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# Brief Reports

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## Induced scalar effects in semileptonic weak interactions

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The size of an induced scalar coupling in semileptonic weak processes arising from quark mass differences, impulse-approximation results, and charged-Higgs-boson exchange is estimated. Although the predicted size of the effect is non-negligible it is apparently below present experimental capability to detect.

Since 1963, when Cabibbo wrote his landmark paper,[1] the analysis of semileptonic weak decays has been based upon this very simple and elegant framework—a current-current weak interaction

$$H_w = \frac{G_\mu}{\sqrt{2}} J_\lambda l^\lambda \ , \tag{1}$$

where $G_\mu \approx 1\times 10^{-5}$ GeV$^{-2}$ is the weak coupling constant observed in muon decay,

$$l^\lambda = \psi_e \gamma^\lambda (1+\gamma_5)\psi_{\nu_e} + \bar{\psi}_\mu \gamma^\lambda (1+\gamma_5)\psi_{\nu_\mu} \tag{2}$$

is the leptonic current, and

$$J_\lambda = \cos\theta_C \bar{u}\gamma_\lambda (1+\gamma_5) d + \sin\theta_C \bar{u}\gamma_\lambda(1+\gamma_5)s \tag{3}$$

is the weak hadronic current. Taking matrix elements between two hyperons one finds

$$\langle B'_{p_2}|J_\lambda|B_{p_1}\rangle = \begin{Bmatrix}\cos\theta_C \\ \text{or} \\ \sin\theta_C\end{Bmatrix} \bar{u}(p_2)\left\{ f_1\gamma_\lambda - i\frac{f_2}{m_1+m_2}\sigma_{\lambda\nu}q^\nu + \frac{f_3}{m_1+m_2}q_\lambda + g_1\gamma_\lambda\gamma_5 - i\frac{g_2}{m_1+m_2}\sigma_{\lambda\nu}q^\nu\gamma_5 + \frac{g_3}{m_1+m_2}q_\lambda\gamma_5 \right\} u(p_1) \ , \tag{4}$$

where $q = p_1 - p_2$, and the various form factors $f_1, \ldots, g_3$ are functions of $q^2$. Analysis of hyperon and $\beta$ decay has conventionally utilized[2] (i) $g_2 = 0$ from $G$ invariance [plus SU(3) symmetry for $\Delta S = 1$] and (ii) $f_3 = 0$ from CVC (conserved vector current) or from $G$ invariance [plus SU(3) symmetry for $\Delta S = 1$].

Recently, in the case of $g_2$, we attempted to estimate how large this form factor might realistically be, taking into account SU(3)- and SU(2)-breaking effects via the quark model.[3] In this note we wish to ask how large the "forbidden" polar-vector term $f_3$ is expected to be in hyperon and nuclear $\beta$ decay. First consider a $\Delta S = 1$ hyperon $\beta$ decay. There are two approaches one can utilize here—one calculating $f_3$ from the current directly, the second utilizing the divergence.

Using the current directly, we can calculate the matrix element of the first moment of the current using the method of Donoghue and Johnson.[4] A simple variant of the result derived in the Appendix to Ref. 3 yields

$$3i\frac{f_3}{m_1+m_2} = \langle B_2,\uparrow| \int d^3x\ \vec{r}\cdot\vec{\nabla}|B_1,\uparrow\rangle = \sigma \ , \tag{5}$$

where $|B,s\rangle$ represent quark-model states for the parent and daughter baryons. In order to evaluate the matrix element, we shall use the MIT bag model,[5] which employs relativistic quarks with wave function

$$\psi_s(x) = \begin{pmatrix} iu(r)\chi_s \\ l(r)\vec{\sigma}\cdot\hat{r}\chi_s \end{pmatrix} \ . \tag{6}$$

 

Here

$$u(r)=Nj_0(pr) \ , \quad l(r)=-N\left(\frac{E-m}{E+m}\right)^{1/2} j_1(pr) \ , \tag{7}$$

$p$ is an eigenvalue of the bag boundary condition (if $m=0$, $p=2.0428/R$), and $N$ is chosen such that

$$1=\int d^3x\, \psi^\dagger(x)\psi(x)=\int d^3x\,(u^2+l^2) \ . \tag{8}$$

For the strangeness-changing current we find then

$$\sigma=-i\int d^3r\, r\,(u_u l_s-u_s l_u) \tag{9}$$

and, thus,

$$f_3=-\tfrac{1}{3}(m_1+m_2)\int d^3x\, r\,(u_u l_s-u_s l_u) \ . \tag{10}$$

We note two interesting features here: (i) $f_3$ vanishes in the SU(3) limit ($m_s=m_u$) as required by symmetry considerations, and (ii) $f_3$ is sensitive to the *same* wave-function integral as is the quark-model calculation of $g_2$.[3]

We can estimate $f_3$, using Eq. (10) and bag wave functions,

$$\frac{f_3}{f_1}\approx-0.1\mu_p^{(0)}\approx-0.3 \ , \tag{11}$$

where $\mu_p^{(0)}$ is the bag-model calculation of the total proton magnetic moment measured in proton magnetons. On the other hand, a nonrelativistic-quark-model estimate gives[3]

$$\frac{f_3}{f_1}\approx-0.7 \ . \tag{12}$$

Provided $g_2/g_1$ can be measured in a hyperon-$\beta$-decay experiment, our quark model predicts

$$\frac{g_2}{g_1}=\frac{f_3}{f_1} \tag{13}$$

which would give then a prediction independent of detailed numerical work.

A second approach to calculation of the induced scalar uses the divergence of the current

$$i\partial^\mu V_\mu=(m_s-m_u)S \ , \tag{14}$$

where $S$ is the scalar density

$$S=\bar{u}s \ . \tag{15}$$

Taking matrix elements and defining

$$\langle B',s'|S|B,s\rangle=\delta_{ss'}\bar{u}(p')u(p)s(q^2) \tag{16}$$

we find

$$(m_1-m_2)f_1(q^2)+\frac{q^2 f_3(q^2)}{m_1+m_2}=(m_s-m_u)s(q^2) \ . \tag{17}$$

At $q^2=0$ we find

$$(m_1-m_2)f_1(0)=(m_s-m_u)s(0) \tag{18}$$

which is a prediction for quark mass splitting

$$m_s-m_u=(m_1-m_2)\frac{f_1(0)}{s(0)} \ . \tag{19}$$

In the nonrelativistic quark model we have

$$f_1(0)/s(0)\approx 1 \tag{20}$$

and

$$m_s-m_u\approx m_1-m_2 \ . \tag{21}$$

However, in the MIT bag model we have $s(0)\approx 0.5$ (Ref. 6) and

$$m_s-m_u\approx 2(m_1-m_2) \ . \tag{22}$$

There may be some evidence for this effect, but this is rather speculative at present.[7]

On the other hand, taking the first moment of Eq. (17) we find

$$\frac{f_3(0)}{f_1(0)}=(m_1{}^2-m_2{}^2)\left(-\frac{f_1'(0)}{f_1(0)}+\frac{s'(0)}{s(0)}\right) \ . \tag{23}$$

Assuming $f_1$ to be dominated by the $K^*(890)$ and $s$ by the $\kappa(1400)$ we find

$$\frac{f_3(0)}{f_1(0)}=(m_1{}^2-m_2{}^2)\left(-\frac{1}{m_{K^*}{}^2}+\frac{1}{m_\kappa{}^2}\right)$$

$$\approx-0.5 \tag{24}$$

which is quite consistent with our other estimate, Eqs. (11) and (12), based upon taking a moment of the current.[8]

Unfortunately, such effects are at present nearly impossible to detect, since any terms in $f_3$ contribute to decay spectra multiplied by[9]

$$\frac{m_l{}^2}{E_l(m_1+m_2)} \ ,$$

where $m_l$ and $E_l$ are the mass and energy of the charged lepton. Thus for electron decay, we have

$$\frac{m_e{}^2}{E_e(m_1+m_2)}\sim 10^{-6} \tag{25}$$

so that such effects are totally obscured. For muon decay, on the other hand, we have

$$\frac{m_\mu{}^2}{E_\mu(m_1+m_2)}\sim 10^{-1} \tag{26}$$

so that scalar contributions should be visible in a high-statistics experiment. Unfortunately current experiments involving muonic decay have very few events.[10]

It is perhaps more interesting then to ask how large one might expect induced scalar effects to be in nuclear $\beta$ decay and muon capture, where a number of high-precision experiments are available.

In this case there is essentially no effect arising from the $m_u, m_d$ mass difference as $\Delta m$ is swamped by the quark energy which enters into all such calculations.[11] However, just as for $g_2$ there will in general be a contribution from the impulse-approximation nuclear current. A straightforward calculation finds[7]

$$f_3(0) = -\left(\frac{2m_N}{3} M_{r\cdot p}\right) g_V \ ,$$
$$f_1(0) = [(M_F + \tfrac{1}{3}\Delta_{\rm nuc}(M_{r\cdot p} - \tfrac{1}{2}\Delta_{\rm nuc} M_{r^2})] \ . \quad (27)$$

Here $A$ is the nuclear mass number, $m_N$ is the nucleon mass,

$$\Delta_{\rm nuc} = m_1 - m_2 \pm \frac{6}{5}\frac{Z\alpha}{R} \mp (m_n - m_p) \quad (28)$$

is the *nuclear* mass difference (i.e., the actual mass difference corrected for the Coulomb energy difference between the two states), $g_V$ is the polar-vector coupling in neutron $\beta$ decay ($g_V = 1$ by CVC), and

$$M_F = \langle \beta || \sum_i \tau_i^{\pm} || \alpha \rangle \ ,$$
$$M_{r\cdot p} = \frac{i}{2m_N} \langle \beta || \sum_i \tau_i^{\pm} (\vec{p}_i \cdot \vec{r}_i + \vec{r}_i \cdot \vec{p}_i) || \alpha \rangle \quad (29)$$

are nuclear matrix elements. Using the Ahrens-Feenberg approximation, we can write[12]

$$-2M_{r\cdot p} \cong \langle \beta || \sum_i \tau_i^{\pm} [H_{\rm nuc}, r_i^2] || \alpha \rangle$$
$$\cong -\Delta_{\rm nuc} \langle \beta || \sum_i \tau_i^{\pm} r_i^2 || \alpha \rangle \equiv -\Delta_{\rm nuc} M_{r^2} \ . \quad (30)$$

Thus

$$f_3(0) \cong -\tfrac{1}{3} m_N \Delta_{\rm nuc} g_r M_{r^2} \quad (31)$$

and we see that $f_3$ vanishes for transitions between members of a common isotopic multiplet ($\Delta_{\rm nuc} = 0$) as required by symmetry considerations. Thus, since the $r^2$ operator carries $\Delta J = 0$, we find $f_3 \neq 0$ only for a Gamow-Teller transition between two states of the same spin. For such a transition, e.g.,

$${}^8\mathrm{Li}(2^+) \to {}^8\mathrm{Be}^*(2.90\ \mathrm{MeV}, 2^+) + e^- + \bar{\nu}_e \ ,$$
$${}^8\mathrm{B}(2^+) \to {}^8\mathrm{Be}^*(2.90\ \mathrm{MeV}, 2^+) + e^+ + \nu_e \ ,$$

we have $f_1(0) = 0$ and

$$\frac{f_3(0)}{g_1(0)} \cong -\frac{1}{3}\frac{g_V}{g_A}\frac{m_N \Delta_{\rm nuc} M_{r^2}}{M_\sigma} \ , \quad (32)$$

when $g_A \simeq 1.25$ is the axial-vector coupling in neutron $\beta$ decay and

$$M_\sigma = \langle \beta || \sum_i \tau_i^{\pm} \vec{\sigma}_i || \alpha \rangle \quad (33)$$

is the Gamow-Teller matrix element. For the $A = 8$ transition listed previously, we have $\Delta_{\rm nuc} \simeq 15$ MeV and

$$\frac{f_3(0)}{g_1(0)} \simeq -\frac{1}{5}\frac{g_V}{g_A} m_N \Delta_{\rm nuc} R^2 \cong 0.4 \ , \quad (34)$$

where $R \simeq 2.5$ fm is the nuclear radius. Thus, not only is $f_3$ small but also since $g_1$ involves a rank 1 tensor $\vec{\sigma}$ while $f_3$ involves a scalar $r^2$, any interference between $g_1$ and $f_3$ can arise *only* in polarization-dependent effects, which are less amenable to high-statistics experiments.[9] Even then the effects are only

$$O\left(\frac{m_e^2}{m_N E_e}\right) \times \frac{f_3}{g_1} \sim 0.1\% \quad (35)$$

which is virtually impossible to detect.

In muon capture the expected scalar effect is somewhat larger

$$O\left(\frac{m_\mu}{m_N}\right) \times \frac{f_3}{g_1} \sim 4\% \ . \quad (36)$$

However, again this will only show up in spin-dependent correlations.

Finally, we note that there exists one additional possible source for a scalar coupling, provided charged Higgs bosons exist. In this case there will exist an effective interaction[13]

$$\mathcal{L}_H = \frac{G}{\sqrt{2}} L D \frac{1}{m_H^2} \ , \quad (37)$$

where $m_H$ is the charged-Higgs-boson mass,

$$L = m_e \bar{e}(1+\gamma_5)\nu_e + m_\mu \bar{\mu}(1+\gamma_5)\nu_\mu \quad (38)$$

is the lepton current, and

$$D = \bar{u}[(m_d - m_u) - (m_d + m_u)\gamma_5] d \cos\theta_C$$
$$+ \bar{u}[(m_s - m_u) - (m_s + m_u)\gamma_5] s \sin\theta_C \quad (39)$$

is the hadronic current. This can be written as an effective induced scalar coupling using the equations of motion for the lepton fields:

$$\mathcal{L}_H = \frac{G}{\sqrt{2}}\frac{1}{m_H^2} D \ i\partial_\mu L^\mu \ . \quad (40)$$

The resulting effective $f_3$ form factor can be written as

$$f_3^{\rm eff}(0) \cong \frac{(m_1 + m_2)(m_s - m_u)}{m_H^2}, \quad \Delta S = 1 \text{ hyperon decay,}$$
$$f_3^{\rm eff}(0) = \frac{2m_N(m_d - m_u)}{m_H^2}, \quad \text{nuclear } \beta \text{ decay} \ . \quad (41)$$

The present lower bound on such a mass is[14]

$$m_H \gtrsim 2 \text{ GeV} ,$$

so

$$\begin{aligned} &f_3^{\text{eff}}(0) \lesssim 0.08, \quad \Delta S = 1 \text{ hyperon decay} , \\ &f_3^{\text{eff}}(0) \lesssim 10^{-3}, \quad \text{nuclear } \beta \text{ decay} . \end{aligned} \tag{42}$$

Since most likely $m_H$ is much heavier than the current 2-GeV lower bound[15] it is likely that these Higgs-boson couplings too are unobservable.

We have thus examined possible origins for induced scalar couplings in semileptonic weak transitions—quark-mass-difference effects, impulse-approximation predictions, and charged-Higgs-boson exchange. Only the first of these was found to produce a sizable effect

$$\frac{f_3(0)}{f_1(0)} \frac{m_\mu{}^2}{E_\mu(m_1+m_2)} \sim 5\% \tag{43}$$

in $\Delta S = 1$ hyperon decays. However, unlike the case of the induced tensor it appears impossible at present to detect the effect of an induced scalar coupling.

At present the only sensitive experimental limit is obtained by a careful analysis of $0^+$-$0^+$ Fermi decays by Szybisz *et al.*, who conclude that[16]

$$\frac{f_3(0)}{f_1(0)} = 0.05 \pm 0.57 . \tag{44}$$

However, we expect a null result in this case, since $\Delta_{\text{nuc}} = 0$.

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