Strings at finite temperature and deconfinement

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We demonstrate two general properties of string models at finite temperature. About zero temperature, the leading decrease in the effective string tension is calculable in any dimension d. For all temperatures, string theories are soluble in an expansion about infinite d, exhibiting a finite temperature T_c at which the effective string tension vanishes, with $T_c \sim d^{-1/2}$.

The existence of a phase transition in finite-temperature QCD between a phase of hadrons and one of "deconfined" quarks and gluons has long been a subject of intense speculation.[1] Such a transition at a finite temperature T_c has been demonstrated in strong-coupling lattice gauge theories.[2] These studies were crucial in pointing out that, for a purely gluonic SU(N) gauge theory, the phase transition could be rigorously characterized by the breaking of a global Z_N symmetry above T_c. This naturally leads to the supposition that the universality class of a d-dimensional SU(N) gauge theory about T_c is the same as that of a $(d-1)$-dimensional Z_N spin system.[2,3] By its nature, this argument does not address such important but nonuniversal relations as that between T_c and quantities in the zero-temperature theory. To answer such questions without resorting to numerical simulations, some sort of effective model for the infrared structure of QCD must be adopted.

In this light, we consider here the properties of flux sheets at finite temperature. We work with a Euclidean theory at a finite temperature T, so out of d dimensions we require all (bosonic) fields to be periodic with period $\beta = T^{-1}$ in one ("time") direction, with no restrictions on the remaining $(d-1)$ ("spatial") dimensions. At $T \neq 0$, confinement is characterized by a flux sheet parallel to the time axis with spatial extent R, as sketched in Fig. 1.

For a pure gauge theory, like Nambu,[4] we neglect the underlying dynamics which leads to creation of the flux sheet, effects from its finite thickness, etc., and calculate only a statistical average W over fluctuating surfaces:

$$W = \int D\bar{\phi}(\bar{s}) \exp\left[-\sigma \int d^2 s \det(\delta_{ab} + \partial_a \bar{\phi}\, \partial_b \bar{\phi})\right] . \tag{1}$$

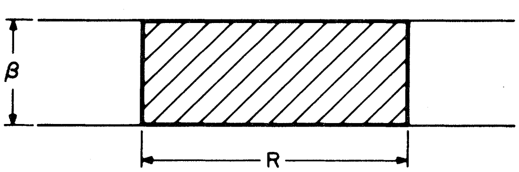

FIG. 1. A flux sheet used to measure confinement at finite temperature.

The functional integral W is a sum over all surfaces with a given boundary, weighted by the exponential of their area times the string tension σ; σ has the dimensionality of (mass)^2. The dimensions of a surface are parametrized by the two-dimensional vector $\bar{s}$ [$\bar{s} = (r,t)$, $a,b = r$ or t]. Ignoring all fluctuations parallel to the flat surface of Fig. 1, the area of an arbitrary surface is found from $\bar{\phi}(\bar{s})$, a $(d-2)$-dimensional vector which measures the deviation from planarity. Along the boundary of the sheet in Fig. 1, $\bar{\phi}(\bar{s})$ must satisfy

$$\bar{\phi}(0,t) = \bar{\phi}(R,t) = 0, \quad \bar{\phi}(r,0) = +\bar{\phi}(r,\beta) \quad . \tag{2}$$

We are only concerned with properties intrinsic to a finite temperature and not the finite spatial extent of the sheet, so we uniformly assume that $R \gg \beta$. Our results can be easily summarized. If $A\,(=R\beta)$ is the area of the flat surface, then for large A,

$$W \sim e^{-\sigma(T)A} \quad , \tag{3}$$

where terms dependent on the linear dimensions of the sheet are ignored. Confinement is indicated when the effective string tension $\sigma(T) \neq 0$,[5] so the temperature T_c at which deconfinement occurs is defined as the lowest temperature such that $\sigma(T_c) = 0$.

We do not detail how we calculate $\sigma(T)$, since this follows easily from similar calculations at zero temperature[6-8] with surfaces greater in time than spatial extent—$\beta \gg R$ instead of $R \gg \beta$—which give an effective string tension $\sigma(R)$. The only care which must be taken is to impose the proper boundary conditions as in Eq. (2).

Before proceeding to our results, we remark that it is possible to define W not only with the action of Eq. (1), but from generalized actions with a more limited group of symmetry transformations.[9] As for similar results with $\sigma(R)$,[7,8] *none* of our results will be affected by this change. Hence our results are truly universal to any string model.

We begin in the limit of low temperature, $T \ll \sqrt{\sigma}$. In this instance, all deviations from planarity will be small, so that in an arbitrary dimension d the action of Eq. (1) can be expanded to lowest order in $\vec{\phi}^2$. Performing the quadratic integration over $\vec{\phi}$ yields

$$\sigma(T) \underset{T \ll \sqrt{\sigma}}{\sim} \sigma - \frac{\pi(d-2)}{6} T^2 + \cdots \quad . \tag{4}$$

This decrease in $\sigma(T)$ is precisely analogous to the universal Coulomb correction to a linear potential found in $\sigma(R)$.[6–8]

As the temperature increases, calculation cannot be performed for arbitrary d. Mimicking a calculation of $\sigma(R)$ by Alvarez,[8] we calculate $\sigma(T)$ about infinite d in a d^{-1} expansion. Around $d=+\infty$, it is possible to expand about a nontrivial stationary point of the action in Eq. (1). The result is

$$\sigma(T) \underset{d\to\infty}{=} \sigma[1-(T/T_c)^2]^{1/2} \quad , \tag{5}$$

with

$$\frac{T_c^{\,2}}{\sigma} \underset{d\to\infty}{=} \frac{3}{\pi d} + O(d^{-2}) \quad . \tag{6}$$

If t denotes the reduced temperature $t=(T_c-T)/T_c$, Eq. (5) predicts a critical exponent ν:

$$\frac{\sigma(T)}{\sigma} \underset{T\to T_c}{\sim} t^{\nu}, \quad \nu \underset{d\to\infty}{=} \tfrac{1}{2} + O(d^{-1}) \quad . \tag{7}$$

The properties of the theory near T_c are of considerable interest. Taking W as a thermodynamic system in its own right, it can be shown that the entropy per unit length diverges as $T\to T_c$ like $t^{-1/2}$. Further, the correlation length for $\vec{\phi}$ fluctuations in the spatial direction vanishes as $t^{+1/2}$.[8] This results because the magnitude of $\vec{\phi}$ fluctuations, $\langle \vec{\phi}^2 \rangle$, and their variation in the time direction, $\langle (\partial_t \vec{\phi})^2 \rangle$, are both finite as $T\to T_c$, while their variation in the spatial direction, $\langle (\partial_r \vec{\phi})^2 \rangle$, diverges as $t^{-1/2}$.[8] In other words, when $T\to T_c$, the surfaces which dominate W are those whose transverse extent is limited but which have infinite area, due to these surfaces "wrinkling" without bound in the spatial direction.[10]

As indicated by the form of Eqs. (6) and (7), we believe that our results are not artifacts of a large-d expansion—that there is a finite T_c for all $d>2$ ($T_c=\infty$ when $d=2$: see below). The calculation of effects next to leading order in d^{-1} for large d is presently underway and are straightforward if tedious. Besides the evident worth of computing the numerical values of corrections in d^{-1} to Eqs. (6) and (7), several questions of principle first arise at this order. These include the possible renormalizability of this apparently nonrenormalizable model, whether the d^{-1} corrections for the model of Eq. (1) and generalized string models[9] differ, and the role of the conformal anomaly.[11]

We note in passing that Eq. (6) has been derived, with *precisely* the same numerical coefficient, in the framework of dual models.[12] It is most unexpected to find that a result derived from counting the degeneracy of dual model states agrees to leading order in d^{-1} with a Euclidean theory of strings.[13]

By Eq. (6), T_c vanishes in the limit of infinite d. A natural question is then: As d is lowered, for what d does T_c first diverge? Because in the string model the dimension d enters *only* as the number of transverse fluctuations ($=d-2$), the T_c^2/σ of Eq. (6) is properly an expansion, order by order, not in d^{-1} but in $(d-2)^{-1}$. The attendant conclusion that $T_c=\infty$ when $d=2$ is true not only for the string theory [where its statement is somewhat misleading since, in $d=2$, $\sigma(T)=\sigma$ for all T anyway], but for $d=2$ gauge theories as well.[14]

More heuristic understanding of the phase transition in SU(N) gauge theories can also be gained. In a bag model of hadronic structure,[15] the string tension is related to the fine-structure constant of the gauge group α and the bag constant b by $\sigma \sim \sqrt{\alpha b}$. Consequently, an effective string tension $\sigma(T)$ could result from an effective bag constant $b(T)$, where from Eq. (7), $b(T)\sim t^{2\nu}$ as $T\to T_c$: the phenomenological implications of such a temperature-dependent bag constant have been discussed before.[16]

Lastly, the predictions of finite-temperature string theories can be compared directly to Monte Carlo simulations of lattice gauge theories. The results are summarized in Table I, where only the leading term of Eq. (6) is used to determine $T_c/\sqrt{\sigma}$ in the string theory. Because the string model does not refer to the origin of the flux sheet, it predicts that $T_c/\sqrt{\sigma}$

TABLE I. $T_c/\sqrt{\sigma}$ for a large-d string theory [Eq. (6)] and from Monte Carlo simulations of SU(N) lattice gauge theories.

d	Theory	$T_c/\sqrt{\sigma}$
3	String	0.56
3	SU(2)	0.70 ±0.05[a]
4	String	0.49
4	SU(2)	0.43 ±0.08[b]
4	SU(3)	0.50 ±0.08[c]

[a] Reference 3(a).
[b] Reference 17.
[c] Reference 18.

should be independent of N for a given d, which is true within the errors for $N=2$ and 3 in $d=4$. The qualitative agreement of the Monte Carlo $T_c/\sqrt{\sigma}$ with the large d string theory is not impressive [even if d is replaced by $d-2$ in Eq. (6)], but the important qualitative conclusion—that $T_c/\sqrt{\sigma}$ increases as d decreases—is borne out strikingly. At present, there are no data sufficiently precise to allow the measurement of ν, Eq. (7). The only Monte Carlo simulations which have measured $\sigma(T)$ near T_c are for a $d=3$ SU(2) gauge theory,[3(a)] where at best all this data indicates is that $\sigma(T)$ does vanish as $T\rightarrow T_c$.

We conclude with the hope that our work will stimulate further Monte Carlo studies to determine $T_c/\sqrt{\sigma}$ and ν as functions of N and d.

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[10]This transition has nothing to do with the roughening transition which occurs in strong-coupling lattice expansions: a flux sheet on a lattice must already have undergone such a transition for the W of Eq. (1) to be appropriate. In fact, it has recently been argued [F. Green, Northeastern Report No. NUB 2556, 1982 (unpublished)] that in a Euclidean formulation of a $d=4$ SU(2) lattice gauge theory, when $T\neq 0$ there is never a roughening transition about strong coupling.

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[13]Indeed, while in the dual model T_c is found from the leading (exponential) growth in the number of states, it is the subleading (powerlike) behavior which determines whether T_c characterizes a second-order phase transition or a true limiting temperature [S. Frautschi, Phys. Rev. D 3, 2821 (1971); N. Cabibbo and G. Parisi, Phys. Lett. 59B, 67 (1975)].

[14]This is evident in the continuum, as with the proper choice of gauge any $d=2$ vector field can be written as a free field. In general (or for the lattice theory), an argument using the correspondence of Ref. 3 can be employed. If T_c were finite, the infrared behavior of the $d=2$ SU(N) gauge theory about T_c could be described by an effective $d=1$ Z_N spin system. However, it is well known that fluctuations will inevitably disorder any $d=1$ system with a discrete global symmetry. Since the Z_N symmetry cannot be broken, T_c must be infinite.

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