PHYSICAL REVIEW D VOLUME 26, NUMBER 2 15 JULY 1982

Walls bounded by strings

T. W. B. Kibble
Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom

G. Lazarides
The Rockefeller University, New York, New York 10021

Q. Shafi
*Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742 and International Center for Theoretical Physics, Trieste, Italy**
(Received 8 March 1982)

Extended structures consisting of walls bounded by strings appear in some unified gauge theories. The Spin(10) grand-unified-theory model provides the simplest example, provided the symmetry breaking proceeds via SU(4)×SU(2)×SU(2). It is shown that the presence of such structures may cause conflict with standard cosmology.

Extended topological structures like magnetic monopoles, strings, and domain walls can appear[1] in spontaneously broken unified gauge theories. It has been pointed out by various authors[1–5] that domain walls occur in any gauge theory in which a discrete symmetry, which is *not* part of the gauge symmetry, is spontaneously broken. The presence of such domain walls has been shown to be cosmologically unacceptable.

In a recent note[6] we introduced a new type of extended structure that is predicted by some unified gauge theories. It consists of a domain wall bounded by a string. In this paper we discuss these objects in more detail and study their cosmological evolution. Such structures can appear, even though *no* external discrete symmetry is imposed on the gauge theory. Their occurrence is intimately related to the symmetry-breaking pattern of the unifying gauge symmetry. The simplest example of a theory that produces such structures is provided by the breaking of Spin(10) (Refs. 7 and 8) to SU(3)×SU(2)×U(1) via SU(4)×SU(2)×SU(2).[9] The first step of symmetry breaking produces topologically stable Z_2 strings. These form boundaries of domain walls, produced when the second breaking occurs. The domain walls separate vacua that are related by charge conjugation, a discrete symmetry which is contained in Spin(10). [Note that *P* and *CP* invariances are not contained in Spin(10). They can thus be broken explicitly so that no *P* or *CP* domain-wall problem arises.] Other examples of theories that produce such extended structures are easily constructed [for instance,[10] $\mathrm{Spin}(4n+6)\rightarrow \mathrm{Spin}(10)\times\mathrm{Spin}(4n-4)\rightarrow \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(2)\rightarrow\cdots$].

The occurrence of domain walls bounded by strings need not, *a priori,* prove catastrophic. Such a domain wall can disappear, in principle, by the production of a hole bounded by a string on its surface, which then expands with the speed of light. However, the probability that the wall disappears through this tunneling process is found to be utterly negligible. The walls, for all practical purposes, are locally stable. They lose energy mainly by interacting with the surrounding medium. The most optimistic assumption, which is to allow the formation only of the smallest possible walls, leads to the result that the walls disappear before dominating the expansion of the Universe. Their presence does not affect standard cosmology in any essential way. Allowing for bigger walls, which would certainly be the case if a certain amount of supercooling precedes the phase transition that produces them, leads to disaster. The walls never enter the horizon but proceed, instead, to collapse into long-lived black holes that dominate the energy density of the Universe.

We consider the following symmetry-breaking pattern[8] of Spin(10):

$$\mathrm{Spin}(10)\underset{\underline{54}}{\overset{M_x}{\longrightarrow}}\mathrm{Spin}(6)\otimes\mathrm{Spin}(4)\underset{\underline{126}}{\overset{M_R}{\longrightarrow}}[\mathrm{SU}(3)\times\mathrm{SU}(2)]\otimes\mathrm{U}(1)\underset{\underline{10}}{\overset{M_W}{\longrightarrow}}\mathrm{SU}(3)\otimes\mathrm{U}(1)\ . \tag{1}$$

One-loop renormalization-group analysis, neglecting Higgs contributions, gives[8,11] ($\sin^2\theta_W=0.23$, $\Lambda=0.1$ GeV)

$$M_x\simeq 1.9\times 10^{15}\ \mathrm{GeV}\ ,\qquad M_R\simeq 4.4\times 10^{13}\ \mathrm{GeV}\ . \tag{2}$$

Let us discuss the first stage of symmetry breaking. The expectation value of the Higgs 54-plet takes the form

$$\langle\phi_{54}\rangle=y\ \mathrm{diag}(2,\ldots,2,-3,\ldots,-3)\ , \tag{3}$$

where 2 occurs 6 times and -3 occurs 4 times. It is certainly left invariant by the subgroup $H_0=\mathrm{Spin}(6)\otimes\mathrm{Spin}(4)$ of Spin(10). Let $T_{ij}=\sigma_{ij}/2=[\Gamma_i,\Gamma_j]/4i$ ($i,j=1,\ldots,10$) be the generators of Spin(10), where Γ_i ($i=1,\ldots,10$) are the generalized Dirac matrices in 10 dimensions. The subgroup H_0 of Spin(10) is then generated by T_{ab} ($1\le a,b\le 6$) and $T_{\alpha\beta}$ ($7\le\alpha,\beta\le 10$). Note that H_0 is not a global direct product of Spin(6) and Spin(4) since these groups have the following nontrivial element in common,

$$\exp(2\pi i T_{ab})=\exp(2\pi i T_{\alpha\beta})=-1\qquad(1\le a\ne b\le 6\ ,\ 7\le\alpha\ne\beta\le 10)\ . \tag{4}$$

This leads[12] to the appearance of topologically stable Z_2 magnetic monopoles.

A closer examination reveals that the stability group of $\langle\phi_{54}\rangle$ is bigger than $\mathrm{Spin}(6)\otimes\mathrm{Spin}(4)$. This may be seen as follows. Consider the following element of Spin(10):

$$\exp(i\theta T_{a\alpha})\ ,\quad 1\le a\le 6\ ,\ 7\le\alpha\le 10\ , \tag{5}$$

which represents a rotation by angle θ in the $a\alpha$ plane. The $a\alpha$ submatrix of $\langle\phi_{54}\rangle$ transforms under this rotation as follows:

$$\begin{bmatrix}2 & 0\\ 0 & -3\end{bmatrix}\to\begin{bmatrix}-\frac{1}{2}+\frac{5}{2}\cos 2\theta & -\frac{5}{2}\sin 2\theta\\ -\frac{5}{2}\sin 2\theta & -\frac{1}{2}-\frac{5}{2}\cos 2\theta\end{bmatrix}. \tag{6}$$

We see that $\langle\phi_{54}\rangle$ is left invariant under

$$\exp(i\pi n T_{a\alpha})\ ,\quad n\in Z\ . \tag{7}$$

It is easy to verify that successive rotations in various $a\alpha$ planes that leave $\langle\phi_{54}\rangle$ invariant can always be written as products of group elements from (7). Since $\exp(2\pi i T_{a\alpha})=-1\in H_0=\mathrm{Spin}(6)\otimes\mathrm{Spin}(4)$ and

$$\exp(i\pi T_{a\alpha})\exp(i\pi T_{\alpha\beta})=\exp(i\pi T_{a\beta})\ ,\quad \alpha\ne\beta\ ,$$
$$\exp(i\pi T_{ba})\exp(i\pi T_{a\alpha})=\exp(i\pi T_{b\alpha})\ ,\quad a\ne b\ , \tag{8}$$

all the group elements in (7) can be reduced by transformations that belong to H_0 to the single group element

$$\exp(i\pi T_{67})=i\sigma_{67}\notin H_0\ . \tag{9}$$

Note that $(i\sigma_{67})\,(i\sigma_{23})=C$, the charge-conjugation operator. Now $i\sigma_{67}$ anticommutes with $i\sigma_{a6}$ ($a=1,2,\ldots,5$) and with $i\sigma_{7\alpha}$ ($\alpha=8,9,10$), and it commutes with all other $i\sigma$'s. It then follows that, for every $g\in H_0$, there exists a $g'\in H_0$ such that

$$g(i\sigma_{67})=(i\sigma_{67})g'\ . \tag{10}$$

Consequently, the stability group H of $\langle\phi_{54}\rangle$ contains not only the elements of H_0 but also the set of elements

$$K=\{g(i\sigma_{67})\ ,g\in H_0\}\ . \tag{11}$$

Since H_0 and K have no elements in common, H is not a connected group. It consists of two connected components, H_0 and K.

The fact that H is not connected implies the appearance of topologically stable Z_2 strings after the first step in the symmetry breaking of Spin(10). This is because

$$\pi_1(\mathrm{Spin}(10)/H)=\pi_0(H)=Z_2\ . \tag{12}$$

Note that these strings are unrelated to the Z_2 monopoles that are also produced in the first stage of the symmetry breaking.

Consider an open string along the z axis. Its thickness is of order ${M_x}^{-1}$ and it has mass per unit length σ of order ${M_x}^2/\alpha_G$. The expectation value $\langle\phi_{54}\rangle$ vanishes along the z axis. For $r=(x^2+y^2)^{1/2}\gg {M_x}^{-1}$, the Higgs $\underline{54}$ lies in the vacuum manifold and may be taken in the form

$$\langle\phi_{54}\rangle(\varphi)=\exp\left[\frac{i\varphi}{2}(T_{23}+T_{67})\right]\langle\phi_{54}\rangle(\varphi=0)\ , \tag{13}$$

where φ is the azimuthal angle. As one moves around the string, the expectation value of the Higgs 54-plet gets transformed by a gauge transformation which continuously interpolates between the identity element and the charge conjugation operator C.

At the second stage of symmetry breaking the 126 acquires a vacuum expectation value which breaks H down to $SU(3)\times SU(2)\times U(1)$. (A Z_2 subgroup of H_0 is also left unbroken by $\langle\phi_{126}\rangle$. This leads to the formation of new Z_2 strings which are unrelated to the Z_2 strings that we previously considered. These new strings are not discussed in this paper.) This transition also breaks the discrete charge-conjugation symmetry and has a profound effect on the Z_2 strings that we previously discussed. The strings become boundaries of domain walls that separate vacua related by charge conjugation. This may be seen as follows. Consider a string that lies along the z axis. Assume that $\langle\phi_{126}\rangle$ lies along the $(\overline{10},1,3)$ direction on the semi-infinite plane $y=0^+, x\geq 0(\varphi=0)$. Then

$$\langle\phi_{126}\rangle(\varphi)=\exp\left[\frac{i\varphi}{2}(T_{23}+T_{67})\right]\langle\phi_{126}\rangle(\varphi=0)\;,\quad 0\leq\varphi\leq 2\pi\;, \tag{14}$$

which means that $\langle\phi_{126}\rangle$ $(\varphi=2\pi)$ lies along the $(10, 3, 1)$ direction, which is the charge conjugate of the $(\overline{10},1,3)$ direction. Thus, in contrast to $\langle\phi_{54}\rangle$, the expectation value of the Higgs 126 does not return to its original value after a full rotation around the string. This implies the existence of a physical domain wall along the $y=0$, $x\geq 0$ semi-infinite plane. The wall is bounded by the string along the z axis and separates vacua related by charge conjugation. Its thickness is of order M_R^{-1} and its mass per unit area μ is of order M_R^3/α_G. Note that $\langle\phi_{126}\rangle=0$ on the midplane of the wall $y=0$, $x\geq 0$. (It should be kept in mind that the breaking of the discrete charge-conjugation symmetry may also produce domain walls without boundaries.)

The domain walls, with or without boundaries, are not topologically stable. This is because charge-conjugation symmetry is part of Spin(10) and can be continuously connected to the identity in the full Spin(10) space. Quantum and/or thermal fluctuations can destroy the walls locally through the creation of holes bounded by Z_2 strings. However, we now show that the probability of hole creation is utterly negligible so that the walls can be taken to be locally stable.

Consider a plane wall along the xy plane at zero temperature. The probability per unit area per unit time to create a hole (bounded by string) on the wall due to quantum fluctuations is given[13] in the semiclassical approximation by

$$\frac{dP}{dA\,dt}\sim\frac{M_R{}^3}{\alpha_G}\exp(-S_0)\;. \tag{15}$$

Here S_0 is the Euclidean action of the bounce solution which corresponds to this tunneling process. The creation and subsequent disappearance of the hole causes the action to change by

$$S=\Sigma\sigma-V\mu\;, \tag{16}$$

where Σ is the invariant area of the closed world surface described by the string and V is the invariant three-volume described by the hole. Minimization of S can be achieved by a spherical world surface of invariant radius $R_0=2\sigma/\mu$. The corresponding change in the action is then

$$S_0=\frac{16\pi\sigma^3}{3\mu^2}\sim\left[\frac{M_x}{M_R}\right]^6\sim 10^{10}\;. \tag{17}$$

Thus,

$$\frac{dP}{dA\,dt}\sim\frac{M_R{}^3}{\alpha_G}\exp(-10^{10})\;, \tag{18}$$

which shows that the probability to create a hole, even on a wall the size of our present Universe, within the age of our Universe is utterly negligible.

The bounce solution considered above also describes the hole creation process at nonzero temperatures, provided $T\lesssim R_0{}^{-1}$. For $T\gg R_0{}^{-1}$ (relevant just after the wall-producing transition at $T_c\sim M_R/g$), one has[14]

$$\frac{dP}{dA\,dt}\sim\frac{M_R{}^3}{\alpha_G}\exp(-E_0/T)\;, \tag{19}$$

where E_0, in our case, is the change in energy due to the presence of a static hole of radius $\frac{1}{2}R_0$ bounded by a Z_2 string. One finds that

$$\frac{E_0}{T}\sim\left[\frac{M_x}{M_R}\right]^3\frac{M_x}{T}\gtrsim 10^6\;. \tag{20}$$

This shows that the probability for hole creation, just after the phase transition that produces the walls, is extremely small. For all practical purposes, the walls are locally stable.

The extended structures we have been considering presumably can be produced in the very early Universe. As the Universe expands and cools after the big bang, it undergoes a series of phase transitions. For $T \gtrsim M_x/g$, the Spin(10) gauge symmetry is unbroken. For $T \sim M_x/g$, Spin(10) breaks down to H and a network of topologically stable Z_2 strings is produced. The scale of the string network grows[3] with the time and becomes of order t at $t \sim t_*^s$, where $t_*^s \simeq 2\times 10^{-35}$ sec as computed[15] from

$$t_*^s \simeq \frac{3\alpha_G \sigma}{32\pi}\frac{M_p^2}{M_x^2}, \quad \sigma \sim \frac{1}{T[\ln(\frac{T}{M_x})]^2} . \tag{21}$$

Here $M_P = 1.2\times 10^{19}$ GeV is the Planck mass. For $t \gtrsim t_*^s$, friction due to the relativisitic gas of particles can be neglected, the strings move with relativistic velocities, and the scale of the string network is essentially given by the size of the horizon.[3]

Let us next discuss the phase transition form H to SU(3)$\times$SU(2)$\times$U(1). Suppose that the phase transition takes place without supercooling at some critical temperature $T_c \sim M_R/g$. For definiteness, we take $T_c \simeq 5\times 10^{13}$ GeV which corresponds to cosmic time $t_c \simeq 6\times 10^{-35}$ sec. Since $t_c > t_*^s$, friction effects on the strings are negligible and the scale of the string network at this phase transition is of order t_c. The phase transition leads to the production of domain walls which terminate on the strings.

The most optimistic assumption one can make is that the production of the walls at t_c forces the string network to rearrange itself in such a way that one ends up with one wall per horizon volume t_c^3, bounded by a closed string of length $\sim t_c$ (closed walls of size $\lesssim t_c$ may also be produced). The walls at t_c are to be thought of as random surfaces with persistence area $l_c^2 \gtrsim (gM_R)^{-2}$. The total area of a typical wall is $\sim t_c^3/l_c$, so that the energy density in the walls at t_c is given by

$$\rho_w(t_c) \simeq \frac{M_R^3}{\alpha_G l_c} . \tag{22}$$

This is smaller than the radiation density $\rho_r(t_c)$.

The persistence area l^2 grows[3] rapidly immediately after the phase transition and ρ_w/ρ_r drops sharply. For $t \gg t_c$, the growth of l^2 is given[3] by

$$l^2 \simeq t^2 \left[\frac{t}{t_*^W}\right] , \tag{23}$$

where

$$t_*^W \simeq \frac{3\alpha_G \eta_0}{32\pi\eta_3}\frac{M_P^2}{M_R^3} \simeq 8.5\times 10^{-32}\text{ sec} , \tag{24}$$

$\eta_0 = 13.25$ is the effective number of massless degrees of freedom which are reflected by the walls, and $\eta_3 = 106.75$ is the effective number of massless degrees of freedom in the SU(3)$\times$SU(2)$\times$U(1) phase. The size of the wall boundary d grows as $t^{1/2}$ while the persistence area l^2 becomes of order d^2 at $t = t_f \simeq (t_*^W t_c)^{1/2} \simeq 2\times 10^{-33}$ sec. For $t_c \ll t \lesssim t_f$,

$$\frac{\rho_w}{\rho_r}(t) \simeq \frac{32\pi}{3\alpha_G}\left[\frac{M_R}{M_P}\right]^3 \left[\frac{t_*^W t}{t_P^2}\right]^{1/2} . \tag{25}$$

At $t = t_f$, the size of a typical wall is equal to $d_f = (t_f t_c)^{1/2} \simeq 3.5\times 10^{-34}$ sec and the ratio of the wall mass to the string mass is given by

$$\frac{M_R^3}{M_x^2} d_f \simeq 12 . \tag{26}$$

Clearly, the wall mass dominates over the string mass.

Due to friction from the surrounding medium, walls of size d and curvature $\sim d^{-1}$ acquire[3] a limiting velocity $v_l \sim t_d/d$, where $t_d \simeq t^2/t_*^W$ is the viscous dissipation time. At $t = t_f$, $v_l t_f \sim d_f$ and the walls can contract to a point within one expansion time. The energy loss due to friction during such a contraction of a typical wall is $\Delta E \sim F d_f$, where

$$F \sim v_l \rho_0 d_f^2 \tag{27}$$

is the force of friction on the wall. Here

$$\rho_0 = \frac{\pi^2}{30}\eta_0 T^4 \tag{28}$$

is the radiation energy density of the massless particle species which are reflected by the walls. It is easy to see, from Eqs. (24), (27), and (28), that

$$\Delta E \sim \frac{M_R^3}{\alpha_G} d_f^2 . \tag{29}$$

This means that the walls lose their energy by friction and disappear at a cosmic time $t \sim t_f$. From Eq. (25), we see that the Universe was never wall dominated. Therefore, in the case where the transition from H to SU(3)$\times$SU(2)$\times$U(1) takes place without supercooling, the presence of the walls bounded by strings does not seem to affect standard cosmology in any essential way.

Matters may become disastrous if the phase transition from H to SU(3)$\times$SU(2)$\times$U(1) proceeds only after supercooling by two or more orders of magnitude. The walls in this case never enter the horizon. They become gravitationally unstable and collapse into long-lived black holes that dominate the energy density of the Universe, thereby destroying standard cosmology. In the model we have been discussing, this scenario occurs for $T_c' \leq 1.4\times10^{11}$ GeV, where T_c' is the temperature at which the transition from H to SU(3) $\times$SU(2)$\times$U(1) takes place. This temperature corresponds to cosmic time $t_c' \geq 1.7\times10^{-33}$ sec. For $T < T_E \simeq 3.2\times10^{13}$ GeV ($t > t_E \simeq 1.5\times10^{-34}$ sec), the vacuum energy density of the phase in which H is unbroken dominates over the radiation energy density and the Universe undergoes an exponential expansion. As a consequence one finds that the scale d_c' of the string network at t_c' is much greater than t_c'. In particular, for $T_c' \leq 1.4\times10^{11}$ GeV, $d_c' \geq 3.4\times10^{-32}$ sec. We assume that the walls produced at t_c' terminate on closed strings of length d_c' and therefore extend over several apparent horizons ($\sim t_c'$). For $t > t_c'$, the length d of the boundary of a wall follows the cosmological expansion. Thus,

$$d \simeq d_c' \left[\frac{t}{t_c'} \right]^{1/2} . \tag{30}$$

The walls do not enter this horizon at least until the cosmic time $t_H \simeq 6.8\times10^{-31}$ sec. Since $t_H > t_*^W$, the persistence area of the walls at t_H is $\sim t_H{}^2$, and there is effectively one wall piece per horizon. This gives

$$\frac{\rho_w}{\rho_r}(t_H) = \frac{32\pi}{3\alpha_G} \left[\frac{M_R}{M_P} \right]^3 \frac{t_H}{t_P} = 1 . \tag{31}$$

For $t > t_H$, the Universe becomes wall dominated. The walls are conformally stretched and one finds[2,4] that

$$\rho_w \propto a^{-1}, \quad a \propto t^2 , \tag{32}$$

where a is the cosmic scale factor. The horizon grows as

$$a(t) \int_{t_H}^{t} \frac{dt'}{a(t')} \sim t^2/t_H . \tag{33}$$

Since the cosmic expansion keeps up with the horizon, the walls never enter the horizon and the Universe remains wall dominated. At $t \simeq 4\times10^{-30}$ sec the wall piece within the horizon enters its Schwarzschild radius and collapse into black holes starts.[5] These black holes have mass greater than 5.6×10^{33} GeV and their lifetime $\tau \gtrsim 10^4$ sec. Clearly, they lead to a cosmology totally different from ours.

To summarize, we have shown that a new type of extended structure, namely walls bounded by strings, is predicted in some unified gauge theories. The presence of such structures may cause conflict with standard cosmology.

ACKNOWLEDGMENTS

We are grateful to A. Everett and A. Vilenkin for drawing our attention to the fact that the walls lose energy predominantly by friction. One of us (Q.S.) wishes to thank Professor W. Greenberg, Professor J. C. Pati, and Professor J. Sucher for hospitality at the Center for Theoretical Physics of the University of Maryland. The research of G. L. was partially supported by the Department of Energy, Grant No. DE-AC02-82ER40033. B000.

---

*Address from May 1, 1982.

[1]See, for instance, T. W. B. Kibble, Phys. Rep. 67, 183 (1980).

[2]Ya. B. Zeldovich, I. Yu. Kobzarev, and L. B. Okun, Zh. Eksp. Teor. Fiz. 67, 3 (1974) [Sov. Phys. JETP 40, 1 (1975)].

[3]T. W. B. Kibble, J. Phys. A 9, 1387 (1976).

[4]G. Lazarides, Q. Shafi, and T. Walsh, Nucl. Phys. B195, 157 (1982); P. Langacker, Phys. Rep. 72, 185 (1981).

[5]A. Vilenkin, Phys. Rev. D 23, 852 (1981).

[6]T. W. B. Kibble, G. Lazarides, and Q. Shafi, Phys. Lett. B (to be published).

[7]H. Georgi, in *Particles and Fields—1975,* proceedings of the 1975 Meeting of the Division of Particles and Fields of the American Physical Society, edited by Carl E. Carlson (AIP, New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) 93, 193 (1975); M. Chanowitz, J. Ellis, and M. K. Gaillard, Nucl. Phys. B128, 506 (1977).

[8]G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. B181, 287 (1981).

[9]J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974).

[10]M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity,* proceedings of the Stony Brook Workshop, edited by P. Van Nieuwenhuizen and D. Z. Freedman (North-Holland, New York, 1979); F. Wilczek and A. Zee, Phys. Rev. D 25, 553 (1982).

[11]F. Bordi, R. Holman, and C. W. Kim, Phys. Lett. 106B, 58 (1981).

[12]G. Lazarides, M. Magg, and Q. Shafi, Phys. Lett. 97B, 87 (1980).

[13]S. Coleman, Phys. Rev. D 15, 2929 (1977).

[14]A. D. Linde, Phys. Lett. 70B, 306 (1977).

[15]A. Everett, Phys. Rev. D 24, 858 (1981).