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# Many-body forces and the structure of hadrons

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A collective-mode description of the dynamical forces among constituents of a hadron by means of distinguishable quasiparticles leads to confinement. Therefore, the confining forces among (Fermi or Bose) constituents are purely a manifestation of the correlations resulting from the statistics of the quasiparticles. Nonetheless, the distinguishable quasiparticles behave in many respects like an ideal Bose (not Fermi) gas.

The proof (or disproof) that quarks are permanently confined in hadrons remains as one of the outstanding problems for a non-Abelian gauge theory. The desire for such a proof is based on the negative results of recent free-quark searches.[1] Of course, the concept that hadrons are composed of more fundamental objects is motivated by the attempt to describe their structure. One important feature of hadrons determined by cross sections is that of hadron size. Now solutions of the classical Euler-Lagrange equation for non-Abelian gauge theories give rise to many types of topological excitations.[2] It is argued that quark confinement is due to a sort of condensation of these topological excitations. Unfortunately, it is not yet clear which one of the already known topological excitations, if any, actually contributes to the confinement mechanism.[2] It is argued also that quark confinement is a consequence of an exponential hadronic spectrum which results in a higher-order phase transition for hadronic matter.[3] The latter confining mechanism occurs in our conception of a particle.[4]

In a model proposed some years ago,[4] an ideal gas of distinguishable quasiparticles constitutes a particle and confinement or localization follows from the strictly classical statistics of the quasiparticles. Thus the dynamical forces confining or localizing the constituents of a hadron are described by collective modes, viz., noninteracting distinguishable quasiparticles, which possess the statistics which gives rise to the Gibbs paradox in statistical mechanics.[4] Now interparticle forces may be viewed as arising wholly from the statistics of noninteracting elementary excitations, which behave as quasiparticles moving in the volume occupied by the body and having definite energies and momenta. For instance, in an ideal gas of elementary excitations with Fermi (Bose) statistics where the antisymmetrical (symmetrical) nature of the states leads to an effective "repulsion" ("attraction") between the particles. Of course, in our case, the remarkable feature is that the dynamical forces among constituents of a hadron, giving rise to confinement or localization, can be described purely by the correlations associated with the statistics of the collective modes of a hadron.

In what follows, we shall find the forces between the confined constituents inside a hadron—either Fermi or Bose constituents—by introducing an explicit duality transformation.[5] Thus, the confined distinguishable quasiparticles are identified with topological excitations which in turn describe the confinement of local excitations, viz., the Fermi or Bose constituents of a hadron. We also find that dynamical variables of distinguishable quasiparticles are related actually to boson dynamical variables and thus provide a field-theoretic foundation to the quantization of our topological excitations.

Consider an ideal gas of distinguishable quasiparticles with masses given by the single-particle hadron-mass spectrum $\rho(m)$. The $i$th type of quasiparticle have mass $m$, momenta $\vec{p}$, energies

$$E_i(\vec{p}) = (c^2p^2 + m^2c^4)^{1/2} \quad ,$$

and their number is not conserved. Thus, the grand canonical partition function with chemical potential $\mu_i = 0$ is given by [4]

$$Z = \prod_i \left\{ 1 - \sum_{\vec{p}} \exp[-E_i(\vec{p})/kT] g_i(\vec{p}) \right\}^{-1} , \qquad (1)$$

where $g_i(\vec{p})$ is the density of levels in the neighborhood of any point $\vec{p}$. The volume $V_0$ of the assembly and the lowest mass $m_0$ of the system determine the maximum temperature $T_0$ which the system can attain[4] and so $T \leq T_0$ with

$$1 = \frac{4\pi V_0 k T_0}{(hc)^3} (m_0 c^2)^2 K_2 \left[ \frac{m_0 c^2}{kT_0} \right] . \qquad (2)$$

where $K_2(x)$ is the modified Bessel function of the second kind with subscript 2. Our model gives rise[4] to $\rho(m) \sim g(m) \exp(mc^2/kT_0)$ as $m \to \infty$ with $g(m) = O(e^{m\epsilon})$ as $m \to \infty$ with $\epsilon > 0$. However, if the asymptotic bootstrap condition of Hagedorn[6] is required, then[4] $\rho(m) \sim m^{-5/2} \exp(mc^2/kT_0)$ as $m \to \infty$. Experimentally it seems that $kT_0 = 160$ MeV and so by (2) the constituents of a hadron are confined[4] in a volume $V_0$ with radius ~1.6 F.[7] Note that the requirement that hadrons have a finite size implies both a maximum temperature and an exponentially rising hadron mass spectrum. Note also that the divergence in the partition function (1) as $T \to T_0$, which gives rise to the confining mechanism, occurs because the number of quasiparticles or topological excitations of at least one given kind can be arbitrarily large. It is interesting that other types of topological excitations which also produce confinement of elementary excitations have this feature in common.[2,3] In our case, the hadron constituents or local excitations are confined because the gauge field or topological excitation is confined.

In order to determine the confining interactions between the constituents, we have to analyze the effect of the statistics of the quasiparticles as contained in (1). Now (1) can be written as

$$Z=\prod_i \sum_{N_i=0}^{\infty} \left\{ \frac{V_0}{h^3} \exp(\mu_i/kT) \times \int \exp[-E_i(\vec{p})/kT]\, d\vec{p} \right\}^{N_i} , \quad (3)$$

where the chemical potentials $\mu_i$ are to be equated to zero since the numbers of quasiparticles are not conserved. The general form of the grand canonical partition function[8] for different types of particles is

$$Z=\sum_{N_1=0}^{\infty}\sum_{N_2=0}^{\infty}\cdots \frac{e^{N_1\mu_1/kT}e^{N_2\mu_2/kT}\cdots}{h^{3(N_1+N_2+\cdots)}N_1!N_2!\cdots} \times\int d\vec{r}_1\cdots d\vec{r}_{N_1+N_2+\cdots}\int d\vec{p}_1\cdots d\vec{p}_{N_1+N_2+\cdots} \times\exp\left\{-\sum_{i=1}^{N_1+N_2+\cdots} E_i(\vec{p})/kT\right\}\exp[-u_{N_1+N_2+\cdots}(\vec{r}_1,\ldots,\vec{r}_{N_1+N_2+\cdots})/kT] \quad (4)$$

where $u_{N_1+N_2+\cdots}(\vec{r}_1,\ldots,\vec{r}_{N_1+N_2+\cdots})$ is the interaction potential of $N_1$ particles of mass $m_1$, etc., and it includes the single-particle outside potential. [Note that for an ideal gas of distinguishable quasiparticles $u_{N_1+N_2+\cdots}\equiv 0$, the factors $N_1!\,N_2!\cdots$ do not appear in the denominator of (4), and so (4) reduces to (3).] We can identify (3) and (4) for all values of the chemical potentials $\mu_i$ provided

$$V_0{}^N=\frac{1}{N_1!N_2!\cdots}\int_{V_0} d\vec{r}_1\cdots\int_{V_0} d\vec{r}_N \exp[-u_N(\vec{r}_1,\ldots,\vec{r}_N)/kT] \quad (5)$$

with $N=N_1+N_2+\cdots$. It is important to remark that the particles newly introduced by (4) as constituents of the hadron, with interparticle interactions given by (5), possess the same masses as the quasiparticles but are actually either fermions or bosons but obey approximate Boltzmann statistics since $kT \gtrsim 140$ MeV.

It is evident that one need not assume, as done in Ref. 4, that the quasiparticle mass spectrum $\rho(m)$ be identical to that of observed physical particles for all values of $m$ in order to obtain the above results and those that follow. One may assume, instead, that the low-mass Fermi or Bose constituents are quarks and gluons and the resulting higher-mass states—recall that $\rho(m)$ must grow exponentially as $m\to\infty$—represent hadronic bound states of such objects. Thus our model is quite consistent with quarks as Fermi constituents of a hadron confined by the dynamical attractions provided by the many-body potentials satisfying (5).

The identification of (3) and (4) represents a duality transformation, albeit not self-dual. It maps a theory with zero coupling constant, an ideal gas of quasiparticles or topological excitations, into another theory with nonzero coupling constants, a theory of quarks interacting with gluons, say. It is important to emphasize that our confining mechanism by means of distinguishable quasiparticles implies that the quasiparticle mass spectrum must grow exponentially as the mass increases. Our model does not fix any specific relationship between the quasiparticle masses and that of observed elementary particles.

We have certainly not solved for $u_N(\vec{r}_1,\ldots,\vec{r}_N)$ with the aid of (5). However, (5) does exclude a single-particle potential or mean field for the confining many-body potentials. Suppose

$$u_N(\vec{r}_1,\ldots,\vec{r}_N)=\sum_{i=1}^{N} u(\vec{r}_i) ,$$

then (5) gives

$$N_1!N_2!\cdots=\left\{\frac{1}{V_0}\int_{V_0} d\vec{r}\exp[-u(\vec{r})/kT]\right\}^N$$

and since the right-hand side depends on $N_1, N_2, \ldots$, etc., only through $N=N_1+N_2+\cdots$, while the left-hand side has explicit $N_1, N_2, \ldots$ dependence; hence, a mean-field potential cannot satisfy (5). Notice that a trivial solution to (5) is a many-body force such that it vanishes outside the confining volume $V_0$ and is given by the constant

$$-u_N(\vec{r}_1, \ldots, \vec{r}_N)/kT = \ln N_1! + \ln N_2! + \cdots \quad (6)$$

inside $V_0$, where $N = N_1 + N_2 + \cdots$. Notice that (6) does not represent a temperature-dependent potential since $kT \simeq 160$ MeV. Also, two constituents of the same mass in the presence of a third constituent of a different mass does not give rise to a three-constituent interaction, the two-body potential is $u_2 \simeq -110$ MeV. However, if all three constituents are identical, then an attractive three-body interaction with uniform strength $-290$ MeV exists inside the confining volume $V_0$. It is interesting that a purely repulsive potential, viz., $u_N(\vec{r}_1, \ldots, \vec{r}_N) > 0$ for $\vec{r}_i$ inside $V_0$, can never be a solution of (5); attractive interactions are needed. Some of these results are reminiscent of the bag model of hadrons.[9]

Our distinguishable quasiparticles or topological excitations are solutions of the classical Klein-Gordon equation. We want to quantize our classical solutions; however, it is clear that the usual quantization of the Klein-Gordon equation will not do. (The process of second quantization is intimately based on the notion of identical particles being indistinguishable and either bosons or fermions.) That is to say, our quasiparticles are distinguishable and, thus, not obviously amenable to the methods of quantum field theory. However, we shall now show that the dynamical variables of distinguishable quasiparticles are related actually to boson dynamical variables.

The probability of finding $n_i(\vec{p})$ quasiparticles of the $i$th type with momentum $\vec{p}$ and so on is given by

$$P = \prod_i \left[1 - \sum_{\vec{p}} g_i(\vec{p}) \exp[-E_i(\vec{p})/kT]\right] \left[\sum_{\vec{p}} n_i(\vec{p})\right]! \prod_{\vec{p}} \frac{\{g_i(\vec{p}) \exp[-E_i(\vec{p})/kT]\}^{n_i(\vec{p})}}{n_i(\vec{p})!} . \quad (7)$$

From (7) we obtain the mean occupation number

$$\bar{n}_i(\vec{p}) = \sum n_i(\vec{p}) P = \frac{g_i(\vec{p}) \exp[-E_i(\vec{P})/kT]}{\left[1 - \sum_{\vec{p}} g_i(\vec{p}) \exp[-E_i(\vec{p})/kT]\right]} , \quad (8)$$

where the sums are over all sets $n_i(\vec{p})$. Similarly for the pair correlation function

$$\overline{n_i(\vec{p}) n_j(\vec{p}\,')} = \sum n_i(\vec{p}) n_j(\vec{p}\,') P = (1 + \delta_{ij}) \bar{n}_i(\vec{p}) \bar{n}_j(\vec{p}\,') + \delta_{ij} \delta_{\vec{p},\vec{p}\,'} \bar{n}_i(\vec{p}) . \quad (9)$$

Notice that there is no correlation between quasiparticles with different masses. However, the correlation between quasiparticles of the same mass and momentum is just as in an ideal Bose gas since

$$\overline{n_i(\vec{p}) n_j(\vec{p}\,')}|_{\mathrm{BE}} = \bar{n}_i(\vec{p}) \bar{n}_j(\vec{p}\,') + \delta_{ij} \delta_{\vec{p},\vec{p}\,'} \bar{n}_i(\vec{p}) [1 + \bar{n}_i(\vec{p})] . \quad (10)$$

Consequently, even though the single-particle distribution (8) is of the Maxwell-Boltzmann type, nevertheless, the two-particle distribution (9) resembles that of an ideal Bose gas. Notice, in particular, that for quasiparticles of the same mass but different momenta, we have that the probability of two quasiparticles being in the same mass state is greater than with Bose statistics. Thus the attraction between constituents is enhanced sufficiently to give confinement. The close analogy to the Bose statistics can be seen further by studying the fluctuations in the total number of quasiparticles of a given type. From (7) one has for the probability $P(N)$ of finding $N$ quasiparticles that

$$P(N) = \sum_{N_1 N_2 \ldots} P_1(N_1) P_2(N_2) \ldots , \quad \sum_i N_i = N ,$$

where the probability $P_i(N_i)$ of finding $N_i$ quasiparticles of the $i$th type is given by the geometric distribution

$$P_i(N_i) = \frac{1}{! + \bar{N}_i} \left[\frac{\bar{N}_i}{1 + \bar{N}_i}\right]^{N_i} \quad (12)$$

with $\bar{N}_i = \sum_{\vec{p}} \bar{n}_i(\vec{p})$. From (12) one has that

$$\overline{N_i N_j} = \bar{N}_i \bar{N}_j + \delta_{ij} \bar{N}_i (1 + \bar{N}_i) . \quad (13)$$

Consequently, from (11) and (12) we have that the total number of quasiparticles of the $i$th type behaves, vis-á-vis other type of quasiparticles, exactly as an ideal Bose gas. More precisely, the set of operators $\hat{N}_i = \sum_{\vec{p}} \hat{n}_i(\vec{p})$ is completely equivalent to the set of occupation number operators $\hat{n}(\vec{p})$ of an assembly of bosons with no interaction between them. Notice that the mass of the quasiparticle is identified with the momentum of the boson. This equivalence may be rather important for obtaining a quantum field theory whose description of the structure of hadrons will result in the model postulated in Ref. 4. However, we have found no indication of this equivalence in current field-theoretic efforts with non-Abelian gauge theories.

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[7]This large radius for hadrons is associated actually with a leveling of hadron-hadron total cross sections at high energies. For $kT_0 = m_{\pi^\pm} c^2$ the radius is 1.87 F and so $\sigma_{\rm tot} = 220$ mb at the square of the total center-of-mass energy of $10^{11}$ $\rm GeV^2$ [M. Alexanian and F. Mejía-Lira, Phys. Rev. D 11, 716 (1975)].

[8]See, for instance, G. E. Uhlenbeck and G. W. Ford, *Lectures in Statistical Mechanics* (American Mathematical Society, Providence, R. I., 1963).

[9]See, for instance, the article of J. Kuti in Ref. 2.