\bar{b} \rightarrow c + (\bar{u}d) \text{ or (leptons)} \rightarrow D^{*+}, D^{*0} \quad (6)$$

Whereas the D^* spectrum of Eq. (5) is hard (dashed curve in Fig. 6), the spectrum of cascade decays is soft, since the energy on the average is equally distributed among the three fermions in the weak b decay of Eq. (6). Folding the c quark spectrum with

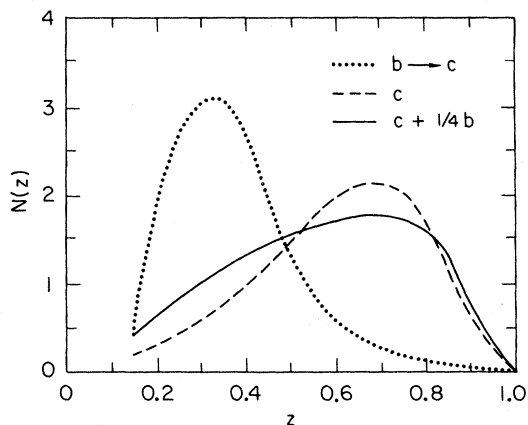


FIG. 6. Charmed-particle spectra in e^+e^- annihilation at 29 GeV. From charm-quark fragmentation (dashed line), from bottom decays (dotted line), and the superposition of both charm production mechanisms in e^+e^- collisions (full line). All distributions are separately normalized to 1.

the $c \rightarrow D^*$ fragmentation function gives the full curve of Fig. 6. In total, the D^* spectrum is slightly softened by the $b \rightarrow c$ cascade since the $b:c$ production ratio is 1:4 at present e^+e^- energies.²²

V. COMPARISON WITH MEASURED INCLUSIVE CROSS SECTIONS

The two experiments which have reported scaling violations in the inclusive cross section cover two different energy intervals. The MARK II group has measured hadron spectra at 6.5 and 29 GeV (Ref. 1) and the TASSO group has several measurements between 12 and 36 GeV (Ref. 2). Both experiments are above the charm threshold, while only the TASSO experiment is also above the bottom threshold.

Let us first consider the perturbative jet evolution. The light quark jets are assumed to evolve due to gluon bremsstrahlung and gluon splitting into quark-antiquark pairs with a Λ parameter of 200 MeV. For the initial fragmentation functions at $Q_0=6.5$ GeV we have chosen $D_q(x) = x^{-1.3}(1-x)^2$ for $x > 0.1$ (in agreement with the measured spectra) and the same form for $D_{\bar{q}}(x)$. For the fragmentation of heavy quarks we have assumed a form according to Eq. (4) with $\epsilon_c = 0.1$ independent of energy. We have repeated the analysis by taking $\epsilon_c = 0.2$ at high energies which yields a very small increase ($\approx 0.5\%$) of the scale-breaking effect. (Such a change could in fact be a consequence of gluon bremsstrahlung.) The result is shown in Fig. 7. The light-meson spectrum in heavy-quark fragmentation has been calculated by folding the $Q \rightarrow H + q$ fragmentation function with the light-quark q fragmentation function. Figure 7(a) shows the measured ra-

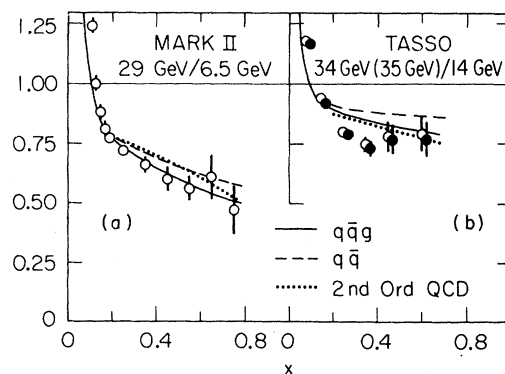


FIG. 7. (a) Ratio of $(1/\sigma)d\sigma/dx$ at 29 and 6.5 GeV for $q=u,d,s,c$, (b) from MARK II. Full line is a Monte Carlo simulation of $q\bar{q}g$ events with $\Lambda=200$ MeV. Dashed line is the Monte Carlo result for $q\bar{q}$ events. Dotted line is the second-order QCD evolution with $\Lambda=200$ MeV. (b) Same as (a) but for the TASSO results at 34 GeV (35 GeV)/14 GeV.

tio of charged-particle yields at 29 and 6.5 GeV in the MARK II experiment, together with the expectation from perturbative quark-jet evolution. Figure 7(b) shows a similar comparison for the TASSO data at 34 GeV (35 GeV) and 14 GeV.

Though these calculations are in reasonable agreement with the data, perturbative jet evolution is not necessarily the only explanation of scale-breaking effects in inclusive e^+e^- annihilation spectra. The predictions of nonperturbative cascade models¹⁰ with occasional hard-gluon bremsstrahlung do describe the data as well (see Fig. 7). While the effect of gluon bremsstrahlung on the ratio of the 6.5- and 29-GeV spectra is marginal compared to finite p_{\perp} and mass effects, gluon bremsstrahlung improves the agreement between this model and the data considerably at higher energies.²³

VI. CONCLUSIONS AND OUTLOOK

Scale-breaking effects in inclusive e^+e^- annihilation spectra are complex phenomena. The onset of scaling effects, aggravated by large masses in heavy-quark production, and perturbative scale breaking due to gluon bremsstrahlung in QCD mix, thus generating the experimentally observed patterns. It is nevertheless very important to study these effects both experimentally and theoretically as the hadron spectra result from the interesting interplay of short- and long-range forces in QCD. Two different models have been pursued to explain the hadron profile of quark and gluon jets. In one approach jets are essentially a perturbative QCD phenomenon, nonperturbative QCD forces being just responsible for a slight rearrangement of color-neutral energy clusters to hadrons. Perturbative jet evolution is then the origin of all scale-breaking effects. In another extreme case, jets in the ~ 10 GeV range are assumed to be a nonperturbative

phenomenon, with gluon bremsstrahlung occasionally developing a third jet in e^+e^- annihilation. In this picture scale-breaking effects are largely kinematic in origin at low energies, due to finite masses and transverse momenta, but they are due to dynamical gluon bremsstrahlung at high energies.

At present energies, data cannot (yet) clearly discriminate between these two models since their algorithmic structures are rather similar. Both are treated as branching processes, with parameters of similar size. When the jet energies are raised, however, the finite mass and p_{\perp} effects in the nonperturbative model disappear, and we are left with moderate scale-breaking effects due to hard, noncollinear gluon bremsstrahlung—in contrast to large scale breaking in perturbative jet models. (More precise measurements of heavy-quark fragmentation functions can eliminate the uncertainties due to copious heavy-quark production in e^+e^- annihilation.)

Of course, the measurement of other observables can help in telling the hadronization models apart.¹³ Transverse-momentum spectra, energy correlations, direct photon distributions, and many other measurements will, together with the inclusive spectra, enable us to resolve this important dynamical problem.

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- ²¹J. M. Yelton *et al.*, Phys. Rev. Lett. **49**, 430 (1982).
- ²² D^{*+} and D^{*0} can only be produced by c and b quarks and not by antiquarks. Since the electroweak forward-backward asymmetry of all quarks has the same sign in the Weinberg-Salam model with $\sin^2\theta_W \simeq 0.23$, the analysis of D^{*} 's is a very clean instrument to look at electroweak interference effects in the heavy-quark sector. The asymmetry is not washed out by quark and antiquark decays into the same final state (as for inclusive μ 's). A cut in x should give a good sample of D^{*} 's from directly produced c quarks. The asymmetries [reflecting directly the quark asymmetries, see, e.g., J. Jersak *et al.*, Phys. Lett. **98B**, 363 (1981)] are large: -10% , -15% , and -22% for a total e^+e^- energy of 30, 36, and 42 GeV, respectively.
- ²³All our calculations have been adjusted to the MARK II data and mapped onto the TASSO data. As can be seen by a direct comparison of the two experiments, the TASSO data at 14 GeV in the interval of $0.2 < x < 0.4$ seem to be higher than expected from a simple interpolation between the low- and high-energy points.