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Effective non-Coulombic power-law potential for the study of light and heavy mesons

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From purely phenomenological considerations we have shown that it is possible to describe successfully the heavy-meson spectra of $c\bar{c}$ and $b\bar{b}$ systems in the framework of an effective non-Coulombic power-law potential in the form $V(r)=V_0+ar^{\nu}$ (with $a,\nu>0$). The nonsingular short-distance behavior of this potential, which is in apparent contradiction with the predictions of quantum chromodynamics, does not pose any problem in explaining the fine-hyperfine splittings, if we prescribe the spin dependence to be generated through this static confining potential in the form of an approximately equal admixture of scalar and vector parts with no contributions from the anomalous quark magnetic moments. This nonrelativistic formalism, when extended to a unified study of the entire meson spectra including the ordinary light and the heavy mesons, gives a very good account of the meson masses, fine-hyperfine splittings, electromagnetic transition rates, and leptonic decay widths without reflecting any inadequacy in the short- and long-range behavior of this simple effective power-law potential.

I. INTRODUCTION

Nonrelativistic potential-model studies of bound heavy-quark—antiquark systems such as $c\bar{c}$ and $b\bar{b}$ representing the heavy-meson families of ψ and Υ, respectively, have proven to be quite successful. Theoretical and phenomenological considerations have led to the suggestions of various forms[1] of static potentials to represent the interquark force. The asymptotic freedom of quantum chromodynamics (QCD) suggests that at very short distances the potential is Coulomb type[2]. At distances exceeding 1 fm, the relativistic string picture[3] makes plausible a confining linear form for the potential. However, for the intermediate regime (quark-antiquark separation distance r in the range $0.1\leq r\leq 1$ fm) in which all the known mesonic levels lie, numerous interpolations with varying degrees of theoretical motivations have been advanced to obtain various forms[4] of the static potential. But one cannot as yet derive from first principles any such potential forms completely using QCD, which is believed to be the underlying theory for quark dynamics.

In such circumstances one often turns towards phenomenology for further understanding of the quark-antiquark force. A straightforward approach to establish the form of the interaction in the region of space occupied by quarkonium levels, without particular regard to what may happen at much shorter and much longer distances, is the application of the inverse-scattering technique.[5] The potential forms, extracted in this manner from the knowledge of the experimentally observed quarkonium levels, would correspond to an intermediate region of the quark-antiquark separation distance relevant to all the known mesonic states. Therefore this technique has the advantage of being free from any theoretical biases about the short- and long-distance behavior of the static potential. Apart from the general confining behavior, one important observation of this analysis about the interquark potential is that the $c\bar{c}$ and $b\bar{b}$ potentials agree well in the interval $0.1\leq r\leq 1$ fm. This agreement provides direct evidence for the fact that the quark-antiquark interaction is independent of quark flavor for interquark separations between 0.1 and 1 fm. This technique, in spite of all its merits, has very little predictive power unless the potential extracted is suitably represented by a simple and compact expression. However, from the observations of this model-independent analysis, if one takes quark confinement and flavor independence as the two basic ingredients of the quark-antiquark forces, then with no theoretical prejudice one can think of a simple empirical power-law potential of the form

$$V(r)=V_0+ar^{\nu}\,. \tag{1.1}$$

This potential may be used to explore the possibility of an adequate description of the $q\bar{q}$ bound states corresponding to various mesonic states. Implications of such a potential have been investigated by Quigg, Rosner, and many other authors.[6] But since the data then available was consistent with a small positive ν close to zero, attention was diverted to a better-looking logarithmic potential,[7] which happens to be a limiting case of the power-law potential. However, the logarithmic potential leads to an exact equality of the mass differences $\Delta M_S(q\bar{q}) = [M_{2S}(q\bar{q}) - M_{1S}(q\bar{q})]$ for all the $q\bar{q}$ systems independent of the constituent quark mass m_q, which is not strictly true according to more accurate experiments. Therefore it is worthwhile to have a fresh look at an effective power-law potential. It has been shown recently[8] that such a potential form with ν close to 0.1 can describe quite satisfactorily the gross features of the ψ and Υ spectra. We have further observed[9] that if one regards this power-law potential as an approximately equal admixture of vector and scalar components with a very small or zero quark anomalous moment, then the fine-hyperfine levels of the $c\bar{c}$ and $b\bar{b}$ system can be reproduced very satisfactorily.

The main purpose of the present work is to show that this relatively simple effective potential model can adequately describe the entire meson spectrum encompassing the ordinary light and new heavy mesons in a unified manner. In this connection we must be reminded of the fact that the usual nonrelativistic Schrödinger-type approach for the heavy quarkoniums has been justified on the basis of large quark masses involved. But the same approach may be unsuitable for the ordinary light mesons due to relativistic complications expected to be significant in these cases. On the other hand, relativistic generalizations attempted in some limited senses by some authors[10] are by no means simple and straightforward. Therefore nonrelativistic potential-model studies are often extended[11] to include light hadrons which gives, if not quantitative, a qualitative understanding. But Stanley and Robson,[12] and also Bhaduri, Cohler, and Nogami[13] have issued cautionary notes in their more recent works in this connection. They have pointed out that the conventional Coulomb potential and the resulting spin-spin hyperfine interaction term taken to have the usual contact form proportional to

$$(\vec{S}_1 \cdot \vec{S}_2)\delta(\vec{r})/m_{q_1}m_{q_2} \tag{1.2}$$

would invalidate the nonrelativistic approximation in addition to making inaccurate the perturbation estimates of the mass splittings for lighter mesons. The required remedy suggested by Bhaduri *et al.*[13] is to take an effective potential with a non-Coulombic part instead of the Coulomb one, in addition to replacing in an *ad hoc* manner the $\delta(\vec{r})$ in the spin-spin hyperfine interaction (Eq. (1.2) by some suitable form factor $f(r,r_0)$. With harmonic or linear potentials, they have tried various forms for $f(r,r_0)$ in order to validate the use of the nonrelativistic approach for light hadrons. Such effective interactions chosen by them, which describe satisfactorily the ground states of light hadrons, would fail to fit the charmonium spectrum simultaneously. On the other hand we will show in this work that it is possible to use the nonrelativistic approach in the framework of our non-Coulombic power-law potential model to obtain a satisfactory description of the entire spectra of heavy mesons simultaneously with at least the ground states of the light mesons. As discussed in Sec. II, spin-spin interaction generated from this static non-Coulombic power-law potential with $\nu \simeq 0.1$ does not possess the contact form [Eq. (2.18)]. Hence quite apart from the values of the velocity parameters $\langle v^2/c^2 \rangle$ of the constituent quark, since this effective power-law potential along with its spin-dependent terms does not possess any objectionable features as those pointed out in Ref. 13 to invalidate the use of nonrelativistic approach, we feel quite optimistic in extending this approach to include light mesons. So we may be justified in believing that a nonrelativistic perturbative treatment for light mesons as well in the framework of the non-Coulombic power-law potential model would not be inaccurate. Nevertheless we would obtain the values of the velocity parameters $\langle v^2/c^2 \rangle$ of the constituent quarks corresponding to light mesons, in order to assess the quantitative significance of our results. For heavy mesons, particularly in the case of charmonium, we will show that our estimates of $\langle v^2/c^2 \rangle$ agree quite well with those of Eichten *et al.*[14] If we would accept the validity of the nonrelativistic approach for the charmonium spectrum, then the range of values for $\langle v^2/c^2 \rangle$, in this case corresponding to the observable levels of this ψ spectrum, can be treated as small enough for a comparison with those obtained in the case of light mesons. Slight departures from this range of values of the velocity parameters for light mesons would not however undermine the qualitative significance of our results. Since for the excited states of light mesons, relativistic corrections would be much larger as compared to the ground states, we would concentrate mainly on the ground state mass splittings for which the nonrelativistic approach may be meaningful.

With the above contention in mind, we feel that a unified nonrelativistic approach to the study of the light and heavy mesons taken together is possible, which would provide a comprehensive picture of the applicability of the simple phenomenological power-law potential. So with a brief review of our earlier works[8,9,20] in this line, we would present here the results of such a study to show clearly the possibility of an effective non-Coulombic power-law description of the quark-antiquark potential leading to a reasonably satisfactory explanation of the light- and heavy-meson spectra.

II. POWER-LAW POTENTIAL AS AN EFFECTIVE QUARK-ANTIQUARK POTENTIAL

In this section we discuss the possibility of describing the meson spectra in the framework of an effective $q\bar{q}$ potential in a power-law form with a suitable prescription for its Lorentz structure in order to generate the necessary spin dependence to explain the fine-hyperfine splittings.

A. The effective static potential

As discussed in Sec. I, we take the static $q\bar{q}$ potential in a simple and effective form

$$V(r)=V_0+ar^{\nu} . \quad (2.1)$$

Now we would like to discuss its applicability in describing successfully the meson spectra in a flavor-independent manner. We will show that from semiclassical (WKB) considerations it is possible to obtain reasonable estimates to the quark masses and also the potential parameters V_0, a, and ν ($a,\nu>0$), which would enable one to describe the $c\bar{c}$ and $b\bar{b}$ spectrum fairly well. This has been reported in our earlier work,[8] a brief account of which is presented here.

For a $q\bar{q}$ bound state formed with the potential as given in Eq. (2.1), a semiclassical solution to the Schrödinger equation exists which would give the binding energy and the absolute value squared of the S-state wave function at the origin as follows[15]:

$$E_{nL}=V_0+(a/m_q^{\nu/2})^{2/(\nu+2)} \times[A(\nu)(n+\tfrac{1}{2}L-\tfrac{1}{4})]^{2\nu/(\nu+2)} , \quad (2.2)$$

$$|\psi_{nS}(0)|^2=\frac{\nu}{2\pi^2(\nu+2)}\{am_q[A(\nu)]^{\nu}\}^{3/(\nu+2)} \times(n-\tfrac{1}{4})^{2(\nu-1)/(\nu+2)} , \quad (2.3)$$

where

$$A(\nu)=2\sqrt{\pi}\,\Gamma(\tfrac{3}{2}+1/\nu)/\Gamma(1+1/\nu) . \quad (2.4)$$

Now taking some experimental inputs from the $c\bar{c}$ and $b\bar{b}$ system such as $(\Delta M_S)_{b\bar{b}}$ and $(\Delta M_S)_{c\bar{c}}$, the spin-averaged mass $M_{1S}(c\bar{c})$, and finally $|\psi_{1S}(0)|_{c\bar{c}}^2$ and $|\psi_{1S}(0)|_{b\bar{b}}^2$, we can obtain from Eqs. (2.2) and (2.3) an estimate of the potential parameters V_0, a, ν, and also the quark-mass parameters m_c and m_b in the following manner. If we denote $(\Delta M_S)_{c\bar{c}}/(\Delta M_S)_{b\bar{b}}$ as D_0 and $|\psi_{1S}(0)|_{b\bar{b}}^2/|\psi_{1S}(0)|_{c\bar{c}}^2$ as W_0, then we get

$$\nu=3\ln D_0/\ln W_0 , \quad (2.5)$$

$$a=(\Delta M_S)_{q\bar{q}}[\tfrac{32}{9}F_{q\bar{q}}(\nu)]^{\nu/3}/[Q(\nu)\{A(\nu)\}^{\nu}] , \quad (2.6)$$

$$m_q=Q(\nu)[\tfrac{3}{2}F_{q\bar{q}}(\nu)]^{2/3}/(\Delta M_S)_{q\bar{q}} , \quad (2.7)$$

$$V_0=M_{1S}(q\bar{q})-2m_q -(a/m_q^{\nu/2})^{2/(\nu+2)}[\tfrac{3}{4}A(\nu)]^{2\nu/(\nu+2)} , \quad (2.8)$$

where

$$Q(\nu)=[(\tfrac{7}{3})^{2\nu/(\nu+1)}-1] , \quad (2.9)$$

$$F_{q\bar{q}}(\nu)=[\pi^2(\nu+2)|\psi_{1S}(0)|_{q\bar{q}}^2/\nu] . \quad (2.10)$$

Since the static potential can only give the spin-averaged masses of the $q\bar{q}$ bound states, the inputs $(\Delta M_S)_{b\bar{b}}$ and $(\Delta M_S)_{c\bar{c}}$ should be taken in an average sense only. Therefore we choose $(\Delta M_S)_{c\bar{c}}=0.605$ GeV and $(\Delta M_S)_{b\bar{b}}=0.568$ GeV, which are somewhat near the corresponding experimental values computed normally with reference to the vector-meson masses. According to the SLAC experiment,[16] if we consider $M_{\eta_c}=2.983$ GeV, we get the spin-averaged mass $M_{1S}(c\bar{c})=3.069$ GeV. Finally we obtain the $|\psi_{1S}(0)|_{q\bar{q}}^2$ values from the experimental leptonic widths expressed in terms of the somewhat corrected Van Royen—Weisskopf formula[17] in the form

$$\Gamma(V_{1S}\to e^+e^-) =\frac{16\pi\alpha^2 e_q^2}{M_{1S}^2(q\bar{q})}|\psi_{1S}(0)|^2\left[1-\frac{8\alpha_s(q\bar{q})}{3\pi}\right]^2 . \quad (2.11)$$

Taking the quark-gluon coupling constant $\alpha_s(c\bar{c})$

$=0.41$ and $\alpha_s(b\bar{b})=0.24$ in conformity with the deep-inelastic scattering data and the requirements of asymptotic freedom, we obtain $|\psi_{1S}(0)|_{c\bar{c}}^2 =0.094$ GeV^3 and $|\psi_{1S}(0)|_{b\bar{b}}^2 =0.5678$ GeV^3. With the above values as the inputs, we obtain the potential parameters and the quark masses m_c and m_b from Eqs. (2.5)–(2.8),

$$(V_0,a,\nu)=(-6.41 \text{ GeV}, 6.08 \text{ GeV}, 0.106) , \tag{2.12}$$

$$(m_c,m_b)=(1.346 \text{ GeV}, 4.759 \text{ GeV}) . \tag{2.13}$$

With these parameters it is possible to calculate the mean mass spectra for the $c\bar{c}$ and $b\bar{b}$ systems using the semiclassical formula (Eq. 2.2) for the binding energy as done in Ref. 2. However, this formula is an approximate one which becomes worse for the excited states, particularly with $L\neq 0$. For example, Eq. (2.2) would predict a degenerate $2S$ and $1D$ level of $q\bar{q}$ bound systems which is not true at least in the case of the known charmonium spectrum. Therefore for exactness we must go for the numerical solutions to the Schrödinger equations with the potential parameters taken around the above-estimated values. We have shown in Ref. 2 that if we vary the quark masses m_c and m_b around the estimated values (2.13), while fixing the potential parameters as obtained in (2.12), then a satisfactory account of the gross features of the mass spectra and leptonic decay widths of the $c\bar{c}$ and $b\bar{b}$ systems can be obtained in a flavor-independent manner. One may argue that in such a phenomenological fit, the short-distance behavior of the actual potential may not be reflected precisely in the gross features of the spectrum. The phenomenological power-law potential may indeed simulate the actual one in a wide range of quark-antiquark separation distances ≥ 0.2 fm appropriate to the average sizes of the heavy mesons of the ψ and Υ families. But for such a potential to stand on its own merit, its short-distance nonsingular behavior, which is in contradiction with QCD, must have to pass a further test in probing quark-antiquark separation distances <0.2 fm. This is possible if it can successfully describe the yet-to-be found heavier meson family of the $t\bar{t}$ system whose average sizes are expected to be of the order of 0.1 fm or even less. Besides, the fine-hyperfine splittings of the quarkonium levels may also reveal the inadequacy, if any, in this non-Coulombic potential, since the short-distance part is believed to play an important role in the hyperfine splittings. However, we have shown[9] that if this potential generates spin dependence through scalar and vector exchanges in almost equal proportions, along with a very small or zero quark anomalous moment, we can describe very satisfactorily the up-to-date data on the fine-hyperfine levels of $c\bar{c}$ and $b\bar{b}$ systems. Therefore we suggest that even if the nonsingular behavior of this potential does not conform to the expectations of QCD, it may have a rightful place to claim its usefulness in the descriptions of heavy-meson spectra. This provides an encouragement to feel optimistic about such an effective power-law potential model that may enable one to describe in a unified manner the entire meson spectra including the light and heavy mesons. If the ordinary light mesons can be described to a reasonable extent in a flavor-independent manner, then it would speak for the appropriateness of its confinement behavior which is not linear as expected from some theoretical points of view.

Therefore a unified description of the meson spectra, encompassing both the light and heavy mesons in the framework of such a power-law potential model, would provide a crucial test for the adequacy of its short-distance as well as the long-distance behavior. With this in view we present here the appropriate spin structure of this potential that would enable us to study the fine-hyperfine splittings and some other relevant aspects of the entire meson spectra.

B. The spin structure of the potential

The quantitative explanation of the fine-hyperfine levels corresponding to the mesonic states depends on the spin structure of the quark-antiquark potential. If we follow the prescription of Ref. 9 in this connection, we can consider the confining potential $V(r)$ given in Eq. (2.1) to be an approximately equal admixture of vector and scalar components with no anomalous moments. Thus we write,

$$V(r)=g_V V(r)+(1-g_V)V(r) , \tag{2.14}$$

where the vector fraction g_V can be adjusted for its value near about 0.5. Such a conclusion was also reached in the phenomenological parametrization of the potential model by Appelquist *et al.*,[18] which was supported by the gauge-invariant formalism of Eichten and Feinberg.[19] In fact we will show in Sec. III that $g_V=0.58$ gives a suitable value of the vector fraction which describes the fine-hyperfine splittings of all mesons.

Now the spin-dependent correction terms generated by this potential can be obtained in the usual manner to the lowest nontrivial order (v^2/c^2)

$$\delta V_{\text{spin}}(r)=A_1(r)\vec{L}\cdot\vec{S}+A_2(r)\vec{S}_1\cdot\vec{S}_2+A_3(r)S_{12}+A_4(r)(\vec{S}_1-\vec{S}_2)\cdot\vec{L}\ , \tag{2.15}$$

where $\vec{S}_1$ and $\vec{S}_2$ are the spins of the individual quarks, $\vec{S}_1=\vec{S}_1+\vec{S}_2$ is their total spin, $\vec{L}$ is the relative orbital angular momenta, and S_{12} is the so-called tensor operator given by

$$S_{12}=3(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})-\vec{S}_1\cdot\vec{S}_2\ . \tag{2.16}$$

Here $A_1(r)$, $A_2(r)$, $A_3(r)$, and $A_4(r)$ are the radially dependent potential functions for the spin-orbit, spin-spin, tensor, and spin-orbit mixing interactions, respectively. The spin-orbit mixing interaction term would not be relevant for the self-conjugate mesons and also for the ground S state of the non-self-conjugate mesons. Therefore excepting $A_4(r)$, the other potential functions, calculated in the usual manner through the standard reduction formulas, can be expressed in the explicit forms

$$A_1(r)=\frac{a\nu}{2m_{q_1}m_{q_2}}(4g_V-1)r^{\nu-2}\ , \tag{2.17}$$

$$A_2(r)=[2a\nu(\nu+1)/3m_{q_1}m_{q_2}]g_V r^{\nu-2}\ , \tag{2.18}$$

$$A_3(r)=[a\nu(2-\nu)/3m_{q_1}m_{q_2}]g_V r^{\nu-2}\ . \tag{2.19}$$

Then the total effective potential including spin-dependent corrections to the lowest order for a mesonic system with quark configuration on $q_1\bar{q}_2$ is given by

$$V_{q_1\bar{q}_2}(r)=V(r)+\delta V_{\text{spin}}(r)\ . \tag{2.20}$$

In fact there should be spin-independent relativistic corrections present along with the spin-dependent $\delta V_{\text{spin}}(r)$ terms. But we do not take into account these possible corrections explicitly for the simple reason that this would only shift the absolute mass scale without any change in the relative mass splittings. However we believe that if the relativistic effects are not very significant, then their average effects can, in some phenomenological sense, be included in the parameterization of the static potential itself by choosing a suitable nonzero value for the parameter V_0 in Eq. (2.1).

C. Level splittings and mass formulas

Now with a perturbation approach to the spin-dependent correction term $\delta V_{\text{spin}}(r)$, one can obtain the fine-hyperfine levels with the mass formulas written conveniently in matrix form:

$$\begin{bmatrix} M(^3S_1) \\ M(^1S_0) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{4} \\ 1 & -\frac{3}{4} \end{bmatrix} \begin{bmatrix} M_{nS} \\ \langle A_2(r)\rangle_s \end{bmatrix}\ , \tag{2.21}$$

$$\begin{bmatrix} M(^3P_2) \\ M(^3P_1) \\ M(^3P_0) \\ M(^1P_1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \frac{1}{4} & -\frac{1}{10} \\ 1 & -1 & \frac{1}{4} & \frac{1}{2} \\ 1 & -2 & \frac{1}{4} & -1 \\ 1 & 0 & -\frac{3}{4} & 0 \end{bmatrix} \begin{bmatrix} M_{nP} \\ \langle A_1(r)\rangle_p \\ \langle A_2(r)\rangle_p \\ \langle A_3(r)\rangle_p \end{bmatrix}\ . \tag{2.22}$$

We must point out here that while (2.21) gives the hyperfine masses of all mesons in S states, (2.22) is meant only for the self-conjugate mesons in the higher orbital state with $L=1$. Here M_{nL} are the spin-averaged masses for the L orbital state of the quark-antiquark bound systems obtained from the exact numerical solution to the Schrödinger equation with the static potential $V(r)$ in Eq. (2.1), and $\langle A_1(r)\rangle_L$, $\langle A_2(r)\rangle_L$, and $\langle A_3(r)\rangle_L$ are the corresponding expectation values of the potential functions. In fact from Eqs. (2.17)–(2.19) it is clear that these quantities depend on the expectation value $\langle r^{\nu-2}\rangle_L$ and also on the parameter g_V.

III. PHENOMENOLOGICAL RESULTS AND DISCUSSIONS

The phenomenological results concerning the light- and heavy-meson spectra are obtained broadly in the usual two-step process. First we solve the nonrelativistic Schrödinger equation numerically with the static potential $V(r)$ as given in Eq. (2.1) to obtain the spin-averaged masses of mesonic states. Then with a perturbative approach to the spin-dependent correction term $\delta V_{\text{spin}}(r)$, we obtain the relevant expectation values required to compute the fine-hyperfine levels from Eqs. (2.21) and (2.22). For doing this we choose to determine the parameters V_0, a, ν, and m_q in the following manner.

A. Fixing of parameters

We already have a preliminary estimate of the potential parameters V_0, a, and ν in Sec. II, which are obtained from the semiclassical results with inputs taken from ψ and Υ spectra. However, we would now like to make a simplifying assumption that the possible spin-independent relativistic corrections can in some phenomenological sense be absorbed in the parameter V_0 of the static potential. In that case we may expect to have different values of $V_0 = V_0(q_1\bar{q}_2)$ for different $q_1\bar{q}_2$ systems. But at the same time we must keep the parameters a and ν the same for all systems in order to maintain an apparent flavor independence of the potential. Now to obtain the potential parameters a and ν and the quark mass parameters m_s, m_c, and m_b, we make a special reference to the spin-averaged masses of the self-conjugate mesons ϕ, ψ, and Υ. With $V_0 = -7.38$ GeV, we obtain

$$(a,\nu) = (6.08 \text{ GeV}, 0.113) , \qquad (3.1)$$
$$(m_s, m_c, m_b) = (623.0, 1854.5, 5215.0) \text{ MeV} .$$

Then in order to obtain the quark masses m_u and m_d, we fit the experimental spin-averaged masses of the $D_c^0(c\bar{u})$ and $D_c^+(c\bar{d})$ systems with m_c, a, and ν fixed as in (3.1). This gives with $V_0(c\bar{u}) = V_0(c\bar{d}) = -7.397$ GeV

$$(m_u, m_d) = (379.31, 385.209) \text{ MeV} . \qquad (3.2)$$

In this manner we fix the light- and heavy-constituent quark masses m_u, m_d, m_s, m_c, and m_b along with the potential parameters a and ν. Then with suitable values of $V_0(q_1\bar{q}_2)$ we can compute the spin-averaged masses of various mesonic states with quark configuration $q_1\bar{q}_2$. For our convenience we have categorized the meson families into two groups depending upon their constituent quark configurations. A quark q_1 and an antiquark $\bar{q}_2$ possessing different flavors would give rise to non-self-conjugate meson families, whereas like-flavored quark-antiquark configurations such as $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, etc. would give rise to self-conjugate meson families.

B. Non-self-conjugate light and heavy mesons

The non-self-conjugate light and heavy mesons such as

$$(\rho^-,\pi^-),(\bar{K}^{*0},\bar{K}^0,K^{*-},K^-) ,$$
$$(F_c^{*+},F_c^+,D_c^{*+},D_c^+,D_c^{*0},D_c^0) ,$$
$$(F_b^{*0},F_b^0,D_b^{*0},D_b^0,D_b^{*-},D_b^-),$$

and E^- correspond, respectively, to the quark-antiquark configurations $d\bar{u}$, $(s\bar{d},s\bar{u})$, $(c\bar{s},c\bar{d},c\bar{u})$, $(b\bar{s},b\bar{d},b\bar{u})$, and $b\bar{c}$. All these mesons, except the heavier b-flavored mesons, have been experimentally observed. A detailed study of all these mesons in their ground state in the same framework as being discussed here has already been reported elsewhere.[20] We must point out here that the parameters used in Ref. 20 are the same as in (3.1) and (3.2). For completeness of our discussion we would like to quote some results of Ref. 20 regarding the computed masses of these mesonic states in comparison with the corresponding experimental values. These are presented in Table I along with the various $V_0(q_1\bar{q}_2)$ values. We find that these parameters $V_0(q_1\bar{q}_2)$ are not significantly different from each other since they lie in a range -7.35 to -7.483 GeV. This may be interpreted as an indication of the fact that the effect due to the relativistic spin-independent corretion terms is not quite significant phenomenologically. We can also notice a very good agreement of the computed hyperfine masses with the corresponding experimental ones.

C. Self-conjugate light and heavy mesons

The self-conjugate quark-antiquark configurations $(1/\sqrt{2})(u\bar{u} \mp d\bar{d})$, $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, and possibly $t\bar{t}$, correspond, respectively, to meson families of (ρ^0,π^0), (ω^0,η), (ϕ,η'), (ψ,η_c), (Υ,η_b) and the yet-to-be observed (ξ,η_t). Our approach here would be to compute first the spin-averaged spectrum of these mesons. Although the fine-hyperfine splittings corresponding to the excited states of the light mesons in this group are not yet identified completely and conclusively, we can still obtain our model predictions for ρ^0 and ϕ, at least for a qualitative picture. On the other hand we would discuss in detail the fine-hyperfine structure of the $c\bar{c}$ and $b\bar{b}$ spectrum.

1. Spin-averaged mass spectra

Following the parametrization discussed in Sec. III A, we solve the Schrödinger equation to obtain the spin-averaged masses of the ground states and also a few excited states of ρ^0, ϕ, ψ, Υ, and ξ. The results are presented in Table II, along with the corresponding potential parameter $V_0(q_1\bar{q}_2)$ values and the quark masses. We must point out here that the quark masses m_c and m_b were obtained as discussed in Sec. III A by making fits to the spin-

TABLE I. Spin-averaged masses $M_0(q_1\bar{q}_2)$ and the hyperfine masses of the experimentally observed non-self-conjugate mesons with the corresponding potential-parameter values $V_0(q_1\bar{q}_2)$.

Quark content $q_1\bar{q}_2$	$V_0(q_1\bar{q}_2)$ (GeV)	$M_0(q_1\bar{q}_2)$ (MeV)	Hyperfine meson symbols	Hyperfine meson masses (MeV)	Experimental mass (MeV)
$d\bar{u}$	-7.483	610.10	ρ^-	759.25	770 ± 5
			π^-	162.63	139.567 ± 0.001
$s\bar{d}$	-7.455	795.80	$\bar{K}^{*0}$	896.41	896 ± 1
			$\bar{K}^0$	493.96	497.7 ± 0.2
$s\bar{u}$	-7.455	791.40	K^{*-}	892.04	892 ± 1
			K^-	489.47	493.669 ± 0.015
$c\bar{s}$	-7.35	2115.0	F_c^{*+}	2149.70	2140 ± 60
			F_c^+	2010.70	2030 ± 60
$c\bar{d}$	-7.397	1973.55	D^{*+}	2013.60	2008.6 ± 1.0
			D_c^+	1853.20	1868.3 ± 0.9
$c\bar{u}$	-7.397	1970.325	D_c^{*0}	2010.40	2006 ± 1.5
			D_c^0	1849.80	1863.1 ± 0.9

averaged ground-state masses $M_{1s}(c\bar{c})=3.069$ GeV and $M_{1s}(b\bar{b})=9.428$ GeV, respectively. But for the light meson $\rho^0(770)$ considered to be a $q\bar{q}$ configuration like $(1/\sqrt{2})(u\bar{u}-d\bar{d})$, we fit its spin-averaged ground-state mass to a quark mass $m_q=m_d=385.21$ MeV. Also for the yet unobserved $t\bar{t}$ spectrum, we have obtained the spectrum by fixing the value of the quark mass m_t arbitrarily at 20 GeV.[21] Our purpose here is not to emphasize the absolute masses of this particular spectrum, which is yet to be discovered experimentally. We would only discuss some qualitative features of the relative level spacings in comparison with other spectra.

From the results in Table II, we obtain the so-called mass differences between the 1S and 2S

TABLE II. The spin-averaged mass spectrum of the light and heavy self-conjugate meson families of ρ^0, ϕ, ψ, Υ, and ξ.

$q\bar{q}$	$(1/\sqrt{2})(u\bar{u}-d\bar{d})$	$(s\bar{s})$	$(c\bar{c})$	$(b\bar{b})$	$(t\bar{t})$
$V_0(q\bar{q})$ (GeV)	-7.483	-7.38	-7.38	-7.38	-7.38
m_q (GeV)	0.385 209	0.6230	1.854	5.215	20.0
L	$M_L(\rho^0)$ (GeV)	$M_L(\phi)$ (GeV)	$M_L(\psi)$ (GeV)	$M_L(\Upsilon)$ (GeV)	$M_L(\xi)$ (GeV)
0	0.6129	1.004	3.069	9.428	38.572
0	1.273	1.649	3.6717	10.001	39.104
0	1.661	2.027	4.029	10.337	39.412
0	1.939	2.299	4.2845	10.579	39.637
1	1.087	1.467	3.518	9.835	38.935
1	1.531	1.899	3.907	10.221	39.295
2	1.401	1.773	3.789	10.109	39.20
3	1.651	2.045	4.262	10.597	

states for ρ^0, ϕ, ψ, Υ, and ξ families

$$\begin{aligned} &M(\rho'-\rho)=0.66 \text{ GeV},\\ &M(\psi'-\psi)=0.603 \text{ GeV},\\ &M(\phi'-\phi)=0.645 \text{ GeV},\\ &M(\Upsilon'-\Upsilon)=0.573 \text{ GeV},\\ &M(\xi'-\xi)=0.532 \text{ GeV}. \end{aligned} \tag{3.3}$$

The corresponding experimental quantities always refer to the vector meson masses rather than the spin-averaged ones. Nevertheless $\Delta M(\psi'-\psi)_{\rm exp}=0.589$ GeV and $\Delta M(\Upsilon'-\Upsilon)_{\rm exp}=0.5585$ GeV compare quite well with the calculated values in (3.3). For the light mesons like ρ' and ϕ', the experimental situations are not very clear yet. There have been several experiments[22] in favor of a $\rho'(1600)$, whereas some other experiments[23] at the same time claim good evidence for a $\rho'(1250)$ as the $2S$ excited state of ρ^0. However, from our calculation we find the $2S$ and $3S$ levels for ρ^0 at the spin-average mass values 1273 and 1661 MeV, respectively. Thus our model gives a $\rho'(1273)$ and a $\rho''(1661)$ as two excited states of ρ^0. However, we would not like to attach too much quantitative significance to these results, since for the excited states of the lighter mesons, the nonrelativistic approach used here may prove completely unsuitable. Finally if we assume that the general nature of the Regge trajectories would not be very much affected by the consideration of spin effects, we may, for simplicity, take these spin-average masses corresponding to different orbital angular momenta to draw Chew-Frautchi plots. Then it can be shown that the leading Regge trajectories are almost linear.

2. *Fine-hyperfine splittings*

The mass values of the fine-hyperfine levels of different mesonic states can be computed from Eqs. (2.21) and (2.22). We find from Eqs. (2.17)–(2.19) that the expectation values of the potential functions $A_1(r)$, $A_2(r)$, and $A_3(r)$ required here depend in their turn on $\langle r^{\nu-2}\rangle_L$ and also on the choice of the parameter g_V. In Table III, we have listed these expectation values $\langle r^{\nu-2}\rangle_L$ for the ground state and a few more excited states of the ρ^0, ϕ, ψ, and Υ systems. We find that the vector fraction $g_V=0.58$ taken to be the same for all the meson states gives results which are quite satisfactory. Table IV gives a detailed account of the fine-hyperfine splittings of $c\bar{c}$ and $b\bar{b}$ bound state families. We find very good agreement of the calculated results and the corresponding experimental mass values. The pseudoscalar partners of ψ and ψ' are found to be at mass values 3.0 and 3.64 GeV, respectively. The experimental evidence of these effects at 2.83 and 3.454 GeV is proved to be controversial in view of the negative evidence for the existence of such effects given by recent experiments.[16] On the other hand, recent experimental results[16] indicate the existence of the $^1S_0$ partner of ψ at a mass value 2.978 GeV, which is quite close to our prediction. We also predict the mass value of the yet-to-be observed $^1P_1$ state of the $c\bar{c}$ system to be 3.503 GeV, which lies between the $^3P_1$ and $^3P_2$ levels.

TABLE III. Calculated values of $\langle r^{\nu-2}\rangle_{nL}$ (in GeV units) for ρ^0, ϕ, ψ, and Υ systems.

$q\bar{q}$ bound state	(ρ^0)	(ϕ)	(ψ)	(Υ)
$1S$	0.295 39	0.350 29	1.010 4	2.552 4
$2S$	0.134 52	0.174 37	0.466 01	1.185 2
$3S$	0.094 625	0.113 27	0.306 57	0.767 72
$4S$	0.074 218	0.085 59	0.231 77	0.574 51
$1P$	0.056 78	0.054 119	0.231 06	0.573 49

Now coming to the $b\bar{b}$ system we find that the vector meson masses of Υ, Υ', Υ'', and Υ''' come out in close agreement with recent experiments.[24] The $^1S_0$ partner of Υ has not yet been observed, which may be expected at a mass value very close to Υ. Our estimate gives $\Delta M(\Upsilon-\eta_b)=27.8$ MeV.

Finally we calculate the fine-hyperfine splittings of the light meson systems ρ^0 and ϕ. The results are presented in Table V, along with the splittings of some other excited states. Of course for the excited states of these mesons, experimental evidence are not conclusively established. Therefore we do not make an attempt to identify these levels with the scarcely known mesons in these mass regions. However, the agreement of the calculated ground-state hyperfine masses with the corresponding experimental ones is found to be better than we would expect in a nonrelativistic model. Particularly in the case of $^1S_0$ states we may find a noticeable departure. This is not very surprising since we have ignored the fact that η' must contain nonorthogonal combinations of u, d, and s quarks which would otherwise complicate the study of these pseudoscalar meson states.

TABLE IV. Fine-hyperfine structure of $c\bar{c}$ and $b\bar{b}$ systems.

$q\bar{q}$ state	$c\bar{c}$ system Predicted mass (GeV)	Experimental mass (GeV)	$b\bar{b}$ system Predicted mass (GeV)	Experimental mass (GeV)
$1\,^1S_0$	3.004	2.983(?)	9.4071	
$1\,^3S_1$	3.091	3.095 ± 0.003	9.4349	9.4345 ± 0.0004
$2\,^1S_0$	3.642		9.9913	
$2\,^3S_1$	3.682	3.684 ± 0.009	10.004	9.993 ± 0.001
$3\,^1S_0$	4.009		10.330	
$3\,^3S_1$	4.036	4.040 ± 0.010	10.339	10.323 ± 0.0007
$4\,^1S_0$	4.27		10.574	
$4\,^3S_1$	4.289	4.417 ± 0.010	10.580	10.546 ± 0.002
$1\,^3P_0$	3.445	3.413 ± 0.005	9.8121	
$1\,^3P_1$	3.501	3.508 ± 0.004	9.8296	
$1\,^3P_2$	3.552	3.554 ± 0.005	9.8455	
$1\,^1P_1$	3.503		9.8303	

3. *Electromagnetic transition widths*

In nonrelativistic approximation, the rates for electric dipole transitions ($E1$) between S- and P-wave states having the same total quark spin are given by[25]

$$\Gamma_\gamma(E1,2\,^3S_1\to1\,^3P_j)=\tfrac{4}{27}(2J+1)\alpha e_q{}^2[M(2\,^3S_1)-M(1\,^3P_j)]^3\,|\,E_{2S,1P}\,|^2$$

$$\Gamma_\gamma(E1,1\,^3P_j\to1\,^3S_1)=\tfrac{4}{9}\alpha e_q{}^2[M(1\,^3P_j)-M(1\,^3S_1)]^3\,|\,E_{1P,1S}\,|^2\,, \quad (3.4)$$

TABLE V. Fine-hyperfine structure of ρ^0 and ϕ systems.

$q\bar{q}$ state	ρ^0 system Predicted mass (GeV)	Experimental mass (GeV)	ϕ system Predicted mass (GeV)	Experimental mass (GeV)
$1\,^1S_0$	0.171	0.13496 ± 0.01	0.804	
$1\,^3S_1$	0.760	$0.770\ \ \pm0.005$	1.070	1.020
$2\,^1S_0$	1.072		1.549	
$2\,^3S_1$	1.34	1.25 (ρ'?)	1.682	
$3\,^1S_0$	1.52		1.962	
$3\,^3S_1$	1.708	1.6 (ρ''?)	2.049	
$1\,^3P_0$	0.672	0.7 (ϵ?)	1.316	
$1\,^3P_1$	0.990		1.432	
$1\,^3P_2$	1.279	1.31 (A_2?)	1.537	1.5(f')
$1\,^1P_1$	1.002		1.436	

where the transition dipole matrix element is

$$E_{\mathrm{if}}=\int_0^\infty dr\, r^3 R_i(r) R_f(r) \tag{3.5}$$

with $R_{i,f}(r)$ being the initial- (final-) state radial wave functions. Using the experimental masses of the $1\,^3S_1$, $2\,^3S_1$, and $1\,^3P_{0,1,2}$ states of the charmonium, we calculate the $S\rightarrow P\ E1$ transition rates according to (3.4). The results of the calculations in our potential model with $g_V=0.58$ are presented in Table VI.

The allowed $M1$ transition rates between $^3S_1$ and $^1S_0$ states of the charmonium spectrum in this model are calculated according to the formula[26]

$$\Gamma(M1, n\,^3S_1\rightarrow n\,^1S_0) = \tfrac{4}{3}\alpha(e_q/m_q)^2[M(n\,^3S_1)-M(n\,^1S_0)]^3\ , \tag{3.6}$$

where a Dirac moment is assumed for the quark. In this calculation (Table VI), we take the predicted mass splitting of the $^3S_1$-$^1S_0$ states in our model. For the $c\bar{c}$ system, the $M1$ transition rates are found quite below the experimental upper limits. Similar calculations on the electromagnetic transition rates for the $b\bar{b}$ spectrum are also presented in Table VI.

4. Leptonic decay widths

Generally the leptonic decay widths of the vector mesons are calculated using the Van Royen—Weisskopf formula [27]

$$\Gamma(V_{ns}\rightarrow e^+e^-)=\frac{16\pi\alpha^2 e_q^{\,2}}{M_V^{\,2}}\,|\,\psi_{ns}(0)\,|^2\ . \tag{3.7}$$

However, this formula should not be trusted too much in its absolute value, since it is affected by correction factors which are not quite certain.[28] In that case, the leptonic decay width ratios given by

$$\frac{\Gamma(V_{ns}\rightarrow e^+e^-)}{\Gamma(V_{1s}\rightarrow e^+e^-)}=\left[\frac{M_v(1s)}{M_v(ns)}\right]^2\frac{|\,\psi_{ns}(0)\,|^2}{|\,\psi_{1s}(0)\,|^2} \tag{3.8}$$

would serve as a meaningful and reliable quantity. Therefore, using the computed values of $|\,\psi_{ns}(0)\,|^2$ and the vector-meson masses, we can calculate these ratios for ρ^0, ϕ, ψ, and Υ states and also for their corresponding radial excitations. These results are presented in Table VII. In case of the heavy vector mesons of the $c\bar{c}$ and $b\bar{b}$ systems, these quantities are found to be in excellent agreement with the corresponding experimental quantities. Particularly in the case of the $b\bar{b}$ system, where we have very recent accurate data,[24] the agreement is quite good.

D. Velocity parameter of the constituent quarks

The quantitative calculations for the meson mass values obtained here in a nonrelativistic perturbative approach are based on an *a priori* assumption that the relativistic corrections are phenomenologically less significant even for the light-quark systems. Therefore in order to assess these quantitative results we have obtained here, we must now look back and determine the validity of this assumption from a detailed calculation of the velocity parameter $\beta_q^{\,2}=\langle v^2/c^2\rangle$ of the individual constituent quarks.

We first obtain the expectation value of the total kinetic energy $\langle T\rangle=\frac{1}{2}\langle r dV/dr\rangle$ for the $q_1\bar{q}_2$ system. This is the total average kinetic energy available to the two constituent quarks in the center-of-

TABLE VI. Calculation of electromagnetic transaction widths Γ_r for $c\bar{c}$ and $b\bar{b}$ systems.

Initial state	Final state	Electromagnetic transitions	$c\bar{c}$ system Predicted Γ_r (keV)	$c\bar{c}$ system Experimental Γ_r (keV)	$b\bar{b}$ system Predicted Γ_r (keV)	$b\bar{b}$ system Experimental Γ_r (keV)
$1\,^3S_1$	$1\,^1S_0$	$M1$	0.83	1.2	0.85×10^{-3}	
$2\,^3S_1$	$2\,^1S_0$	$M1$	0.08	5.7	0.08×10^{-3}	
$2\,^3S_1$	$1\,^3P_0$	$E1$	49.88	16 ± 9	1.56	
$2\,^3S_1$	$1\,^3P_1$	$E1$	40.98	16 ± 8	3.50	
$2\,^3S_1$	$1\,^3P_2$	$E1$	27.53	16 ± 9	4.38	
$1\,^3P_0$	$1\,^3S_1$	$E1$	156.43		24.57	
$1\,^3P_1$	$1\,^3S_1$	$E1$	342.67		28.15	
$1\,^3P_2$	$1\,^3S_1$	$E1$	470.40		31.69	

TABLE VII. The absolute square of the wave function $|\psi_{nS}(0)|^2$ at the origin and the ratio of leptonic decay widths of light and heavy vector mesons.

$q\bar{q}$ state	$\|\psi_{nS}(0)\|^2$ (GeV^3)	Predicted $\frac{\Gamma(nS\to e^+e^-)}{\Gamma(1S\to e^+e^-)}$	Experimental $\frac{\Gamma(nS\to e^+e^-)}{\Gamma(1S\to e^+e^-)}$
ρ	0.008 74	1	1
ρ'	0.004 97	0.181	
ρ''	0.003 58	0.081	
ρ'''	0.002 81	0.048	
ϕ	0.017 41	1	1
ϕ'	0.009 79	0.227	
ϕ''	0.007 04	0.110	
ϕ'''	0.005 64	0.069	
ψ	0.081 98	1	1
ψ'	0.046 29	0.398	0.45 ± 0.09
ψ''	0.033 43	0.239	0.156
ψ'''	0.026 91	0.170	0.102
Υ	0.356 29	1	1
Υ'	0.201 89	0.504	0.44 ± 0.06, 0.31 ± 0.06
Υ''	0.145 06	0.339	0.35 ± 0.04, 0.32 ± 0.04
Υ'''	0.115 64	0.258	0.20 ± 0.06

mass sytem. In that case the individual share of the average kinetic energy available to each constituent quark would be given by

$$\langle T_{q_i}\rangle = \frac{m_{q_j}}{(m_{q_1}+m_{q_2})}\langle T\rangle\ ,\quad j\neq i=1,2\ . \tag{3.9}$$

Now we can define a parameter

$$\epsilon_{q_i} = \frac{\langle T_{q_i}\rangle}{m_{q_i}}\ , \tag{3.10}$$

which is the ratio of the individual average kinetic energy and the rest mass energy m_{q_i} of the constituent quark q_i. $\epsilon_{q_i}<1$ would indicate the validity of taking a Schrödinger-type approach in a nonrelativistic limit with spin-dependent and relativistic effects treated as perturbations. The importance of the spin-independent relativistic effects can be judged from the velocity parameters $\beta_{q_i}^{\ 2}=\langle v^2/c^2\rangle$ of each individual quark, which can be calculated from the relation

$$\beta_{q_i}^{\ 2} = 1-1/(1+\epsilon_{q_i}^{\ 2})^2\ . \tag{3.11}$$

The results of these calculations are presented for the ground states of some light and heavy mesons in Table VIII.

We find that in the case of heavy mesons such as ψ and Υ, $\beta_c^{\ 2}=0.169$ and $\beta_b^{\ 2}=0.062$, which implies clearly the nonrelativistic motion of the constituent quarks. The same can also be found to be true even for several excited states of these heavy mesons. Now coming to the lighter meson ϕ in its ground state, we find that $\beta_S^{\ 2}=0.414$. Since $\beta_S^{\ 2}$ for the strange quark in the ground state of ϕ is in the same range as in the 4S excited state of the

TABLE VIII. Calculation of velocity parameters for the constituent quarks in some of the light and heavy meson states.

$q_1\bar{q}_2$ states	$\langle T\rangle$ (GeV)	ϵ_{q_1}	ϵ_{q_2}	$\beta_{q_1}^{\ 2}$	$\beta_{q_2}^{\ 2}$
ρ^0	0.391	0.508	0.508	0.56	0.56
K^0	0.387	0.237	0.621	0.347	0.619
ϕ	0.381	0.306	0.306	0.414	0.414
D_c^+	0.381	0.035	0.818	0.067	0.698
F_c^+	0.373	0.051	0.448	0.094	0.523
ψ	0.359	0.097	0.097	0.169	0.169
Υ	0.340	0.033	0.033	0.062	0.062

charmonium,[13] we may regard it as marginally nonrelativistic. However for other mesons such as ρ^0, K^0, D_c^+, F_c^+, we find $\beta_q{}^2 \leq 0.698$ and hence we presume that in these cases the relativistic effects may be significant, although with varying degrees. Therefore our quantitative results even for the ground states of these mesons should be taken with caution. But if we do not put emphasis on the absolute mass scale, the relative mass splittings in these cases will not be significantly altered with the explicit inclusion of the relativistic corrections. Then the overall systematics obtained here for the entire meson spectra, with our initial assumption of treating the relativistic effects to be less significant, can, at the expense of certain quantitative details, reflect the underlying unified mechanism responsible for the light- and heavy-quark—antiquark bound states representing various mesons.

IV. CONCLUSION

We conclude that an effective non-Coulombic power-law potential, generating spin dependence through scalar and vector exchanges in almost equal proportions, can describe the meson spectroscopy in a sufficiently unified manner encompassing both the heavy- and the light-quark systems. The overall systematics of our phenomenological predictions for the meson masses and the leptonic decay widths do not reflect any inadequacy in the short- or long-distance behavior of the simple power-law potential. Therefore we can argue that even if this phenomenological potential does not possess certain behaviors expected from the theoretical approaches, it definitely simulates the actual potential in a wide range of the quark-antiquark separation distance probed in this analysis of the entire meson spectra.

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