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Backward-scattering sum rules and hadronic production of charmed particles

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Backward-scattering superconvergence sum rules are written for the elastic meson-baryon process $D + N \to D + N$ by considering the intercept of the charmed-baryon Regge trajectory to be smaller than -1. Hadronic couplings of charmed particles are then determined by evaluating the sum rules with the known charmed-baryon states and are compared with the previous values. In a toy calculation, we use these couplings to explore the order of suppression for the exclusive charm production cross section in comparison to strange-particle production in pp and πp interactions.

I. INTRODUCTION

One of the most important discoveries[1,2] of the last decade has been the experimental detection of charmed particles. Like strangeness, charm is now regarded as a property of matter. So far only a few calculations[3-14] have been done for the determination of the hadronic coupling constants of charmed particles. However, the values of the couplings differ by considerable amounts in all these calculations. In the past, we[15-17] have also used the complementary tools of PCAC (partial conservation of axial-vector current) and superconvergence sum rules for obtaining the hadronic parameters of charmed particles. In this paper, we deduce the superconvergence sum rules for the process

$$D + N \to D + N$$

in the backward direction. Saturating them with the known charmed states we can determine the values of the coupling constants of charmed particles.

Similarly, many theoretical calculations have recently been reported on the hadroproduction of charmed particles. Mostly the theoretical ideas are based either on perturbative quantum chromodynamics (QCD) or on purely phenomenological reaction mechanisms. Barger and Phillips have exploited[18] the similarity of associated production of charmed particles with that of strange particles and predicted the cross section ~1 nb for two-body exclusive charm production processes such as $\pi^- p \to D^- C_0^+$. They have used SU(4) symmetry for the determination of residue values and they have further taken the universal slope 0.9 GeV^{-2} for the Regge trajectories and the intercept $\alpha_{D^*}(0) = -3.3$ for the D^* trajectory. They find that the cross section for charm production should be suppressed by a factor ~10^{-5} in comparison to the cross section for strange-particle production at CERN ISR energies. In this paper, we present a calculation for the suppression factor by taking the modified values of the coupling constants as determined from the backward-scattering superconvergence sum rules.

II. SUM RULES

If an amplitude $f(\nu)$, which is crossing odd and satisfies the unsubtracted dispersion relation, has the asymptotic behavior $\nu^{-1-\epsilon}$, with $\epsilon > 0$ as $\nu \to \infty$, it must satisfy the superconvergence relation[19]

$$\int_{-\infty}^{\infty} \mathrm{Im} f(\nu)\, d\nu = 0 , \qquad (1)$$

where $\nu = s - u/4m$ and s, u are the Mandelstam variables. The asymptotic behavior of the amplitudes A and B for the process (4) in the backward direction is given[20] as follows:

$$A^{C_1} \sim s^{\alpha' C_1(0) - 1/2} , \qquad (2)$$

$$B^{C_1} \sim s^{\alpha C_1(0) - 1/2} . \qquad (3)$$

If we consider the linear combinations like $A^{C_1} - m_N B^{C_1}$, the combination[21] will have the following high-energy behavior:

$$A^{C_1} - m_N B^{C_1} \sim s^{\alpha C_1(0) - 3/2} \qquad (4)$$

for the Regge trajectory C_1 in the u channel. We take the slope of the trajectory C_1 as 0.33 as obtained[18] from the nonparallel nature of the trajectories. The trajectory function can be written as

$$\alpha_{C_1}(u) = \tfrac{1}{2} + 0.33(u - m_{C_1}{}^2) . \qquad (5)$$

We, therefore, get $\alpha_{C_1}(0) = -1.44$. Now from Eqs. (2), (3), and (4) we find that A^{C_1}, B^{C_1}, and $A^{C_1} - m_N B^{C_1}$ satisfy the requirement of superconvergence, and the corresponding sum rules are

$$\int_{-\infty}^{\infty} \mathrm{Im} A^{C_1} ds = 0 , \qquad (6)$$

$$\int_{-\infty}^{\infty} \mathrm{Im}\, B^{C_1} ds = 0\,, \tag{7}$$

$$\int_{-\infty}^{\infty} \mathrm{Im}(A^{C_1} - m_N B^{C_1}) ds = 0\,. \tag{8}$$

For numerical evaluation of the sum rules (6)–(8), we take the pole-term contributions from C_0, C_1, and C_1^* in the s channel and from the ρ state in the t channel. Using the Lagrangians[22] for the interaction vertices and the isospin projection operators[23] as well as isospin crossing matrices,[24] we get the following relations between the masses and the coupling constants after evaluating these sum rules:

$$\left(\frac{2m_N^2 + 2m_D^2 - m_\rho^2}{2m_N}\right) g^T_{\rho NN} g_{\rho DD} - (m_{C_0} - m_N) g_{DC_0N}{}^2 - (m_{C_1} - m_N) g_{DC_1N}{}^2 - \left[(m_{C_1} + m_N)\left(m_D^2 - \frac{m_{C_1^*}{}^2}{2} + \frac{m_N^2}{3} + \frac{2}{3}E^2\right) + \tfrac{1}{3}(m_{C_1^*}{}^2 - m_N^2 - m_D^2)(E + m_N)\right] \frac{g_{DC_1^*N}{}^2}{m_D^2} = 0\,, \tag{9}$$

$$2g_{\rho DD}(g^V_{\rho NN} - g^T_{\rho NN}) + g_{DC_0N}{}^2 + g_{DC_1N}{}^2 + \left(\frac{m_{C_1^*}{}^2}{2} + \frac{m_N^2}{3} - m_D^2 - \tfrac{2}{3}E^2 + \tfrac{2}{3}m_N E\right)\frac{g_{DC_1^*N}{}^2}{m_D^2} = 0\,, \tag{10}$$

$$\left[\left(\frac{2m_N^2 + 2m_D^2 - m_\rho^2}{2m_N}\right) g^T_{\rho NN} - 2m_N(g^V_{\rho NN} - g^T_{\rho NN})\right] g_{\rho DD} - m_{C_0} g_{DC_0N}{}^2 - m_{C_1} g_{DC_1N}{}^2 - \left[(m_{C_1^*} + m_N)\left(m_D^2 - \frac{m_{C_1^*}{}^2}{2} + \frac{m_N^2}{3} + \tfrac{2}{3}E^2\right) + \tfrac{1}{3}(m_{C_1^*}{}^2 - m_N^2 - m_D^2)(E + m_N) + m_N\left(\frac{m_{C_1^*}{}^2}{2} + \frac{m_N^2}{3} - m_D^2 - \tfrac{2}{3}E^2 + \tfrac{2}{3}m_N E\right)\right] \frac{g_{DC_1^*N}{}^2}{m_D^2} = 0\,. \tag{11}$$

We use[25] $g^V_{\rho NN}{}^2/4 = 2.5$, $g^T_{\rho NN}/g^V_{\rho NN} = 6.6$, $g_{\rho\pi\pi}/g_{\rho KK} = 1.42$, and $g_{\rho DD}/g_{\rho KK} = 1.21$ (Ref. 8) and, from the above relations we get the following values of the coupling constants:

$$\frac{g_{DC_0N}{}^2}{4\pi} = 19.3\,,\quad \frac{g_{DC_1N}{}^2}{4\pi} = 6.2\,,\quad \frac{g_{DC_1^*N}{}^2}{4\pi m_D^2} = 5.2\,. \tag{12}$$

The above values can be compared with those obtained[17] from forward-scattering superconvergence sum rules. We find that the changes in the values of these couplings are not very significant. However, these changes can very well be overlooked in view of the large uncertainties present in the input couplings. Also the value of $g_{DC_0N}/\sqrt{4\pi}$ (~4.39) is in agreement with the value (~4.0) obtained[12,13] from the SU(4)-invariant interaction and also with the value (~4.02) determined[14] from SU(4) breaking.

III. SUPPRESSION OF CHARM PRODUCTION IN HADRONIC INTERACTIONS

Let us consider the following processes:

$$p\bar{p} \to D\bar{D}\,,\quad p\bar{p} \to K\bar{K}\,, \tag{13}$$

$$p\bar{p} \to C_0\bar{C}_0\,,\quad p\bar{p} \to \Lambda\bar{\Lambda}\,, \tag{14}$$

$$\pi p \to D^* C_0\,,\quad \pi p \to K^*\Lambda\,. \tag{15}$$

To calculate the suppression, we consider the s and t dependence of a typical dual amplitude

$$T(s,t) = -\beta\,\frac{\Gamma(1-\alpha(t))\Gamma(1-\alpha(s))}{\Gamma(1-\alpha(t)-\alpha(s))}\,, \tag{16}$$

where β is a constant and $\alpha(s)$, $\alpha(t)$ are the Regge trajectories. The above amplitude contains an infinite series of poles in both s and t arising due to the poles of the Γ functions whenever the argument passes through a positive integer. The asymptotic behavior of the amplitude is given as

$$T(s,t) \sim -\beta\Gamma(1-\alpha(t))(-\alpha' s)^{\alpha(t)}\,. \tag{17}$$

Barger and Phillips have taken[18] exact-SU(4) coefficients β which are the same for strange and charmed particles. Since the SU(4) symmetry is badly broken, we assume that the ratio of the residue functions is proportional to the ratio of the coupling constants involved in the charmed- and strange-particle productions. The extrapolation of a Regge residue function directly to the pole may give errors of factors of 2 or 3 for the coupling. However, these errors can be minimized by taking the ratios of the coupling constants of different reactions with similar Regge trajectories. For the process (14) we find that the leading Regge trajectories can be taken as D and K, respectively. We thus find the ratio

$$\frac{d\sigma/dt(p\bar{p} \to C_0\bar{C}_0)}{d\sigma/dt(p\bar{p} \to \Lambda\bar{\Lambda})} = \left(\frac{g_{DC_0N}{}^2}{g_{K\Lambda N}{}^2}\right)^2 \left[\frac{\Gamma(1-\alpha_D(t))}{\Gamma(1-\alpha_K(t))}\right]^2 \times (\alpha'_s s)^{2[\alpha_D(t) - \alpha_K(t)]}\,, \tag{18}$$

where α'_s is the slope of the s-channel Regge trajectory. For comparison of the cross sections, the Γ functions in Eq. (18) can be omitted because

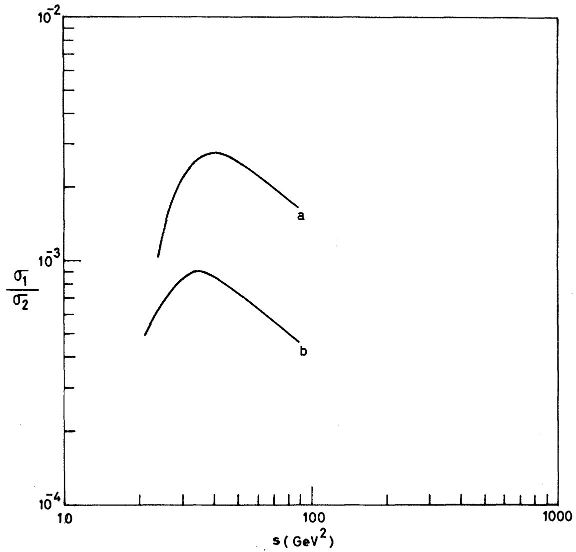

FIG. 1. Variation of suppression with energy for charm production relative to strange-particle production (a) $\sigma_1(p\bar{p} \to C_0\bar{C})/\sigma_2(p\bar{p} \to \Lambda\bar{\Lambda})$, (b) $\sigma_1(\pi p \to D^*C_0)/\sigma_2(\pi p \to K^*\Lambda)$.

they make little difference. Thus our calculation cannot be regarded as very rigorous and unambiguous at this stage. However, since we are interested in the order of magnitude, the above omissions would not affect our results seriously. For obtaining the ratio of total cross sections we integrate Eq. (18) and find the ratio as follows:

$$\frac{\sigma(p\bar{p} \to C_0\bar{C}_0)}{\sigma(p\bar{p} \to \Lambda\bar{\Lambda})} = \frac{\alpha'_K}{\alpha'_D}\left(\frac{g_{DC_0N}{}^2}{g_{K\Lambda N}{}^2}\right)^2 (\alpha'_s s)^{2[\alpha_D(t_{\min})-\alpha_K(t_{\min})]}\,. \tag{19}$$

The terms involving $t_{\max}$ have been dropped since they are negligible except right at threshold. Similarly, the ratio of cross sections for the processes (13) and (15) can be written as

$$\frac{\sigma(p\bar{p} \to D\bar{D})}{\sigma(p\bar{p} \to K\bar{K})} = \frac{\alpha'_\Lambda}{\alpha'_{C_0}}\left(\frac{g_{DC_0N}{}^2}{g_{K\Lambda N}{}^2}\right)^2 (\alpha'_s s)^{2[\alpha_{C_0}(t_{\min})-\alpha_\Lambda(t_{\min})]}\,, \tag{20}$$

$$\frac{\sigma(\pi p \to D^*C_0)}{\sigma(\pi p \to K^*\Lambda)} = \frac{\alpha'_K}{\alpha'_D}\,\frac{g_{\pi DD^*}\,g_{DC_0N}{}^2}{g_{\pi KK^*}\,g_{K\Lambda N}} (\alpha'_s s)^{2[\alpha_D(t_{\min})-\alpha_K(t_{\min})]}\,, \tag{21}$$

where C_0 and D have been taken as the leading Regge trajectories for the processes $p\bar{p} \to D\bar{D}$ and $\pi p \to D^*C_0$, respectively.

IV. RESULTS AND DISCUSSION

We can use our values of the couplings obtained above to predict the order of suppression of charm production relative to strange-particles production for the processes (13)–(15). The Regge trajectories can be given by the general relation

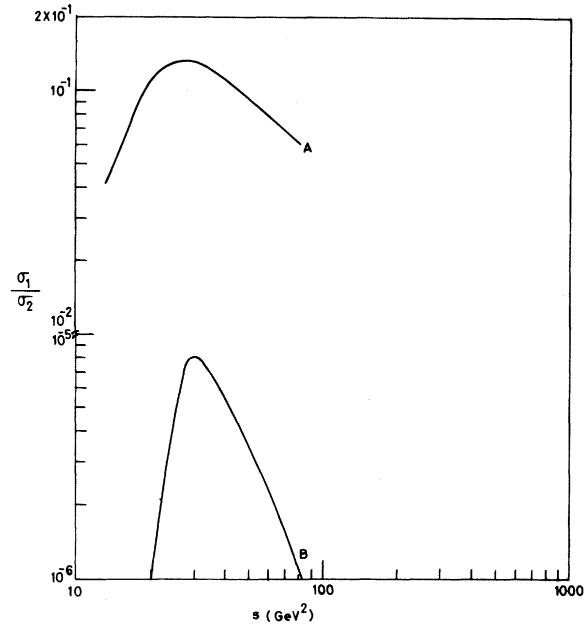

FIG. 2. Variation of suppression with energy for charm production relative to strange-particle production, for example, $\sigma_1(p\bar{p}\to D\bar{D})/\sigma_2(p\bar{p}\to K\bar{K})$. Our calculation is represented by the curve A, and the curve B represents the calculation of Barger and Phillips.

$$\alpha(t) = J + \alpha'(t - m^2)\,, \tag{22}$$

where m is the mass and J is the spin of the exchanged pole. We have used $\alpha' = 0.90$ for strange-particles trajectories and $\alpha' = 0.33$ for charmed-particles trajectories. Using the value of the coupling g_{DC_0N} as given in Eq. (12) and $g_{K\Lambda N}{}^2/4 = 10$, $g_{\pi KK^*}{}^2/4 = 0.84$ (Ref. 25), $g_{\pi DD^*}{}^2/4 = 0.4$ (Ref. 17) the suppression of charm production relative to strange-particles production can be calculated from Eqs. (19)–(21). The results have been shown in Figs. 1 and 2. For comparison, we have also plotted the data of Barger and Phillips in Fig. 2.

We find that the suppression factor for charm-production processes is ~10^{-3} in comparison to strange-particles production. This factor is quite reasonable as we notice that the experimental data for the inclusive charm production represent the cross section in the microbarn range.

In conclusion, we have presented a phenomenological numerical estimate for the cross sections regarding charm production in hadronic interactions. However, a phenomenological S matrix approach is no substitute for a formal dynamical theory like QCD. Therefore, in the absence of an adequate formal theory for such an exclusive process, we can hope that the study presented here will provide a qualitative as well as a semiquantitative understanding of the hadronic productions of all the charmed particles.

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