First-order phase transition in the SU(3) gauge theory at finite temperature

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We argue that the phase transition which separates the low-temperature confining phase from the high-temperature nonconfining phase in the SU(3) gauge theory (without quarks) is first order.

Quantum chromodynamics in the absence of quarks is expected to possess a phase transtion at finite temperature separating a confining low-temperature phase from a nonconfining high-temperature phase. Perturbative analysis[1] and Monte Carlo data[2] support this expectation. Unfortunately, almost no quantitative information concerning the nature of this transition is available. (Perturbation theory is completely inapplicable near the transition, and currently available Monte Carlo data do not have sufficiently high statistics to enable one to extract more than a rough estimate of the transition temperature.) In this paper, we argue that the finite-temperature transition should be first order. This prediction is specific to the SU(3) pure gauge theory in $3+1$ dimensions.[3] Our argument is based on a few qualitative features of gauge-theory dynamics, plus certain standard results from the renormalization-group treatment of critical phenomena.[4]

The partition function may be represented as a functional integral over gauge fields which are periodic in (Euclidean) time with period $\beta \equiv 1/T$,

$$Z=\int_{A_\mu(\beta,\vec{x})=A_\mu(0,\vec{x})} \mathfrak{D}A \times \exp\left[-\frac{1}{g^2}\int_0^\beta dt \int d^3x\, \tfrac{1}{4}\,\hat{\mathrm{tr}}\, F_{\mu\nu}{}^2\right] . \tag{1}$$

The presence or absence of confinement in the finite-temperature gauge theory is directly determined by the realization of a $Z(3)$ global symmetry. This symmetry is generated by the action of local gauge transformations which are periodic up to an arbitrary element of the center, $U(\beta,\vec{x})=zU(0,\vec{x})$, $z=e^{i2\pi n/3}$, modulo strictly periodic local gauge transformations. The "Wilson line,"

$$L(\vec{x}) \equiv \mathrm{tr}\, P \exp\left[\int_0^\beta dt\, A_0(t,\vec{x})\right] ,$$

transforms as $L \to zL$, and its expectation may be regarded as an order parameter for the $Z(3)$ global symmetry.[5] Physically, the Wilson line expectation determines the free energy of a single static quark relative to the vacuum, via

$$e^{-\beta F_q} = \langle L(\vec{x})\rangle \tag{2}$$

and the correlation function of two Wilson lines yields the static quark-antiquark free energy,

$$\exp[-\beta F_{q\bar{q}}(\vec{x}-\vec{y})] = \langle L(\vec{x})L(\vec{y})^\dagger\rangle \ . \tag{3}$$

At low temperatures, non-Abelian gauge theories are believed to be in a confining phase in which the static quark-antiquark potential rises linearly, $F_{q\bar{q}}(\vec{x})=\sigma|\vec{x}|$. Equivalently, the correlation function (3) decays exponentially, $\langle L(0)L(\vec{x})^\dagger\rangle \sim e^{-\beta\sigma|\vec{x}|}$, the order parameter vanishes, $\langle L(\vec{x})\rangle = 0$, and the global $Z(3)$ symmetry is unbroken. Furthermore, expectations of spacelike Wilson loops,

$$W_c \equiv \mathrm{tr}\, P \exp\left[\int_c \vec{A}\cdot d\vec{x}\right] ,$$

are believed to decay with an area law for large loops, $\langle W_c\rangle \sim \exp[-\kappa(\text{area})]$. Note that at nonzero temperature, the coefficient of the area law, κ, is *not* related to the physical string tension, σ. Finally, the confining phase is believed to possess a nonzero mass gap which is reflected, for example, in the exponential decay of the connected correlation of two well-separated spacelike Wilson loops.

To understand the high-temperature behavior of the theory, it is convenient to rescale the time variable, $t=\beta\tau$, and the timelike component of the gauge field, $A_0 \to A_0/\beta$, yielding

$$Z=\int_{A_\mu(1,\vec{x})=A_\mu(0,\vec{x})} \mathfrak{D}A_\mu \times \exp\left[-\frac{1}{g^2 T}\int_0^1 d\tau \int d^3x\, \hat{\mathrm{tr}}\left(\tfrac{1}{2}T^2\vec{E}^2+\tfrac{1}{2}\vec{B}^2\right)\right] . \tag{4}$$

One sees that the contribution of any configuration with a nonzero electric field becomes arbitrarily small at $T \rightarrow \infty$. Consequently, at high temperatures the functional integral is highly peaked about configurations with zero electric field,

$$\vec{E} \equiv \partial \vec{A}/\partial \tau - \vec{D} A_0 = 0 \quad , \tag{5}$$

that is, configurations in which the spatial gauge fields are static up to a gauge transformation. Condition (5) plus the basic periodicity condition, $\vec{A}(\tau=1) = \vec{A}(\tau=0)$, may easily be shown to imply that the "twist"

$$\Omega(\vec{x}) \equiv P \exp\left[\int_0^\beta dt\, A_0(t, \vec{x})\right]$$

is covariantly constant,

$$\vec{D}\Omega(\vec{x}) \equiv \vec{\partial}\Omega(\vec{x}) + [\vec{A}(\vec{x}), \Omega(\vec{x})] = 0 \quad . \tag{6}$$

For almost all configurations of the spatial gauge field, this condition in turn implies that the twist is a constant element of the center, $\Omega(\vec{x}) = z \epsilon Z(3)$.[6] Thus, at high temperatures the only significant contribution to the functional integral comes from the neighborhood of configurations which are static (up to a gauge transformation) and for which the twist is a constant element of the center.

Since the Wilson line $L(\vec{x})$ is simply the trace of the twist $\Omega(\vec{x})$, this result shows that for sufficiently high temperature the order parameter $\langle L(\vec{x})\rangle$ will have a nonzero expectation, and the global $Z(3)$ symmetry will be spontaneously broken. This establishes the existence of a high-temperature nonconfining phase in the theory.

The fact that essentially static configurations dominate the functional integral implies that the high-temperature behavior of the spatial gauge fields in the $(3+1)$-dimensional theory is determined by the dynamics of the zero-temperature three-dimensional SU(3) pure gauge theory (with coupling g^2T). Three-dimensional non-Abelian gauge theories are believed to generate a nonzero mass gap and area-law behavior for Wilson loops.[7] Therefore, spacelike Wilson loops in the high-temperature $(3+1)$-dimensional theory should continue to have area-law behavior. Thus, the behavior of the spatial gauge fields shows no qualitative change between low and high temperature. We emphasize that this area-law behavior for spacelike loops does not imply physical quark confinement in the high-temperature theory.[8]

Since the global $Z(3)$ symmetry is unbroken at low temperatures and broken at high temperatures, there must be a phase transition separating the confining and nonconfining phases. We would like to argue that if this transition is continuous then the critical behavior will be due to long-range fluctuations in the Wilson line $L(\vec{x})$. Specifically, integrating out the spatial gauge fields should produce only short-range interactions in the resulting effective theory for the order parameter $L(\vec{x})$.[9] The basis for this suggestion is the aforementioned fact that the behavior of the spatial gauge fields shows no qualitative change between low and high temperature. At low temperature, integrating out the spatial gauge fields will clearly produce an effective theory in which the range of interactions is on the order of the zero-temperature correlation length. At high temperature, integrating out the spatial fields should produce interactions whose range is on the order of the inverse mass gap in the three-dimensional gauge theory [i.e., $O((g^2T)^{-1})$]. Since the behavior of the spatial gauge fields is qualitatively similar in both extremes, it is at least consistent to expect similar behavior at all temperatures.

In sum, integrating out the spatial gauge fields should produce an effective three-dimensional theory for the Wilson line which has short-range interactions and is invariant under the $Z(3)$ symmetry, $L(\vec{x}) \rightarrow zL(\vec{x})$. If, as a function of temperature, this theory has a second-order transition, then by applying standard renormalization-group ideas[4] one could, at least in principle, locate the renormalization-group fixed point which governs the critical behavior. However, in the space of three-dimensional $Z(3)$-symmetric theories, no stable renormalization-group fixed point is known. (A $Z(3)$-symmetric theory of a single complex scalar field $L(\vec{x})$ may be regarded as a U(1)-invariant theory in the presence of perturbations, such as $\mathrm{Re}[L(\vec{x})^3]$, which break the symmetry down to $Z(3)$. $L(\vec{x})^3$ is a sufficiently low dimension operator that an arbitrarily small-L^3 perturbation will always destabilize the U(1)-invariant fixed point.[4]) Therefore $Z(3)$-invariant theories in three dimensions are believed always to have first-order transitions.[10]

This absence of a stable fixed point leads us to predict that the original $(3+1)$-dimensional SU(3) gauge theory must have a first-order finite-temperature phase transition as well. Clearly this argument is not a proof. However, it does provide the simplest possible description of the transition consistent with the current understanding of gauge-theory dynamics and critical phenomena. We cannot yet say anything about the expected size of the latent heat at the transition, or address similar detailed questions. This requires precise quantitative control over the theory on both sides of the transition; such control is totally lacking today.

Finally, we say a few words about the effects of quarks. Imagine decreasing the mass of the quarks from infinity down to zero. Since the pure SU(3) gauge-theory transition is predicted to be first order, this transition will certainly be stable for sufficiently large quark mass, $m_{\mathrm{cr}} < m < \infty$. (This need not have been true if the pure gauge-theory transition were second order.) A nonzero critical mass, m_{cr}, may exist such that for $m < m_{cr}$ the transition becomes continuous, or perhaps disappears altogether. (In the presence of dynamical quarks with nonzero mass, no symmetry exists whose breakdown guarantees the existence of a phase transition.) Alternatively, a first-order transition may remain all the way down to zero mass, where a phase transition associated with chiral symmetry must be present. The order of this transition is currently unknown.

If the finite-temperature transition remains first order in the presence of light quarks, then one might hope to see some experimental signatures of the transition (such as metastability of the plasma phase) in heavy-ion collisions.[11]

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[1]See, for example, D. J. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys. 53, 43 (1981).

[2]K. Kajantie, C. Montonen, and E. Pietarinen, Z. Phys. C 9, 253 (1981).

[3]For a fuller discussion of the expected critical behavior in this and other gauge theories, see B. Svetitsky and L. Yaffe, Nucl. Phys. B (to be published).

[4]For reviews, see, for example, P. Pfeuty and G. Toulouse, *Introduction to the Renormalization Group and Critical Phenomena,* (Wiley, New York, 1977), or *Phase Transitions and Critical Phenomena,* edited by C. Domb and M. S. Green (Academic, New York, 1976), Vol. 6.

[5]For further discussion of this material see Ref. 3, or L. D. McLerran and B. Svetitsky, Phys. Rev. D 24, 450 (1981).

[6]For details, see Ref. 3.

[7]R. P. Feynman, Nucl. Phys. B188, 479 (1981).

[8]The preceding discussion of the high-temperature phase is completely consistent with perturbative results. Direct examination of finite-temperature perturbation theory shows one that (a) all nonstatic components of the gauge field behave like massive fields with large masses $[O(T)]$ whose only significant effects are a number of finite renormalizations [such as replacing g^2 by the running coupling $g^2(T)$], and (b) the static component of A_0 acquires a mass of order (gT) due to Debye screening (Ref. 1). Therefore, the long-distance behavior of the theory is controlled by the dynamics of the static components of the spatial gauge fields. This portion of the theory is precisely a three-dimensional pure gauge theory with coupling g^2T. Since the scale g^2T which governs the dynamics of the spatial gauge fields is much less than the masses of A_0 and of the nonstatic components, these latter fields decouple as $T \to \infty$, and for high temperature produce corrections to the three-dimensional pure gauge dynamics suppressed by powers of g. [High temperature always means $T >> \Lambda$ so that $g^2(T) << 1$.] See Ref. 1 for further discussion.

[9]Note that after integrating out the spatial gauge fields, gauge invariance requires the resulting theory to depend only on symmetric functions of the eigenvalues of $\Omega(\vec{x})$. For SU(3) any such function may be expressed in terms of the Wilson line $L(\vec{x}) = \mathrm{tr}\,\Omega(\vec{x})$.

[10]Further evidence for this comes from study of the three-state Potts model. See S. J. Knak Jensen and O. G. Mouritsen, Phys. Rev. Lett. 43, 1736 (1979); J. B. Kogut and D. K. Sinclair, Solid State Commun. 41, 187 (1982); F. Fucito and A. Vulpiani, Phys. Lett. 89A, 33 (1982); F. Y. Wu, Rev. Mod. Phys. 54, 235 (1982).

[11]R. Anishetty, P. Koehler, and L. McLerran, Phys. Rev. D 22, 2793 (1980).