Radiative contributions to quark and lepton masses in grand unified theories

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The mass perturbation and the evolution equation of fermion masses are studied in different gauge models on the basis of the Dyson-Schwinger equation. In theories with spontaneous symmetry breaking, we recover the results obtained using the renormalization-group method for momentum scales well above symmetry breaking and obtain new results for energies close to, or well below, the symmetry-violating scale, where spontaneous symmetry breaking gives an important contribution to the self-mass of a fermion. We examine the contribution to self-masses in the $SU(3)\times U(1)_{em}$ gauge theory and in the $SU(3)\times SU(2)\times U(1)$ model and discuss the relevance of embedding the standard component model in the structure of SU(5). For asymptotically free grand unified models, the self-masses are individually finite with no constraint on the number of quark or lepton flavors. Within this context we examine the possibility that the proton-neutron mass difference is determined by the very short distance scale associated with the unified theory of the strong, weak, and electromagnetic interactions.

I. INTRODUCTION

The description of the fundamental interactions based on the theory of gauge fields brings a new perspective to the old unsolved problem of the self-mass of hadrons.[1] Non-Abelian gauge theories such as quantum chromodynamics (QCD) and grand unified models which are asymptotically free can give new and unexpected answers to the typical questions encountered in the study of the asymptotic behavior of quantum electrodynamics. In a recent paper[2] (hereafter referred to as I) the problem of the ultraviolet behavior of self-masses was examined in the context of the Dyson-Schwinger equation for the self-energy in QCD. An integral equation for the quark running mass equivalent to the renormalization-group equation[3] (RGE) was obtained, and it was shown that the lowest-order electromagnetic perturbation to the self-energy leads to a finite mass shift, provided that the number of quark flavors is greater than or equal to eleven. This result contradicts the conclusions of Weinberg[4] and Collins[5] which are based on the operator-product expansion[6] (OPE) of the Cottingham formula,[1] expressed as the one-photon loop integrated over the virtual forward Compton amplitude $T_\mu{}^\mu$ of a proton, computed to all orders in the strong interaction. Since the leading operators in the short-distance expansion of $T_\mu{}^\mu$ are renormalization-group invariants, one obtains the usual logarithmically divergent result corresponding to the bare mass in the Lagrangian.

The traditional calculation of self-energies suffers from two major difficulties: the self-mass of a fermion and corrections to mass differences are infinite; and the sign of the lowest-order electromagnetic contribution to the self-energy tends to make the u quark heavier than the d quark (proton heavier than neutron). Regardless of the ultimate origin of the mass, these problems should be answered unambiguously in order to compute the radiative contribution to fermion masses or to mass differences. If the leading contribution is absorbed into a Coleman-Glashow tadpole,[5] an arbitrary parameter is introduced in order to impose a cutoff in the photon momentum so that nothing is gained at a fundamental level. Another approach to overcoming the divergence is to introduce a dynamical mass insertion.[7,8] In this case, the form factor associated with the wave function of the bound fermion pair leads to an ultraviolet cutoff. Although this is a very interesting possibility, it suffers from the inherent problems of the hypercolor models.[9] In renormalizable gauge theories with spontaneous symmetry breaking (SSB), there are certain natural mass relations at the tree level of symmetry breaking, such as $m_d = m_e$ in SU(5).[10] For such models the zeroth-order contribution to some masses, or mass differences, vanishes as a consequence of the symmetry content of the theory, but the sum of higher-order effects from the unbroken symmetries is finite in each order of perturbation theory since any counterterms in the Lagrangian also obey the symmetry relation.[11] However, in the standard model of the electroweak interactions, where the fermions are contained in complex representations of $SU(2)\times U(1)$, a mechanism of cancellation of electromagnetic divergences by the weak interactions does not exist for the u-d mass difference.[12] In grand unified models[13] m_u and m_d are largely unrelated, unless extra symmetries are introduced in a rather arbitrary way. Even in the SO(10) model, where all the fermions in a given generation are in the same irreducible representation, m_u and m_d are generated at the tree level by different components of the Higgs representation.

In the OPE of the Compton amplitude, the leading operator $Q_f{}^2\bar{\psi}_f\psi_f$ is not a flavor singlet, and unless some very restrictive model is found a cancellation of infinities for the $\Delta I=1$ mass differences, such as proton-neutron, is not guaranteed. We consider here another possibility, namely, that the electromagnetic and weak contributions to the self-energy of a fermion are individually finite. As stated above, the lowest-order perturbation of the Dyson-Schwinger equation for the self-energy in the combined QED-QCD theory leads to a convergent result if $n_f>\frac{21}{2}$. How can these conflicting results be reconciled? Recently, Dine[14] extended the results of I to models possessing natural mass relations and SSB and found that the prescription described in I agrees with the conventional methods for models with zeroth-order relations among fermion masses. In the absence of such zeroth-order symmetry relations the problem is not so well understood. Based on renormalization-group methods, Kiskis and West[15] have noticed that the convergent solution found in I forces the bare parameters in the QCD Lagrangian to have a dependence on the electric charge. On the other hand, Craigie, Narison, and Riazuddin[16] have shown that the reordering of summation and integration of the photon loop relative to the gluon loop plays a crucial role in the electromagnetic mass-shift calculation. In fact, in the Cottingham formula, the photon loop integration is done last, and the quark mass is renormalized only by the strong interactions[2] with a cancellation of the QCD effects.[4] If we impose the condition that the result be independent of the order of integration, the results of I are found[16]; namely, a finite mass perturbation with the arbitrarily large cutoff in the Weisskopf divergence being replaced by the QCD scale parameter Λ, thus ensuring a sign reversal for $p>\Lambda$.

In any case, the electromagnetic perturbation to quark running masses found in I leads to a value of the mass shift that is rather too small to account for the observed p-n mass difference. In his work Dine[14] found that quark masses and natural mass relations acquire large corrections from high-energy symmetry-breaking scales. It is thus of interest to examine the problem of the u-d quark mass difference in grand unified theories of the strong, electromagnetic, and weak interactions since the mass shift can receive important contributions from the very short distance scale of grand unification. Furthermore, if the theory is embedded in a grand unified group which is asymptotically free, the constraint on a minimum number of flavors and the difficulties pointed out in Ref. 15 are no longer present.

In this paper we derive the evolution equation for fermion masses for different gauge models from the Dyson equation for the self-energy. In theories where the symmetry is spontaneously broken, we find that our results are identical with the renormalization-group results of Buras, Ellis, Gaillard, and Nanopoulos[17] (BEGN) and Elias,[18] for energies well above the symmetry-violating scale. To include the effects of symmetry breaking, we study the lowest-order perturbation of a fermion running mass by the electroweak interaction and the baryon- and lepton-number-violating interactions of grand unification which, as found by Dine,[14] is logarithmically sensitive to the symmetry-violating scale. A closed-form solution for the mass perturbation is found for the various models considered. The advantage of the method of calculation discussed here is that the solution is not limited to the high-energy range (well above SSB), and the various components of the anomalous dimension of the fermion mass operator are easily identified.

In Sec. II we review briefly the standard approach for computing mass differences based on the OPE and outline the main results of Ref. 2. We consider the electromagnetic contribution of order α to a QCD quark running mass and write the $SU(3)\times U(1)_{em}$ evolution equation. The results of Sec. II can be extended without difficulty to gauge models where the left- and right-handed fields are not necessarily in the same representation. As a specific example we consider in Sec. III the standard $SU(3)\times SU(2)\times U(1)$ model. The contribution to self-masses from the SU(2) sector is obviously zero in the Landau gauge, since only the left-handed fermions are coupled to the gauge fields. At large momentum scales SSB is ignored, and the BEGN[17] renormalization-group result [apart from the SU(2) factor] follows from the solution of the Dyson equation, written as an evolution equation in terms of the SU(3) and U(1) running coupling constants. To include the effects of symmetry breaking, we write the U(1) Lagrangian in terms of gauge fields of definite mass A^{μ} and Z^{μ} to obtain the lowest-order perturbation from the electroweak interaction.

The effects of the interaction of the baryon- and lepton-number-violating currents with the superheavy gauge bosons characteristic of grand unified theories are examined in Sec. IV. The calculations are carried out explicitly in the simplest theory of grand unification based on SU(5).[10] At momentum scales well above M_X we find evolution equations for the masses of the electron, u- and d-quark equivalent to the RGE,[18] and obtain an expression for the self-energy of the quarks which is valid above and below M_X. We give an estimate of the radiative contribution to the light-quark-mass ratio m_d/m_u renormalized relative to the QCD scale Λ. At low energies, the self-energy is largely dominated by the contribution from spontaneous symmetry breaking at the grand unification scale. A discussion of the results is presented in Sec. V.

II. THE SELF-ENERGY IN QCD

We begin our discussion with the study of the self-energy in the combined QED-QCD theory. Using the Cottingham formula and the OPE, it is not difficult to show that the ultraviolet electromagnetic contribution to a hadronic mass difference contains the usual QED divergence with all the effects from the strong interactions cancelling out.[4,5] The Cottingham formula[1] gives the lowest-order electromagnetic contribution to the hadron mass M,

$$\delta M=\frac{e^2}{2M}\int\frac{id^4q}{(2\pi)^4}\frac{T_{\mu}{}^{\mu}(q^2,p\cdot q)}{q^2+i\epsilon}\,, \tag{2.1}$$

where

$$T_{\mu\nu}=\int d^4x e^{iq\cdot x}\langle p\,|\,j_{\mu}(x)j_{\nu}(0)\,|\,p\rangle \tag{2.2}$$

is the virtual Compton amplitude for the scattering of a photon of momentum q of a hadronic target of momentum p. We expand the product of currents in (2.2) into a complete set of operators $\mathcal{O}^i$ for $q^2 \gg p^2$ and $p\cdot q$,

$$\delta M=\frac{3\alpha}{4\pi M}\int_{p^2}^{\infty}\frac{dq^2}{q^2}\sum_i C_i(q^2)\langle p\,|\,\mathcal{O}^i_{\mu\mu}\,|\,p\rangle\,, \tag{2.3}$$

where $C_i(q^2)$ is the Wilson coefficient which satisfies the RGE. The scalar operators of lowest dimensionality (leading operators) are $Q_f{}^2 m_f\bar{\psi}_f\psi_f$ and $G^i_{\mu\nu}G^{i\mu\nu}$, where Q_f and m_f are quark charge and mass, ψ is the matter field, and $G^i_{\mu\nu}$ is the gauge field tensor. These operators have no anomalous dimension. The corresponding coefficient C_i behaves as a constant for large q^2 and the self-energy diverges logarithmically.[4,5,15] The gluon contribution G^2 is a flavor singlet and cancels when computing mass differences. The higher-order operators have anomalous dimensions, but are suppressed by inverse powers of q^2. Thus, the large-q^2 contribution to mass differences is computed in this framework as if quarks were decoupled from the strong interactions.

Let us remark that for $\Delta I=2$ mass differences such as $m_{\pi^+}-m_{\pi^0}$ or $m_{\Sigma^+}+m_{\Sigma^-}-2m_{\Sigma^0}$, there is a neat cancellation for large q^2 of the radiative contribution due to the flavor-dependent divergent terms arising from the leading operator $Q^2m\bar{\psi}\psi$ in the OPE, and the mass shift is computed successfully in terms of a dispersion integral over low-lying resonances.[1] For $\Delta I=1$ mass differences such as m_p-m_n the large-q^2 cancellation is absent. This is the origin of the subtraction term in the dispersion integral in the Cottingham formula, which has made impossible any progress along this line.[1] It can also be noted that quark masses at the tree level of symmetry breaking do not contribute to the $\Delta I=2$ mass differences, which contain the same average number of quarks.

Let us consider the ultraviolet behavior of the self-energy in the context of the Dyson-Schwinger equation and examine the lowest-order electromagnetic perturbation to a quark mass in QCD. We write a renormalized Dyson equation in terms of the running coupling constant $g(p^2)$ and the running mass $m(p^2)$ which is defined as the pole in

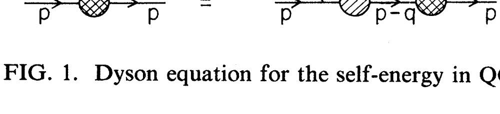

FIG. 1. Dyson equation for the self-energy in QCD.

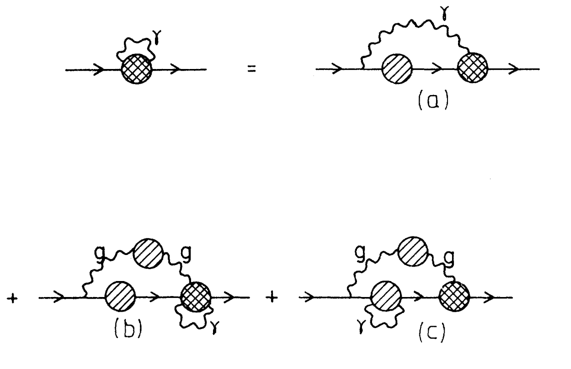

FIG. 2. Dyson equation for calculating the lowest-order electromagnetic perturbation to a quark mass in QCD.

the renormalized off-shell fermion propagator. The resulting integral for the running mass, after Wick rotation, is expected to be convergent in the deep Euclidean region, at least for asymptotically free theories. In the leading-logarithm approximation for g,[19] the contribution from the region of integration $q^2 \gg p^2$ and $p\cdot q$, for large p^2, in the Landau gauge is given by the homogeneous equation[2] (Fig. 1)

$$m(p^2)=\frac{3}{4\pi}C_F\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_3(q^2)m(q^2)\ , \tag{2.4}$$

provided that in the limit as $p^2\to\infty$, $m(p^2)$ vanishes. C_F is the quadratic Casimir operator of the fermion representation and α_3 is the QCD running coupling constant for large q^2,

$$\alpha_3(q^2)=\frac{4\pi}{\beta_0\ln(q^2/\Lambda^2)}\ , \tag{2.5}$$

with $\beta_0=11-\frac{2}{3}n_f$ and where Λ sets the scale of the strong interactions. We can express (2.4) as an evolution equation,

$$\frac{dm(p)}{m(p)}=\gamma_f(p)\frac{dp}{p}\ , \tag{2.6}$$

where

$$\gamma_f=-3g^2C_F/8\pi^2\ , \tag{2.7}$$

is the anomalous dimension of the fermion mass operator. This expression is identical with the result obtained by the renormalization-group equation.[3] Integration of (2.6) gives the well-known result

$$m(p^2)=m(M^2)\left[\frac{\alpha_3(p^2)}{\alpha_3(M^2)}\right]^{\gamma}\ , \tag{2.8}$$

where $\gamma=3C_F/\beta_0$. To order α, the electromagnetic perturbation $\delta m(p^2)$ to the QCD running mass $m(p^2)$ has the following form[2] for large p^2 (Fig. 2):

$$\delta m(p^2)=\frac{3}{4\pi}Q_q{}^2\alpha\int_{p^2}^{\infty}\frac{dq^2}{q^2}m(q^2) +\frac{3}{4\pi}C_F\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_3(q^2)\delta m(q^2)\ . \tag{2.9}$$

We can write Eq. (2.9) in operator notation as follows:

$$\delta m=K+A\delta m\ . \tag{2.10}$$

This type of equation has been extensively used in the study of feedback mechanisms.[1] K is the driving term which contains the contribution from the emission and reabsorption of virtual photons [Fig. 2(a)]. The integral operator A describes the mechanism of "feedback" by the strong interactions: the modifications in the strong forces due to electromagnetism, which are proportional to δm itself. In our calculation this term corresponds to the modification of the QCD vertex function and propagators [Figs. 2(b) and 2(c)], and is determined for large p^2 to lowest order in the electromagnetic interaction. In the usual approach[4,5] the integral operator A is zero.

If $\gamma>1$ ($n_f>11$) the driving term in the integral equation (2.9) is convergent and the solution is[2]

$$\delta m(p^2)=-\frac{3}{4\pi}\alpha Q_q{}^2 m(p^2)\ln\left[\frac{p^2}{\Lambda^2}\right]+Cm(p^2)\ . \tag{2.11}$$

We set C equal to zero since this term has no dependence on α and should be absorbed in the definition of $m(p^2)$ which is a free parameter in the standard model. This choice for the constant C

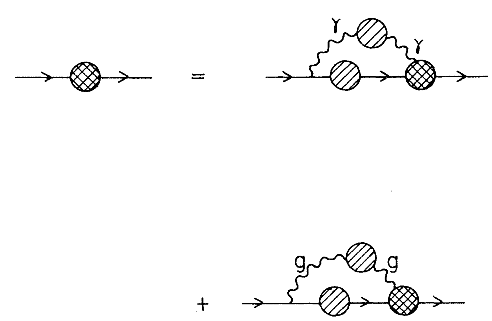

FIG. 3. Dyson equation for computing the evolution equations of quark running masses in $SU(3)_{\rm color}\times U(1)_{\rm em}$.

has been questioned in Refs. 14 and 15. An alternative method of solution would be to include appropriate boundary conditions for the fermion masses at some grand unified scale and evolve the masses to lower energies using the renormalization-group equations. However, in the present paper we are mainly concerned with the radiative contribution from SSB which has not been included in the usual RGE approach.

The $SU(3)\times U(1)_{em}$ evolution equation for the running mass is obtained from the integral equation corresponding to the diagrams illustrated in Fig. 3,

$$m(p^2)=\frac{3}{4\pi}\int_{p^2}^{\infty}\frac{dq^2}{q^2}[Q_q{}^2\alpha(q^2)+C_F\alpha_3(q^2)]m(q^2)\ , \tag{2.12}$$

where $\alpha(q^2)$ is the running coupling constant of QED,

$$\alpha(q^2)=\frac{\alpha(\mu^2)}{1-K_f\dfrac{\alpha(\mu^2)}{3\pi}\ln\left[\dfrac{q^2}{\mu^2}\right]}\ ,\quad K_f=\sum_n Q_n{}^2\ . \tag{2.13}$$

Equation (2.12) can be expressed as an evolution equation of the form (2.6) with

$$\gamma_f=-3e_q{}^2/8\pi^2-3g^2C_F/8\pi^2\ , \tag{2.14}$$

to first order in α and α_3. Integration of Eq. (2.6) corresponding to γ_f given by (2.14) yields[2]

$$m(p^2)=m(M^2)\left[\frac{\alpha(p^2)}{\alpha(M^2)}\right]^{-\frac{9Q_q{}^2}{4K_f}}\left[\frac{\alpha_3(p^2)}{\alpha_3(M^2)}\right]^{\gamma}\ . \tag{2.15}$$

III. ELECTROWEAK CONTRIBUTIONS TO FERMION MASSES

In this section we extend the results of Sec. II to groups having complex fermion representations. As a specific example, we consider the standard QCD-electroweak theory based on the group $SU(3)\times SU(2)\times U(1)$. At sufficiently high momentum scales where the effects of spontaneous symmetry breaking can be ignored, the fermion mass dimension γ_f of a fermion f corresponding to the gauge group G_i, has the form

$$\gamma_f=-3g_i{}^2C^{(i)}_{f_{LR}}/8\pi^2 \tag{3.1}$$

TABLE I. Values of the Casimir operator $C^{(i)}_{f_{LR}}$ for various gauge models relevant to the calculation of self-energies well above the spontaneous symmetry-breaking scale.

$C^{(i)}_{f_{LR}}$ \ f	e	ν	u	d
$C^{(1)}_{em}$	1	0	$\frac{4}{9}$	$\frac{1}{9}$
$C^{(1)}$	$\frac{3}{10}$	0	$\frac{1}{15}$	$-\frac{1}{30}$
$C^{(2)}$	0	0	0	0
$C^{(3)}$	0	0	$\frac{4}{3}$	$\frac{4}{3}$
$C^{(5)}$	$\frac{9}{5}$	0	$\frac{12}{5}$	$\frac{9}{5}$

to second order in g. $C^{(i)}_{f_{LR}}$ is a Casimir operator which corresponds to a loop diagram with a mass insertion in the fermion propagator, attached to a left-handed fermion at one end, and to a right-handed one at the other end, in order to change helicity.[20] (In QCD $C^{(3)}_{f_{LR}}=C_F=\frac{4}{3}$.) We calculate $C^{(i)}_{f_{LR}}$ for a given flavor, as a sum over all the contributions to the self-energy due to the interaction of the fermion fields with the gauge bosons of G_i. The values of this operator are listed in Table I for the various models considered in this paper. To illustrate the method discussed here we derive first the $SU(3)\times SU(2)\times U(1)$ evolution equation for the fermion masses well above SSB, and later determine how the expression (3.1) is modified in the presence of symmetry breaking. As mentioned in Sec. I, the SU(2) sector does not contribute to the self-mass (in the Landau gauge) since only the left-handed fields are coupled to the gauge bosons. The relevant contributions to the Dyson equation for the running mass are obtained from the U(1) interaction Lagrangian,[21]

$$\mathscr{L}_1=\frac{g_1}{\sqrt{60}}(4\bar{u}_R\gamma_\mu u_R+\bar{u}_L\gamma_\mu u_L-2\bar{d}_R\gamma_\mu d_R+\bar{d}_L\gamma_\mu d_L-6\bar{e}_R\gamma_\mu e_R-3\bar{e}_L\gamma_\mu e_L-3\bar{\nu}_L\gamma_\mu\nu_L)B^\mu \tag{3.2}$$

with $\psi_{L,R}=\frac{1}{2}(1\mp\gamma_5)\psi$. For large p^2 we obtain the following integral equation for the u and d quark running masses (Fig. 4):

$$m_{u,d}(p^2)=\frac{3}{4\pi}\int_{p^2}^{\infty}\frac{dq^2}{q^2}[C^{(1)}_{(u,d)_{LR}}\alpha_1(q^2)+C_F\alpha_3(q^2)]m_{u,d}(q^2)\ , \tag{3.3}$$

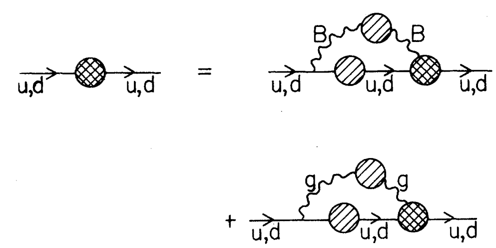

FIG. 4. Dyson equation for calculating the evolution equations of quark running masses in SU(3)×SU(2)×U(1).

where $\alpha_1(q^2)$ is the U(1) running coupling constant

$$\alpha_1(q^2)=\frac{\alpha_1(\mu^2)}{1-\frac{n_f}{6\pi}\alpha_1(\mu^2)\ln(q^2/\mu^2)} . \tag{3.4}$$

The corresponding equation for the electron (Fig. 5) is given by

$$m_e(p^2)=\frac{3}{4\pi}C^{(1)}_{e_{LR}}\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_1(q^2)m_e(q^2) . \tag{3.5}$$

We can express Eqs. (3.3) and (3.5) in the form of an evolution equation,

$$\frac{dm_f(p)}{m_f(p)}=\gamma_f(p)\frac{dp}{p} , \tag{3.6}$$

with

$$\gamma_{u,d}=-3g_1{}^2C^{(1)}_{(u,d)_{LR}}/8\pi^2-3g_3{}^2C_F/8\pi^2 \tag{3.7}$$

and

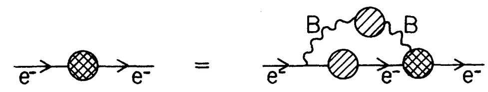

FIG. 5. Dyson equation for calculating the running mass of the electron in SU(2)×U(1).

$$\gamma_e=-3g_1{}^2C^{(1)}_{e_{LR}}/8\pi^2 . \tag{3.8}$$

Direct integration of (3.6) gives

$$m_u(p^2)=m_u(M^2)\left[\frac{\alpha_1(p^2)}{\alpha_1(M^2)}\right]^{-3/10n_f}\left[\frac{\alpha_3(p^2)}{\alpha_3(M^2)}\right]^{\gamma} , \tag{3.9}$$

$$m_d(p^2)=m_d(M^2)\left[\frac{\alpha_1(p^2)}{\alpha_1(M^2)}\right]^{3/20n_f}\left[\frac{\alpha_3(p^2)}{\alpha_3(M^2)}\right]^{\gamma} , \tag{3.10}$$

and

$$m_e(p^2)=m_e(M^2)\left[\frac{\alpha_1(p^2)}{\alpha_1(M^2)}\right]^{-27/20n_f} , \tag{3.11}$$

which are identical with the BEGN results of Ref. 17 except for the SU(2) factor in that reference. To include the effects of symmetry breaking, we write the U(1) Lagrangian in terms of mass eigenstates A_μ and Z_μ:

$$\mathscr{L}_1=g\sin\theta_W(-\bar{e}\gamma_\mu e+\tfrac{2}{3}\bar{u}\gamma_\mu u-\tfrac{1}{3}\bar{d}\gamma_\mu d)A^\mu+\frac{g}{\cos\theta_W}[\tfrac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L-\tfrac{1}{2}\bar{e}_L\gamma_\mu e_L+\tfrac{1}{2}\bar{u}_L\gamma_\mu u_L-\tfrac{1}{2}\bar{d}_L\gamma_\mu d_L$$
$$+\sin^2\theta_W(\bar{e}\gamma_\mu e-\tfrac{2}{3}\bar{u}\gamma_\mu u+\tfrac{1}{3}\bar{d}\gamma_\mu d)]Z^\mu . \tag{3.12}$$

The corresponding integral equations for the interaction Lagrangian (3.12) for large p^2 in the 't Hooft-Landau gauge are given by

$$m_u(p^2)=\frac{3}{4\pi}Q_u{}^2\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha(q^2)m_u(q^2)+\frac{3}{4\pi}(\tfrac{4}{9}\sin^2\theta_W-\tfrac{1}{3})\int_{p^2}^{\infty}\frac{dq^2}{q^2+M_Z{}^2}\alpha_{g'}(q^2)m_u(q^2)$$
$$+\frac{3}{4\pi}C_F\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_3(q^2)m_u(q^2) , \tag{3.13}$$

$$m_d(p^2)=\frac{3}{4\pi}Q_d^2\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha(q^2)m_d(q^2)+\frac{3}{4\pi}(\tfrac{1}{9}\sin^2\theta_W-\tfrac{1}{6})\int_{p^2}^{\infty}\frac{dq^2}{q^2+M_Z^2}\alpha_{g'}(q^2)m_d(q^2)$$
$$+\frac{3}{4\pi}C_F\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_3(q^2)m_d(q^2)\,, \tag{3.14}$$

and

$$m_e(p^2)=\frac{3}{4\pi}\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha(q^2)m_e(q^2)+\frac{3}{4\pi}\left[\sin^2\theta_W-\tfrac{1}{2}\right]\int_{p^2}^{\infty}\frac{dq^2}{q^2+M_Z^2}\alpha_{g'}(q^2)m_e(q^2)\,, \tag{3.15}$$

where

$$\tan\theta_W=g'/g,\quad e=g\sin\theta_W\,. \tag{3.16}$$

From the above equations we derive the anomalous dimension γ_f in the presence of SSB. In the SU(3)×SU(2)×U(1) model it has the form

$$\gamma_u(p)=-\frac{3}{8\pi^2}e_u^2-\frac{3}{8\pi^2}(\tfrac{4}{9}\sin^2\theta_W-\tfrac{1}{3})g'^2\frac{p^2}{p^2+M_Z^2}-\frac{3}{8\pi^2}C_Fg_3^2\,, \tag{3.17}$$

$$\gamma_d(p)=-\frac{3}{8\pi^2}e_d^2-\frac{3}{8\pi^2}(\tfrac{1}{9}\sin^2\theta_W-\tfrac{1}{6})g'^2\frac{p^2}{p^2+M_Z^2}-\frac{3}{8\pi^2}C_Fg_3^2\,, \tag{3.18}$$

and

$$\gamma_e(p)=-\frac{3}{8\pi^2}e^2-\frac{3}{8\pi^2}(\sin^2\theta_W-\tfrac{1}{2})g'^2\frac{p^2}{p^2+M_Z^2}\,. \tag{3.19}$$

For momentum scales well above symmetry breaking, $p^2 \gg M_Z^2$, we recover Eqs. (3.3) and (3.5) which reproduce the BEGN results, provided that we set $\alpha_{g'}=\frac{3}{5}\alpha_1$. The common factor $\frac{3}{5}$ has its origin in the hypercharge generator, which is normalized to $\sqrt{3/5}y$ in the U(1) current in (3.2). We solve Eqs. (3.13) and (3.14) to lowest order in α and $\alpha_{g'}$. Since the SU(3) color sector of the theory couples identically to the different quarks, the radiative contribution to the u-d mass shift is determined to lowest order in the electroweak interaction from the mass perturbation, $\delta m(p^2)$, to the SU(3) running mass component which we henceforth will label $m^s(p^2)$. We write the corresponding equations as follows:

$$\delta m_u(p^2)=\frac{3}{4\pi}Q_u^2\alpha\int_{p^2}^{\infty}\frac{dq^2}{q^2}m_u^s(q^2)+\frac{3}{4\pi}(\tfrac{4}{9}\sin^2\theta_W-\tfrac{1}{3})\alpha_{g'}\int_{p^2}^{\infty}\frac{dq^2}{q^2+M_Z^2}m_u^s(q^2)$$
$$+\frac{3}{4\pi}C_F\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_3(q^2)\delta m_u(q^2)\,, \tag{3.20}$$

$$\delta m_d(p^2)=\frac{3}{4\pi}Q_d^2\alpha\int_{p^2}^{\infty}\frac{dq^2}{q^2}m_d^s(q^2)+\frac{3}{4\pi}(\tfrac{1}{9}\sin^2\theta_W-\tfrac{1}{6})\alpha_{g'}\int_{p^2}^{\infty}\frac{dq^2}{q^2+M_Z^2}m_d^s(q^2)$$
$$+\frac{3}{4\pi}C_F\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_3(q^2)\delta m_d(q^2)\,. \tag{3.21}$$

The solution to the above equations is given by

$$\delta m_u(p^2)=-\frac{3}{4\pi}Q_u^2\alpha m_u^s(p^2)\ln\left[\frac{p^2}{\Lambda^2}\right]-\frac{3}{4\pi}(\tfrac{4}{9}\sin^2\theta_W-\tfrac{1}{3})\alpha_{g'}m_u^s(p^2)\ln\left[\frac{p^2+M_Z^2}{\Lambda^2}\right]\,, \tag{3.22}$$

$$\delta m_d(p^2)=-\frac{3}{4\pi}Q_d^2\alpha m_d^s(p^2)\ln\left[\frac{p^2}{\Lambda^2}\right]-\frac{3}{4\pi}(\tfrac{1}{9}\sin^2\theta_W-\tfrac{1}{6})\alpha_{g'}m_d^s(p^2)\ln\left[\frac{p^2+M_Z^2}{\Lambda^2}\right]\,. \tag{3.23}$$

At low energies $p^2 \ll M_Z{}^2$ ($\sin^2\theta_W \simeq 0.2$) we obtain

$$\delta m_u(p^2) \simeq \frac{3}{20\pi}\alpha_1 m_u^s(p^2)\ln\left[\frac{M_Z{}^2}{\Lambda^2}\right], \tag{3.24}$$

$$\delta m_d(p^2) \simeq \frac{3}{40\pi}\alpha_1 m_d^s(p^2)\ln\left[\frac{M_Z{}^2}{\Lambda^2}\right]. \tag{3.25}$$

This result exhibits an important feature, since the sign of the self-energy is opposite to that obtained in Eq. (2.11). This example shows that the sign of the radiative contribution to a fermion mass depends on the group content of the theory. To ensure the ultraviolet convergence of the self-energy integrals, the number of flavors have again to be restricted by the condition $\frac{21}{2} < n_f < \frac{33}{2}$.

IV. MASS PERTURBATION IN SU(5)

The embedding of the standard SU(3)×SU(2)×U(1) component model in the structure of a larger underlying group has important consequences for the fermion mass spectrum. The appearance of additional gauge-boson degrees of freedom, associated with the new symmetry generators, is expected to give a significant radiative contribution to the self-energy of a fermion due to SSB at the grand unification scale. In this section we give our results for the SU(5) model of Georgi and Glashow,[10] but the method can be easily generalized to other models.

We first derive the evolution equation for fermion masses. In the energy region $p^2 \gg M_X{}^2$, the effects of symmetry breaking can be ignored, and the different interactions, which behave very much alike, can be regrouped into a single expression for the running mass, in order to build the SU(5) anomalous dimension of the fermion mass operator. To include the effects of symmetry breaking at the grand unified mass M_X we next study the lowest-order mass perturbation, $\delta m(p^2)$, in the SU(5) running coupling constant α_5.

We write the SU(5) interaction Lagrangian in terms of a single coupling constant g_5 as follows[13]:

$$\begin{aligned}\mathscr{L}_5 = {} & g_5\sum_{i=1}^{8}(\bar u\gamma^\mu\tfrac{1}{2}\lambda^i u+\bar d\gamma^\mu\tfrac{1}{2}\lambda^i d)A^i_\mu + g_5\sum_{i=1}^{3}\left[(\bar u\,\bar d)_L\gamma^\mu\tfrac{1}{2}\tau^i\begin{bmatrix}u\\ d\end{bmatrix}_L+(\bar\nu\,\bar e^-)_L\gamma^\mu\tfrac{1}{2}\tau^i\begin{bmatrix}\nu\\ e^-\end{bmatrix}_L\right]W^i_\mu\\ &+\frac{g_5}{\sqrt{60}}(4\bar u_R\gamma^\mu u_R+\bar u_L\gamma^\mu u_L-2\bar d_R\gamma^\mu d_R+\bar d_L\gamma^\mu d_L-6\bar e^-_R\gamma^\mu e^-_R-3\bar e^-_L\gamma^\mu e^-_L-3\bar\nu_L\gamma^\mu\nu_L)B_\mu\\ &+\frac{g_5}{\sqrt{2}}(\epsilon_{abd}\bar u^c_{dL}\gamma^\mu u_{bL}+\bar d_a\gamma^\mu e^+)\bar X^a_\mu+\text{H.c.}+\frac{g_5}{\sqrt{2}}(\epsilon_{abd}\bar u^c_{dL}\gamma^\mu d_{bL}-\bar u_{aL}\gamma^\mu e^+_L+\bar d_{aR}\gamma^\mu\nu^c_R)\bar Y^a_\mu+\text{H.c.}\end{aligned} \tag{4.1}$$

The first three terms in (4.1) are the conventional SU(3), SU(2), and U(1) interactions above M_X, and the last two describe the interaction of the lepton- and baryon-number-violating currents with the superheavy X and Y bosons which carry color, charge, and flavor quantum numbers ($Q_X = \frac{4}{3}$, $Q_Y = \frac{1}{3}$). To determine which diagrams in the above Lagrangian give a nonvanishing contribution to the Dyson equation for the self-mass, we expand the complete fermion propagator,

$$\left\langle 0\left|T\left[\psi(x)\bar\psi(x')\exp\left[i\int\mathscr{L}(x)dx\right]\right]\right|0\right\rangle=\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-x')}\{iS(p)+iS(p)[-i\Sigma(p)]iS(p)+\cdots\} \tag{4.2}$$

in terms of α_5. As noted previously, the only nonvanishing contributions to the self-energy $\Sigma(p)$ from the SU(3)×SU(2)×U(1) sector of the theory are due to the color and the U(1) interactions. As regards the interaction with the X and Y bosons, it follows from the particular form of the SU(5) Lagrangian (4.1), that the only nonvanishing products of field operators are the following (recall that $\bar\psi_{aL}\gamma^\mu\psi_{bL} = -\bar\psi^c_{bR}\gamma^\mu\psi^c_{aR}$):

$$\int d^4y\int d^4z\langle 0\,|\,T\{u(x)[\bar u_{cR}(y)]_\alpha[u^c_{dR}(y)]_\beta[\bar u^c_{bL}(z)]_\gamma[u_{aL}(z)]_\delta\,\bar u(x')\}\,|\,0\rangle\,, \tag{4.3}$$

$$\int d^4y\int d^4z\langle 0\,|\,T\{d(x)[\bar d_{bR}(y)]_\alpha[e^+_R(y)]_\beta[\bar e^+_L(z)]_\gamma[d_{aL}(z)]_\delta\,\bar d(x')\}\,|\,0\rangle\,, \tag{4.4}$$

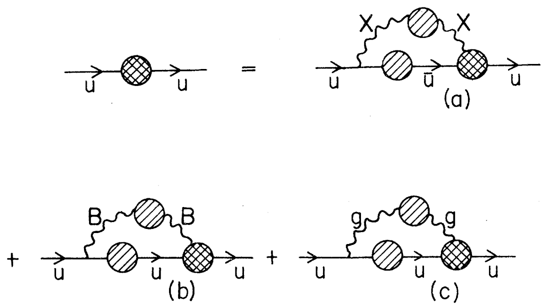

FIG. 6. Dyson equation for computing the u-quark running mass in SU(5). The superheavy gauge bosons of grand unification are denoted by X, the gauge bosons of U(1) by B, and g denotes the gauge bosons of the SU(3) sector.

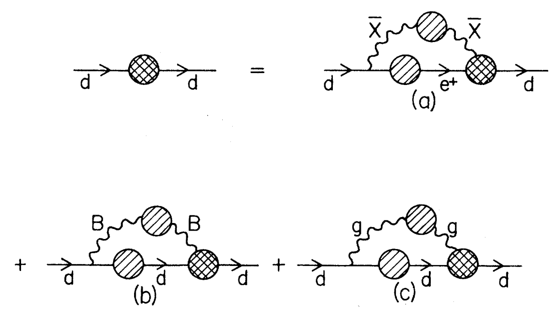

FIG. 7. Dyson equation for the d-quark running mass in SU(5).

and

$$\int d^4y \int d^4z \langle 0 | T\{e^+(x)[\bar{e}^+_R(y)]_\alpha [d_{bR}(y)]_\beta [\bar{d}_{aL}(z)]_\gamma [e^+_L(z)]_\delta \bar{e}^+(x')\} | 0\rangle \ . \tag{4.5}$$

The relevant self-energy diagrams for the u, d, and e^+ in SU(5) are indicated in Figs. 6–8. The corresponding Dyson equations for large p^2 in the 't Hooft-Landau gauge are given by

$$m_u(p^2) = \frac{3}{4\pi}\left[\int_{p^2}^{\infty} \frac{dq^2}{q^2+M_X{}^2}\alpha_5(q^2) m_{\bar{u}}(q^2) + \int_{p_2}^{\infty}\frac{dq^2}{q^2}\left[C^{(1)}_{u_{LR}} + C_F\right]\alpha_5(q^2) m_u(q^2)\right] , \tag{4.6}$$

$$m_d(p^2) = \frac{3}{4\pi}\left[\tfrac{1}{2}\int_{p^2}^{\infty} \frac{dq^2}{q^2+M_{\bar{X}}{}^2}\alpha_5(q^2) m_{e^+}(q^2) + \int_{p^2}^{\infty}\frac{dq^2}{q^2}\left[C^{(1)}_{d_{LR}} + C_F\right]\alpha_5(q^2) m_d(q^2)\right] , \tag{4.7}$$

and

$$m_{e^+}(p^2) = \frac{3}{4\pi}\left[\tfrac{3}{2}\int_{p^2}^{\infty} \frac{dq^2}{q^2+M_X{}^2}\alpha_5(q^2) m_d(q^2) + \int_{p^2}^{\infty}\frac{dq^2}{q^2} C^{(1)}_{e^+_{LR}}\alpha_5(q^2) m_{e^+}(q^2)\right] . \tag{4.8}$$

Note that the first term in Eq. (4.8) is a factor of 3 greater than the first term in Eq. (4.7) whereas Eq. (4.6) is a factor 2 greater. These factors have their origin in the different intermediate states. For example, the factor 3 in Eq. (4.8) corresponds to the three color states of the quark propagator [Fig. 8(a)]. The occurrence of the factor 2 in Eq. (4.6) can be explained as follows. Suppose that the initial u quark in Fig. 6(a) is in a red state $|R\rangle$. It then follows from color conservation that the only possible color states of the intermediate X boson and u^c antiquark, $|X,u^c\rangle$, are the $|\bar{Y},\bar{B}\rangle$ and the $|\bar{B},\bar{Y}\rangle$ states, thus yielding the factor 2.

We can write the above equations in a form equivalent to the RGE (Ref. 18) above the grand unification mass (GUM) if we set $m_d = m_e$. At first sight this is surprising since we are not invoking a particular Higgs-boson structure, but it is compatible with the model since the left- or right-handed components of d and e stand on equal footing in the SU(5) multiplets. As a result, the anomalous dimension of d and e are degenerate in SU(5).[18] For $p^2 \gg M_X{}^2$

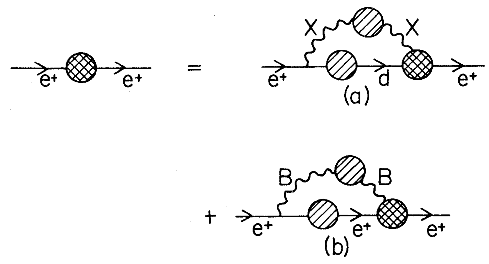

FIG. 8. Dyson equation for the electron running mass in SU(5).

$$m_f(p^2)=\frac{3}{4\pi}C_{f_{LR}}^{(5)}\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_5(q^2)m_f(q^2)\ , \tag{4.9}$$

where

$$C_{u_{LR}}^{(5)}=1+\tfrac{1}{15}+\tfrac{4}{3}=\tfrac{12}{5}\ ,$$

$$C_{d_{LR}}^{(5)}=\tfrac{1}{2}-\tfrac{1}{30}+\tfrac{4}{3}=\tfrac{9}{5}\ ,$$

$$C_{e_{LR}}^{(5)}=\tfrac{3}{2}+\tfrac{3}{10}=\tfrac{9}{5}\ .$$

Alternatively, we can write Eq. (4.9) as an evolution equation,

$$\frac{dm_f(p)}{m_f(p)}=\gamma_f^{(5)}(p)\frac{dp}{p}\ , \tag{4.10}$$

with

$$\gamma_f^{(5)}(p)=-3g_5{}^2C_{f_{LR}}^{(5)}/8\pi^2\ . \tag{4.11}$$

This expression is valid for $p^2 \gg M_X{}^2$. The correct expression for the anomalous dimension $\gamma_f^{(5)}$ above M_X including the effects of SSB at the grand unification scale is derived from Eqs. (4.6)–(4.8) and is given by

$$\gamma_u^{(5)}(p)=-\frac{3}{8\pi^2}g_5{}^2\left[C_{u_{LR}}^{(1)}+C_F+p^2/(p^2+M_X{}^2)\right]\ , \tag{4.12}$$

$$\gamma_d^{(5)}(p)=-\frac{3}{8\pi^2}g_5{}^2\left[C_{d_{LR}}^{(1)}+C_F+\tfrac{1}{2}p^2/(p^2+M_X{}^2)\right]\ , \tag{4.13}$$

and

$$\gamma_e^{(5)}(p)=-\frac{3}{8\pi^2}g_5{}^2[C_{e_{LR}}^{(1)}+\tfrac{3}{2}p^2/(p^2+M_X{}^2)]\ . \tag{4.14}$$

Finally, we evaluate the radiative contribution to the u-d quark mass difference from SSB at the grand unification scale. To lowest order in α_5 it is determined by the contributions of the U(1) and the lepton- and baryon-number-violating interactions to the mass perturbation, $\delta m(p^2)$, of the SU(3) running mass component, $m^s(p^2)$.[22] The dominant contribution to the mass shift comes from the M_X scale where $m_d=m_e$ and the running fine structure constants α_1, α_2, α_3 of the group $SU(3)\times SU(2)\times U(1)$ approach a common value, although the contributions from the different subgroups differ according to the particular weight factor of the Casimir operator $C^{(i)}$. For SU(2), $C^{(2)}$ vanishes in the Landau gauge, whereas the contribution from U(1) is rather small (see Table I).[23] We obtain the following Dyson equation for $\delta m(p^2)$ (Figs. 9 and 10):

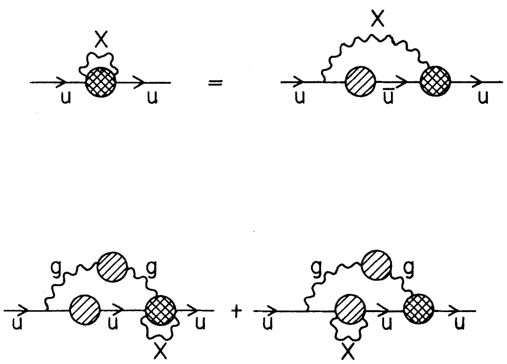

FIG. 9. Relevant diagrams for the Dyson equations to compute the lowest-order mass perturbation in $\alpha_{\rm GUM}$ to the u quark in SU(5).

$$\delta m_u(p^2)\simeq\frac{3}{4\pi}\alpha_{\rm GUM}\int_{p^2}^{\infty}\frac{dq^2}{q^2+M_X{}^2}m_u^s(q^2)+\frac{3}{4\pi}C_F\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_s(q^2)\delta m_u(q^2)\ , \tag{4.15}$$

$$\delta m_d(p^2)\simeq\frac{3}{8\pi}\alpha_{\rm GUM}\int_{p^2}^{\infty}\frac{dq^2}{q^2+M_X{}^2}m_d^s(q^2)+\frac{3}{4\pi}C_F\int_{p^2}^{\infty}\frac{dq^2}{q^2}\alpha_s(q^2)\delta m_d(q^2)\ , \tag{4.16}$$

with $\alpha_s=\alpha_3$, for $p^2<M_X{}^2$ and $\alpha_s=\alpha_5$ for $p^2>M_X{}^2$. Here we have factored out the X-uu^c and the $\bar{X}$-de^+ vertex functions from the driving terms in the integral equations, and replaced them by the SU(5) coupling constant at the SSB scale, $\alpha_{\rm GUM}=\alpha_5(M_X{}^2)$, since the most important contribution to the integrals appearing above comes from the neighborhood of $M_X{}^2$. Note that since $\alpha_5(q^2)m(q^2)$ decreases faster than a logarithm without restriction on a minimum number of flavors, the driving terms are well defined at infinity. The solution to Eqs. (4.15) and (4.16) is given by

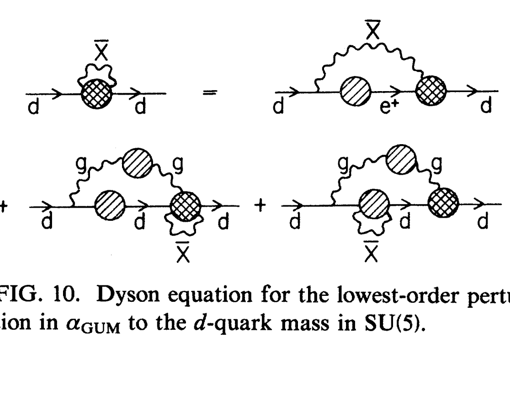

FIG. 10. Dyson equation for the lowest-order perturbation in $\alpha_{\rm GUM}$ to the d-quark mass in SU(5).

$$\delta m_u(p^2) = -\frac{3}{4\pi}\alpha_{\text{GUM}} m_u^s(p^2) \ln\left[\frac{p^2+M_X{}^2}{\Lambda^2}\right], \tag{4.17}$$

$$\delta m_d(p^2) = -\frac{3}{8\pi}\alpha_{\text{GUM}} m_d^s(p^2) \ln\left[\frac{p^2+M_X{}^2}{\Lambda^2}\right] \tag{4.18}$$

up to corrections of the order of $\ln\ln(M_X{}^2/\Lambda^2)$ from the momentum dependence of the SU(5) running coupling constant in the driving terms in Eqs. (4.15) and (4.16). If we identify the strong components of the mass by

$$m(p^2) = m^s(p^2) + \delta m(p^2), \tag{4.19}$$

we find the following ratio for $p^2 \ll M_X{}^2$:

$$\frac{m_d/m_u}{m_d^s/m_u^s} = \frac{1-(3/4\pi)\alpha_{\text{GUM}}\ln\left[\dfrac{M_X{}^2}{\Lambda^2}\right]}{1-(3/8\pi)\alpha_{\text{GUM}}\ln\left[\dfrac{M_X{}^2}{\Lambda^2}\right]}$$

$$= 1.35 \pm 0.05 . \tag{4.20}$$

Note that also that $m_u(M_X{}^2) \neq m_d(M_X{}^2)$.

The above result is valid for values of Λ between 0.2 and 0.5 GeV, $M_X = 10^{14}$ to 10^{16} GeV, and $\alpha_{\text{GUM}} = 0.0244$.[24] If we further assume that the u- and d-quark masses are degenerate in the absence of the electroweak and the baryon-number violating interactions (this would be the case if the u-d mass difference had its origin entirely in radiative corrections) we obtain the following ratio of light quark masses ($m_u^s = m_d^s$):

$$m_d/m_u = 1.35 . \tag{4.21}$$

This result is to be compared with the current mass ratio[25] $1.5 < m_d/m_u < 2$. If we set $m_d = 7$ MeV we obtain $m_u = 5.2$ MeV. Spontaneous symmetry breaking at the grand unification scale gives an important contribution to the u-d mass difference.[26]

V. SUMMARY AND CONCLUSIONS

We have studied the radiative contributions to self-energies and fermion mass differences for various gauge models in the framework of renormalized Dyson-Schwinger equations which are equivalent to the renormalization-group equations. Our main results are the following.

(1) The Dyson-Schwinger equations yield ultraviolet convergent self-masses if the theory is embedded in an asymptotically free grand unified group.

(2) We have shown how to include symmetry-breaking effects in such models. Spontaneous symmetry breaking at the grand unification scale gives an important contribution to fermion masses.

(3) The sign of the self-energy depends on the structure of the underlying gauge group.

To make further progress the zeroth-order relation between fermion masses (in particular for the u and d masses) should be better understood. Finally, it would be of interest to extend the calculations presented in this paper to larger groups, such as E_6, containing light and superheavy fermions in the same representation, and to asymptotically free family groups, such as SU(7) or SU(9), in order to calculate the radiative contribution to fermion masses and mass relations among the different families of fermion generations.

Note added in proof: It is worth stressing that the solution for the self-energy given by Eq. (2.11) with the constant C equal to zero is not equivalent to the solution required by the boundary conditions imposed by the renormalization-group equations, which evolve the running masses from one energy scale to another. To compare with the RGE, the QCD running mass [Eq. (2.8)] is subtracted from Eq. (2.15). Expanding the resulting expression to first order in α gives for C

$$C(M^2) = \frac{3}{4\pi}\alpha Q^2 \ln\left[\frac{M^2}{\mu^2}\right],$$

$$\mu^2 = \Lambda^2, \text{ for } \delta m(M^2) = 0 ,$$

if the boundary conditions are specified at $p^2 = M^2$. The solution with $C = 0$ is equivalent to setting the QCD parameter Λ as the natural scale of the problem. On the other hand, if we take M as the ultraviolet scale, $M \to \infty$, we recover the usual OPE-Cottingham result.

A numerical analysis of coupled equations of the type of Eqs. (4.6)–(4.8) with RGE boundary conditions and natural mass relations for various grand unified models is now in progress.

ACKNOWLEDGMENT

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[26]During the completion of this work I became aware of a related paper by P. H. Frampton and K. Kang [Hadron. J. 3, 814 (1980)]. This paper calculates the mass shift corresponding to the bare mass, thus obtaining infinite results as discussed above. Furthermore, I do not obtain the cancellations mentioned in their work, which does not include appropiate mass insertions to change helicity. I wish to thank Dr. P. Langacker for calling my attention to this paper.