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Twist-4 effects in electroproduction: Canonical operators and coefficient functions

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The interpretation of observed scaling violations in leptoproduction is complicated by the possible presence of significant higher-twist effects. We refine the machinery of the operator-product expansion sufficiently for a study of twist-4 effects. In particular, we introduce and review the advantages of a special, "canonical" basis. We demonstrate that the canonical basis is adequate for the necessary twist-4 perturbative calculations, and calculate the operators' tree-level coefficient functions in electroproduction. Our results establish a framework within which careful analysis of more accurate data can provide information regarding correlations among the constituents of the proton.

I. INTRODUCTION

There is a growing realization that the scaling violations seen in leptoproduction at low and even moderately large Q^2 may be due in part to higher-twist effects.[1] This poses a difficult problem for theorists because the importance of higher-twist effects is affected by nonperturbative features of the hadron bound state for which we have no entirely satisfactory model. Also the parton-model intuition applicable to leading twist has not been extended to twist 4 or beyond. In two previous publications[2,3] we have suggested and outlined a program for the evaluation of twist-4 effects in leptoproduction based on the operator-product expansion (OPE).[4] The OPE is particularly attractive because it allows a separation of the perturbative and nonperturbative pieces of the calculation and organizes the perturbative piece in a manageable, if somewhat formal, fashion.

In earlier papers[2,3] we described at some length the physical foundations of our program, outlined the steps necessary to extend the classic twist-2 analysis[5] to twist 4, and combined our perturbative results with nonperturbative information from the bag model to calculate the twist-4 corrections to the lowest nonsinglet moment of the electroproduction structure function F_2. In the course of that work it became clear to us that it is necessary to demonstrate the adequacy of a special "canonical" operator basis for twist-4 perturbative calculations. Furthermore, the canonical operators' coefficient functions are the critical link connecting experiment with the sort of bound-state information available in simple relativistic quark models.

Therefore, we will describe in detail in this paper the construction of the twist-4, spin-n canonical operator basis and the calculation of the operators' coefficient functions in tree approximation in the OPE of two electromagnetic currents.[6] Some of the results are not new but have been included for completeness. Discussions of the other elements in an OPE analysis of twist-4 effects, namely of anomalous dimensions[7] and matrix elements, will be given elsewhere. With these important applications in mind we have taken pains to cast our results in terms convenient for both such calculations.

The remainder of the paper is organized as follows: In Sec. II we define a canonical basis of twist-4 operators and list those members which occur naturally in the calculation of the coefficient functions in tree level. We also show that a consistent treatment of the renormalization-group equations requires us to keep all tree-level coefficient functions. Section III contains the most important details of the coefficient-function calculations. It is divided into a number of subsections which treat the contributions from Figs. 1 and 3. The major results are Eqs. (1), (2), and (46). In Sec. IV we present our conclusions. The more technical details of the calculation and related issues are presented in a series of appendices. In Appendix A we list the elements of the complete canonical basis. In Appendix B we present the calculation leading to Eq. (2). We prove in Appendix C that the spin-averaged single-particle forward matrix elements of any traceless, "not-totally-symmetric" operator vanish. Appendices D, F, and G contain many of the technical details in the derivation of Eq. (46). In Appendix E we present algorithms to relate any twist-4 totally symmetric noncanonical operator to twist-4 canonical operators.

II. OPERATOR BASIS

We have argued, building on the work of Politzer,[10] that the current product should be expanded in a basis composed of operators which (a) are totally symmetric, (b) are traceless (in all index pairs), and (c) have no contracted derivatives. We will call such operators "canonical." The advantages of this basis have been described at length in Refs. 3 and 10. In brief, the set of all twist-4 operators is an overcomplete set, because the single-particle forward matrix elements of operators which vanish by the naive operator equations of motion are zero.[11] The set of canonical operators is complete, and better tailored to the bound-state problem for two closely related reasons. First, the matrix elements of canonical operators are interpretable as generalized longitudinal-momentum distributions in an infinite-momentum frame; and second, in practice the evaluation of such matrix elements in a model quantized on the light-cone makes use of the model wave functions only, and not the model Hamiltonian. The complete set of twist-4 canonical operators is given in Appendix A.

Not all of the operators listed there actually appear in the expansion of the current product at tree level. The ones which arise naturally in the calculation are listed below with the appropriate flavor structure. For convenience an arbitrary lightlike four-vector Δ^μ has been contracted with all free indices. Terms necessary to make an operator traceless are proportional to $g_{\alpha\beta}$. Consequently, they vanish when contracted with Δ. It should be remembered that the completely symmetrized operators should be traceless as well. Define $d \equiv i\Delta\cdot D$, $f^\beta \equiv F^{\rho\beta}\Delta_\rho$, and ${}^*f^\beta \equiv {}^*F^{\rho\beta}\Delta_\rho = (\epsilon^{\rho\beta\sigma\tau}F_{\sigma\tau})\Delta_\rho$. We use the abbreviated notation $\Delta\cdot O_n \equiv \Delta^{\mu_1}\cdots\Delta^{\mu_n}O_{n,\mu_1\cdots\mu_n}$. Also $F^{\mu\nu}=\tau_a F_a^{\mu\nu}$, where τ_a is a generator of SU(3) color normalized to $\mathrm{tr}(\tau_a\tau_b)=\frac{1}{2}\delta_{ab}$, and $D^\mu=\partial^\mu+ig\tau_a A_a^\mu$. Q is the charge matrix of the up, down, and strange quarks. The relevant operators are

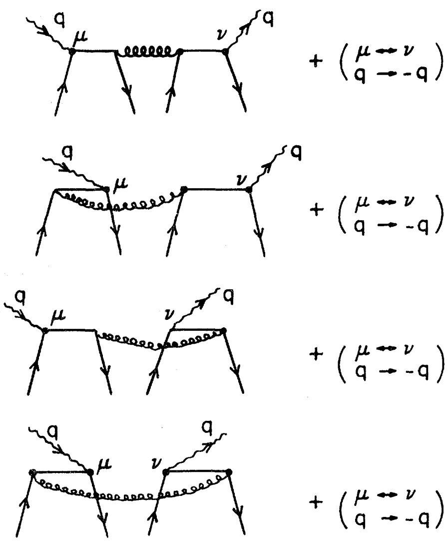

FIG. 1. The tree graphs required for the calculation of $Y^{\mu\nu}_{T=4}$.

$$\Delta\cdot O_n^{1(k,l)} = g(\bar\psi Q\overleftarrow{d}^{\,l}\overrightarrow{d}^{\,k}\not{\Delta}\tau_a\psi)(\bar\psi Q\overrightarrow{d}^{\,n-2-k-l}\not{\Delta}\tau_a\psi)\ ,$$

$$\Delta\cdot O_n^{2(k,l)} = g(\bar\psi Q\overleftarrow{d}^{\,l}\overrightarrow{d}^{\,k}\not{\Delta}\gamma_5\tau_a\psi)(\bar\psi Q\overrightarrow{d}^{\,n-2-k-l}\not{\Delta}\gamma_5\tau_a\psi)\ ,$$

$$\Delta\cdot O_n^{3(k)} = \bar\psi d^{k}\,{}^*\!\not{f}\gamma_5 d^{n-1-k}Q^2\psi\ ,$$

$$\Delta\cdot O_n^{4(k)} = i\bar\psi d^{k}\not{f}d^{n-1-k}Q^2\psi\ ,$$

$$\Delta\cdot O_n^{5(k,l)} = g\bar\psi d^{k}f^{\beta}d^{l}f_{\beta}d^{n-3-k-l}\not{\Delta}Q^2\psi\ ,$$

$$\Delta\cdot O_n^{6(k,l)} = ig\epsilon^{\alpha\beta\delta\lambda}\Delta_\delta\bar\psi d^{k}f_{\alpha}d^{l}f_{\beta}d^{n-3-k-l}\gamma_\lambda\gamma_5 Q^2\psi\ ,$$

$$\Delta\cdot O_n^{7(k)} = g(\bar\psi Q^2\overleftarrow{d}^{\,k}\overrightarrow{d}^{\,n-2-k}\not{\Delta}\tau_a\psi)(\bar\psi\not{\Delta}\tau_a\psi)\ .$$

In operators 3, 4, 5, and 6 the derivatives act on all fields to the right. Operator 7 differs from a special case of operator 1 only in its flavor structure. Here the subscript n is the actual spin of the operator.

Before proceeding to the calculation of the coefficient functions, it is important to address a problem involving the consistent treatment of orders in perturbation theory.[2,12] In an abbreviated notation, collectively call $O_1'=(\bar\psi\psi)^2$ and $O_2=\bar\psi F\psi$ (not including $\bar\psi FF\psi$ operators). The coefficient function $C_1'(q^2/\mu^2,g)$, of O_1', is given by graphs such as Fig. 1. It is order g^2, while $C_2(q^2/\mu^2,g)$ is order g. One is tempted to ignore $C_1'(1,\bar g)$ relative to $C_2(1,\bar g)$ because $\bar g$ is small at large Q^2. This procedure is not correct however. The standard renormalization-group analysis yields a result of the form

$$\langle p\,|\,O_i\,|\,p\,\rangle_{(\mu^2)}\left[T\exp\left[-\int_{\bar g(\mu^2)}^{\bar g(Q^2)}dg'\frac{\bar\gamma(g')}{\beta(g')}\right]\right]_{ij}C_j(1,\bar g(Q^2))\ .$$

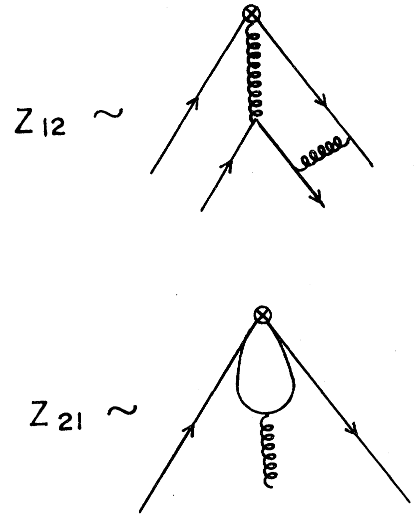

FIG. 2. Typical graphs relevant to the operator-mixing problem as described in Sec. II.

To leading order the coefficient functions $C_j(1,\bar{g})$ are of the form $C_j(1,\bar{g})=\bar{g}^{n_j}c_j$ where c_j are calculable constants. In general the leading q^2 dependence will be a logarithm raised to a calculable power determined in part by the C_j's and in part by the exponential of the anomalous dimension matrix, and will multiply a linear combination of coefficients c_j and a linear combination of operator matrix elements $\langle p\,|\,O_i\,|\,p\,\rangle_{(\mu^2)}$. At first glance c_1 will not appear in this linear combination. However, a closer look at the operator mixing problem shows both that the $(\bar{\psi}\psi)^2$ operators cannot be ignored in the calculation of the anomalous dimension matrix and that the coefficient c_1 cannot be ignored under the circumstances at hand. The renormalization constant Z_{12} is order g^3, and Z_{21} is order g, as can be seen from inspection of the relevant diagrams shown in Fig. 2. A direct calculation of the dominant logarithm is made difficult by the complexity of

$$T\exp\left[-\int_{\bar{g}(\mu^2)}^{\bar{g}(Q^2)}\frac{\tilde{\gamma}'(g')}{\beta(g')}dg'\right],$$

because of the form of

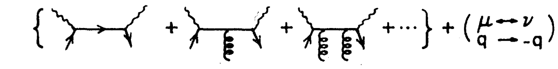

FIG. 3. The tree graphs required for the calculation of $X^{\mu\nu}_{T=4}$.

$$\tilde{\gamma}'\sim\begin{bmatrix} g^2\gamma^{0'}_{11} & g^3\gamma^{0}_{12}\\ g\gamma^{0}_{21} & g^2\gamma^{0}_{22}\end{bmatrix}.$$

The canonical form of the mixing matrix (i.e., proportional to g^2 in lowest order) can be regained by a redefinition of the four-quark operators $O_1=g(\bar{\psi}\psi)^2$. Of course, the dominant logarithm itself is independent of such conventions. Only the path to its calculation is made clearer. Afterward,

$$\tilde{\gamma}\sim g^2\begin{bmatrix} \gamma^{0}_{11} & \gamma^{0}_{12}\\ \gamma^{0}_{21} & \gamma_{22}\end{bmatrix}\quad \text{where } \gamma^{0}_{11}=\gamma^{0'}_{11}+\beta_0$$

and $C_1(q^2/\mu^2,g)$ is order g. It is clear then that the weighting of the dominant logarithm will involve both c_1 and c_2. Incidentally, the matrix element of O_1 is suppressed by a factor of $g(\mu^2)$ relative to that of O_2. This factor would be significant if the renormalization scale μ^2 were large. However, in the scheme we have outlined in Ref. 3, μ^2 is around 1 GeV^2, and the $(\bar{\psi}\psi)^2$ matrix elements should be kept. It is evident that the same general considerations apply to the $\bar{\psi}FF\psi$ operators as to the $(\bar{\psi}\psi)^2$ operators.

III. COEFFICIENT FUNCTIONS

The contributions at tree level to the coefficient functions naturally divide into those arising from the graphs of Figs. 1 and 3. We write

$$-i\int d^4x\, e^{iq\cdot x}T[J_\mu(x)J_\nu(0)]=X_{\mu\nu}+Y_{\mu\nu}\,, \qquad (1)$$

where $X_{\mu\nu}$ ($Y_{\mu\nu}$) is the contribution from the graphs of Fig. 3 (Fig. 1). Let the twist-4 pieces of $X_{\mu\nu}$ and $Y_{\mu\nu}$ be denoted by $X^{T=4}_{\mu\nu}$ and $Y^{T=4}_{\mu\nu}$.

A. Calculation of $Y^{T=4}_{\mu\nu}$

A calculation of $Y^{T=4}_{\mu\nu}$ has been given in Ref. 12. We repeat the calculation briefly in Appendix B in order to express the coefficient functions in a form more suitable for our purposes. The result is

$$Y_{\mu\nu}^{T=4}=-\frac{g}{q^6}\sum_{\substack{n=2\\ \text{(even)}}}^{\infty}\left[\frac{2}{q^2}\right]^{n-2}T_{\mu\nu}^{\mu_1\mu_2}q^{\mu_3}\cdots q^{\mu_n}$$

$$\times\sum_{k=0}^{n-2}\sum_{l=0}^{n-2-k}\left\{O_{n,\mu_1\cdots\mu_n}^{1(k,l)}\left[\frac{n!}{k!l!(n-1-k-l)!}\left[\frac{1}{n-k}-\frac{1}{n-l}\right]\right.\right.$$

$$\left.+(-)^{k+l}\frac{(l+k+1)!}{l!k!}\left[\frac{1}{k+1}-\frac{1}{l+1}\right]\right]$$

$$+O_{n,\mu_1\cdots\mu_n}^{2(k,l)}\left[\frac{n!}{k!l!(n-1-k-l)!}\left[\frac{1}{n-k}+\frac{1}{n-l}\right]\right.$$

$$\left.\left.+(-)^{k+l}\frac{(l+k+1)!}{k!l!}\left[\frac{1}{k+1}+\frac{1}{l+1}\right]\right]\right\}, \tag{2}$$

where

$$T_{\mu\nu}^{\mu_1\mu_2}=q^2g_\mu^{\mu_1}g_\nu^{\mu_2}-(g_\mu^{\mu_1}q_\nu+g_\nu^{\mu_1}q_\mu)q^{\mu_2}+g_{\mu\nu}q^{\mu_1}q^{\mu_2}\,. \tag{3}$$

In anticipation of the day that the operator mixing problem can be handled properly, we give the flavor-SU(3) Clebsch-Gordan decomposition of $\bar\psi Q\psi\bar\psi Q\psi$ (Ref. 13) in terms of the isospins 1 and 2, $I_z=0$, $Y=0$ operators:

$$\bar\psi Q\psi\bar\psi Q\psi=O_{I=2}^{27}+O_{I=1}^{27}+O_{I=1}^{8_s}+\text{isospin singlets}\,, \tag{4}$$

where

$$O_{I=2}^{27}=\tfrac{1}{6}[(\bar uu\bar uu+\bar dd\bar dd)-(\bar du\bar ud+\bar ud\bar du+\bar uu\bar dd+\bar dd\bar uu)]\,, \tag{5}$$

$$O_{I=1}^{27}=\tfrac{1}{10}[(\bar uu\bar uu-\bar dd\bar dd)+(\bar ds\bar sd+\bar sd\bar ds+\bar dd\bar ss+\bar ss\bar dd)-(\bar us\bar su+\bar su\bar us+\bar uu\bar ss+\bar ss\bar uu)]\,, \tag{6}$$

$$O_{I=1}^{8_s}=\tfrac{1}{15}\{(\bar uu\bar uu-\bar dd\bar dd)+[-\tfrac{3}{2}(\bar ds\bar sd+\bar sd\bar ds)+(\bar dd\bar ss+\bar ss\bar dd)]$$

$$-[-\tfrac{3}{2}(\bar us\bar su+\bar su\bar us)+(\bar uu\bar ss+\bar ss\bar uu)]\}\,. \tag{7}$$

Later we will need the decomposition of $\bar\psi Q^2\psi\bar\psi\psi=\tilde O_{I=1}^{8}+$ isospin singlets. Here

$$\tilde O_{I=1}^{8}=\tfrac{1}{6}(\bar uu-\bar dd)(\bar uu+\bar dd+\bar ss)\,. \tag{8}$$

B. Calculation of $X_{T=4}^{\mu\nu}$

The calculation of $X_{T=4}^{\mu\nu}$ requires more effort. From Fig. 3 one establishes the operator identification[14]

$$X^{\mu\nu}=\bar\psi\gamma^\mu\frac{1}{\not q+\not\Pi}\gamma^\nu\psi+\left[\begin{matrix}\mu\leftrightarrow\nu\\ q\to-q\end{matrix}\right]. \tag{9}$$

Here $\Pi^\mu=iD^\mu$. For convenience the quarks fields will be omitted hereafter. We will often recall their presence and use the quark equation of motion $\not\Pi\psi=0$ as necessary. To maintain manifest charge-conjugation symmetry, we write

$$X^{\mu\nu}=\tfrac{1}{2}\left\{[\gamma^\mu(\not q+\not\Pi)\Lambda\gamma^\nu+\gamma^\mu\Lambda(\not q+\not\Pi)\gamma^\nu]+\left[\begin{matrix}\mu\leftrightarrow\nu\\ q\to-q\end{matrix}\right]\right\}, \tag{10}$$

where $\Lambda=(q^2+2q\cdot\Pi+\not\Pi^2)^{-1}$. The expansion of Λ as

$$\Lambda=\frac{1}{q^2}\left[\Lambda_0-\Lambda_0\frac{\not{\Pi}^2}{q^2}\Lambda_0+\cdots\right] \tag{11}$$

with $\Lambda_0=(1+2q\cdot\Pi/q^2)^{-1}$ leads rather directly to an expansion in twist. It can be shown that all twist-2 operators are contained in the first term and all twist-4 operators are in the first two terms. Combining Eqs. (10) and (11), one can rewrite the relevant piece of $X^{\mu\nu}$, defined to be $\tilde{X}^{\mu\nu}$, as

$$\tilde{X}^{\mu\nu}=\frac{1}{2q^2}\left\{2\gamma^\mu\not{q}\gamma^\nu\Lambda_0+(\gamma^\mu\not{\Pi}\gamma^\nu\Lambda_0+\Lambda_0\gamma^\mu\not{\Pi}\gamma^\nu)\right.$$
$$\left.-\frac{1}{q^2}[\Lambda_0\gamma^\mu(\not{q}\not{\Pi}^2+\not{\Pi}^2\not{q})\gamma^\nu\Lambda_0+(\gamma^\mu\not{\Pi}\Lambda_0\not{\Pi}^2\Lambda_0\gamma^\nu+\gamma^\mu\Lambda_0\not{\Pi}^2\Lambda_0\not{\Pi}\gamma^\nu)]\right\}+\begin{bmatrix}\mu\leftrightarrow\nu\\ q\to-q\end{bmatrix}. \tag{12}$$

It is advantageous to simplify the products of Dirac matrices as far as possible without introducing F's. To this end we use the identity

$$\gamma^\mu(\not{q}\not{\Pi}\not{\Pi}+\not{\Pi}\not{\Pi}\not{q})\gamma^\nu=A^{\mu\nu}+B^{\mu\nu}+C^{\mu\nu}, \tag{13}$$

where

$$B^{\mu\nu}=2i[-(\epsilon^{\alpha\mu\beta\lambda}\Pi_\alpha q_\beta\gamma_\lambda\gamma_5)\Pi^\nu+\Pi^\nu(\epsilon^{\alpha\mu\beta\lambda}\Pi_\alpha q_\beta\gamma_\lambda\gamma_5)]+(\mu\leftrightarrow\nu), \tag{14}$$

$$C^{\mu\nu}=2\{-g^{\mu\nu}[(q\cdot\Pi)\not{\Pi}+\not{\Pi}(q\cdot\Pi)]-\not{\Pi}(q^\mu\gamma^\nu+q^\nu\gamma^\mu-g^{\mu\nu}\not{q})\not{\Pi}+q^\mu(\not{\Pi}\Pi^\nu+\Pi^\nu\not{\Pi})+q^\nu(\not{\Pi}\Pi^\mu+\Pi^\mu\not{\Pi})\}, \tag{15}$$

and $A^{\mu\nu}$ are terms antisymmetric in μ and ν, whose effects vanish when averaged over target spin. After use of the quark equation of motion (dropping further spin-dependent terms),

$$\tilde{X}^{\mu\nu}=X_{\rm I}^{\mu\nu}+X_{\rm II}^{\mu\nu}+X_{\rm III}^{\mu\nu}+X_{\rm IV}^{\mu\nu}, \tag{16}$$

where

$$X_{\rm I}^{\mu\nu}=-\frac{1}{2q^2}\{2(g^{\mu\nu}\not{q}-q^\mu\gamma^\nu-q^\nu\gamma^\mu)\Lambda_0-[(\Pi^\mu\gamma^\nu+\Pi^\nu\gamma^\mu)\Lambda_0+\Lambda_0(\Pi^\mu\gamma^\nu+\Pi^\nu\gamma^\mu)]\}+\begin{bmatrix}\mu\leftrightarrow\nu\\ q\to-q\end{bmatrix}, \tag{17}$$

$$X_{\rm II}^{\mu\nu}=-\frac{1}{2q^4}\Lambda_0B^{\mu\nu}\Lambda_0+\begin{bmatrix}\mu\leftrightarrow\nu\\ q\to-q\end{bmatrix}, \tag{18}$$

$$X_{\rm III}^{\mu\nu}=-\frac{1}{2q^4}\Lambda_0C^{\mu\nu}\Lambda_0+\begin{bmatrix}\mu\leftrightarrow\nu\\ q\to-q\end{bmatrix}, \tag{19}$$

$$X_{\rm IV}^{\mu\nu}=-\frac{1}{q^4}(\Pi^\mu\Lambda_0\not{\Pi}^2\Lambda_0\gamma^\nu+\gamma^\mu\Lambda_0\not{\Pi}^2\Lambda_0\Pi^\nu)+\begin{bmatrix}\mu\leftrightarrow\nu\\ q\to-q\end{bmatrix}. \tag{20}$$

Before proceeding with the analysis of $X_{T=4}^{\mu\nu}$, several general comments are in order. The first concerns certain operators which have appeared and which cannot be symmetrized in all indices. One such not-totally-symmetric operator can be extracted directly from $X_{\rm II}^{\mu\nu}$:

$$\Pi^{\mu_1}\cdots\Pi^{\mu_i}[\Pi^{\mu_{i+1}}\epsilon^{\beta\mu_{i+2}\mu_{i+3}\lambda}\Pi_\beta\gamma_\lambda\gamma_5-\epsilon^{\beta\mu_{i+1}\mu_{i+2}\lambda}\Pi_\beta\gamma_\lambda\gamma_5\Pi^{\mu_{i+3}}]\Pi^{\mu_{i+4}}\cdots\Pi^{\mu_n}.$$

It appears to have a sufficient number of free indices to contribute as a twist-2 operator; however, restrictive symmetry conditions forbid the single-particle forward matrix element from containing a term proportional to $p^{\mu_1}\cdots p^{\mu_n}$. The operator will lead to twist-4 effects at most, though its twist is not manifest. Similar operators arise out of $X_{\rm I}^{\mu\nu}$ after projecting out the totally symmetrized twist-2 operator. In evaluating the matrix elements of not-totally-symmetric operators, one cannot assign the value $+$ to all free indices. Consequently, such matrix elements lack the physical interpretation of generalized longitudinal-momentum distributions in an infinite-momentum frame. Perhaps more importantly, the calculation of these matrix elements in a model quantized on the light-cone makes unnecessary use of the model's transverse-momentum distributions or Hamiltonian, typically less reliable features than its longitudinal-momentum distributions. The problem is avoidable because *traceless* not-totally-symmetric operators do not contribute in leptoproduction. In Appendix C it is proven that their spin-averaged single-particle forward matrix elements vanish. As a result, any not-totally-symmetric operator can be replaced by the terms which must be subtracted from it to render it traceless. We will refer to these somewhat loosely as "its traces." The traces of a not-totally-symmetric operator can always be expressed in terms of totally symmetric operators of lower spin. A simple example can be found at the end of Appendix C.

In practice the structure of $X^{\mu\nu}$ in the external indices μ and ν is often made clearer if one imagines taking forward matrix elements. Neglecting terms of twist 6 or higher the matrix element of a totally symmetric twist-4 operator is

$$\langle p \mid O_{T=4}^{\mu_1 \cdots \mu_n} \mid p \rangle = A_n p^{\mu_1} \cdots p^{\mu_n} .$$

To the same order one can make a substitution such as

$$g^{\mu}_{\mu_1} q_{\mu_2} \cdots q_{\mu_n} O_{T=4}^{\mu_1 \cdots \mu_n} = \frac{p^{\mu}}{p \cdot q} q_{\mu_1} \cdots q_{\mu_n} O_{T=4}^{\mu_1 \cdots \mu_n} . \quad (21)$$

As is well known, there are two gauge-invariant tensors which can be constructed out of $p^\mu p^\nu$, $p^\mu q^\nu + p^\nu q^\mu$, $g^{\mu\nu}$, and $q^\mu q^\nu$. Using substitutions such as Eq. (21), we can identify the coefficients of these different tensors directly in the OPE. To save labor we have chosen to drop twist-4 $p^\mu p^\nu$ terms. At the end of the calculation we will be able to reconstruct the full gauge-invariant form, with one highly nontrival check on the algebra.

It is now possible to give an outline of the procedure that we have followed in our computation of $X^{\mu\nu}_{T=4}$. As stated before all twist-2 pieces lie in $X^{\mu\nu}_{\mathrm{I}}$. We write $X^{\mu\nu}_{\mathrm{I}}$ in terms of the totally symmetrized twist-2 operators and not-totally-symmetric operators of the sort discussed above. Care must be taken to construct traceless twist-2 operators. Of course, the twist-2 result is well known and the twist-2 operator can be dropped. In light of our discussion of not-totally-symmetric operators, it is clear that for our purposes $X^{\mu\nu}_{\mathrm{I}}$ can be replaced by its traces. The same holds for $X^{\mu\nu}_{\mathrm{II}}$, as it contains only not-totally-symmetric operators. $X^{\mu\nu}_{\mathrm{III}}$ can be put in canonical form by commuting the factors of Π to the appropriate ends and using $\not{\Pi}\psi = 0$. Finally, since $X^{\mu\nu}_{\mathrm{IV}}$ produces only $p^\mu p^\nu$ twist-4 terms, it can be ignored from now on.

1. Traces of $X^{\mu\nu}_{\mathrm{I}}$

Expanding Λ_0 in Eq. (17) gives

$$X^{\mu\nu}_{\mathrm{I}} = -\frac{1}{q^2}\left[g^{\mu\nu} \sum_{\substack{n=1 \\ (\mathrm{odd})}}^{\infty} \left[\frac{-2}{q^2}\right]^n q^\alpha q^{\mu_1} \cdots q^{\mu_n} K^{(n-1)}_{\mu_1 \cdots \mu_n;\alpha} - 2q^\mu \sum_{\substack{n=1 \\ (\mathrm{odd})}}^{\infty} \left[\frac{-2}{q^2}\right]^n g^{\nu\alpha} q^{\mu_1} \cdots q^{\mu_n} K^{(n-1)}_{\mu_1 \cdots \mu_n;\alpha} - 2 g^{\mu\alpha} g^{\nu\beta} \sum_{\substack{n=0 \\ (\mathrm{even})}}^{\infty} \left[\frac{-2}{q^2}\right]^n q^{\mu_1} \cdots q^{\mu_n} H^{(n)}_{\mu_1 \cdots \mu_n;\alpha\beta} \right] + (\mu \leftrightarrow \nu) , \quad (22)$$

where

$$K^{(n-1)}_{\mu_1 \cdots \mu_n;\alpha} = \gamma_\alpha S[\Pi_{\mu_1} \cdots \Pi_{\mu_n}] , \quad (23)$$

$$H^{(n)}_{\mu_1 \cdots \mu_n;\alpha\beta} = \tfrac{1}{4}\{ \gamma_\alpha \Pi_\beta S[\Pi_{\mu_1} \cdots \Pi_{\mu_n}] + S[\Pi_{\mu_1} \cdots \Pi_{\mu_n}] \gamma_\alpha \Pi_\beta \} + (\alpha \leftrightarrow \beta) . \quad (24)$$

As discussed above, both K and H are to be replaced by their traces when dropping the well-known twist-2 term. Keeping only twist-4 pieces,

$$q^\alpha q^{\mu_1} \cdots q^{\mu_n} K^{(n-1)}_{\mu_1 \cdots \mu_n;\alpha} \Rightarrow \frac{q^2}{2(n+1)^2}\left[\frac{n}{2}(n^2-1) q \cdot K^{(n-1)\rho;}_{\rho} + n(n+1) q \cdot K^{(n-1);\rho}_{\rho} \right] \quad (25)$$

and

$$q^{\mu}g^{\nu\alpha}q^{\mu_1}\cdots q^{\mu_n}K^{(n-1)}_{\mu_1\cdots\mu_n;\alpha}\Longrightarrow q^2\left[\frac{q^{\mu}p^{\nu}}{p\cdot q}\right]\frac{n(n-1)}{2(n+1)^2}\left[\left[\frac{n+3}{2}\right]q\cdot K_{\rho}^{(n-1)\rho;}-q\cdot K_{\rho}^{(n-1);\rho}\right]$$
$$+q^{\mu}q^{\nu}\frac{n}{(n+1)^2}\left[-\frac{(n-1)}{2}q\cdot K_{\rho}^{(n-1)\rho;}+nq\cdot K_{\rho}^{(n-1);\rho}\right] \tag{26}$$

by Eq. (D3). The notation used here is established in Appendix D. The arrow in Eqs. (25) and (26) denotes the procedure of replacing the operator by its traces. Similarly, by Eq. (D5)

$$g^{\mu\alpha}g^{\nu\beta}q^{\mu_1}\cdots q^{\mu_n}H^{(n)}_{\mu_1\cdots\mu_n;\alpha\beta}\Longrightarrow g^{\mu\nu}\frac{n}{2(n+2)^2}[-2nq\cdot H_{\rho}^{(n);\rho}+(n-1)q\cdot H_{\rho}^{(n)\rho;}]$$
$$+\left\{\left[\frac{q^{\mu}p^{\nu}}{p\cdot q}\right]\frac{n}{(n+2)^2}[2nq\cdot H_{\rho}^{(n);\rho}-(n-1)q\cdot H_{\rho}^{(n)\rho;}]+(\mu\leftrightarrow\nu)\right\}. \tag{27}$$

The explicit forms for the traces of H and K are

$$\Delta\cdot K_{\rho}^{(n);\rho}=\frac{1}{(n+1)}\sum_{k=0}^{n}d^k\not{\Pi}d^{n-k}, \tag{28}$$

$$\Delta\cdot K_{\rho}^{(n)\rho;}=\frac{2}{n(n+1)}\not{\Delta}\sum_{k=0}^{n-1}\sum_{l=0}^{k}d^l\Pi_{\beta}d^{k-l}\Pi^{\beta}d^{n-1-k}, \tag{29}$$

$$\Delta\cdot H_{\rho}^{(n);\rho}=\frac{1}{4n}\left\{2\sum_{k=0}^{n}d^k\not{\Pi}d^{n-k}+\not{\Delta}\left[\sum_{k=0}^{n-1}(\Pi_{\beta}d^k\Pi^{\beta}d^{n-1-k}+d^k\Pi_{\beta}d^{n-1-k}\Pi^{\beta})\right]\right\}, \tag{30}$$

$$\Delta\cdot H_{\rho}^{(n)\rho;}=\frac{1}{n(n-1)}\not{\Delta}\left[2\sum_{k=0}^{n-1}\sum_{l=0}^{k}d^l\Pi_{\beta}d^{k-l}\Pi^{\beta}d^{n-1-k}-\sum_{k=0}^{n-1}(\Pi_{\beta}d^k\Pi^{\beta}d^{n-1-k}+d^k\Pi_{\beta}d^{n-1-k}\Pi^{\beta})\right], \tag{31}$$

where $d=\Delta\cdot\Pi$ and all Π's act to the right. The factors involving n arise from the partial cancellation of factorials used in symmetrization. Only three of the traces are independent. We choose to eliminate $\Delta\cdot H_{\rho}^{(n)\rho;}$:

$$\Delta\cdot H_{\rho}^{(n)\rho;}=\frac{n+1}{n-1}\Delta\cdot K_{\rho}^{(n)\rho;}-\frac{4}{n-1}\Delta\cdot H_{\rho}^{(n);\rho}+\frac{2(n+1)}{n(n-1)}\Delta\cdot K_{\rho}^{(n);\rho}. \tag{32}$$

Combining various results gives the form for $X_1^{\mu\nu}$,

$$X_1^{\mu\nu}=\sum_{\substack{n=2\\(\text{even})}}^{\infty}\left[\frac{-2}{q^2}\right]^{n+1}\frac{1}{(n+2)^2}\left\{-g^{\mu\nu}\left[\frac{n(n+1)(n+4)}{2}q\cdot K_{\rho}^{(n)\rho;}+(n+1)(n+4)q\cdot K_{\rho}^{(n);\rho}-2n(n+2)q\cdot H_{\rho}^{(n);\rho}\right]\right.$$
$$+\left[\frac{p^{\nu}q^{\mu}+p^{\mu}q^{\nu}}{p\cdot q}\right]\left[\frac{n(n+1)(n+8)}{2}q\cdot K_{\rho}^{(n)\rho;}\right.$$
$$\left.-(n+1)(n-4)q\cdot K_{\rho}^{(n);\rho}-4n(n+2)q\cdot H_{\rho}^{(n);\rho}\right]$$
$$\left.+4\left[\frac{q^{\mu}q^{\nu}}{q^2}\right]\left[-\frac{n(n+1)}{2}q\cdot K_{\rho}^{(n)\rho;}+(n+1)^2q\cdot K_{\rho}^{(n);\rho}\right]\right\}. \tag{33}$$

At this stage much of the preliminary combinatoric manipulations have been completed. The central problem, still to be addressed directly, is the issue of reexpressing the traces in terms of canonical operators.

In Appendix E we present a proof by exhaustion that any twist-4 noncanonical operator can be traded for canonical ones. The algorithms described there are not the most efficient ones at the tree level. For example, it is easier in practice to do the tracings in a way which naturally pairs any term containing $D_\beta F^{\beta\rho}$ with another containing $-F^{\beta\rho}D_\beta$ to form a commutator $[D_\beta,F^{\beta\rho}]=g\tau_a\bar{\psi}\gamma^\rho\tau_a\psi$ by the gluon equation of motion. The methods we have followed are described below. Most of the actual calculations have been relegated to Appendices F and G.

The treatment of the traces of K and H is greatly simplified through the use of the generating function $g(\alpha)\equiv 1/1-d\alpha$. Equations (28)–(30) may be rewritten as

$$\Delta\cdot K_{\cdot\rho}^{(n);\rho}=\frac{1}{(n+1)}\left\{\frac{1}{n!}\left[\frac{\partial}{\partial\alpha}\right]^n[g(\alpha)\not{\Pi}g(\alpha)]\right\}\Bigg|_{\alpha=0}, \tag{34}$$

$$\Delta\cdot K_{\rho}^{(n)\rho;}=\frac{2\not{\Delta}}{n(n+1)}\left\{\frac{1}{(n-1)!}\left[\frac{\partial}{\partial\alpha}\right]^{n-1}[g(\alpha)\Pi_\beta g(\alpha)\Pi^\beta g(\alpha)]\right\}\Bigg|_{\alpha=0}, \tag{35}$$

$$\Delta\cdot H_{\rho}^{(n);\rho}=\frac{1}{4n}\left\{\frac{1}{n!}\left[\frac{\partial}{\partial\alpha}\right]^n 2[g(\alpha)\not{\Pi}g(\alpha)] + \frac{1}{(n-1)!}\left[\frac{\partial}{\partial\alpha}\right]^{n-1}\not{\Delta}[\Pi_\beta g(\alpha)\Pi^\beta g(\alpha)+g(\alpha)\Pi_\beta g(\alpha)\Pi^\beta]\right\}\Bigg|_{\alpha=0}. \tag{36}$$

The manipulations needed to eliminate contracted derivatives can be performed relatively easily on these generating functions. The identity

$$[g(\alpha),\Pi^\beta]=\alpha g(\alpha)z^\beta g(\alpha),$$

where

$$z^\beta=[d,\Pi^\beta]=-ig\Delta_\sigma F^{\sigma\beta}=-igf^\beta,$$

based on the operator identity

$$[1/A,B]=-\frac{1}{A}[A,B]\frac{1}{A},$$

facilitates the calculations.

As a simple example consider $g(\alpha)\not{\Pi}g(\alpha)$. Operating symmetrically gives

$$2g(\alpha)\not{\Pi}g(\alpha)=\alpha[g(\alpha)\not{z}g^2(\alpha)-g^2(\alpha)\not{z}g(\alpha)].$$

Performing the appropriate projection, i.e., applying $1/n!(\partial/\partial\alpha)^n$, yields

$$\sum_{k=0}^{n}d^k\not{\Pi}d^{n-k}=\tfrac{1}{2}\sum_{k=0}^{n-1}(n-1-2k)d^k\not{z}\,d^{n-1-k}. \tag{37}$$

Note that there is an unavoidable ambiguity in writing this operator because of the identity $\sum_{r=0}^{n}d^r\not{z}\,d^{n-r}=0$, obtained by moving $\not{\Pi}$ all the way to the right in $\not{\Pi}d^{n+1}$. We have chosen to write the operator in a form which has manifest charge-conjugation properties.

The calculations of

$$g(\alpha)\Pi_\beta g(\alpha)\Pi^\beta g(\alpha)$$

and

$$[\Pi_\beta g(\alpha)\Pi^\beta g(\alpha)+g(\alpha)\Pi_\beta g(\alpha)\Pi^\beta]$$

are more laborious; their details are in Appendix F, as is the final form for $X_{\rm I}^{\mu\nu}$.

2. Traces of $X_{\rm II}^{\mu\nu}$

The general outline of the calculation of the traces of $X_{\rm II}^{\mu\nu}$ is quite similar to that for $X_{\rm I}^{\mu\nu}$. Expanding Λ_0 in Eq. (18) gives

$$X_{\rm II}^{\mu\nu}=-\frac{2i}{q^4}\sum_{\substack{n=2\\ \text{(even)}}}^{\infty}\left[\frac{-2}{q^2}\right]^{n-1}q_{\mu_1}\cdots q_{\mu_{n-1}}q_\alpha J^{\mu_1\cdots\mu_{n-1};\alpha;\mu\nu}, \tag{38}$$

where

$$q_{\mu_1}\cdots q_{\mu_{n-1}}J^{\mu_1\cdots\mu_{n-1};\alpha;\mu\nu}=\sum_{k=0}^{n-1}(q\cdot\Pi)^k(\Pi^\nu\epsilon^{\beta\mu\alpha\lambda}\Pi_\beta\gamma_\lambda\gamma_5-\epsilon^{\beta\mu\alpha\lambda}\Pi_\beta\gamma_\lambda\gamma_5\Pi^\nu)(q\cdot\Pi)^{n-1-k}+(\mu\leftrightarrow\nu). \tag{39}$$

Because J is a not-totally-symmetric operator, it can be replaced by its traces. By Eq. (D7),

$$q_{\mu_1}\cdots q_{\mu_{n-1}}q_\alpha J^{\mu_1\cdots\mu_{n-1};\alpha;\mu\nu} \Longrightarrow -\frac{g^{\mu\nu}}{(n+2)}\{(n-1)q\cdot J^{(n)\rho;;}{}_\rho+(n+1)q\cdot J^{(n);\rho;}{}_\rho\}$$
$$+\frac{q^\mu p^\nu+q^\nu p^\mu}{p\cdot q}\frac{1}{(n+2)}[2(n-1)q\cdot J^{(n)\rho;;}{}_\rho+nq\cdot J^{(n);\rho;}{}_\rho]\,. \tag{40}$$

The explicit forms for the traces of J are

$$\Delta\cdot J^{(n);\rho;}{}_\rho=\sum_{k=0}^{n-1}d^{k*}z\gamma_5 d^{n-1-k}\,, \tag{41}$$

$$\Delta\cdot J^{(n)\rho;;}{}_\rho=-\frac{1}{(n-1)}\Delta_\mu\left[2\sum_{k=1}^{n-1}\sum_{l=0}^{k-1}d^{k-1-l}\Pi_\rho d^{l+1}\epsilon^{\sigma\mu\rho\lambda}\Pi_\sigma\gamma_\lambda\gamma_5 d^{n-1-k}\right.$$
$$-\sum_{k=1}^{n-1}\sum_{l=0}^{k-1}d^{k-1-l}\Pi_\rho d^l\epsilon^{\sigma\mu\rho\lambda}\Pi_\sigma\gamma_\lambda\gamma_5 d^{n-k}$$
$$\left.+\sum_{k=0}^{n-2}\sum_{l=0}^{n-2-k}d^{k+1}\epsilon^{\sigma\mu\rho\lambda}\Pi_\sigma\gamma_\lambda\gamma_5 d^l\Pi_\rho d^{n-2-k-l}\right]\,. \tag{42}$$

Equation (42) can be rewritten in terms of a generating function:

$$\Delta\cdot J^{(n)\rho;;}{}_\rho=-\frac{1}{(n-1)}\left\{\frac{1}{(n-2)!}\left[\frac{\partial}{\partial\alpha}\right]^{n-2}\epsilon^{\rho\sigma\mu\lambda}\Delta_\mu\gamma_\lambda\gamma_5\right.$$
$$\left.\times[2g(\alpha)\Pi_\rho g(\alpha)d\Pi_\sigma g(\alpha)-dg(\alpha)\Pi_\rho g(\alpha)\Pi_\sigma g(\alpha)-g(\alpha)\Pi_\rho g(\alpha)\Pi_\sigma g(\alpha)d]\right\}\,. \tag{43}$$

In Appendix G further details of the calculation are given. Also, an explicit canonical form is presented for $X_{\rm II}^{\mu\nu}$, combined with the result of the next subsection.

3. Calculation of $X_{\rm III}^{\mu\nu}$

The treatment of $X_{\rm III}^{\mu\nu}$ involves no tracing, and is quite easy as a result. In the usual way, expanding Λ_0 in Eq. (19) gives

$$X_{\rm III}^{\mu\nu}=\frac{1}{q^4}\left[g^{\mu\nu}-2\left[\frac{q^\mu p^\nu}{q\cdot p}\right]\right]\sum_{\substack{n=2\\ (\text{even})}}^{\infty}\left[-\frac{2}{q^2}\right]^{n-1}q\cdot I^{(n)}+(\mu\leftrightarrow\nu)\,, \tag{44}$$

where

$$\Delta\cdot I^{(n)}=\sum_{k=0}^{n-1}d^k(d\not\Pi+\not\Pi d-\not\Pi\not\Delta\not\Pi)d^{n-1-k}\,.$$

The standard manipulations give

$$\Delta\cdot I^{(n)}=\sum_{k=0}^{n-1}(n-1-2k)d^k z\,d^{n-1-k}-\not\Delta\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)d^k z_\rho d^l z^\rho d^{n-3-k-l}$$
$$+i\epsilon^{\rho\sigma\nu\lambda}\gamma_\lambda\Delta_\nu\gamma_5\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)d^k z_\rho d^l z_\sigma d^{n-3-k-l}\,. \tag{45}$$

The expression of $X_{T=4}^{\mu\nu}$ is completed upon combining Eqs. (F7) and (G5). The result is

$$X^{\mu\nu}_{T=4}=g\sum_{\substack{n=2\\ (\text{even})}}^{\infty}\left[-\frac{2}{q^2}\right]^{n+1}\frac{1}{(n+2)^2}$$

$$\times\Bigg\{\left[\frac{q^\mu q^\nu}{q^2}-g^{\mu\nu}\right]\Bigg[-(n+1)\sum_{k=0}^{n-1}q\cdot O_n^{3(k)}-2(n+1)\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{5(k,l)}$$

$$-4\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(k+1)(n-2-k-l)q\cdot O_n^{5(k,l)}$$

$$+2(n+1)\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{6(k,l)}$$

$$+2\sum_{k=0}^{n-2}(-)^k(k+1)(n-1-k)q\cdot O_n^{7(k)}\Bigg]+\left[g^{\mu\nu}-\frac{p^\mu q^\nu+p^\nu q^\mu}{p\cdot q}+\frac{q^2p^\mu p^\nu}{(p\cdot q)^2}\right]$$

$$\times\Bigg[\frac{n(n-1)}{4}\sum_{k=0}^{n-1}q\cdot O_n^{3(k)}-\frac{(5n+4)}{2}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{5(k,l)}$$

$$-(n+8)\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(k+1)(n-2-k-l)q\cdot O_n^{5(k,l)}+\frac{(5n+4)}{2}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{6(k,l)}$$

$$+\frac{(n+8)}{2}\sum_{k=0}^{n-2}(-)^k(k+1)(n-1-k)q\cdot O_n^{7(k)}\Bigg]\Bigg\}. \tag{46}$$

As in the Appendices, we have used a notation in which any group of terms should be ignored if the upper limit on the group's summation is negative. In particular, the expression for $X^{\mu\nu}_{n=2,T=4}$ is in agreement with Eq. (11) of Ref. 3 and Eq. (2.17) of Ref. 16.[15] The coefficient functions themselves can be read off from Eqs. (2) and (46) immediately.

The final results of these lengthy calculations deserve some discussion. Note first of all that the coefficients of the three tensors have combined in the necessary gauge-invariant fashion in Eq. (46). Accordingly, we feel justified in restoring the $p^\mu p^\nu$ terms as required by gauge invariance. The coefficient of $q^\mu q^\nu/q^2-g^{\mu\nu}$ in Eq. (46) contributes to W_L', while the coefficient of

$$g^{\mu\nu}-\frac{p^\mu q^\nu+p^\nu q^\mu}{p\cdot q}+p^\mu p^\nu q^2/(p\cdot q)^2$$

in Eq. (46) and the corresponding term in Eq. (2) contribute to W_T'. The relations of W_L' and W_T' to the standard definitions of the transverse and longitudinal structure functions are[17]

$$W_T=W_1\,,$$
$$W_T'=\frac{\nu^2}{Q^2}W_2\,, \tag{47}$$
$$W_L=\left[1+\frac{\nu^2}{Q^2}\right]W_2-W_1=W_L'+W_2\,.$$

As a second observation, it is quite surprising that the operator $\Delta\cdot O_n^{4(k)}$ canceled out from both the contribution to W_L' and to W_T'. We have no physical insight into this.

Third, our results are remarkably simple in a particular sense. If one redefines the operators of the spin-n basis to include the combinatoric factors which depend on k or l, there are only five spin-n operators in $X^{\mu\nu}_{n,T=4}$, and perhaps four more in $Y^{\mu\nu}_{n,T=4}$. The same five operators of $X^{\mu\nu}_{n,T=4}$ appear in the contributions to W_L' and W_T', and to W_3 as shown by Iijima.[6] Pursuing this observation a little further, it appears that the twist-4 contributions to W_T' will be greater than or roughly equal to those to W_L'. The suggestion rests on a comparison of the n dependence of the coefficient functions of the nine suitably defined operators. Clearly, the conjecture could be evaded through cancellations after the evaluation of matrix elements or through higher-order corrections. Both possibilities are under investigation presently.[18]

As a final, more speculative remark, it is conceivable that there exists a suitably defined subclass of spin-n twist-4 operators, including these nine, which closes under renormalization at least to one loop. There are certainly some simple features of the one-loop anomalous-dimension matrix. For example, four-quark operators do not mix into $\bar{\psi}F\psi$ operators, as all the counterterms arising from the relevant graphs come from operators which can be related to four-quark operators by the gluon equation of motion.[19]

IV. CONCLUSION

The examination of higher-twist effects in leptoproduction has two primary motivations. The first is to refine the predictions of perturbative QCD as far as possible. The second, perhaps more interesting, object is to establish a vehicle by which experiment can provide more detailed information regarding the structure of the proton. We have proposed a framework for the study of twist-4 effects which is well suited for the bound-state problem. It is based on the operator-product expansion, and makes use of a canonical basis of operators. Proton matrix elements of canonical operators probe the constituent wave functions on the light-cone, and are not sensitive to the Hamiltonian of a model used in approximate calculations. More precise data on structure functions at moderate Q^2 should make possible a study of correlations of constituents inside the proton.

Note added. After the completion of this paper we received a paper by S. P. Luttrell and S. Wada [Nucl. Phys. B197, 290 (1982)], in which the coefficient functions were calculated in a basis of operators with contracted derivatives.

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APPENDIX A

The complete canonical twist-4 nonsinglet operator basis is given explicitly below, in a form suitable for anomalous-dimension calculations. We use the notation of Sec. II:

$$\Delta\cdot Q_n^{1(k,l)}=g\bar{\psi}_R \not{\Delta}\overleftarrow{d}^{\,l}\vec{d}^{\,k}\psi_R\bar{\psi}_R\not{\Delta}\vec{d}^{\,n-2-k-l}\psi_R\ ,$$

$$\Delta\cdot Q_n^{2(k,l)}=g\bar{\psi}_R\tau_a \not{\Delta}\overleftarrow{d}^{\,l}\vec{d}^{\,k}\psi_R\bar{\psi}_R\not{\Delta}\vec{d}^{\,n-2-k-l}\tau_a\psi_R\ ,$$

$$\Delta\cdot Q_n^{3(k,l)}=g\bar{\psi}_R \not{\Delta}\overleftarrow{d}^{\,l}\vec{d}^{\,k}\psi_R\bar{\psi}_L\not{\Delta}\vec{d}^{\,n-2-k-l}\psi_L\ ,$$

$$\Delta\cdot Q_n^{4(k,l)}=g\bar{\psi}_R\tau_a \not{\Delta}\overleftarrow{d}^{\,l}\vec{d}^{\,k}\psi_R\bar{\psi}_L\not{\Delta}\vec{d}^{\,n-2-k-l}\tau_a\psi_L\ ,$$

$$\Delta\cdot Q_n^{5(k,l)}=g\bar{\psi}_L \not{\Delta}\overleftarrow{d}^{\,l}\vec{d}^{\,k}\psi_L\bar{\psi}_L\not{\Delta}\vec{d}^{\,n-2-k-l}\psi_L\ ,$$

$$\Delta\cdot Q_n^{6(k,l)}=g\bar{\psi}_L\tau_a \not{\Delta}\overleftarrow{d}^{\,l}\vec{d}^{\,k}\psi_L\bar{\psi}_L\not{\Delta}\vec{d}^{\,n-2-k-l}\tau_a\psi_L\ ,$$

$$\Delta\cdot Q_n^{7(k)}=\bar{\psi}\overleftarrow{d}^{\,k}{}^*f\gamma_5\vec{d}^{\,n-1-k}\psi\ ,$$

$$\Delta\cdot Q_n^{8(k)}=i\bar{\psi}\overleftarrow{d}^{\,k}f\vec{d}^{\,n-1-k}\psi\ ,$$

$$\Delta\cdot Q_n^{9(k,l)}=g\bar{\psi}\overleftarrow{d}^{\,k}f_a^\alpha(\vec{d}^{\,l}f_\alpha)_a\vec{d}^{\,n-3-k-l}\not{\Delta}\psi\ ,$$

$$\Delta\cdot Q_n^{10(k,l)}=igf_{abc}\bar{\psi}\overleftarrow{d}^{\,k}f_a^\alpha(\vec{d}^{\,l}f_\alpha)_b\vec{d}^{\,n-3-k-l}\not{\Delta}\tau_c\psi\ ,$$

$$\Delta\cdot Q_n^{11(k,l)}=gd_{abc}\bar{\psi}\overleftarrow{d}^{\,k}f_a^\alpha(\vec{d}^{\,l}f_\alpha)_b\vec{d}^{\,n-3-k-l}\not{\Delta}\tau_c\psi\ ,$$

$$\Delta\cdot Q_n^{12(k,l)}=ig\bar{\psi}\overleftarrow{d}^{\,k}{}^*f_a^\alpha(\vec{d}^{\,l}f_\alpha)_a\vec{d}^{\,n-3-k-l}\not{\Delta}\gamma_5\psi\ ,$$

$$\Delta\cdot Q_n^{13(k,l)}=gf_{abc}\bar{\psi}\overleftarrow{d}^{\,k}{}^*f_a^\alpha(\vec{d}^{\,l}f_\alpha)_b\vec{d}^{\,n-3-k-l}\not{\Delta}\gamma_5\tau_c\psi\ ,$$

$$\Delta\cdot Q_n^{14(k,l)}=igd_{abc}\bar{\psi}\overleftarrow{d}^{\,k}{}^*f_a^\alpha(\vec{d}^{\,l}f_\alpha)_b\vec{d}^{\,n-3-k-l}\not{\Delta}\gamma_5\tau_c\psi\ .$$

Here, $f_a^\alpha=\Delta_\beta F_a^{\beta\alpha}$ and $f=\tau_a f_a$. Covariant derivatives which act on f's alone are in the adjoint rather than the fundamental representation. We have followed Gottlieb[8] and Owaka[9] by writing the four-quark operators in a helicity basis. Flavor has been ignored. It is important to note that only four-quark operators can transform as flavor 27.[8] The canonical operators of the form

$$Q^{(j,k,l)}=\epsilon_{\alpha\beta\delta\lambda}\Delta^\delta\bar{\psi}d^jf^\alpha d^kf^\beta d^l\gamma^\lambda\gamma_5\psi$$

have not been listed for the following reason.[20] Rewriting

$$f^\alpha=\tfrac{1}{4}\epsilon^{\alpha\rho\sigma\tau}\Delta_\rho{}^*F_{\sigma\tau},\quad Q^{(j,k,l)}$$

$$=\tfrac{1}{2}\bar{\psi}d^{j*}f^\beta d^kf_\beta d^l\not{\Delta}\gamma_5\psi+(\text{twist }6)\ . \qquad (\text{A1})$$

This form of $Q^{(i,j,k)}$ already appears in the list. The same substitution for f_a^α reveals that operators of the form

$$g^2\mathscr{C}_{abcd}\epsilon_{\alpha\beta\delta\lambda}\Delta^\delta\bar{\psi}\overleftarrow{d}^{\,j}f_a^\alpha(\vec{d}^{\,k}f^\beta)_b(\vec{d}^{\,l}f^\lambda)_c \times\overleftarrow{d}^{\,n-4-j-k-l}\tau_d\not{\Delta}\gamma_5\psi$$

are actually twist 6.

There are other identities among canonical operators. The identity $\sum_{i=0}^{n} d^i f d^{n-i}=0$ was discussed in regard to Eq. (37). Okawa[9] noted another identity among flavor-27 four-quark operators. Under a simultaneous Dirac, color, and flavor Fierz transformation Q^1 (Q^5) operators can be expressed in terms of Q^2 (Q^6) operators. Unfortunately, this trick fails for the corresponding octet operators [see Eqs. (6) and (7)]. There may well exist more relations among canonical operators.

APPENDIX B

The calculation of $Y^{\mu\nu}$ begins with the contribution to the scattering amplitude of the graphs of Fig. 1 and their associates:

$$T^{\mu\nu}=\int d^4x\, e^{iq\cdot x}\langle p_2p_4\,|\,T[J^\mu(x)J^\nu(0)]\,|\,p_1p_3\rangle$$
$$=-ig^2\left[\bar q(p_2)Q\tau_a\left[\gamma^\rho\frac{1}{\not p_1+\not q}\gamma^\mu+\gamma^\mu\frac{1}{\not p_2-\not q}\gamma^\rho\right]q(p_1)\bar q(p_4)Q\tau_a\left[\gamma^\nu\frac{1}{\not p_4+\not q}\gamma_\rho+\gamma_\rho\frac{1}{\not p_3-\not q}\gamma^\nu\right]q(p_3)\right.$$
$$\left.+\begin{bmatrix}\mu\leftrightarrow\nu\\ q\to -q\end{bmatrix}\right]$$
$$+[p_2\leftrightarrow p_4 \text{ if the flavor of particle 2 is that of particle 4}]\,. \tag{B1}$$

The set of terms above which is indicated as obtained by crossing is identical to that obtained by the operation $(p_1\leftrightarrow p_3, p_2\leftrightarrow p_4)$. There are no $q^\mu q^\nu$ terms arising in twist 4, so that the full answer is transverse and can be reconstructed from the coefficient of $g^{\mu\nu}$ using gauge invariance. The advantage of selecting the $g^{\mu\nu}$ terms is that the procedure automatically projects out a totally symmetrized twist-4 operator. Define the $g^{\mu\nu}$ twist-4 terms of $T^{\mu\nu}$ to be $g^{\mu\nu}T_{T=4}$. Also define $y_i=-2p_i\cdot q/q^2$, $w^\mu_{a,ij}=\bar q(p_i)Q\tau_a\gamma^\mu q(p_j)$, and $v^\mu_{a,ij}=\bar q(p_i)Q\gamma^\mu\gamma_5\tau_a q(p_j)$.

Dropping terms which lead to target-spin-dependent effects and using momentum conservation, $g^{\mu\nu}T_{T=4}$ is given below in the case where the two incident quarks have different flavors:

$$g^{\mu\nu}T_{T=4}=-ig^2\frac{g^{\mu\nu}}{q^6}\left[w_{a,43}\cdot q\,w_{a,21}\cdot q\right.$$
$$\times\left[\frac{1}{1-y_1}\frac{1}{1-y_1+y_2}\frac{1}{1-y_1+y_2-y_3}-\frac{1}{1+y_2}\frac{1}{1+y_2-y_1}\frac{1}{1+y_2-y_1-y_3}\right.$$
$$\left.+\frac{1}{1+y_2}\frac{1}{1+y_2-y_1}\frac{1}{1+y_3}-\frac{1}{1-y_1}\frac{1}{1-y_1+y_2}\frac{1}{1+y_3}\right]$$
$$+v_{a,21}\cdot q\,v_{a,43}\cdot q\left[\frac{1}{1-y_1}\frac{1}{1-y_1+y_2}\frac{1}{1-y_1+y_2-y_3}\right.$$
$$+\frac{1}{1+y_2}\frac{1}{1+y_2-y_1}\frac{1}{1+y_2-y_1-y_3}+\frac{1}{1+y_2}\frac{1}{1+y_2-y_1}\frac{1}{1+y_3}$$
$$\left.\left.+\frac{1}{1-y_1}\frac{1}{1-y_1+y_2}\frac{1}{1+y_3}\right]\right]+(y_i\to -y_i)\,. \tag{B2}$$

The identity

$$\frac{1}{1-x}\frac{1}{1-x-y}\frac{1}{1-x-y-z}=\sum_{j,k,l=0}^{\infty}\frac{(j+k+l+2)!}{k!\,l!\,(j+1)!\,(j+l+2)}x^k y^l z^j \tag{B3}$$

may be verified by the application of the following sequence of six consecutive operations from the right, beginning with differentiation with respect to z, p times:

$$\left[\left[\frac{\partial}{\partial x}\right]^m\bigg|_{x=0}\right]\left[\left[\frac{\partial}{\partial y}\right]^n\bigg|_{y=0}\right]\left[\left[\frac{\partial}{\partial z}\right]^p\bigg|_{z=0}\right].$$

With the aid of this identity,

$$g^{\mu\nu}T_{T=4}=-ig^2\frac{g^{\mu\nu}}{q^6}\sum_{\substack{j,k,l=0\\(j+k+l\text{ even})}}^{\infty}\left[\frac{2}{q^2}\right]^{j+k+l}\Big\langle p_2p_4\Big|\Big\{\bar\psi Q\tau_a \not q(iq\cdot\overleftarrow{\partial})^l(iq\cdot\overrightarrow{\partial})^k\psi\bar\psi\not q(iq\cdot\overrightarrow{\partial})^jQ\tau_a\psi$$
$$\times\left[\frac{(j+k+l+2)!}{k!l!(j+1)!}\left[\frac{1}{j+l+2}-\frac{1}{j+k+2}\right]+(-)^j\frac{(k+l+1)!}{k!l!}\left[\frac{1}{k+1}-\frac{1}{l+1}\right]\right]$$
$$+\bar\psi Q\tau_a\not q\gamma_5(iq\cdot\overleftarrow{\partial})^l(iq\cdot\overrightarrow{\partial})^k\psi\bar\psi\not q\gamma_5(iq\cdot\overrightarrow{\partial})^jQ\tau_a\psi$$
$$\times\left[\frac{(j+k+l+2)!}{k!l!(j+1)!}\left[\frac{1}{j+l+2}+\frac{1}{j+k+2}\right]+(-)^j\frac{(k+l+1)!}{k!l!}\left[\frac{1}{k+1}+\frac{1}{l+1}\right]\right]\Big\}\Big|p_1p_3\Big\rangle.$$

(B4)

The final result is independent of any condition on the flavors of the incident quarks. Equation (2) of the text follows on the restoration of gauge invariance.

APPENDIX C

Here we prove the theorem that only one rank-n, traceless (in all index pairs) tensor can be constructed from the metric $g_{\alpha\beta}$, and a single four-vector p_β, and *that tensor is totally symmetric.* An immediate corollary is that the *forward, single-particle matrix element of any traceless operator with no totally symmetric part vanishes.*[21]

The proof is quite simple. After giving it we will illustrate it with an example. First we need a lemma: There is a unique, rank-n traceless, *symmetric* tensor constructed from $g_{\alpha\beta}$ and p_β, call it $S^n_{\mu_1\cdots\mu_n}$. This result is well known and is easily proven by direct construction: start with $p_{\mu_1}\cdots p_{\mu_n}$ and subtract symmetric combinations of terms with successively more factors of $g_{\alpha\beta}$ to render the result traceless. Note the following:

(i) $S^2_{\mu_1\mu_2}=p_{\mu_1}p_{\mu_2}-\frac{1}{4}p^2g_{\mu_1\mu_2}$ is the unique traceless rank-2 tensor constructed from $g_{\alpha\beta}$ and p_β regardless of symmetry, up to a multiplicative constant.

(ii) $S^n_{\mu_1\cdots\mu_n}p^{\mu_1}\cdots p^{\mu_n}\equiv S^n\cdot p\neq 0$ [can be seen directly or see (v) below].

Now suppose there exists a tensor $T^n_{\mu_1}\cdots\mu_n$ constructed from $g_{\alpha\beta}$ and p_β which has no totally symmetric part. By hypothesis

$$\text{(iii)}\quad p^{\mu_1}\cdots p^{\mu_n}T^n_{\mu_1\cdots\mu_n}=0$$

and

$$\text{(iv)}\quad g^{\mu_j\mu_k}T^n_{\mu_1\cdots\mu_m}=0$$

for any j and k. Note there must be some k, such that

$$\text{(v)}\ p^{\mu_k}T^n_{\mu_1\cdots\mu_n}\neq 0\,.$$

Otherwise $T^n\equiv 0$. This last remark is the only nontrivial step in the proof. It follows because T^n lives in a vector space spanned by vectors which are products of p_β's and $g_{\alpha\beta}$'s. If (v) were false then T^n would be orthogonal to all vectors in the space and therefore be zero.

Now define

$$T^{n-1}_{\mu_1\cdots\mu_{n-1}} \equiv p^{\mu_n} T^n_{\mu_1\cdots\mu_n} , \quad \text{(C1)}$$

where we have relabeled indices so the index k which appears in (v) is called n. By (iv) T^{n-1} is traceless. In addition T^{n-1} has no totally symmetric part. To prove this suppose T^{n-1} had a totally symmetric part. By our lemma it must be proportional to S^{n-1}:

$$T^{n-1}_{\mu_1\cdots\mu_{n-1}} = CS^{n-1}_{\mu_1\cdots\mu_{n-1}} + R^{n-1}_{\mu_1\cdots\mu_{n-1}} , \quad \text{(C2)}$$

where R^{n-1} has no totally symmetric part. By (2) $p^{\mu_1}\cdots p^{\mu_{n-1}} T^{n-1}_{\mu_1\cdots\mu_{n-1}} = C(S^{n-1}\cdot p) \neq 0$. But this contradicts (3) unless $C=0$. So T^{n-1} is traceless, with no totally symmetric part. Now continue this process constructing T^{n-k} from T^{n-k+1} until $n-k=2$. We now have a tensor $T^2_{\mu_1\mu_2}$ which is traceless with no totally symmetric part contradicting (1). Thus our hypothesis that there exists a nontrivial T^n is wrong and the proof is complete.

To illustrate this theorem and the notation of replacing an operator by its traces we present a simple example. Let $O_{[\alpha\beta][\delta\gamma]}$ be an operator antisymmetric in its first and second index pairs. Symmetry dictates the form of its forward matrix elements to be

$$\langle p \mid O_{[\alpha\beta][\gamma\delta]} \mid p \rangle = A^{(2)}(p_\alpha p_\gamma g_{\beta\delta} + p_\beta p_\delta g_{\alpha\gamma} - p_\beta p_\gamma g_{\alpha\delta} - p_\alpha p_\delta g_{\beta\gamma}) . \quad \text{(C3)}$$

If $O_{[\alpha\beta][\gamma\delta]}$ is a traceless operator we are forced to conclude $A^{(2)}=0$ (e.g., contract the indices α and γ), in accord with our theorem. If $O_{[\alpha\beta][\gamma\delta]}$ is not a traceless operator we proceed as follows: Define $\bar{O}_{[\alpha\beta][\gamma\delta]}$ by

$$\bar{O}_{[\alpha\beta][\gamma\delta]} = O_{[\alpha\beta][\gamma\delta]} - (O_{\alpha\gamma} g_{\beta\delta} + O_{\beta\delta} g_{\alpha\gamma} - O_{\beta\gamma} g_{\alpha\delta} - O_{\alpha\delta} g_{\beta\gamma}) . \quad \text{(C4)}$$

If we require $\bar{O}_{[\alpha\beta][\gamma\delta]}$ to be traceless we can solve for the operator $O_{\alpha\beta}$:

$$O_{\alpha\beta} = \tfrac{1}{2} O_{[\alpha\lambda][\beta\kappa]} g^{\lambda\kappa} - \tfrac{1}{12} g_{\alpha\beta} O_{[\sigma\tau]}{}^{[\sigma\tau]} . \quad \text{(C5)}$$

Because of our theorem the operator $\bar{O}_{[\alpha\beta][\gamma\delta]}$ drops out of the OPE analysis of leptoproduction. We may therefore replace the not-totally-symmetric operator $O_{[\alpha\beta][\gamma\delta]}$ by its traces:

$$O_{[\alpha\beta][\gamma\delta]} \to O_{\alpha\gamma} g_{\beta\delta} + O_{\beta\delta} g_{\alpha\gamma} - O_{\beta\gamma} g_{\alpha\delta} - O_{\alpha\delta} g_{\beta\gamma} . \quad \text{(C6)}$$

Finally note that if the operator $O_{\alpha\beta}$ were not-totally-symmetric we could repeat this procedure until, in the end $O_{[\alpha\beta][\gamma\delta]}$, or any other not-totally-symmetric operator, is replaced by totally symmetric (and traceless) operators of lower rank.

APPENDIX D

In this appendix we calculate the terms, needed in the main body of the text, which must be subtracted from certain not-totally-symmetric operators to make them traceless. Three forms of not-totally-symmetric operators will be considered in turn. By convention an operator is totally symmetric in any subset of indices not separated by a semicolon, and is traceless if barred. Indices which should be omitted in operators are enclosed by parentheses.

Consider an operator of the form $O^{\mu_1\cdots\mu_n;\alpha}$:

$$\bar{O}^{\mu_1\cdots\mu_n;\alpha} = O^{\mu_1\cdots\mu_n;\alpha} - \sum_{i<j} g^{\mu_i\mu_j} M^{\mu_1\cdots(\mu_i)\cdots(\mu_j)\cdots\mu_n\alpha} - \sum_i g^{\mu_i\alpha} N^{\mu_1\cdots(\mu_i)\cdots\mu_n} + \cdots . \quad \text{(D1)}$$

There are two different equations which arise through the contraction of any one pair of indices. For convenience an arbitrary null vector Δ^μ has been contracted with all free indices which remain:

$$0 = \Delta^{n-1}\cdot O^\rho{}_\rho{}^; - 2n\Delta^{n-1}\cdot M - 2\Delta^{n-1}\cdot N , \quad 0 = \Delta^{n-1}\cdot O_\rho{}^{;\rho} - (n-1)\Delta^{n-1}\cdot M - (n+3)\Delta^{n-1}\cdot N . \quad \text{(D2)}$$

Here, for example, $\Delta^{n-1}\cdot O^\rho{}_\rho{}^; = \Delta_{\mu_1}\cdots\Delta_{\mu_{n-1}} O^{\mu_1\cdots\mu_{n-2}\rho}{}_\rho{}^{;\mu_{n-1}}$. In the text, where the spin of the operator is specified, we write $\Delta\cdot O_\rho^{(n-1)\rho;}$ for $\Delta^{n-1}\cdot O_\rho{}^{\rho;}$.

The solutions are

$$\Delta^{n-1}\cdot M=\frac{1}{(n+1)^2}\left[\frac{(n+3)}{2}\Delta^{n-1}\cdot O^{\rho}{}_{\rho}{}^{;}-\Delta^{n-1}\cdot O^{\rho;}{}_{\rho}\right],$$
$$\Delta^{n-1}\cdot N=\frac{1}{(n+1)^2}\left[-\frac{(n-1)}{2}\Delta^{n-1}\cdot O^{\rho}{}_{\rho}{}^{;}+n\Delta^{n-1}\cdot O^{\rho;}{}_{\rho}\right]. \quad \text{(D3)}$$

The second form of operator of interest is $O^{\mu_1\cdots\mu_n;\alpha\beta}$:

$$\bar{O}^{\mu_1\cdots\mu_n;\alpha\beta}=O^{\mu_1\cdots\mu_n;\alpha\beta}-g^{\alpha\beta}P^{\mu_1\cdots\mu_n}-\left[\sum_i g^{\mu_i\alpha}Q^{\mu_1\cdots(\mu_i)\cdots\mu_n\beta}+(\alpha\leftrightarrow\beta)\right]$$
$$-\sum_{i<j}g^{\mu_i\mu_j}R^{\mu_1\cdots(\mu_i)\cdots(\mu_j)\cdots\mu_n\alpha\beta}. \quad \text{(D4)}$$

In the case required $O^{;\rho}{}_{\rho}\cdot\Delta=0$, and $\Delta\cdot R$ is not needed. The expressions for $\Delta\cdot P$ and $\Delta\cdot Q$ are

$$\Delta^{n}\cdot P=\frac{1}{2}\frac{1}{(n+2)^2}[-2n^2\Delta^{n}\cdot O^{\rho;}{}_{\rho}+n(n-1)\Delta^{n}\cdot O^{\rho}{}_{\rho}{}^{;}],$$

$$\Delta^{n}\cdot Q=\frac{1}{(n+2)^2}[2n\Delta^{n}\cdot O^{\rho;}{}_{\rho}-(n-1)\Delta^{n}\cdot O^{\rho}{}_{\rho}{}^{;}]. \quad \text{(D5)}$$

The final case is that of $O^{\mu_1\cdots\mu_n;\alpha;\mu\nu}$:

$$\bar{O}^{\mu_1\cdots\mu_n;\alpha;\mu\nu}=O^{\mu_1\cdots\mu_n;\alpha;\mu\nu}-[g^{\mu\alpha}A^{\mu_1\cdots\mu_n\nu}+(\mu\leftrightarrow\nu)]-\sum_i g^{\mu_i\alpha}B^{\mu_1\cdots(\mu_i)\cdots\mu_n\mu\nu}$$
$$-\left[\sum_i g^{\mu_i\mu}C^{\mu_1\cdots(\mu_i)\cdots\mu_n\alpha\nu}+(\mu\leftrightarrow\nu)\right]-g^{\mu\nu}D^{\mu_1\cdots\mu_n\alpha}$$
$$-\sum_{i<j}g^{\mu_i\mu_j}E^{\mu_1\cdots(\mu_i)\cdots(\mu_j)\cdots\mu_n\alpha\mu\nu}. \quad \text{(D6)}$$

In the case required $\Delta\cdot O^{\rho}{}_{\rho}{}^{;;}=0$, $\Delta\cdot O^{;;\rho}{}_{\rho}=-2\Delta\cdot O^{;\rho}{}_{;\rho}$, and $\Delta\cdot O^{\rho;}{}_{\rho}{}^{;}=-2\Delta\cdot O^{\rho;;}{}_{\rho}$. Also $\Delta\cdot B$ and $\Delta\cdot E$ are not needed:

$$\Delta^{n+1}\cdot A=\frac{1}{2(n+3)}[n\Delta^{n+1}\cdot O^{\rho;;}{}_{\rho}+(n+2)\Delta^{n+1}\cdot O^{;\rho;}{}_{\rho}],$$
$$\Delta^{n+1}\cdot C=\frac{1}{2(n+3)}[3\Delta^{n+1}\cdot O^{\rho;;}{}_{\rho}+\Delta^{n+1}\cdot O^{;\rho;}{}_{\rho}], \quad \text{(D7)}$$
$$\Delta^{n+1}\cdot D=-\frac{1}{(n+3)}[n\Delta^{n+1}\cdot O^{\rho;;}{}_{\rho}+(n+2)\Delta^{n+1}\cdot O^{;\rho;}{}_{\rho}].$$

APPENDIX E

In the main body of the text the particular noncanonical operators which appear in tree level are eliminated in favor of canonical ones through a wide variety of tricks. It is important to know, though, that any twist-4 noncanonical operator, which may arise under renormalization, or in higher order, can be reexpressed in terms of canonical operators. The proof that we present for this is by construction.

Operators which are manifestly twist-4 four-quark operators can be readily put in canonical form. The only possible operation needed is the elimination of extra γ matrices by successive application of three-γ identities.

Operators of the form $\bar{\psi}\cdots\psi$ (with no more quark fields) may be more difficult to put in canonical form. By chirality they must contain an odd number of γ matrices excluding γ_5 matrices. Repeated application of three-γ identities can be used to eliminate all but one γ matrix. Give the generic label O_1 to any operator with one γ matrix and no ϵ tensor or γ_5 matrix, and O_2 to any with one γ matrix, one ϵ tensor, and one γ_5 matrix. In the operators, all covariant derivatives may be reworked so that they act to the right.

Consider first the operators of the sort O_1. If the γ matrix index is contracted internally, the noncanonical operators can be expressed in terms of $\bar\psi(\Delta\cdot\Pi)^i \not\Pi(\Delta\cdot\Pi)^j\psi$, which is easily handled. Otherwise, the basic noncanonical operators are of the form $\bar\psi(\Delta\cdot\Pi)^i\Pi_\alpha(\Delta\cdot\Pi)^j\Pi^\alpha(\Delta\cdot\Pi)^k\not\Delta\psi$. These can be reexpressed as canonical operators, and

$$\bar\psi(\Delta\cdot\Pi)^i\Pi_\alpha f^\alpha(\Delta\cdot\Pi)^l\not\Delta\psi,$$

$$\bar\psi(\Delta\cdot\Pi)^i f^\alpha\Pi_\alpha(\Delta\cdot\Pi)^l\not\Delta\psi,$$

and

$$+\bar\psi(\Delta\cdot\Pi)^i\Pi^2(\Delta\cdot\Pi)^{l+1}\not\Delta\psi\ .$$

The first operator can be related to the third through the identity

$$\Pi_\alpha f^\alpha=\frac{i}{g}\Pi_\alpha[\Pi^\beta,\Pi^\alpha]\Delta_\beta$$

$$=\frac{i}{2g}\{\,[\Pi_\alpha,[\Pi^\beta,\Pi^\alpha]]+(\Pi^\beta\Pi^2-\Pi^2\Pi^\beta)\,\}\Delta_\beta\ .$$

A similar trick relates the second to the third. Finally, the identity $\Pi^2=\not\Pi\not\Pi-\frac{1}{4}[\gamma^\alpha,\gamma^\beta][\Pi_\alpha,\Pi_\beta]$ can be used to transform the third into a canonical form.

The operators of the form O_2 may be treated along the same lines. The noncanonical operators with the γ-matrix index contracted internally are related to

$$\epsilon_{\alpha\beta\delta\lambda}\Delta^\delta\bar\psi(\Delta\cdot\Pi)^i\Pi^\alpha(\Delta\cdot\Pi)^j\Pi^\beta(\Delta\cdot\Pi)^k\gamma^\lambda\gamma_5\psi\ .$$

Moving Π^α to the left and Π^β to the right will produce noncanonical operators of the form

$$\epsilon_{\alpha\beta\delta\lambda}\Delta^\delta\bar\psi\Pi^\alpha\gamma^\lambda\gamma_5(\Delta\cdot\Pi)^l f^\beta(\Delta\cdot\Pi)^m\ ,$$

$$\epsilon_{\alpha\beta\delta\lambda}\Delta^\delta\bar\psi(\Delta\cdot\Pi)^l f^\alpha(\Delta\cdot\Pi)^m\Pi^\beta\gamma^\lambda\gamma_5\psi\ ,$$

and

$$\epsilon_{\alpha\beta\delta\lambda}\Delta^\delta\bar\psi\Pi^\alpha\gamma^\lambda\gamma_5(\Delta\cdot\Pi)^n\Pi^\beta\psi\ .$$

One may use the identities (G3) to rewrite these in terms of operators of type 1. The noncanonical operators without an internally contracted γ matrix are related to

$$\epsilon_{\alpha\beta\delta\lambda}\Delta^\lambda\bar\psi(\Delta\cdot\Pi)^i\Pi^\alpha(\Delta\cdot\Pi)^j\Pi^\beta(\Delta\cdot\Pi)^k\Pi^\delta(\Delta\cdot\Pi)^l\not\Delta\gamma_5\psi\ .$$

Moving Π^α to the left and Π^β to the right as far as Π^δ, noncanonical operators arise which are of the form

$$\epsilon_{\alpha\beta\delta\lambda}\Delta^\delta\bar\psi(\Delta\cdot\Pi)^l f^\alpha(\Delta\cdot\Pi)^m f^\beta(\Delta\cdot\Pi)^n\Pi^\delta(\Delta\cdot\Pi)^p\not\Delta\gamma_5\psi,$$

$$\epsilon_{\alpha\beta\delta\lambda}\Delta^\delta\bar\psi\Pi^\alpha(\Delta\cdot\Pi)^l f^\beta(\Delta\cdot\Pi)^m\Pi^\delta(\Delta\cdot\Pi)^p\not\Delta\gamma_5\psi\ ,$$

and

$$\epsilon_{\alpha\beta\delta\lambda}\Delta^\delta\bar\psi\Pi^\alpha(\Delta\cdot\Pi)^l F^{\beta\delta}(\Delta\cdot\Pi)^m\not\Delta\gamma_5\psi\ .$$

Moving Π^δ to the right in the first of these, and application of identities (G3) enables all of these to be reexpressed as canonical and type 1 operators.

APPENDIX F

The calculations of the quantities $\Delta\cdot K_p^{(n)p;}$ and $\Delta\cdot H_p^{(n);p}$, defined in Eqs. (35) and (36), are addressed in detail in this appendix. As emphasized in the main body of the text, the generating functions provide a considerable short-cut. Define

$$c_{n+1}=\frac{1}{n!}\left\{\left[\frac{\partial}{\partial\alpha}\right]^n[g(\alpha)\Pi_\beta g(\alpha)\Pi^\beta g(\alpha)]\right\}\Bigg|_{\alpha=0}\ . \qquad \text{(F1)}$$

Moving the Π_β's together symmetrically in the generating function gives

$$2g(\alpha)\Pi_\beta g(\alpha)\Pi^\beta g(\alpha)=g(\alpha)\{\,g(\alpha)\Pi_\beta+[\Pi_\beta,g(\alpha)]\,\}\Pi^\beta g(\alpha)+g(\alpha)\Pi_\beta\{\,\Pi^\beta g(\alpha)+[g(\alpha),\Pi^\beta]\,\}g(\alpha)\ . \qquad \text{(F2)}$$

A little more algebra yields

$$g(\alpha)\Pi_\beta g(\alpha)\Pi^\beta g(\alpha)=\tfrac{1}{2}\{\,g^2(\alpha)\Pi^2 g(\alpha)+g(\alpha)\Pi^2 g^2(\alpha)+\alpha g^2(\alpha)[\Pi_\beta,z^\beta]g^2(\alpha)\ -2\alpha^2 g^2(\alpha)z_\beta g(\alpha)z^\beta g^2(\alpha)\,\}\ . \qquad \text{(F3)}$$

Performing the appropriate projection gives

$$\not\Delta c_{n+1}=\frac{\not\Delta}{2}\left\{(n+2)\sum_{k=0}^{n} d^k\Pi^2 d^{n-k}+\sum_{k=0}^{n-1}(k+1)(n-k)d^k[\Pi_\beta,z^\beta]d^{n-1-k}\right.$$
$$\left.-2\sum_{k=0}^{n-2}\sum_{l=0}^{n-k-2}(k+1)(n-1-k-l)d^k z_\beta d^l z^\beta d^{n-2-k-l}\right\}. \tag{F4}$$

(We have used a notation in which any group of terms should be ignored if the upper limit on the group's summation is negative, i.e., $c_1=\Pi^2$.)

The troublesome term remaining is

$$d_{n+1}=\not\Delta\sum_{k=0}^{n} d^k\Pi^2 d^{n-k}.$$

Its generating function is $\not\Delta g(\alpha)\Pi^2 g(\alpha)$. Using the relation $\Pi^2=\not\Pi\not\Pi-\frac{1}{4}[\gamma_\rho,\gamma_\beta][\Pi^\rho,\Pi^\beta]$,

$$2\not\Delta g(\alpha)\Pi^2 g(\alpha)=g(\alpha)(\not\Delta\not\Pi\not\Pi+\not\Pi\not\Pi\not\Delta)g(\alpha)-\tfrac{1}{4}g(\alpha)(\not\Delta[\gamma_\rho,\gamma_\beta]+[\gamma_\rho,\gamma_\beta]\not\Delta)[\Pi^\rho,\Pi^\beta]g(\alpha). \tag{F5}$$

The Π's can be moved easily to the appropriate end after rewriting $(\not\Delta\not\Pi\not\Pi+\not\Pi\not\Pi\not\Delta)=-2\not\Pi\not\Delta\not\Pi+2(d\not\Pi+\not\Pi d)$ in the first term on the right-hand side of (F5). The second term leads directly to a canonical form after the use of the standard three-γ identity. The appropriate projection gives the canonical form of $\not\Delta d_{n+1}$. Then, $\not\Delta c_{n+1}$ can be written as

$$\not\Delta c_{n+1}=\not\Delta\sum_{k=0}^{n-2}\sum_{l=0}^{n-2-k}\left[-\left[\frac{n+2}{2}\right](l+1)-(k+1)(n-1-k-l)\right]d^k z_\beta d^l z^\beta d^{n-2-k-l}$$
$$+\left[\frac{n+2}{2}\right]\sum_{k=0}^{n}(n-2k)d^k \not z d^{n-k}+i\left[\frac{n+2}{4}\right]\sum_{k=0}^{n} d^{k*}\not z\gamma_5 d^{n-k}$$
$$+i\left[\frac{n+2}{2}\right]\epsilon^{\rho\sigma\delta\lambda}\Delta_\delta\gamma_\lambda\gamma_5\sum_{k=0}^{n-2}\sum_{l=0}^{n-2-k}(l+1)d^k z_\rho d^l z_\sigma d^{n-2-k-l}$$
$$+\frac{\not\Delta}{2}\sum_{k=0}^{n-1}(k+1)(n-k)d^k[\Pi_\beta,z^\beta]d^{n-k-1}. \tag{F6}$$

Identical techniques can be employed to rewrite

$$\not\Delta\sum_{k=0}^{n-1}(\Pi_\beta d^k\Pi^\beta d^{n-1-k}+d^k\Pi_\beta d^{n-1-k}\Pi^\beta)$$

in its canonical form.

Combining the previous results of this appendix and Eqs. (33)—(37) gives

$$X_1^{\mu\nu}=g\sum_{\substack{n=2\\(\text{even})}}^{\infty}\left[\frac{-2}{q^2}\right]^{n+1}\frac{1}{(n+2)^2}$$
$$\times\left[-g^{\mu\nu}\left\{\frac{n^2+4n+2)}{4}\sum_{k=0}^{n-1}q\cdot O_n^{3(k)}-\frac{n(n+2)}{8}(q\cdot O_n^{3(0)}+q\cdot O_n^{3(n-1)})\right.\right.$$
$$-\frac{(n+2)^2}{2}\sum_{k=0}^{n-1}(n-1-2k)q\cdot O_n^{4(k)}+\frac{n(n+2)}{4}(q\cdot O_n^{4(0)}-q\cdot O_n^{4(n-1)})$$
$$-\frac{(n+4)}{2}\sum_{k=0}^{n-2}(k+1)(n-1-k)(-)^k q\cdot O_n^{7(k)}+\frac{n(n+2)}{4}\sum_{k=0}^{n-2}(-)^k q\cdot O_n^{7(k)}$$

$$+(n+4)\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}\left[\left[\frac{n+1}{2}\right](l+1)+(k+1)(n-2-k-l)\right]q\cdot O_n^{5(k,l)}$$

$$-\frac{n(n+2)}{2}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}q\cdot O_n^{5(k,l)}-\frac{(n^2+4n+2)}{2}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{6(k,l)}\Bigg\}$$

$$+4(n+1)\frac{q^\mu q^\nu}{q^2}\Bigg\{-\tfrac{1}{4}\sum_{k=0}^{n-1}q\cdot O_n^{3(k)}+\frac{1}{2(n+1)}\sum_{k=0}^{n-2}(k+1)(n-1-k)(-)^k q\cdot O_n^{7(k)}$$

$$-\frac{1}{(n+1)}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}\left[\left[\frac{n+1}{2}\right](l+1)+(k+1)(n-2-k-l)\right]q\cdot O_n^{5(k,l)}$$

$$+\tfrac{1}{2}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{6(k,l)}\Bigg\}$$

$$+\frac{(p^\nu q^\mu+p^\mu q^\nu)}{p\cdot q}\Bigg\{\frac{(n^2+7n+4)}{4}\sum_{k=0}^{n-1}q\cdot O_n^{3(k)}-\frac{n(n+2)}{4}(q\cdot O_n^{3(0)}+q\cdot O_n^{3(n-1)})$$

$$-\frac{(n+2)^2}{2}\sum_{k=0}^{n-1}(n-1-2k)q\cdot O_n^{4(k)}+\frac{n(n+2)}{2}(q\cdot O_n^{4(0)}-q\cdot O_n^{4(n-1)})$$

$$-n(n+2)\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}q\cdot O_n^{5(k,l)}$$

$$+(n+8)\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}\left[\left[\frac{n+1}{2}\right](l+1)+(k+1)(n-2-k-l)\right]q\cdot O_n^{5(k,l)}$$

$$-\frac{(n^2+7n+4)}{2}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{6(k,l)}+\frac{n(n+2)}{2}\sum_{k=0}^{n-2}(-)^k q\cdot O_n^{7(k)}$$

$$-\frac{(n+8)}{2}\sum_{k=0}^{n-2}(k+1)(n-1-k)(-)^k q\cdot O_n^{7(k)}\Bigg\}\Bigg]. \tag{F7}$$

APPENDIX G

In this appendix we continue with the calculation of $X_{\rm II}^{\mu\nu}$ at the point of Eq. (43). By the definition of $g(\alpha)$, $g(\alpha)d=(1/\alpha)[g(\alpha)-1]$. Then, Eq. (43) may be rewritten

$$\Delta\cdot J^{(n)\rho;;}{}_{\rho}=-\frac{1}{(n-1)}\left[\frac{1}{(n-2)!}\left[\frac{\partial}{\partial\alpha}\right]^{n-2}\right.$$

$$\left.\times\left\{-\frac{1}{\alpha}g(\alpha)^{*}z\gamma_5 g(\alpha)+\frac{1}{\alpha}\epsilon^{\rho\sigma\mu\lambda}\Delta_\mu\gamma_\lambda\gamma_5[\Pi_\rho g(\alpha)\Pi_\sigma g(\alpha)+g(\alpha)\ \Pi_\rho g(\alpha)\Pi_\sigma]\right\}\right]\Bigg|_{\alpha=0}. \tag{G1}$$

The two noncanonical terms are treated in quite a similar manner. One begins as in Appendix F, e.g.,

$$\frac{2}{\alpha}\epsilon^{\rho\sigma\mu\lambda}g(\alpha)\Pi_\rho g(\alpha)\Pi_\sigma\gamma_\lambda\gamma_5=\frac{1}{\alpha}\epsilon^{\rho\sigma\mu\lambda}\{\ g(\alpha)[g(\alpha)\Pi_\rho-\alpha g(\alpha)z_\rho g(\alpha)]\Pi_\sigma$$

$$+g(\alpha)\Pi_\rho[\Pi_\sigma g(\alpha)+\alpha g(\alpha)z_\sigma g(\alpha)]\ \}\gamma_\lambda\gamma_5\ . \tag{G2}$$

The procedure is hindered at this point by the fact that the z and the Π in the second and fourth terms do not have the same index. The resolution of the difficulty is to move the Π to the appropriate end and use either of the identities

$$\epsilon^{\alpha\beta\mu\lambda}\gamma_\lambda\Pi_\beta\gamma_5\psi=-i(\Pi^\alpha\gamma^\mu-\Pi^\mu\gamma^\alpha)\psi,\quad \epsilon^{\alpha\beta\mu\lambda}\bar{\psi}\gamma_\lambda\Pi_\alpha\gamma_5=-i\bar{\psi}(\Pi^\beta\gamma^\mu-\Pi^\mu\gamma^\beta)\ . \tag{G3}$$

The further manipulations are straightforward, as the z and Π naturally pair to form a commutator.

The result is

$$\begin{aligned}\frac{2}{\alpha}\epsilon^{\rho\sigma\mu\lambda}\Delta_\mu\left[\Pi_\rho g(\alpha)\Pi_\sigma g(\alpha)+g(\alpha)\Pi_\rho g(\alpha)\Pi_\sigma\right]\gamma_\lambda\gamma_5 &= \frac{1}{2\alpha}[g^2(\alpha)^*z+2g(\alpha)^*zg(\alpha)+{}^*zg^2(\alpha)]\gamma_5+2\alpha\epsilon^{\rho\sigma\mu\lambda}\Delta_\mu g(\alpha)z_\rho g^2(\alpha)z_\sigma g(\alpha)\gamma_\lambda\gamma_5\\ &\quad -i[g^2(\alpha)\not{z}g(\alpha)+g(\alpha)\not{z}g^2(\alpha)]d+id[g^2(\alpha)\not{z}g(\alpha)+g(\alpha)\not{z}g^2(\alpha)]\\ &\quad -i\{g^2(\alpha)[\Pi^\sigma,z_\sigma]g(\alpha)+g(\alpha)[\Pi^\sigma,z_\sigma]g^2(\alpha)\}\not{\Delta}\\ &\quad +2i\alpha\{g^2(\alpha)z_\rho g(\alpha)z^\rho g(\alpha)+g(\alpha)z_\rho g^2(\alpha)z^\rho g(\alpha)+g(\alpha)z_\rho g(\alpha)z^\rho g^2(\alpha)\}\not{\Delta}\ . \end{aligned} \tag{G4}$$

After making the proper projection, and combining with Eqs. (38), (40), and (41) we obtain

$$\begin{aligned} X_{\rm II}^{\mu\nu}+X_{\rm III}^{\mu\nu}=\frac{g}{2}\sum_{\substack{n=2\\ \text{(even)}}}^{\infty}\left[\frac{-2}{q^2}\right]^{n+1}\Bigg[g^{\mu\nu}\Bigg\{&-\sum_{k=0}^{n-1}(n-1-2k)q\cdot O_n^{4(k)}+\frac{n}{2(n+2)}(q\cdot O_n^{4(0)}-q\cdot O_n^{4(n-1)})\\ &+\frac{(n+\frac{3}{2})}{(n+2)}\sum_{k=0}^{n-1}q\cdot O_n^{3(k)}-\frac{n}{4(n+2)}(q\cdot O_n^{3(n-1)}+q\cdot O_n^{3(0)})\\ &+\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{5(k,l)}-\frac{n}{n+2}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}q\cdot O_n^{5(k,l)}\\ &-\frac{(n+1)}{(n+2)}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{6(k,l)}+\frac{n}{2(n+2)}\sum_{k=0}^{n-2}(-)^k q\cdot O_n^{7(k)}\Bigg\}\\ -\frac{(p^\mu q^\nu+p^\nu q^\mu)}{p\cdot q}\Bigg\{&-\sum_{k=0}^{n-1}(n-1-2k)q\cdot O_n^{4(k)}+\frac{n}{(n+2)}[q\cdot O_n^{4(0)}-q\cdot O_n^{4(n-1)}]\\ &+\frac{(n+1)}{(n+2)}\sum_{k=0}^{n-1}q\cdot O_n^{3(k)}-\frac{n}{2(n+2)}[q\cdot O_n^{3(n-1)}+q\cdot O_n^{3(0)}]\\ &+\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{5(k,l)}-\frac{2n}{(n+2)}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}q\cdot O_n^{5(k,l)}\\ &-\frac{n}{n+2}\sum_{k=0}^{n-3}\sum_{l=0}^{n-3-k}(l+1)q\cdot O_n^{6(k,l)}\\ &+\frac{n}{n+2}\sum_{k=0}^{n-2}(-)^k q\cdot O_n^{7(k)}\Bigg\}\Bigg]\ . \end{aligned} \tag{G5}$$

[1]The difficulty in distinguishing scale breaking due to logarithms from that of powers of Q^2 at low and moderate Q^2 was pointed out by L. F. Abbott, W. B. Atwood, and R. M. Barnett, Phys. Rev. D 22, 582 (1980); P. W. Duke and R. G. Roberts, Phys. Lett. 94B, 417 (1980); A. Donnachie and P. V. Landshoff, *ibid.* 95B, 437 (1980). For a discussion of the evidence for the presence of power-law corrections at such Q^2, see the review talk of J. Drees, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn,* edited by W. Pfeil (Physikalisches Institut, Universität Bonn, Bonn, 1981), pp. 474—507.

[2]R. L. Jaffe and M. Soldate, in *Perturbative Quantum Chromodynamics,* proceedings of the Tallahassee Conference, 1981, edited by D. W. Duke and J. F. Owens (AIP, New York, 1981), pp. 60—72.

[3]R. L. Jaffe and M. Soldate, Phys. Lett. 105B, 467 (1981).

[4]We assume that the OPE is not spoiled at twist 4 by nonperturbative effects or difficulties with incomplete cancellation of infrared divergences.

[5]For reviews of the twist-2 analysis see H. D. Politzer, Phys. Rep. 14C, 129 (1974); A. Peterman, *ibid.* 53C, 157 (1979); A. J. Buras, Rev. Mod. Phys. 52, 199 (1980).

[6]See B. Iijima (unpublished) for the tree-level coefficient functions of neutrino production.

[7]References 8 and 9 contain some of the necessary anomalous-dimension calculations.

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[11]W. Zimmermann, *Lectures on Elementary Particles and Quantum Field Theory,* 1970 Brandeis Summer Institute in Theoretical Physics, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, 1970); a proof for non-Abelian gauge theories is given in S. Joglekar, Ann. Phys. (N.Y.) 108, 223 (1977); 109, 210 (1977).

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[14]A De Rújula, H. Georgi, and H. D. Politzer, Ann. Phys. (N.Y.) 103, 315 (1977).

[15]In particular, the sign of this result is in agreement with Ref. 3. The bag-model estimate of the spin-2, twist-4 correction to F_2^{ep-en} was calculated correctly in Ref. 3; however, in contradiction to our statement there the sign of the result is not that which would account for the observed scaling violations.

[16]After the completion of our work we received copies of the work of E. V. Shuryak and A. I. Vainshtein, Phys. Lett. 105B, 65 (1981) and Novosibirsk Report No. 81-77 (unpublished).

[17]In Ref. 3 we mistakenly identified W_L with W_L', and W_T with W_2.

[18]We thank S. Brodsky for a suggestion regarding potentially important higher-order corrections.

[19]See Ref. 16 for the spin-2 calculation; M. Soldate (unpublished) for spin n.

[20]We thank B. Iijima for this observation.

[21]We thank B. Iijima for pointing out an error in the original version of this proof.