PHYSICAL REVIEW D VOLUME 26, NUMBER 4 15 AUGUST 1982

Extended Becchi-Rouet-Stora invariance for gravity via local OSp(4/2) supersymmetry

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(Received 24 February 1982)

The supersymmetry OSp (4/2) provides a natural classification group for gauge fields and their fictitious partners, needed for gauge fixing and restoration of unitarity. When the gravitational field and its adjuncts are so treated, with appropriate geometrical restrictions on the geometry of the superspace, a natural extended Becchi-Rouet-Stora (BRS) symmetry emerges for gravity in which the ghosts appear on an equal footing. The resulting action, invariant under gravitational BRS transformations and their duals, differs from the conventional asymmetric form. The validity of the new BRS identities is verified at the tree and one-loop levels.

I. INTRODUCTION

Although of recent origin, the concept of Becchi-Rouet-Stora (BRS) invariance in gauge theory has proved extraordinarily fruitful. Taken in conjunction with the fictitious field equations, the global BRS symmetry of the action is strictly equivalent to the original local gauge invariance,[1] and it has greatly facilitated the renormalization program of unified models as well as shedding new light on the unitarity of the theories. Soon after the discovery[2] of the original BRS symmetry for Yang-Mills theory, it became apparent that the Lagrangian admitted another "dual" BRS invariance[3] in which the role of ghost and antighost were essentially interchanged. The question then arose as to whether gravity, with its own BRS symmetry,[4] also admitted a dual invariance.[5] It became clear that simple "trial and error" methods, based on the analogy with vector mesons, would not work[5] and the indications where that radical modifications of the ghost Lagrangian[6] would be needed to permit the dual transformations.

In parallel with these developments, the ideas of supersymmetry proved beneficial to the development of gauge theory.[7] It becomes possible to incorporate the Yang-Mills field and its ghost (+ antighost) into a natural gauge supervector[8] by extending space-time to a six-dimensional superspace, admitting an OSp(4/2) geometrical supergroup [see also (Ref. 9)]. The associated supertranslation and Sp(2) invariance of the Lagrangian then finds its natural expression in the ordinary plus dual BRS symmetry of the theory, where the fictitious fields enter on a completely equal footing, a situation which is reflected in the "Hermitian" limit of other treatments.[10,11] The symmetrical OSp(4/2) Lagrangian differs from that of the traditional treatment by a BRS variation[11] which produces a renormalizable, four-ghost, coupling among other effects. Nevertheless, the on-shell S matrix is deemed equivalent to the conventional one as the changes merely imply field redefinitions. It is worth pointing out that, although the superfield approach to BRS symmetry differs from the description in terms of the intrinsic gauge geometry,[12] nevertheless (in the Yang-Mills case) the extended BRS transformations agree.[11]

This supersymmetric version of gauge theory has led us to reconsider the notion of dual BRS variations for gravity. In a recent letter,[13] we have demonstrated that a supersymmetric OSp(4/2) treatment of gravity successfully admits dual BRS transformations and, as anticipated from the Sp(2) structure, the ghost and antighost enter in a totally symmetrical manner. The ghost fields interactions with the gravitational field (and an auxiliary field) are nontrivial and could not have been guessed beforehand. In this paper we would like to expose the details of the construction which, for lack of space, were suppressed in the letter. We shall go further and establish fully and verify properly the ensuing BRS identities for the Green's functions of the new action.

In Sec. II is given the geometrical concepts underlying the local OSp(4/2) formalism, with particular emphasis on the sechbein and "spin connection" aspects. The restrictions needed to ensure flatness in the spinorial directions—a kind of dimensional reduction in the fermionic degrees of freedom—are determined in Sec. III. This is the heart of the paper and we are able to establish the BRS transformations in terms of the restricted metric. The associated action, gravitational plus ghost, is constructed in the next section; it is of course invariant under the extended BRS variations. The gauge-fixing term in general contains an arbitrary scalar weight p; we take advantage of the weight $p=0$ in order to simplify the Lagrangian and arrive at a relatively simple action, associated with the de Donder gauge. Next the new BRS identities are derived and explicitly verified at the tree level in Sec. V. Section VI establishes the correctness of the self-energy identities to one-loop level, and finally Sec. VII contains a discussion of the S-matrix equivalence of this work with the conventional formalism. There are three Appendixes: the first is very short and merely sets out our notation; the second shows how the new action is BRS invariant, without reference to earlier work; and the third lists the Feynman rules required to check the identities of Sec. VI.

II. SUPERSPACE GEOMETRY AND LOCAL OSp(4/2) SUPERSYMMETRY

Our development of local OSp(4/2) supersymmetry will follow the formalism of Wess.[14] The geometrical arena is that of six-dimensional superspace, parametrized by local coordinates $X^M=(x^\mu,\theta^m)$ and equipped with a structure group OSp(4/2). We consider a basis of local one-forms (see Appendix for notational details),

$$E^A(X)=dX^M E_M{}^A ,$$

which transforms into

$$E'^A=E^B U_B{}^A \tag{1}$$

under local OSp(4/2) transformations. [These are simply the extension of Weyl's local SL(2,C) transformations for gravity into superspace.] The spin connection ϕ is a Lie-algebra-valued one-form,

$$\phi=E^A\phi_A=dX^M E_M{}^A\phi_A=dX^M\phi_M\equiv\tfrac{1}{2}\phi_M{}^{AB}J_{AB} , \tag{2}$$

wherein J_{AB} are the local superalgebra generators. Taking matrix elements of (2) and using (A.1), we have simply $(\phi)_A{}^B=\phi_A{}^B\equiv dX^M\phi_{MA}{}^B$. From this point of view an OSp(4/2) transformation engenders the change

$$\phi_A{}^B\to\phi'_A{}^B=U^{-1}{}_A{}^C(\phi_C{}^D U_D{}^B+dU_C{}^B) . \tag{3}$$

The dynamical fields are the sechsbein $E_M{}^A$ and the spin connection $\phi_{MA}{}^B$ and, under general coordinate transformations, they naturally behave as supervectors,

$$E'_M{}^A(X')=\frac{\partial X^N}{\partial X'^M}E_N{}^A,\quad \phi'_M{}^{AB}(X')=\frac{\partial X^N}{\partial X'^M}\phi_N{}^{AB} . \tag{4}$$

They are of fundamental importance for treating fermionic matter. Secondary quantities are the torsion two-form,

$$T^A=dE^A-E^B\phi_B{}^A , \tag{5}$$

the metric tensor,

$$G_{MN}=E_M{}^A[AN]E_{NA} \tag{6}$$

and the curvature two-form

$$R_A{}^B=d\phi_A{}^B-\phi_A{}^C\phi_C{}^B .$$

For the case of pure gravity, the torsion can be made to vanish (Riemannian geometry) and one obtains relations between E and ϕ which can be solved algebraically as

$$\begin{aligned}\phi_{LMN}=\tfrac{1}{2}[&(\partial_L E_M{}^A-[LM]\partial_M E_L{}^A)E_{NA}[AN]-[LM][LN](\partial_M E_N{}^A-[MN]\partial_N E_M{}^A)E_{LA}[AL]\\&+[LN][MN](\partial_N E_L{}^A-[LN]\partial_L E_N{}^A)E_{MA}[AM]] .\end{aligned} \tag{7}$$

Under these circumstances the covariant derivative for matter fields (in some representation), $(D\psi)^i=d\psi^i-\psi^j(\phi)_j{}^i$, reduces for tensor fields to the usual rule involving the Christoffel affinity, namely,

$$\begin{aligned}D_M\psi^N&=\partial_M\psi^N+\psi^L\Gamma_{LM}{}^N ,\\D_M\psi_N&=\partial_M\psi_N-\Gamma_{MN}{}^L\psi_L\end{aligned} \tag{8}$$

with

$$\Gamma_{MN}{}^{L}=\tfrac{1}{2}(\partial_M G_{NK}+[MN]\partial_N G_{MK} - [KM][KN]\partial_K G_{MN})G^{KL}\ , \quad (9)$$

where G^{KL} is the inverse of the normal covariant metric G_{KL}. The curvature then assumes the conventional form (see Also Ref. 15)

$$R_{KLM}{}^{N}=\partial_K\Gamma_{LM}{}^{N}-[KL]\partial_L\Gamma_{KM}{}^{N} -[ML]\Gamma_{KM}{}^{R}\Gamma_{RL}{}^{N} +[KL][KM]\Gamma_{LM}{}^{R}\Gamma_{RK}{}^{N}\ . \quad (10)$$

It is possible to reexpress (9) and (10) directly in terms of sechsbein field, but that is needless for our purposes.

III. FERMIONIC FLATNESS AND EXTENDED BRS SYMMETRY

The general superspace formalism describes more component fields than are required for conventional gauge fixing and ghost compensation of the gravitational field. We know,[4] *a priori*, that we can get by with $g_{\mu\nu}$ the ghost field ω_μ, the antighost $\bar{\omega}_\mu$, plus possibly an auxiliary field B_μ. Therefore we shall impose constraints which restrict the dependence of the superfields on the fermionic variables, constraints which are reminiscent of dimensional reduction,[16] and which eventually lead us to the minimum number of desired fields. We achieve this end by imposing flatness in all θ directions.

Specifically we *assume* that for suitable coordinates, the (contravariant) curvature tensor attains the form

$$0=R^{\kappa\lambda\mu n}=R^{\kappa\lambda mn}=R^{\kappa lmn}=R^{klmn}\ , \quad (11)$$

with no restriction on $R^{\kappa\lambda\mu\nu}$. Consider then two coordinate systems (X) and (X_0) differing infinitesimally, $X=X_0+\Lambda(X_0)$, such that (11) holds in both. Since

$$R^{KLMN}(X)=R^{KLMN}(X_0)+R^{KLMP}(X_0)\partial_P\Lambda^N(X_0) +R^{KLPN}(X_0)\partial_P\Lambda^M(X_0)[MN]+\cdots\ ,$$

it follows from (11) that $R^{\kappa\lambda\mu\rho}\partial_\rho\Lambda^n=0$. Since R is arbitrary over ordinary space-time, Λ^n must be independent of the argument x over the region. Hence the most general finite coordinate transformation over superspace compatible with (11) is

$$x^\mu=\xi^\mu(x_0)+\theta_0{}^m\omega_m{}^\mu(x_0)+\tfrac{1}{2}\theta_0{}^2B^\mu(x_0)\ , \qquad \theta^m=\xi^m(\theta_0)\ . \quad (12)$$

However, $\xi^m(\theta_0)$ and $\xi^\mu(x_0)$ correspond, respectively, to a trivial θ reparametrization and an ordinary general coordinate transformation. Therefore, in addition to these, the most general fermion-boson mixing transformation is

$$x^\mu=x_0{}^\mu+\theta^m\omega_m{}^\mu(x_0)+\tfrac{1}{2}\theta^2B^\mu(x_0)\ , \qquad \theta^m=\theta_0{}^m\ . \quad (13)$$

Being locally flat, superspace admits canonical coordinates such that

$$G^{\mu\nu}(X)=\eta^{\mu\nu},\quad G^{\mu n}(X)=0\ ,\quad G^{mn}(X)=\epsilon^{mn} \quad (14)$$

for X in the neighborhood of a given point X_0. Allowing then for ordinary general coordinate transformations on $G^{\mu\nu}$ we may consider coordinates X_0 such that

$$G^{\mu\nu}(X_0)=g^{\mu\nu}(x_0)\ ,\quad G^{\mu m}(X_0)=0\ , \qquad G^{mn}(X_0)=\epsilon^{mn} \quad (15)$$

which are sufficient to guarantee (11). From this starting point, the most general coordinate system remaining compatible with (11) comes from the transformation law

$$G^{MN}(X)=G^{LK}(X_0)\frac{\partial X^M}{\partial X_0{}^K}\frac{\partial X^N}{\partial X_0{}^L}[LM]$$

with the (dimensionally) restricted variations (13). We thereby establish

$$G^{\mu\nu}=g^{\mu\nu}-\omega^\mu_m\omega^{m\nu}+\theta^m[(g^{\lambda\nu}\partial_\lambda+B^\nu)\omega_m{}^\mu+(g^{\lambda\mu}\partial_\lambda+B^\mu)\omega_m{}^\nu-\omega_m{}^\lambda\partial_\lambda(g^{\mu\nu}-\omega^\mu_n\omega^{n\nu})]$$
$$+\tfrac{1}{2}\theta^2[B^\lambda\partial_\lambda(g^{\mu\nu}-\omega^\mu_m\omega^{m\nu})+(B^\mu\partial_\lambda g^{\lambda\nu}+B^\nu\partial_\lambda g^{\lambda\mu})+2B^\mu B^\nu-(B^\mu\omega^\nu_m+B^\nu\omega^\mu_m)\partial_\lambda\omega^{m\lambda}$$
$$+(\partial_\kappa\omega^{m\mu})g^{\kappa\lambda}(\partial_\lambda\omega^\nu_m)+\partial_\kappa\omega^{m\kappa}(g^{\lambda\mu}\partial_\lambda\omega^\nu_m+g^{\lambda\nu}\partial_\lambda\omega^\mu_m)+\omega^{m\kappa}(\tfrac{1}{2}\omega^\lambda_m\partial_\kappa+\partial_\kappa\omega^\lambda_m)\partial_\lambda(g^{\mu\nu}-\omega^\mu_m\omega^{m\nu})]\ , \quad (16a)$$

$$G^{m\nu}=\omega^{m\nu}+\theta_n(\omega^{n\lambda}\partial_\lambda\omega^{m\nu}+B^\nu\epsilon^{mn})+\tfrac{1}{2}\theta^2[(B^\lambda\partial_\lambda+\tfrac{1}{2}\omega^{n\lambda}\omega^\kappa_n\partial_\lambda\partial_\kappa+\omega^{n\lambda}\partial_\lambda\omega^\kappa_n\partial_\kappa)\omega^{m\nu}+\omega^{m\lambda}\partial_\lambda B^\nu]\ , \quad (16b)$$

$$G^{mn}=\epsilon^{mn}\ , \quad (16c)$$

where the arguments on the right are x, not x_0.

Now amongst the θ reparametrizations compatible with (12) are the supertranslations,

$$x'^{\mu}=x^{\mu}\ ,\quad \theta'^{m}=\theta^{m}+\epsilon^{m}\ . \tag{17}$$

This means that the composition of (17) and (12) should be the same as (13) with different functions ω'^{μ}_{m} and B'^{μ} succeeding a general coordinate transformation $x'=x+\lambda(x)$. It is straightforward to verify that this is indeed correct and, for infinitesimal ϵ, the field variations read

$$\begin{aligned} &\lambda^{\mu}(x)=\epsilon^{a}\omega_{a}{}^{\mu}(x)\ ,\\ &\delta\omega_{a}{}^{\mu}(x)=\epsilon_{a}B^{\mu}(x)-\epsilon^{b}\omega_{b}{}^{\nu}(x)\partial_{\nu}\omega_{a}{}^{\mu}(x)\ ,\\ &\delta B^{\mu}(x)=-\epsilon^{a}\omega_{a}{}^{\nu}(x)\partial_{\nu}B^{\mu}(x)\ . \end{aligned} \tag{18}$$

A check on these transformation rules comes directly from the sechsbein variation,

$$\delta E_{A}{}^{M}(x,\theta)=E_{A}{}^{M}(x,\theta+\epsilon)-E_{A}{}^{M}(x,\theta)\ , \tag{19}$$

where the coordinates corresponding to (15) are

$$\begin{aligned} &E_{\alpha}{}^{\mu}(X_0)=e_{\alpha}{}^{\mu}(x_0),\quad E_{a}{}^{\mu}(X_0)=E_{\alpha}{}^{m}(X_0)=0\ ,\\ &E_{a}{}^{m}(X_0)=\delta_{a}{}^{m}\ . \end{aligned} \tag{15$'$}$$

The traditional notation for ghosts and antighosts is recovered by putting

$$\epsilon_1,\epsilon_2=\epsilon,\bar{\epsilon};\quad \omega_1{}^{\mu},\omega_2{}^{\mu}=\omega^{\mu},\bar{\omega}^{\mu};\ldots .$$

The gravitational extended BRS variations (18) can then be recognized in the form

$$\begin{aligned} &\delta\omega^{\mu}=-\bar{\epsilon}(\omega^{\lambda}\partial_{\lambda}\omega^{\mu})+\epsilon(B^{\mu}+\bar{\omega}^{\lambda}\partial_{\lambda}\omega^{\mu})\ ,\\ &\delta\bar{\omega}^{\mu}=\bar{\epsilon}(B^{\mu}-\omega^{\lambda}\partial_{\lambda}\bar{\omega}^{\mu})+\epsilon(\bar{\omega}^{\lambda}\partial_{\lambda}\bar{\omega}^{\mu})\ ,\\ &\delta B^{\mu}=-\bar{\epsilon}\omega^{\lambda}\partial_{\lambda}B^{\mu}+\epsilon\bar{\omega}^{\lambda}\partial_{\lambda}B^{\mu}\ ,\\ &\delta g^{\mu\nu}=\bar{\epsilon}(g^{\mu\lambda}\partial_{\lambda}\omega^{\nu}+g^{\lambda\nu}\partial_{\lambda}\omega^{\mu}-\omega^{\lambda}\partial_{\lambda}g^{\mu\nu})\\ &\qquad -\epsilon(g^{\mu\lambda}\partial_{\lambda}\bar{\omega}^{\nu}+g^{\lambda\nu}\partial_{\lambda}\bar{\omega}^{\mu}-\bar{\omega}^{\lambda}\partial_{\lambda}g^{\mu\nu})\ ,\\ &\delta e_{\alpha}{}^{\mu}=\bar{\epsilon}(e_{\alpha}{}^{\lambda}\partial_{\lambda}\omega^{\mu}-\omega^{\lambda}\partial_{\lambda}e_{\alpha}{}^{\mu})\\ &\qquad -\epsilon(e_{\alpha}{}^{\lambda}\partial_{\lambda}\bar{\omega}^{\mu}-\bar{\omega}^{\lambda}\partial_{\lambda}e_{\alpha}{}^{\mu})\ . \end{aligned} \tag{20}$$

Above, $\bar{\epsilon}$ is associated with the ordinary BRS transformations while ϵ provides the dual- or anti-BRS transformations. The analogy with the Yang-Mills case[8] is striking and complete.

IV. GAUGE FIXING AND BRS-INVARIANT ACTIONS

Given the restricted nature of the transformations (12) or (13), we only consider actions which are supertranslation and Sp(2) invariant, as well as being generally covariant in the ordinary sense before gauge fixing. The construction of these follows standard superfield techniques. The first part of the action is chosen to reduce to the usual Einstein form and is written

$$W_0=(2K^2)^{-1}\int d^6X X^2\sqrt{-G}\,R^{MN}G_{NM}\ , \tag{21}$$

where $G\equiv S\,\det G_{KL}=\mathrm{ber}G_{KL}$, and $R_{MN}=R_{M}{}^{K}{}_{KN}$. The Einstein action falls out of (21), remembering that $\sqrt{-G}\,d^6X$ is fully OSp(4/2) invariant and that the factor X^2 does not destroy translation invariance, but is just there to select the appropriate superfield component.

For the gauge-fixing piece we look for an action which breaks general coordinate invariance. Since we wish to retain the flat or Minkowskian symmetry (rather than proceed to an axial gauge where even this is abandoned) we have the luxury of introducing the flat metric I_{KL}. Given these premises, we will take the (two-parameter) gauge-breaking action to be

$$W_1=(2\xi K^2)^{-1}\int d^6X(\sqrt{-G}\,)^{1-p}G^{KL}I_{LK}\ . \tag{22}$$

W_1 may be simplified by moving back to the standard coordinates (x_0,θ_0). The pth power of the Jacobian enters and the result is

$$\begin{aligned} 2\xi K^2\mathscr{L}_1=&(-g)^{(1-p)/2}[2g^{\lambda\nu}\partial_{\lambda}B^{\mu}+2B^{\nu}B^{\mu}-g^{\kappa\lambda}(\partial_{\kappa}\omega^{\mu a})(\partial_{\lambda}\omega^{\nu}{}_{a})]\eta_{\mu\nu}\\ &+(-g)^{(1-p)/2}p\{-2(\partial_{\lambda}\omega^{\lambda a})(B^{\mu}\omega_{a}{}^{\nu}+g^{\mu\lambda}\partial_{\lambda}\omega_{a}{}^{\nu})+(g^{\mu\nu}+\omega^{\mu a}\omega_{a}{}^{\nu})[\partial_{\lambda}B^{\lambda}+\tfrac{1}{2}(\partial_{\kappa}\omega^{a\lambda})(\partial_{\lambda}\omega_{a}{}^{\kappa})]\}\eta_{\mu\nu}\\ &+(-g)^{(1-p)/2}p^2\{(g^{\mu\nu}+\omega^{\mu a}\omega_{a}{}^{\nu})[-\tfrac{1}{2}(\partial_{\lambda}\omega^{a\lambda})(\partial_{\kappa}\omega_{a}{}^{\kappa})]\}\eta_{\mu\nu}\ . \end{aligned} \tag{23}$$

The $p=1$ form was given in our letter,[13] but the $p=0$ case is an elegant alternative which also has the correct ingredients for gauge fixing. When $p\to 0$ the full Lagrangian simplifies to

$$\mathscr{L}=\sqrt{-g}\,K^{-2}\{R+\xi^{-1}[g^{\lambda\nu}\partial_{\lambda}B^{\mu}+B^{\nu}B^{\mu}-g^{\kappa\lambda}(\partial_{\kappa}\bar{\omega}^{\mu})(\partial_{\lambda}\omega^{\nu})]\eta_{\mu\nu}\} \tag{24}$$

and the invariance of the action under the full

BRS transformations (20) is verified, *ab initio*, in Appendix B, as confirmation of our initial derivation. The case of arbitrary p is also mentioned there.

Observe the symmetrical way in which ω and $\bar{\omega}$ enter above, in contrast to the conventional treatment.

Make one last simplification of (24). Rescale the auxiliary and ghost fields (without effect on BRS transformations).

$$B \rightarrow B\xi K^2 \ , \quad \omega \rightarrow K\omega\xi^{1/2}$$

and pick on the gauge parameter $\xi = -1$. Then

$$\mathscr{L} = \sqrt{-g}\,\{R/K^2 + \eta_{\mu\nu}[g^{\lambda\mu}\partial_\lambda B^\mu - \tfrac{1}{4}B^\mu B^\nu - g^{\kappa\lambda}(\partial_\kappa \bar{\omega}^\mu)(\partial_\lambda \omega^\nu)]\} \ . \tag{25}$$

Elimination of the auxiliary field B reveals that one is working in the de Donder gauge. On the other hand, the limit $\xi \rightarrow 0$ in (24) amounts to the harmonic gauge, the gravitational counterpart of the Landau gauge for vectors. Such a limit yields Nakanishi's[6] Lagrangian.

V. THE GAUGE IDENTITIES

We shall now derive the gauge identities for Green's functions in the new symmetrical version (25). We shall only examine the BRS variations and the identities for them since the dual identities can be similarly derived and just amount to conjugation. In fact, because of the inherent symmetry between the ghost and antighost, the information content in either set is the same. The graviton field ϕ in (25) may be defined via

$$\sqrt{-g}\,g^{\mu\nu} = \eta^{\mu\nu} + K\phi^{\mu\nu} \tag{26}$$

whereupon the BRS variation on the gravitational field transcribes into

$$\delta\phi^{\mu\nu} = \bar{\epsilon} D^{\mu\nu}{}_\lambda \omega^\lambda \ , \tag{27}$$

$$D^{\mu\nu}{}_\lambda = -\partial_\lambda \phi^{\mu\nu} - \phi^{\mu\nu}\partial_\lambda + (\delta^\mu_\lambda \phi^{\kappa\nu} + \delta^\nu_\lambda \phi^{\kappa\mu})\partial_\kappa + K^{-1}(\delta^\mu_\lambda \eta^{\nu\kappa} + \delta^\nu_\lambda \eta^{\mu\kappa} - \eta^{\mu\nu}\delta^\kappa_\lambda)\partial_\kappa \ . \tag{28}$$

Add to the action the following source terms

$$\mathscr{L}_S = -J_{\mu\nu}\phi^{\mu\nu} - \bar{J}_\mu \omega^\mu - J_\mu \bar{\omega}^\mu - K_\mu B^\mu - I_\mu(B^\mu - \omega^\lambda \partial_\lambda \bar{\omega}^\mu) - I_{\mu\nu} D^{\mu\nu}{}_\lambda \omega^\lambda + \bar{I}_\mu \omega^\lambda \partial_\lambda \omega^\mu + L_\mu \omega^\lambda \partial_\lambda B^\mu \ , \tag{29}$$

including the composite source couplings to I and L; the latter will have vanishing BRS variations due to nilpotency. Begin with the vacuum generating functional

$$Z[J_{\mu\nu}, \ldots, L_\mu] = \int (d\phi\, dB\, d\omega\, d\bar{\omega}) \exp\left[i \int d^{2l}x\, (\mathscr{L} + \mathscr{L}_S \right] \ .$$

The BRS invariance of all[17] but the source terms yields the primitive identity

$$\int d^{2l}z \left[-J^z_{\mu\nu} \frac{\delta}{\delta I^z_{\mu\nu}} + \bar{J}^z_\mu \frac{\delta}{\delta \bar{I}^z_\mu} + J^z_\mu \frac{\delta}{\delta I^z_\mu} + K^z_\mu \frac{\delta}{\delta L^z_\mu} \right] Z = 0 \ . \tag{30}$$

As usual, pass to the effective action by going over to the connected vacuum functional $iW = \ln Z$ and taking the Legendre transform

$$\Gamma[\phi^{\mu\nu}, \omega^\mu, \bar{\omega}^\mu, B^\mu; I_{\mu\nu}, I_\mu, \bar{I}_\mu, L_\mu] = W[J_{\mu\nu}, \ldots, L_\mu] + \int d^{2l}x\, (J_{\mu\nu}\phi^{\mu\nu} + \bar{J}_\mu \omega^\mu + J_\mu \bar{\omega}^\mu + K_\mu B^\mu) \ . \tag{31}$$

[In (31) we have taken the liberty of using the same symbol for the classical fields as for the corresponding functional variables; it should not cause any confusion.] The gauge identity (30) is reexpressed as

$$\int d^{2l}z \left[\frac{\delta\Gamma}{\delta\phi^{\mu\nu}_z} \frac{\delta\Gamma}{\delta I^z_{\mu\nu}} + \frac{\delta\Gamma}{\delta\omega^\mu_z} \frac{\delta\Gamma}{\delta \bar{I}^z_\mu} + \frac{\delta\Gamma}{\delta\bar{\omega}^\mu_z} \frac{\delta\Gamma}{\delta I^z_\mu} - \frac{\delta\Gamma}{\delta B^\mu_z} \frac{\delta\Gamma}{\delta L^z_\mu} \right] = 0 \ . \tag{32}$$

Its elegant form has been guaranteed by the extra source terms incorporated in Eq. (29).

Note that (32) is rather general and does not lean on the particular values $p = 0$ and $\xi = -1$ adopted at the end of Sec. IV. However, in our attempts to verify some consequences of the BRS identity we will be obliged, in the interests of computational simplicity, to assume these special values of p and ξ. The special Lagrangian (25) does not contain very many terms, leads to simple Feynman rules (Figs. 1 and 2, Appendix C) for perturbative calcu-

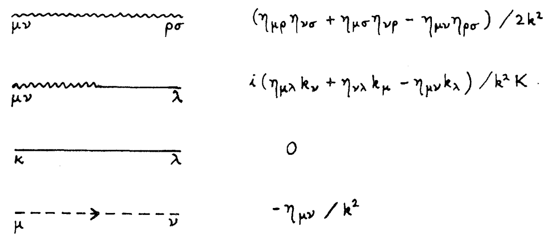

FIG. 1. Propagators for graviton ϕ (wavy line), auxiliary field B (solid line), and ghost field ω (dashed line).

lations, and means that our verification of (32) at tree and one-loop level will not be too arduous. For the rest of this section, we shall content ourselve with checking some identities at tree level.

One has to operate on (32) with enough fields to leave zero ghost number. First, take $\delta^2/\delta\phi^{\rho\sigma}\delta\omega^\lambda$. Then we expect that

$$\int d^{2l}z \left[\frac{\delta^2\Gamma}{\delta\phi_z^{\mu\nu}\delta\phi_x^{\rho\sigma}} \frac{\delta^2\Gamma}{\delta I_{\mu\nu}^z\delta\omega_y^\lambda} + \frac{\delta^2\Gamma}{\delta\overline{\omega}_z^\mu\delta\omega_y^\lambda} \frac{\delta^2\Gamma}{\delta I_\mu^z\delta\phi_x^{\rho\sigma}} - \frac{\delta^2\Gamma}{\delta B_z^\mu\delta\phi_x^{\rho\sigma}} \frac{\delta^2\Gamma}{\delta L_\mu^z\delta\omega_y^\lambda} \right] = 0 \,. \quad (33)$$

The satisfaction of this relation is given diagrammatically in Fig. 3 at the classical level, using the Feynman rules of Appendix C. Second, operate on (32) with $\delta^2/\delta B^\kappa\delta\omega^\lambda$ to yield

$$\int d^{2l}z \left[\frac{\delta^2\Gamma}{\delta\phi_z^{\mu\nu}\delta B_x^\kappa} \frac{\delta^2\Gamma}{\delta I_{\mu\nu}^z\delta\omega_y^\lambda} + \frac{\delta^2\Gamma}{\delta\overline{\omega}_z^\mu\delta\omega_y^\lambda} \frac{\delta^2\Gamma}{\delta I_\mu^z\delta B_x^\kappa} - \frac{\delta^2\Gamma}{\delta B_z^\mu\delta B_x^\kappa} \frac{\delta^2\Gamma}{\delta L_\mu^z\delta\omega_y^\lambda} \right] = 0 \,. \quad (34)$$

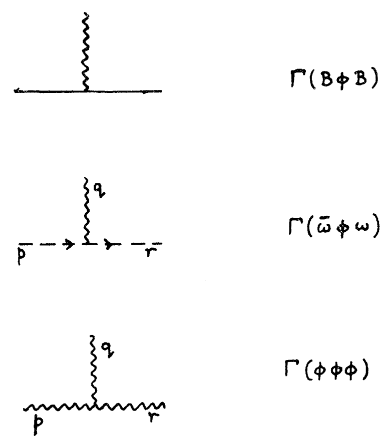

FIG. 2. Primary three-point vertices. Algebraic values of Γ are quoted in (C7).

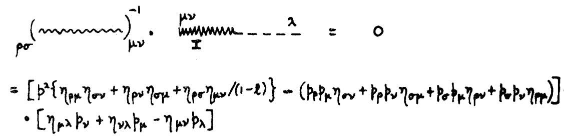

FIG. 3. Tree-level satisfaction of identity (33).

Again this is readily seen to be true at tree level; see Fig. 4. We have tested other identities (by operating with $\delta^2/\delta\omega^\kappa\delta\omega^\lambda\delta\overline{\omega}^\rho$, etc.) at tree level and find that they are all obeyed, giving us reasonable confidence in the whole construction.

VI. ONE-LOOP VERIFICATION

The real test of (32) lies in the quantum corrections. We shall therefore provide a check of (33) and (34) at the one-loop level since these identities are the only ones not involving full three-point (or higher) vertex corrections, which are notoriously unmanageable in gravity. For the self-energies in (33) we can also lean on previous research[18] to lessen our labor. The work is nevertheless considerable, so much so that we would contend that the demonstration is totally nontrivial. Nakanishi and Yamagishi[19] have checked the "choral" invariances associated with the $\xi=0$ Lagrangian, complementing our work.

For obvious reasons we shall rely on dimensional continuation as the regularization method of choice. That is, we continue our basic superalgebra to OSp($2l/2$) and then proceed to the limit $l\rightarrow 2$ in order to expose the proper infinities; remark that the ordinary space-time is being extended, not the fermionic dimensions. There is nothing we have developed in previous sections which does not generalize in an obvious way to a full $(2l+2)$-dimensional superspace. No obstacles occur which prevent our replacement of d^6X by $d^{2l+2}X$ and d^4x by $d^{2l}x$. Certainly the BRS variations (20) are equally valid when l is arbitrary.

The graviton self-energy,

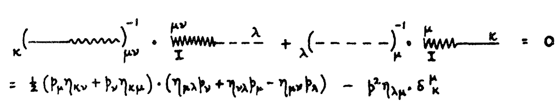

FIG. 4. Tree-level satisfaction of identity (34).

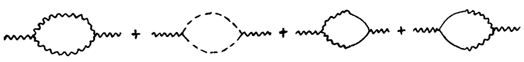

FIG. 5. Graviton self-energy due to intermediate gravitons, ghosts, and auxiliary fields. Tadpole graphs are neglected here and subsequently.

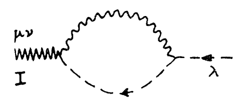

FIG. 6. One-loop contribution to $\delta^2\Gamma/\delta I_{\mu\nu}\delta\omega^\lambda$.

$$\frac{\delta^2\Gamma}{\delta\phi^{\mu\nu}\delta\phi^{\kappa\lambda}} \equiv [d_{\kappa\mu}d_{\lambda\nu}+d_{\kappa\nu}d_{\lambda\mu}-2d_{\kappa\lambda}d_{\mu\nu}/(2l-1)]\pi_1+(d_{\kappa\mu}e_{\lambda\nu}+d_{\kappa\nu}e_{\lambda\mu}+e_{\kappa\mu}d_{\lambda\nu}+e_{\kappa\nu}d_{\lambda\mu})\pi_2$$
$$+e_{\kappa\lambda}e_{\mu\nu}\pi_3+d_{\kappa\lambda}d_{\mu\nu}\pi_4+(e_{\kappa\lambda}d_{\mu\nu}+d_{\kappa\lambda}e_{\mu\nu})\pi_5\ , \tag{35}$$

with

$$d_{\mu\nu}(p)\equiv\eta_{\mu\nu}-p_\mu p_\nu/p^2\equiv\eta_{\mu\nu}-e_{\mu\nu}(p)\ ,$$

receives three distinct kinds of contributions, depicted in Fig. 5: from intermediate gravitons, from intermediate auxiliary fields, and from intermediate ghosts. The intermediate graviton contribution to it has been isolated in Ref. 18 and for us it is sufficient to abstract a subset of their results, translated into our decomposition, viz.

$$\pi_2^{\rm grav}=p^4I/16(2l-1)\ ,\quad \pi_3^{\rm grav}-(2l-1)\pi_5^{\rm grav}=p^4I(7l-5)/8(l-1)\ ,$$
$$\pi_5^{\rm grav}-(2l-1)\pi_4^{\rm grav}=p^4I(5l-3)/8(l-1)(2l-1) \tag{36}$$

with

$$I=\frac{K^2(-p^2)^{l-2}\Gamma(2-l)\Gamma(l-1)\Gamma(l-1)}{(4\pi)^l\Gamma(2l-2)}\ . \tag{37}$$

The auxiliary field contributions must, however, be worked out from scratch. The answer is

$$\frac{\delta^2\Gamma^{\rm aux}}{\delta\phi^{\kappa\lambda}\delta\phi^{\mu\nu}}=\frac{p^4I}{16(l-1)^2}\left[\frac{5l-3}{2l-1}d_{\kappa\lambda}d_{\mu\nu}+(d_{\kappa\lambda}e_{\mu\nu}+d_{\mu\nu}e_{\kappa\lambda})\frac{l^2+3l-2}{2l-1}+(l+1)e_{\kappa\lambda}e_{\mu\nu}\right]\ . \tag{38}$$

Last but not least there is contribution from intermediate ghosts which, for our ghost-symmetric Lagrangian, turns out to equal

$$\frac{\delta^2\Gamma^{\rm ghost}}{\delta\phi^{\kappa\lambda}\delta\phi^{\mu\nu}}=-\frac{lp^4I}{8(4l^2-1)}[d_{\kappa\mu}d_{\lambda\nu}+d_{\kappa\nu}d_{\lambda\mu}+d_{\kappa\lambda}d_{\mu\nu}+(2l+1)(e_{\kappa\lambda}d_{\mu\nu}+e_{\mu\nu}d_{\kappa\lambda})+(4l^2-1)e_{\kappa\lambda}e_{\mu\nu}]\ . \tag{39}$$

From (38) and (39) we note the relevant combinations,

$$\pi_2^{\rm aux}=\pi_2^{\rm ghost}=0\ ,$$
$$\pi_3^{\rm aux}-(2l-1)\pi_5^{\rm aux}=-p^4I(l+3)/16(l-1)\ ,$$
$$\pi_5^{\rm aux}-(2l-1)\pi_4^{\rm aux}=-p^4I(9l-5)/16l-1)(2l-1)\ , \tag{40}$$
$$\pi_3^{\rm ghost}-(2l-1)\pi_5^{\rm ghost}=\pi_5^{\rm ghost}-(2l-1)\pi_4^{\rm ghost}=0\ .$$

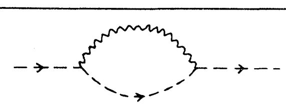

FIG. 7. Ghost self-energy in one-loop order, zero by dimensional regularization.

The next vertex function of interest is

$$\frac{\delta^2\Gamma}{\delta I_{\mu\nu}\delta\omega^\lambda}\equiv p_\lambda(d^{\mu\nu}F+e^{\mu\nu}G)+(p^\nu d^\mu{}_\lambda+p^\mu d^\nu{}_\lambda)H\ . \tag{41}$$

In one-loop order, using the effective interaction

$$I_{\mu\nu}D^{\mu\nu}{}_\lambda\omega^\lambda=\partial_\lambda I_{\mu\nu}\phi^{\mu\nu}\omega^\lambda+2I_{\mu\nu}\phi^{\lambda\nu}\partial_\lambda\omega^\mu\ ,$$

we have calculated (Fig. 6) the scalar functions of (41) to be

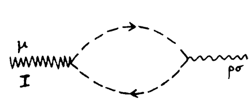

FIG. 8. One-loop contribution to $\delta^2\Gamma/\delta I_\mu\delta\phi^{\rho\sigma}$.

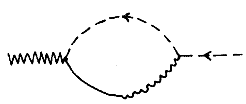

FIG. 9. One-loop contribution to $\sigma\delta\Gamma/\delta L_\mu \delta\omega^\lambda$.

$$F=\tfrac{1}{2}p^2 I,\quad G=-\tfrac{1}{2}p^2 I,\quad H=0\ . \tag{42}$$

Next there is the ghost self-energy $\delta^2\Gamma/\delta\bar{\omega}^\mu\delta\omega^\lambda$. We have discovered that the one-loop correction (Fig. 7) vanishes identically by dimensional regularization. We do not know whether the result persists in higher loop order but remark that it has a character similar to the last equation (40). The one-loop contribution (Fig. 8) to

$$\frac{\delta^2\Gamma}{\delta I_\mu \delta\phi^{\rho\sigma}} \equiv p^\mu(d_{\rho\sigma}F'+e_{\rho\sigma}G')+(p_\rho d_\sigma{}^\mu+p_\sigma d_\rho{}^\mu)H' \tag{43}$$

has likewise had to be computed *ab initio*. We find

$$\begin{aligned} &F'=-p^2I/16(2l-1),\\ &G'=-p^2I/16,\quad H'=0\ . \end{aligned} \tag{44}$$

For the remainder it is not necessary to determine $\delta^2\Gamma/\delta B^\mu\delta\phi^{\rho\sigma}$ beyond the tree level because $\delta^2\Gamma/\delta L_\mu\delta\omega^\lambda$ is *purely* one loop. Let

$$\frac{\delta^2\Gamma}{\delta L_\mu \delta\omega^\lambda}=d^\mu{}_\lambda D+e^\mu{}_\lambda E\ . \tag{45}$$

Via the effective interaction $L_\mu\omega^\lambda\partial_\lambda B^\mu$ we derive the vertex rule

$$\Gamma[L_\mu(p)B^\kappa(q)\omega^\lambda(r)]=iq^\lambda\delta_\mu{}^\kappa$$

and go on (Fig. 9) to evaluate

$$D=-p^4I/8(2l-1)\ ,\quad E=-p^4I/8\ . \tag{46}$$

Putting all these computations together, the identity (34) devolves upon finding if the relations

$$\begin{aligned} &2\pi_2+D=0,\quad G'=(2l-1)F'\ ,\\ &\pi_5-(2l-1)\pi_4+\frac{p^2(F+G)}{2(1-l)}+p^2F'=0 \end{aligned} \tag{47}$$

are obeyed. By adding (36) to (40) and comparing

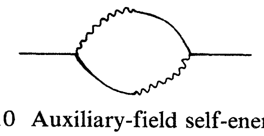

FIG. 10 Auxiliary-field self-energy.

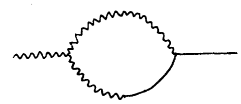

FIG. 11. Mixed graviton—auxiliary-field self-energy.

with (42), (44), and (46) the reader can convince himself that indeed they are. In fact the complex character of the manipulations and the almost miraculous combinations in (47) convince us that no serious error has occurred in our work.

The second identity (34) is perhaps easier to check. There are just three new vertex functions of interest,

$$\frac{\delta^2\Gamma}{\delta I_\mu \delta B^\lambda}\equiv d^\mu{}_\lambda D''+e^\mu{}_\lambda E''\ , \tag{48}$$

the auxiliary field self-energy,

$$\frac{\delta^2\Gamma}{\delta B^\mu \delta B^\kappa}=d_{\mu\kappa}D'+e_{\mu\kappa}E'\ , \tag{49}$$

and the mixing self-energy,

$$\begin{aligned} \frac{\delta^2\Gamma}{\delta\phi^{\mu\nu}\delta B^\kappa}=&\,p_\kappa d_{\mu\nu}F''+p_\kappa e_{\mu\nu}G''\\ &+(p_\mu d_{\kappa\nu}+p_\nu d_{\kappa\mu})H''\ . \end{aligned} \tag{50}$$

The first quantum loop makes D'' and E'' equal to zero because there is no *direct* interaction between auxiliary field and ghosts. For the rest we have evaluated (Figs. 10 and 11)

$$D'=p^2I/4(2l-1),\quad E'=p^2I/4 \tag{51}$$

and

$$\begin{aligned} &F''=-p^2I(9l-5)/8(l-1)(2l-1)\ ,\\ &G''=p^2I(l-5)/8(l-1),\\ &H''=p^2I/8(2l-1)\ . \end{aligned} \tag{52}$$

The one-loop level of identity (34) requires that

$$\begin{aligned} &p^2H+p^2H''+D=0\ ,\\ &p^2G+\tfrac{1}{2}p^2[G''-(2l-1)F'']+E=0 \end{aligned} \tag{53}$$

be obeyed. Again this is borne out by the computations.

VII. S-MATRIX EQUIVALENCE

With the introduction of a multiplier B field we may write the *conventional* gauge-fixing and

Faddeev-Popov ghost terms of gravity, [we have needed to use g rather than $\tilde{g}$ as defining the graviton field here in order to enable a comparison with (57) below] as

$$\mathscr{L}=\partial_\mu g^{\mu\nu}B_\nu+\tfrac{1}{2}\xi K^2 B^\mu B_\mu-\bar{\omega}^\nu\partial^\mu\mathscr{D}_{\mu\nu\lambda}\omega^\lambda \,. \tag{54}$$

This Lagrangian is invariant under the BRS transformation[4]

$$\delta g_{\mu\nu}=\bar{\epsilon}\mathscr{D}_{\mu\nu\lambda}\omega^\lambda,\quad \delta\omega^\lambda=-\bar{\epsilon}\omega^\mu\partial_\mu\omega^\lambda\,,$$
$$\delta\bar{\omega}^\nu=\bar{\epsilon}B^\nu,\quad \delta B^\mu=0\,. \tag{55}$$

We want to compare this with the Lagrangian (23) having $p=1$, corresponding to the gauge-fixing term chosen in a recent letter,[13] up to an integration by parts. After appropriate partial integration of (23) the resulting Lagrangian is invariant under the transformations (20), to which we can add the extra transformations

$$\delta B^\mu_- =\delta(B^\mu-\omega^\lambda\partial_\lambda\bar{\omega}^\mu)=0\,,$$
$$\delta B^\mu_+ =\delta(B^\mu+\bar{\omega}^\lambda\partial_\lambda\omega^\mu)=0 \tag{56}$$

in the BRS case and dual BRS case, respectively. The gauge-fixing Lagrangian (23) becomes

$$\begin{aligned}\mathscr{L}_1=&\partial_\rho B_\mu g^{\rho\mu}-\tfrac{1}{2}\partial_\rho\omega^{\mu a}g^{\rho\sigma}\partial_\sigma\omega_{\mu a}+\tfrac{1}{2}\xi B^\mu B_\mu K^2-\tfrac{1}{2}(B^\rho\partial_\rho+\tfrac{1}{2}\omega^{\rho a}\omega_a^\sigma\partial_\rho\partial_\sigma+\omega^{\rho a}\partial_\rho\omega_a^\sigma\partial_\sigma)g^{\mu\nu}\eta_{\mu\nu}\\ &-\tfrac{1}{2}K^2\xi(B^\rho\partial_\rho\omega^{\mu a}\cdot\omega_{\mu a}+\tfrac{1}{2}\omega^{\rho b}\omega^\sigma{}_b\partial_\rho\partial_\sigma\omega^{\mu a}\omega_{\mu a}+\omega^{\rho b}\partial_\rho\omega^\sigma{}_b\partial_\sigma\omega^{\mu a}\cdot\omega_{\mu a}\\ &-\tfrac{1}{2}\omega^{\rho b}\partial_\rho\omega^{\mu a}\cdot\omega^\sigma{}_b\partial_\sigma\omega_{\mu a}-\omega^{\rho a}\partial_\rho B^\mu\omega_{\mu a}-B^\mu\omega^{\rho a}\partial_\rho\omega_{\mu a})+\omega^{\sigma a}\partial_\sigma g^{\rho\mu}\partial_\rho\omega_{\mu a}+g^{\rho\mu}\omega^{\sigma a}\partial_\sigma\partial_\rho\omega_{\mu a}\,.\end{aligned} \tag{57}$$

If we rewrite $\mathscr{L}_1$, in terms of the B_- field, then $\mathscr{L}_2$ and $\mathscr{L}$ are invariant under the usual BRS transformations, i.e., set (55) (just exchange B and B_-). *Hence the two Lagrangians differ by a term, that is itself invariant under the BRS transformation.*

We can now invoke an argument of Zinn-Justin[11] which states that if we add a term to the action which is invariant under BRS transformation, it is equivalent to a redefinition of the field coupled to the source. Further, Lee[20] shows that generating functionals differing only in the terms coupled to their sources, lead to the same S matrix. We find then, that the OSp(4/2) Lagrangian (57) and the conventional choice (54) *have the same S matrix.* Nakanishi[21] has also shown this to be true for a variant of the $\xi=0$ Lagrangian. In Ref. 5 conventional gauge fixing in gravity was shown not to be dual BRS invariant *in general.* In that work the gauge-fixing and Faddeev-Popov terms were

$$\mathscr{L}_2=\partial_\mu\phi^{\mu\nu}B^\nu+\tfrac{1}{2}\xi B^\mu B_\mu K^2-\bar{\omega}^\nu\partial_\nu D_{\mu\nu\lambda}\omega^\lambda\,, \tag{58}$$

and maximal cancellation of terms in the dual BRS transformation was found for

$$\delta\phi_{\mu\nu}=-\epsilon D_{\mu\nu\lambda}\omega^\lambda,\quad \delta\bar{\omega}^\nu=\epsilon\bar{\omega}^\mu\partial_\mu\bar{\omega}^\nu\,,$$
$$\delta\omega^\lambda=\epsilon(B^\omega+\omega^\rho\partial_\rho\bar{\omega}^\lambda+\bar{\omega}^\rho\partial_\rho\omega^\lambda)\,,\quad \delta B^\mu=0\,, \tag{59}$$

the leftover part of (58) under the dual transformations (59) being

$$\delta\mathscr{L}_2=\epsilon\partial_\rho\phi^{\rho\nu}[\partial_\lambda\bar{\omega}^{B^\nu}-\bar{\omega}^\lambda\partial_\nu B^\nu]\,. \tag{60}$$

In Ref. 5 it was pointed out that $\delta\mathscr{L}_2$ is identically zero in the longitudinal limit $\xi\to\infty$; however, there is a second limit in which $\delta\mathscr{L}_2$ is zero, namely, $\xi\to 0$, which corresponds to the harmonic gauge. The B equation of motion is

$$\xi K^2 B^\mu=-\partial_\mu\phi^{\mu\nu}\,. \tag{61}$$

Therefore as ξ tends to zero, we find that $\partial_\mu\phi^{\mu\nu}=0$ so the offending piece (60) vanishes and indeed the variations (59) leave $\mathscr{L}_2$ invariant. If the other limit, ξ tending to infinity, is taken we find instead that $B^\mu=0$, and $\delta\mathscr{L}_2=0$ again, as previously reported.

It is worth remarking that the $\xi=0$ limit in the Lagrangian $\mathscr{L}_1$ also reduces it to the harmonic gauge:

$$\begin{aligned}\mathscr{L}_1(\xi=0)=&\partial_\rho B_\mu g^{\rho\mu}-\tfrac{1}{2}B^\rho\partial_\rho g^{\mu\nu}\eta_{\mu\nu}-\tfrac{1}{2}\partial_\rho\omega^{\mu a}g^{\rho\sigma}\partial_\sigma\omega_{\mu a}+\omega^{\sigma a}\partial_\sigma g^{\rho\mu}\partial_\rho\omega_{\mu a}+g^{\rho\mu}\omega^{\sigma a}\partial_\sigma\partial_\rho\omega_{\mu a}\\ &-\tfrac{1}{2}(\tfrac{1}{2}\omega^{\rho a}\omega^\sigma{}_a\partial_\rho\partial_\sigma+\omega^{\rho a}\partial_\rho\omega^\sigma{}_a\partial_\sigma)g^{\mu\nu}\eta_{\mu\nu}\,.\end{aligned} \tag{62}$$

Here the B equation of motion leads to

$$\partial_\rho g^{\rho\mu}+\tfrac{1}{2}\partial^\mu g^{\rho\sigma}\eta_{\rho\sigma}=0\ , \qquad (63)$$

which is the linearized version of the harmonic gauge expressed in terms of the field $g^{\mu\nu}$. That the $\xi=0$ limit reduces the $p=0$ version of (23) to the conventional harmonic gauge of (58) can also be shown trivially. Hence the $p=0$ gauge-fixing choice and conventional gravity are one and the same in the harmonic gauge.

The gauge-fixing choice $\mathscr{L}_1$ is obtained from the second variation of $g^{\mu\nu}+\omega^{\mu a}\omega^\nu{}_a$.[13] It is interesting to note by the Zinn-Justin argument that dropping the terms which arise from the variation of $\omega^{\mu a}\omega_{\mu a}$ will still lead us to a theory with the same S matrix as the original. Doing this, we find that this gauge-fixing choice is nothing more than expression (62). So if we wish to deal with gravity in a *general* covariant gauge, and retain extended BRS the invariance, *the* $\omega^{\mu a}\omega_{\mu a}$ *variations are essential.*

APPENDIX A

Inhomogeneous OSp(4/2) is the group of all superlinear transformations preserving the distance

$$(X-Y)^2\equiv(X-Y)^A I_{AB}(X-Y)^B$$

between points in superspace. Taking $X^A=(x^\alpha,\theta^a)$, where

$$\alpha,\beta,\gamma,\ldots=0,1,2,3$$

are Lorentz indices, and

$$a,b,c,\ldots=1,2$$

are symplectic indices, we have

$$X^2=x^\alpha\eta_{\alpha\beta}x^\beta+\theta^a\epsilon_{ab}\theta^b\ ,$$

where $\eta_{\alpha\beta}$ is the Minkowskian (symmetric), and ϵ_{ab} is the symplectic (antisymmetric), part of I_{AB}. In the text, early capitals A, B, C,... are reserved for tangent space indices, while world indices are denoted by late capitals K, L, M,.... Thus

$$\kappa\lambda\mu,\ldots=0,1,2,3$$

are Lorentz world indices, and

$$k,l,m,\ldots-1,2$$

are symplectic world indices. Finally there is the matter of sign factors $[AB]$, $[MN]$,.... We define $[\mu\nu]=[\mu n]=[m\nu]=1$ and $[mn]=-1$ corresponding to the rule that two fermionic quantities anticommute (with commutation in all other cases). With this notation, the matrix elements of the infinitesimal generators J_{AB} of OSp(4/2) are

$$(J_{AB})_M{}^N=I_{MA}\delta_B{}^N-[AB]I_{MB}\delta_A{}^N\ , \qquad (A1)$$

and they satisfy

$$[J_{AB},J_{CD}]=I_{BC}J_{AD}-[AB]I_{AC}J_{BD}-[CD]I_{BD}J_{AC}+[AB][CD]I_{AD}J_{BC}\ . \qquad (A2)$$

Conversion between tangent space indices and world indices is made via the sechsbein $E_M{}^A$ and its inverse $E_A{}^M$:

$$E_M{}^A E_A{}^N=\delta_M{}^N,\ \ E_A{}^M E_M{}^B=\delta_A{}^B\ . \qquad (A3)$$

Tangent indices may be raised and lowered by means of the flat metric I_{AB}, and world indices by the Riemannian metric

$$G_{MN}=E_M{}^A[AN]E_{NA}\ , \qquad (A4)$$

and their respective inverses.

Finally, exterior derivatives of one-forms are obtained through the rule

$$dE^A=dX^M dX^N\partial_N E_M{}^A\ , \qquad (A5)$$

with $dX^M dX^N=-[MN]dX^N dX^M$, $X^M dX^N=[MN]dX^N X^M$, etc.

APPENDIX B

We would like to demonstrate directly that the ghost-symmetric action

$$\int d^{2l}x\,(2\tilde g^{\mu\lambda}\partial_\lambda B^\nu-2B^\mu B^\nu\sqrt{-g}-\tilde g^{\kappa\lambda}\partial_\kappa\omega^{\mu a}\partial_\lambda\omega^\nu{}_a)\eta_{\mu\nu}\ , \qquad (B1)$$

where

$$\tilde g^{\mu\nu}=\sqrt{-g}\,g^{\mu\nu}\ , \qquad (B2)$$

is invariant under the extended BRS transformations,

$$\begin{aligned}\delta g^{\mu\nu}&=\epsilon^a(\partial_\lambda\omega^\mu{}_a g^{\lambda\nu}+\partial_\lambda\omega^\nu{}_a g^{\nu\lambda}-\omega^\lambda{}_a\partial_\lambda g^{\mu\nu})\ ,\\ \delta\omega^{\mu a}&=\epsilon^b(B^\mu\delta^a{}_b-\omega^\lambda{}_b\partial_\lambda\omega^{\mu a})\ ,\\ \delta B^\mu&=-\epsilon^a\omega^\lambda{}_a\partial_\lambda B^\mu\ .\end{aligned} \qquad (B3)$$

For the proof we first note that[18] in $2l$ dimensions,

$$\begin{aligned}\delta g &= g\tilde{g}_{\mu\nu}\delta\tilde{g}^{\mu\nu}/(l-1)\\ &= g\tilde{g}_{\mu\nu}\epsilon^a(-\omega^\lambda{}_a\partial_\lambda\tilde{g}^{\mu\nu}+\partial_\lambda\omega^\mu{}_a\tilde{g}^{\nu\lambda}\\ &\qquad +\partial_\lambda\omega^\nu_a\tilde{g}^{\mu\lambda}-\partial_\lambda\omega^\lambda{}_a\tilde{g}^{\mu\nu})/(l-1)\\ &= -\epsilon^a(\omega^\lambda{}_a\partial_\lambda g+2g\partial_\lambda\omega^\lambda{}_a)\ .\end{aligned}$$

Therefore

$$\begin{aligned}\delta(-g)^{1/2} &= -\tfrac{1}{2}(-g)^{-1/2}\cdot-\epsilon^a(\omega^\lambda{}_a\partial_\lambda g+2g\partial_\lambda\omega^\lambda{}_a)\\ &= -\epsilon^a\partial_\lambda[\omega^\lambda{}_a(-g)^{1/2}]\ .\end{aligned}$$

Hence

$$\begin{aligned}\delta[(-g)^{1/2}B^2] &= -\epsilon^a\{\partial_\lambda[\omega^\lambda{}_a(-g)^{1/2}]B^2\\ &\qquad +2\eta_{\mu\nu}(-g)^{1/2}B^\mu\omega^\lambda{}_a\partial_\lambda B^\nu\}\\ &= -\epsilon^a\partial_\lambda[\omega^\lambda{}_a B^2(-g)^{1/2}] \qquad \text{(B4)}\end{aligned}$$

is a total derivative. We have only to establish that the remaining part of (B1) leads again to a pure divergence. This is straightforward, and we obtain

$$\delta(2\tilde{g}^{\mu\lambda}\partial_\lambda B^\nu-\tilde{g}^{\kappa\lambda}\partial_\kappa\omega^{\mu a}\partial_\lambda\omega^\nu{}_a)\eta_{\mu\nu} = -\epsilon^a\partial_\lambda(\omega^\lambda{}_a 2\tilde{g}^{\mu\rho}\partial_\rho B^\nu-\omega^\lambda{}_a\tilde{g}^{\kappa\rho}\partial_\kappa\omega^{\mu b}\partial_\rho\omega^\nu{}_b)\eta_{\mu\nu} \qquad \text{(B5)}$$

where $\partial^\kappa=\eta^{\kappa\lambda}\partial_\lambda$ in (B5). Taken together, (B4) and (B5) show that the action is invariant under the extended set of transformations (20). The generalization to the Lagrangian $\mathscr{L}$ in (23) is straightforward but tedious, and we find that

$$\delta\{\mathscr{L}\} = -\epsilon^a\partial_\lambda\{\omega^{a\lambda}\mathscr{L}\}\ , \qquad \text{(B6)}$$

again establishing the extended BRS invariance from scratch.

APPENDIX C

The bilinear parts of (25), comprising the gravitational and auxiliary field, read

$$\begin{aligned}\mathscr{L}_2 &= \tfrac{1}{2}[p^2\phi_{\mu\nu}(p)\phi^{\mu\nu}(-p)-2p_\mu p_\nu\phi^\lambda{}_\mu(p)\phi_\lambda{}^\nu(-p)-p^2\phi_\mu{}^\mu(p)\phi_\nu{}^\nu(-p)/(2l-2)]\\ &\quad -iKp^\nu\phi_{\mu\nu}(p)B^\mu(-p)-\tfrac{1}{4}K^2B_\mu(p)B^\mu(-p)\ ,\end{aligned} \qquad \text{(C1)}$$

where the metric now is purely Minkowskian. Adding source terms

$$\mathscr{L}_{S2} = -\phi_{\mu\nu}(p)J^{\mu\nu}(-p)-B_\mu(p)J^\mu(-p)$$

we can derive the propagators of the theory, in standard fashion, by inverting the equations of motion,

$$\begin{aligned}&k^2\phi_{\mu\nu}(k)-k_\mu k^\lambda\phi_{\nu\lambda}(k)-k_\nu k^\lambda\phi_{\mu\lambda}(k)-k^2\phi_\mu{}^\mu(k)/(2l-2)-\tfrac{1}{2}iK[k_\mu B_\nu(k)+k_\nu B_\mu(k)]=J_{\mu\nu}(k)\ ,\\ &iKk^\nu\phi_{\mu\nu}(k)-\tfrac{1}{2}K^2B_\mu(k)=J_\mu(k)\ .\end{aligned} \qquad \text{(C2)}$$

By manipulating the left-hand side of (C2) appropriately, we can reduce the field-current equations to

$$Kk^2B_\nu=i(2k^\mu J_{\mu\nu}-k_\nu J_\mu{}^\mu)\ ,\quad k^2\phi_{\mu\nu}=(J_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}J_\lambda{}^\lambda)-iK^{-1}(k_\mu J_\nu+k_\nu J_\mu-\eta_{\mu\nu}k\cdot J)\ . \qquad \text{(C3)}$$

Then without further ado we may read off the gravitational propagators

$$\begin{aligned}&\langle\phi_{\mu\nu},\phi_{\rho\sigma}\rangle=\tfrac{1}{2}(k^2)^{-1}(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\rho\sigma}),\\ &\langle\phi_{\nu\mu},B_\lambda\rangle=-\langle B_\lambda,\phi_{\mu\nu}\rangle=i(Kk^2)^{-1}(k_\nu\eta_{\mu\lambda}+k_\mu\eta_{\nu\lambda}-k_\lambda\eta_{\mu\nu}),\quad \langle B_\kappa,B_\lambda\rangle=0\end{aligned} \qquad \text{(C4)}$$

confirming that we have chosen the de Donder gauge. These rules appear in Fig. 1, where they are supplemented by the ghost propagator

$$\langle\omega_\mu,\bar{\omega}_\nu\rangle=-\eta_{\mu\nu}/k^2\ . \qquad \text{(C5)}$$

It only remains to give the Feynman rules for the vertices, which arise directly from (25) and (27),

$$\begin{aligned}\mathscr{L} &= \mathscr{L}_2-K\phi^{\kappa\lambda}\partial_\kappa\bar{\omega}^\mu\partial_\lambda\omega_\mu-\tfrac{1}{4}K^2B_\mu B^\mu(e^{\mathrm{Tr}\ln(1+K\phi)/(2l-2)}-1)\\ &\quad +K\phi^{\mu\nu}[\tfrac{1}{2}\partial_\mu\phi_{\kappa\lambda}\partial_\nu\phi^{\kappa\lambda}-\partial_\lambda\phi_{\mu\kappa}\partial^\lambda\phi_\nu{}^\kappa+\partial_\kappa\phi_{\mu\lambda}\partial^\lambda\phi_\nu{}^\kappa+(2\partial_\lambda\phi_{\mu\nu}\partial^\lambda\phi_\kappa{}^\kappa-\partial_\mu\phi_\lambda{}^\lambda\partial_\nu\phi_\kappa{}^\kappa)/4(l-1)]+\mathscr{L}_{>3\text{gravitons}}\ .\end{aligned} \qquad \text{(C6)}$$

We may read off the three-point vertices (Fig. 2),

$$\Gamma[B_\mu \phi_{\kappa\lambda} B_\nu] = -K^3 \eta_{\kappa\lambda}\eta_{\mu\nu}/4(l-1) ,$$
$$\Gamma[\overline{\omega}_\mu(p)\phi_{\kappa\lambda}(q)\omega_\nu(r)] = -\tfrac{1}{2}K\eta_{\mu\nu}(p_\kappa r_\lambda + p_\lambda r_\kappa) , \qquad \text{(C7)}$$
$$\Gamma[\phi_{\kappa\lambda}(p)\phi_{\mu\nu}(q)\phi_{\rho\sigma}(r)] = -\tfrac{1}{4}K\sum(p_\rho q_\sigma + p_\sigma q_\rho)(\eta_{\kappa\mu}\eta_{\lambda\nu} + \eta_{\kappa\nu}\eta_{\lambda\mu}) - \tfrac{1}{8}K(p^2+q^2+r^2)\sum \eta_{\rho\mu}\eta_{\kappa\sigma}\eta_{\nu\lambda}$$
$$-\tfrac{1}{4}K\sum(\eta_{\rho\kappa}q_\lambda + \eta_{\rho\lambda}q_\kappa)(\eta_{\sigma\mu}p_\nu + \eta_{\sigma\nu}p_\mu)$$
$$+\frac{K}{4(l-1)}\sum[(p_\rho q_\sigma + p_\sigma q_\rho)\eta_{\kappa\lambda}\eta_{\mu\nu} + r^2\eta_{\rho\sigma}(\eta_{\kappa\mu}\eta_{\lambda\nu} + \eta_{\kappa\nu}\eta_{\lambda\mu})] ,$$

the summations being taken over *distinct* permutations of indices and momenta. The higher point vertices are not used in the text and have therefore been suppressed in (C6).

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[17]That the measure is extended BRS invariant may be shown in the following way. Delbourgo and Medrano and Stelle[4] showed quite early on that the measure is invariant under the BRS transformations, $\delta\phi_{\mu\nu} = \overline{\epsilon}D_{\mu\nu\lambda}\omega^\lambda$, $\delta\omega^\mu = -\overline{\epsilon}\omega^\lambda\partial_\lambda\omega^\mu$, $\delta\overline{\omega} = \overline{\epsilon}\partial_\nu\phi^{\nu\mu}/\alpha$. Or in terms of the B-field formalism, $\delta\phi_{\mu\nu} = \overline{\epsilon}D_{\mu\nu\lambda}\omega^\lambda$, $\delta\omega^\mu = -\overline{\epsilon}\omega^\lambda\partial_\lambda\omega^\mu$, $\delta\overline{\omega} = \overline{\epsilon}B^\mu_+$, $\delta B^\mu_+ = 0$, which is precisely the BRS transformation (20) if we set $B^\mu_+ = B^\mu - \omega^\lambda\partial_\lambda\overline{\omega}^\mu$. Hence by Sp(2) symmetry the full set (20) leaves the measure $(d\phi\, d\omega\, d\overline{\omega}\, dB)$ invariant.

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