Hadronic-energy correlations at the Z^0

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The energy-energy correlations produced in $e^-e^+ \to Z^0 \to$ hadrons are investigated. We show that this process measures exactly the same hadronic structure functions determined by $e^-e^+ \to \gamma \to$ hadrons, but that the two processes differ in their dependence on "external" angles when the initial leptons are polarized. Measurements of the energy-energy correlation at the Z^0 are important since the nonperturbative hadronization corrections to the perturbative QCD reaction are not significant at this high energy.

The fundamental theory of hadronic physics, quantum chromodynamics (QCD), is asymptotically free. Hence hadronic processes at high energy can be computed in perturbation theory provided that they are free of mass singularities. The energy-weighted cross sections for the production of quarks and gluons in electron-positron annihilation are devoid of such mass singularities by virtue of the energy weighting and the absence of hadrons in the initial state. Particularly promising is the study of the energy-energy correlation at finite opening angles.[1–3] Since the basic quark-antiquark reaction produces energy correlations only in back-to-back orientations, there is no zeroth-order background, and the measurement of correlations at finite angles may provide a precise confirmation of the QCD theory.

The energy-energy correlation cross section is defined as follows. A large number N of hadron producing e^+e^- collisions are observed in the center-of-mass frame with an individual collision labeled by A with $A=1,2,\ldots,N$. In each collision one measures the energies dE_A and dE_A' which are carried by hadrons into the solid angles $d\Omega$ and $d\Omega'$ that lie in the directions $\hat{r}$ and $\hat{r}'$ relative to the collision point. The geometry is illustrated in Fig. 1. The energy-energy correlation cross section is defined by

$$\frac{1}{\sigma}\frac{d^2\Sigma}{d\Omega\, d\Omega'}=\frac{1}{N}\sum_{A=1}^{N}\left[\frac{dE_A}{W\,d\Omega}\right]\left[\frac{dE_A'}{W\,d\Omega'}\right], \qquad (1)$$

where σ is the total hadronic cross section and W is the total energy.

For energies well below the Z^0 mass, the cross section is dominated by the production of hadrons by virtual photons as shown in Fig. 2. This cross section is proportional to the virtual-photon spin density matrix which contains angular momentum two and zero. The coupling of this density matrix to the final hadronic system gives the energy-energy cross section an angular dependence on the detection directions $\hat{r}$ and $\hat{r}'$ with respect to external directions such as those defined by the colliding-beam axis ($\hat{l}$) or a transverse direction ($\hat{b}$) defined, for example, by a magnetic field. Since the angular momentum in the photon density matrix is limited, so is the complexity of this angular dependence. As shown in Ref. 2, there are, in fact,

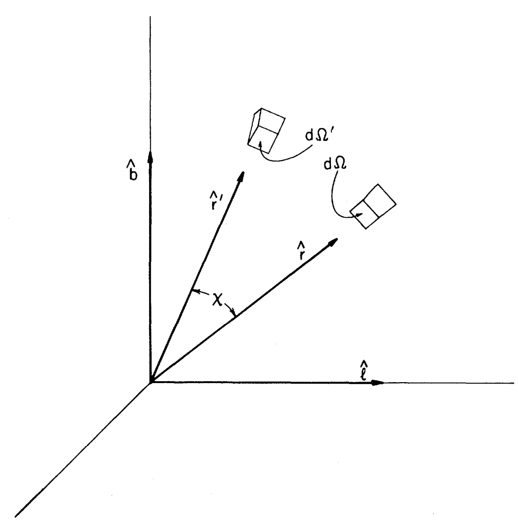

FIG. 1. Geometry for the energy-energy correlation experiment. The energies are measured which pass into the solid angles $d\Omega, d\Omega'$ lying in the directions $\hat{r}$ and $\hat{r}'$ with respect to the interaction point. The opening angle between these directions is denoted by χ. The incident electron and positron are along the directions $\hat{l}$ and $-\hat{l}$, respectively. The unit vector $\hat{b}$ is perpendicular to the beam direction.

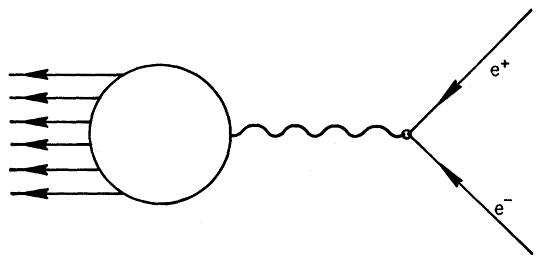

FIG. 2. At lower energies the electron-positron annihilate into a virtual photon, indicated by the wavy line, which produces the final hadronic state. The same graph represents the Z^0 resonance production, but with the wave line now representing virtual Z^0 exchange.

only three independent terms in the energy-energy cross section which depend upon these external angles. To illustrate this angular dependence, we consider the case where the electrons are perfectly polarized in the transverse direction $\hat{b}$ and the positrons perfectly polarized in the opposite direction $-\hat{b}$, where

$$\frac{1}{\sigma}\frac{d^2\Sigma}{d\Omega\, d\Omega'}=\frac{3}{8\pi}\{\mathscr{A}(\chi)[2-(\hat{r}\cdot\hat{b})^2-(\hat{r}'\cdot\hat{b})^2] +\mathscr{B}(\chi)[\cos\chi-(\hat{r}\cdot\hat{b})(\hat{r}'\cdot\hat{b})] +\mathscr{C}(\chi)\}\ . \tag{2}$$

The three structure functions $\mathscr{A}$, $\mathscr{B}$, and $\mathscr{C}$ depend only upon the angle χ between the detection directions $\hat{r}$ and $\hat{r}'$ with

$$\cos\chi=\hat{r}\cdot\hat{r}'\ . \tag{3}$$

The specific forms of the structure functions derived[2] in first-order perturbation theory for the production of quarks and gluons are given by

$$\mathscr{A}^{(1)}(\chi)=\frac{\alpha_S(W)}{12\pi^2}\frac{1}{1-\zeta}\left[\frac{(3-4\zeta)}{\zeta^5}\ln(1-\zeta)+\frac{3}{\zeta^4}-\frac{5}{2\zeta^3}-\frac{1}{\zeta^2}\right], \tag{4a}$$

$$\mathscr{B}^{(1)}(\chi)=\frac{\alpha_S(W)}{12\pi^2}\frac{1}{1-\zeta}\left[\frac{4(3-\zeta)(1-\zeta)}{\zeta^5}\ln(1-\zeta)+\frac{12}{\zeta^4}-\frac{10}{\zeta^3}\right], \tag{4b}$$

and

$$\mathscr{C}^{(1)}(\chi)=0\ , \tag{4c}$$

where

$$\zeta=\tfrac{1}{2}(1-\cos\chi)\ , \tag{5}$$

and $\alpha_S(W)$ is the usual running strong coupling at energy W.

At finite energy, this perturbative result must be corrected for the nonperturbative hadronization of the quarks and gluons into the observed hadrons. A simple model[2] corrects the $\mathscr{A}$ coefficient by the addition of

$$\mathscr{A}^{(\mathrm{qf})}(\chi)=\frac{C\langle h_\perp\rangle}{4\pi W\sin^3\chi} \tag{6}$$

as the leading term in an expansion in powers of $1/W$ [the superscript (qf) refers to quark fragmentation]. Here C is the coefficient of the (assumed) logarithmic rise in the total multiplicity in e^--e^+ collisions, and $\langle h_\perp\rangle$ is the average transverse momentum of a hadron produced in the hadronization process. There are no leading $[O(1/W)]$ hadronization corrections to the $\mathscr{B}(\chi)$ and $\mathscr{C}(\chi)$ coefficients. With sufficiently high energies the perturbative contribution, which behaves as $\alpha_S(W)\sim 1/\ln W$, dominates over the nonperturbation correction, which decreases as $1/W$. At lower energies, the nonperturbative background can be diminished by measuring the asymmetry[1,4] about $\chi=90°$ [since $\mathscr{A}^{(\mathrm{qf})}(\chi)$ is symmetric to $O(1/W)$] or by separating out the structure functions $\mathscr{B}(\chi)$ and $\mathscr{C}(\chi)$ from the data[5] [since they have no hadronization corrections to $O(1/W)$]. Although the data[4] at $W=30$ GeV are in good agreement with the first-order perturbative calculation [Eqs. (4)] supplemented by the nonperturbative corrections of the simple hadronization model [Eq. (10)], with the perturbative asymmetry needing no hadronization correction, the leading nonperturbative correction is substantial at this energy[5] and measurements need to be made at a much higher energy to secure a definitive test of the theory. This is our motivation for discussing the energy-energy correlations when the total e^--e^+ energy, W, equals the mass of the Z^0 neutral weak boson, $M_Z\simeq 90$ GeV. The measurement of the energy correlations at the Z^0 is advantageous not only in reducing the hadronization corrections by a factor of $\frac{1}{3}$; since the cross section is very large, it may be possible at this energy to gather a good statistical sample and measure the separate structure functions $\mathscr{A}(X)$, $\mathscr{B}(X)$, and $\mathscr{C}(X)$.

Since we are concerned solely with testing QCD, we shall compute the energy-energy correlation cross section only on resonance where $W=M_Z$. In this case the Z^0 exchange amplitude is purely imaginary relative to the γ exchange amplitude (which can be seen explicitly from the discussion below), and so there is no Z^0-γ interference. The remaining squared γ amplitude is of order $(\Gamma_Z/M_Z)^2 \approx 10^{-3}$ relative to the squared Z amplitude. Thus the photon-exchange background is negligible, and the process can be considered to proceed entirely from Z^0 exchange, with the Z^0 replacing the γ previously used in the graph of Fig. 2. The energy-energy correlation cross section arising from Z^0 exchange is computed by an energy-weighted phase-space integral over a squared amplitude that has the form

$$|T|^2 \propto \mathscr{L}_{\mu\nu} \left| \frac{1}{W^2-M_Z{}^2+iM_Z\Gamma_Z} \right|^2 H_Z^{\mu\nu} . \quad (7)$$

The factor $\mathscr{L}_{\mu\nu}$ represents the square of the lepton weak neutral current, the Z^0 propagator factor, as we have just mentioned, will be evaluated only at $W=M_Z$, and the factor $H_Z^{\mu\nu}$ involves the square of the hadronic contribution to the weak neutral current.

Let us first consider the structure of the hadronic tensor defined by

$$H_Z^{\mu\nu}=\sum \langle 0|J^\mu|f+\rangle\langle f+|J^\nu|0\rangle , \quad (8)$$

where J^μ is the hadronic weak-neutral-current operator. The sum in Eq. (8) is over all flavors, families, color components, and spins in the outgoing (colorless) final state $\langle f+|$. Since particles and antiparticles are equally energy weighted, we may take this sum to include an average in which the particles and antiparticles are interchanged. Thus the final state is effectively invariant under charge conjugation. On the other hand, the vector (J_V^μ) and axial-vector (J_A^μ) pieces of the full neutral current (J^μ) transform with a relative minus sign under charge conjugation. Hence there is no vector—axial-vector interference, and we have

$$H_Z^{\mu\nu}=\sum \langle 0|J_V^\mu|f+\rangle\langle f+|J_V^\nu|0\rangle + \sum \langle 0|J_A^\mu|f+\rangle\langle f+|J_A^\nu|0\rangle . \quad (9)$$

We are concerned with the high-energy limit in which hadronic energy cross sections can be computed by viewing the process as one which involves massless quarks.[6] Since we are concerned with the high-energy limit of an asymptotically free theory, we need in fact consider only graphs contributing to the squared matrix element (9) which have the connected topology of the lowest-order graph. That is, we need consider only graphs where a quark line runs from one current to the other current and neglect those graphs where a current produces a quark pair which annihilates into gluons that eventually connect the other current.[7] Hence in our limit, and with a sum over the quark spin components, the squared matrix element for an axial-vector current is identical to the squared matrix element for the corresponding vector current. Moreover, all quark types are equivalent both for the weak neutral current and for the electromagnetic current except for overall numerical factors. We conclude that in our limit the hadronic tensor $H_Z^{\mu\nu}$ for the weak neutral current is proportional to the hadronic tensor $H_\gamma^{\mu\nu}$ for the electrical current,

$$H_Z^{\mu\nu} \propto H_\gamma^{\mu\nu} . \quad (10)$$

A final comment on the structure of $H^{\mu\nu}$ needs to be made. The electromagnetic or weak current is left invariant by the CPT reflection. This operation transforms matrix elements into their complex conjugates, interchanges particles and antiparticles, and also interchanges incoming ($|-\rangle$) and outgoing ($|+\rangle$) states. The distinction between the outgoing final state which must be used in Eq. (8) and the use of an incoming state arises from the effects of the interactions amongst the particles in the final state. With the neglect of these interactions, the CPT invariance of Eq. (8) (which involves an average over the interchange of particle and antiparticle) implies that the hadronic tensor is symmetric,

$$H^{\mu\nu}=H^{\nu\mu} . \quad (11)$$

Since the QCD theory is asymptotically free, we may neglect the final-state interactions which are of higher order in the coupling $\alpha_S(W)$ and take the hadronic tensor to be symmetrical as shown in Eq. (11).

We turn now to discuss the leptonic tensor $\mathscr{L}_{\mu\nu}$ which appears in Eq. (7). It is the squared matrix element for the leptonic neutral current. We shall assume that the standard theory is valid so that this current is proportional to

$$j_\mu=\bar{e}\gamma_\mu[(1-4\sin^2\theta_W)+i\gamma_5]e , \quad (12)$$

where θ_W is the Weinberg angle with $\sin^2\theta_W \simeq 0.23$. We are considering the electron-positron annihilation produced from oppositely directly beams of equal energy. In circular storage rings, the electrons and positrons may become equally but oppositely polarized along the transverse direction of the magnetic field. This polarization comes about from the small spin dependence of the synchrotron radiation. The Z^0 resonance may be produced, however, by a single-pass collider such as that proposed by SLAC. In such a machine the electron and positron may have separate and independent polarizations. Typically the positron would be unpolarized and the electron may have a longitudinal polarization. Thus we shall treat the general case where the electrons and positrons have arbitrary but uncorrelated polarizations—where we have an arbitrary but factored form $\rho\bar{\rho}$ for the spin density matrix. The polarization formalism is reviewed in the Appendix. The polarization of the electron moving in the direction $\hat{l}$ may be described by a spatial vector $\vec{s}$ which is convenient to decompose into a part $\vec{s}_\perp$ perpendicular to $\hat{l}$ and a part s_L parallel to $\hat{l}$. Similarly, the polarization of the positron moving in the direction $-\hat{l}$ is described by $\underline{\vec{s}}$ which is decomposed into a part $\underline{\vec{s}}_\perp$ perpendicular to $\hat{l}$ and a part $\underline{s}_L$ parallel to $-\hat{l}$. The magnitudes of these vectors are bounded by unity,

$$0 \leq \vec{s}^{\,2} = \vec{s}_\perp^{\,2} + s_L^{\,2} \leq 1 \ , \tag{13a}$$

$$0 \leq \underline{\vec{s}}^{\,2} = \underline{\vec{s}}_\perp^{\,2} + \underline{s}_L^{\,2} \leq 1 \ . \tag{13b}$$

With $\vec{s}=0$ (or $\underline{\vec{s}}=0$) the electron (or positron) is unpolarized. With the bound saturated, one has perfect polarization. For example, an electron with right-handed helicity is described by $\vec{s}_\perp = 0$, $s_L = +1$. In terms of the notation used previously,[1–3,5] the polarization of the electrons and positrons in a storage ring is given by $\vec{s} = \underline{P}\hat{b}$ and $\underline{\vec{s}} = -\underline{P}\hat{b}$, where $\hat{b}$ is the unit vector along the direction of the transverse magnetic field. With this spin notation in hand, we can now write down the result sketched in the Appendix for the expectation value of the square of the current given in Eq. (12):

$$\begin{aligned}\mathscr{L}_{(kl)} = {} & (1-4\sin^2\theta_W)^2 \tfrac{1}{2}[(\delta_{kl}-\hat{l}_k\hat{l}_l)(1-\vec{s}_\perp\cdot\underline{\vec{s}}_\perp - s_L\underline{s}_L) - s_{\perp k}\underline{s}_{\perp l} - \underline{s}_{\perp k}s_{\perp l}] \\ & + (1-4\sin^2\theta_W)[(\delta_{kl}-\hat{l}_k\hat{l}_l)(s_L-\underline{s}_L)] + \tfrac{1}{2}[(\delta_{kl}-\hat{l}_k\hat{l}_l)(1-\vec{s}_\perp\cdot\underline{\vec{s}}_\perp - s_L\underline{s}_L) + s_{\perp k}\underline{s}_{\perp l} + \underline{s}_{\perp k}s_{\perp l}] \ .\end{aligned} \tag{14}$$

Here we have written only the spatial components since $\mathscr{L}_{\mu\nu}$ vanishes if one of its indices is a time component (which follows from current conservation). Moreover, we have set down only the symmetrical part of $\mathscr{L}_{kl}$ since this is all we need to contract with the symmetrical H_{kl} in Eq. (7).

The energy-energy correlation cross section for the exchange of a virtual photon given in Eq. (2) corresponds to setting $\theta_W = 0$ and $\vec{s} = \hat{b} = -\underline{\vec{s}}$ in Eq. (14). Hence the energy-energy correlation cross section for Z^0 exchange, normalized by the diversion of the total cross section for Z^0 exchange, is obtained from Eq. (2) by making the replacement

$$\hat{b}_k\hat{b}_l \rightarrow \mathscr{N}^{-1}\mathscr{L}_{kl} \ . \tag{15}$$

Since $\hat{b}^2 = \hat{b}_k\hat{b}_k = 1$, the normalizing factor $\mathscr{N}$ is determined by

$$\begin{aligned}\mathscr{N} &= \mathscr{L}_{kk} \\ &= (1-4\sin^2\theta_W)^2(1-s_L\underline{s}_L) \\ &\quad + 2(1-4\sin^2\theta_W)(s_L-\underline{s}_L) + (1-s_L\underline{s}_L) \ .\end{aligned} \tag{16}$$

It is a straightforward matter to substitute Eq. (15) into Eq. (2) and thereby compute the general form for the energy-energy correlation at the Z^0. The result, however, is rather lengthy and cumbersome while in practice much simpler forms suffice. If one of the leptons has no transverse polarization we have

$$\frac{1}{\sigma}\frac{d^2\Sigma}{d\Omega\, d\Omega'} = \frac{3}{8\pi}\{\mathscr{A}(\chi)\tfrac{1}{2}[2+(\hat{l}\cdot\hat{r})^2+(\hat{l}\cdot\hat{r}')^2] + \mathscr{B}(\chi)\tfrac{1}{2}[\cos\chi + (\hat{l}\cdot\hat{r})(\hat{l}\cdot\hat{r}')] + \mathscr{C}(\chi)\} \ , \tag{17}$$

which does not depend upon the weak angle θ_W, and which is identical to the result for the cross section produced by virtual photon exchange with an unpolarized lepton. Even for general polarization a simpler form is adequate since $\sin^2\theta_W \simeq \frac{1}{4}$. Neglecting $(1-4\sin^2\theta_W)$ gives

$$\frac{1}{\sigma}\frac{d^2\Sigma}{d\Omega\, d\Omega'}=\frac{3}{8\pi}\Big(\mathscr{A}(\chi)\{1+\tfrac{1}{2}[(\hat{l}\cdot\hat{r})^2+(\hat{l}\cdot\hat{r}')^2](1-\vec{s}_\perp\cdot\underline{\vec{s}}_\perp-s_L\underline{s}_L) +\vec{s}_\perp\cdot\underline{\vec{s}}_\perp-(\vec{s}_\perp\cdot\hat{r})(\underline{\vec{s}}_\perp\cdot\hat{r})-(\vec{s}_\perp\cdot\hat{r}')(\underline{\vec{s}}_\perp\cdot\hat{r}')\} +\mathscr{B}(\chi)\tfrac{1}{2}[(\hat{l}\cdot\hat{r})(\hat{l}\cdot\hat{r}')(1-\vec{s}_\perp\cdot\underline{\vec{s}}_\perp-s_L\underline{s}_L) -(\vec{s}_\perp\cdot\hat{r})(\underline{\vec{s}}_\perp\cdot\hat{r}')-(\underline{\vec{s}}_\perp\cdot\hat{r})(\vec{s}_\perp\cdot\hat{r}')+\cos\chi(\vec{s}_\perp\cdot\underline{\vec{s}}_\perp+1)]+\mathscr{C}(\chi)\Big) . \tag{18}$$

This is identical with the cross section from virtual-photon exchange except that the sign of the transverse polarization of one of the leptons is reversed, say $\underline{\vec{s}}_\perp \to -\underline{\vec{s}}_\perp$. This sign reversal appears because with $\sin^2\theta_W=\frac{1}{4}$ the weak neutral current has only an axial-vector part. Inserting the values for $\mathscr{A}(\chi)$, $\mathscr{B}(\chi)$, and $\mathscr{C}(\chi)$ given by Eqs. (4) into Eq. (17) or Eq. (18) yields the first-order QCD prediction for the energy-energy correlation at the Z^0. The result is corrected for the hadronization process by adding to $\mathscr{A}(\chi)$ the form given in Eq. (6).

To summarize, the process $e^-e^+\to Z^0 \to$(hadronic energy correlations) measures the same structure functions as measured in $e^-e^+\to\gamma \to$(hadronic energy correlations). For practical purposes, the normalized cross sections are essentially identical. However, the hadronization background, which can only be estimated with a phenomenological model, is much smaller at the Z^0 than at $W=30$ GeV. Moreover, the large cross section at the Z^0 may enable the separation of the structure functions $\mathscr{A}(\chi)$, $\mathscr{B}(\chi)$, and $\mathscr{C}(\chi)$ which would provide a precise test of the fundamental theory of quantum chromodynamics.

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APPENDIX

The lepton tensor $\mathscr{L}_{\mu\nu}$ is formed from the square of the leptonic current given in Eq. (12),

$$\mathscr{L}_{\mu\nu}=\frac{1}{W^2}\sum_{\lambda\underline{\lambda}\lambda'\underline{\lambda}'}\bar{v}_{\underline{\lambda}}(\underline{p})\gamma_\mu[(1-4\sin^2\theta_W)+i\gamma_5]u_\lambda(p)\,\rho_{\lambda\lambda'}\underline{\rho}_{\underline{\lambda}\underline{\lambda}'}\bar{u}_{\lambda'}(p)\gamma_\nu[(1-4\sin^2\theta_W)+i\gamma_5]v_{\underline{\lambda}'}(\underline{p}) . \tag{A1}$$

Here $v_{\underline{\lambda}}(\underline{p})$ is the antiparticle spinor for the positron of spin $\underline{\lambda}$ and momentum $\underline{p}$, $u_\lambda(p)$ is the corresponding spinor for the electron, and $\rho_{\lambda\lambda'}$, $\underline{\rho}_{\underline{\lambda}\underline{\lambda}'}$ are the spin density matrices for the electron and positron. In the nonrelativistic limit, these density matrices are $\frac{1}{2}(1+\vec{P}\cdot\vec{\sigma})$ and $\frac{1}{2}(1-\underline{\vec{P}}\cdot\vec{\sigma})$ with $P^2\le 1$ and $\underline{P}^2\le 1$. The minus sign for the positron enters because its spinors appear in a transposed order. Using $\vec{\sigma}=\vec{\gamma}i\gamma_5\gamma^0$ with $\gamma^{0'}=+1$ ($\gamma^{0'}=-1$) for an electron (positron) at rest, it is easy to see that we have the relativistic forms

$$\sum_{\lambda\lambda'}u_\lambda(p)\rho_{\lambda\lambda'}\bar{u}_{\lambda'}(p)=\tfrac{1}{2}[1+\gamma^\mu P_\mu i\gamma_5](m-\gamma p) \tag{A2}$$

and

$$\sum_{\underline{\lambda}\underline{\lambda}'}v_{\underline{\lambda}'}(\underline{p})\underline{\rho}_{\underline{\lambda}\underline{\lambda}'}\bar{v}_{\underline{\lambda}}(\underline{p}) =-\tfrac{1}{2}[1+\gamma^\mu\underline{P}_\mu i\gamma_5](m+\gamma\underline{p}) , \tag{A3}$$

where

$$P^\mu P_\mu\le 1,\quad p_\mu P^\mu=0 \tag{A4}$$

and

$$\underline{P}^\mu\underline{P}_\mu\le 1,\quad \underline{p}_\mu\underline{P}^\mu=0 . \tag{A5}$$

Inserting Eqs. (A2) and (A3) into Eq. (A1) produces a trace over the Dirac matrices which is straightforward to compute. We are, however, interested in the high-energy limit where, effectively, $-p^2=-\underline{p}^2=m^2\to 0$. In this limit the longitudinal and time components of the polarization fourvectors may be badly behaved. To rectify this we write for the electron

$$\vec{P}=\vec{s}_\perp+\hat{p}\frac{p^0}{m}s_L\ , \tag{A6a}$$

where $\vec{s}_\perp$ are the transverse polarization components perpendicular to the electron's spatial momentum $\vec{p}$ and s_L is a renormalized longitudinal polarization. With this decomposition, the orthogonality condition $p_\mu P^\mu=0$ gives

$$s^0=\frac{|\vec{p}|}{m}s_L\ . \tag{A6b}$$

The constraint that $P^\mu P_\mu \leq 1$ now yields

$$\vec{s}_\perp{}^2+s_L{}^2\leq 1\ . \tag{A7}$$

Similarly for the positron we write

$$\underline{\vec{P}}=\underline{\vec{s}}_\perp+\underline{\hat{p}}\frac{p^0}{m}\underline{s}_L \tag{A8a}$$

with

$$\underline{s}^0=\frac{|\underline{\vec{p}}|}{m}\underline{s}_L \tag{A8b}$$

and

$$\underline{\vec{s}}_\perp{}^2+\underline{s}_L{}^2\leq 1\ . \tag{A9}$$

These are the polarization vectors which are employed in the text.

With the redefinitions (A6) and (A8) of the polarization vectors we may take the limit $m\to 0$ and secure

$$\sum_{\lambda\lambda'} u_\lambda(p)\rho_{\lambda\lambda'}\bar{u}_{\lambda'}(p) =-\tfrac{1}{2}[1+\vec{\gamma}\cdot\vec{s}_\perp i\gamma_5+s_L i\gamma_5]\gamma p \tag{A10}$$

and

$$\sum_{\underline{\lambda\lambda'}} v_{\underline{\lambda}'}(\underline{p})\rho_{\underline{\lambda}\underline{\lambda}'}\bar{v}_{\underline{\lambda}}(\underline{p}) =-\tfrac{1}{2}[1+\vec{\gamma}\cdot\underline{\vec{s}}_\perp i\gamma_5-\underline{s}_L i\gamma_5]\gamma \underline{p}\ . \tag{A11}$$

These limits simplify the trace and yield the result quoted in Eq. (14) of the text.

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