PHYSICAL REVIEW D VOLUME 26, NUMBER 1 1 JULY 1982

Rapid Communications

The Rapid Communications section is intended for the accelerated publication of important new results. Manuscripts submitted to this section are given priority in handling in the editorial office and in production. A Rapid Communication may be no longer than 3½ printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but, because of the rapid publication schedule, publication is not delayed for receipt of corrections unless requested by the author.

Ultrahigh-energy cross section from study of longitudinal development of air showers

R. W. Ellsworth
Department of Physics, George Mason University, Fairfax, Virginia 22030

T. K. Gaisser and Todor Stanev*
Bartol Research Foundation of The Franklin Institute, University of Delaware, Newark, Delaware 19711

G. B. Yodh
Department of Physics, University of Maryland, College Park, Maryland 20742
(Received 1 March 1982)

We present calculations of the type that will be necessary for interpretation of large cosmic-ray experiments that measure longitudinal profiles of individual showers. A primary goal of such experiments is to determine both cross section and composition around 10^{18} eV.

Study of longitudinal development of air showers initiated by cosmic rays of ultrahigh energy has recently become feasible through study of time structure of Cherenkov light from air showers,[1,2] and in the last few months by observations of scintillation light from nitrogen fluorescence in the atmosphere induced by traversal of large air showers.[3] The last technique will make it possible to study a large sample of events ($\sim 10^4$) with energy around 10^9 GeV in the next several years. Individual shower profiles can in principle be measured, in particular, the distribution of effective shower starting points, shower maxima, and shower demise for fixed total track length (i.e., total deposited energy). In this paper, we discuss the capabilities and limitations of experiments that measure shower profiles for determining the asymptotic behavior of the proton-proton total cross sections up to $\sqrt{s} \sim 100$ TeV.[4] We also discuss the sensitivity of these techniques to primary composition of cosmic rays of about 10^{18} eV. We conclude that it will probably be possible to measure the proton cross section at these energies if it is less than ~ 120 mb and to place a lower bound otherwise.

The use of air showers to determine cross section and composition simultaneously has a long history.[5] Results have been limited by problems of fluctuations (both intrinsic and instrumental) in the presence of a steep primary energy spectrum. Although these problems are still present in the type of experiment discussed here, the ability to measure longitudinal profiles of individual showers is a substantial improvement over the situation in the classic air-shower experiment in which the cascade is sampled at one depth only.

If the actual shower starting points x_0 could be measured, then the cross sections on air nuclei of the different components in the incident cosmic-ray beam and their relative weights could be unfolded directly from the measured attenuation of the primary beam. In any indirect experiment, however, x_0 cannot be measured. Our task, therefore, is to find out to what extent measurable distributions, such as depth of quarter maximum ($x_{1/4}$) or depth of maximum (x_m), reflect the fundamental interaction lengths and the composition despite the existence of fluctuations in shower development which may also contribute significantly to these distributions.

This problem was investigated in Ref. 6 with emphasis on fragmentation and pion production in collisions of nuclear projectiles. It was found that, as expected, the tail of the $x_{1/4}$ distribution reflected the input cross section for protons and that the portion of the distribution with small values of $x_{1/4}$ is sensitive to the fraction of heavy primaries. The calculations of Ref. 6, however, are based on a simplified model for nucleon showers and consider only one, energy-dependent $\sigma_{p\text{-air}}$. Moreover, only results for $x_{1/4}$ were reported in that paper since that distribution should resemble the distribution of shower starting points more closely than x_m. We have since learned[7] that determination of $x_{1/4}$ with Fly's Eye[3] is at present limited due to problems arising from lack of light, contamination due to Cherenkov light, atmospheric scattering, and other effects. We therefore present results here for x_m, based on a variety of assumptions for $\sigma_{p\text{-air}}$ and using a rather detailed Monte Carlo treatment of development of showers initiated by protons and nucleons of nuclei. Fragmentation and pion production by nuclear projectiles are treated as in Ref. 6. Momentum distributions of secondaries produced in collisions of nucleons and pions are obtained by simply scaling the measured distributions from $\sqrt{s} = 20-60$ GeV to cosmic-ray energies, as in Ref. 8.[9]

Figure 1 shows distributions of x_m for two arbitrary compositions.[10] The region $600 \leq x_m \leq 750$ g/cm^2 is particularly sensitive to abundance of heavy primaries relative to protons, whereas the region $x_m \geq 750$ g/cm^2 reflects primarily protons. We therefore expect that the tail of the x_m distribution may reflect the proton cross section. Accordingly, we define an effective attenuation of the maximum by fitting the deep portion (> 760 g/cm^2) of the distribution to $\exp(-x_m/\Lambda_m)$. In the example shown here the interaction lengths for protons and α's are $\lambda_{p\text{-air}} \sim 40$ g/cm^2 and $\lambda_{\alpha\text{-air}} \sim 35$ g/cm^2 (corresponding to $\sigma_{pp}^{\text{tot}} \sim 130$ mb at $\sqrt{s} = 25$ TeV).[11] If the proton cross section is indeed this large, measuring it will be possible only if the concentration of α primaries is not too large (i.e., only if $N_\alpha < N_p$).[12] The extent to which heavy primaries may interfere with determination of $\lambda_{p\text{-air}}$ is illustrated by noting that Λ_m is decreased by 13% for the heavier composition in Fig. 1 and by 7% for the composition with 55% protons relative to the case for pure protons. For p-air cross sections in the range of 500 mb, a 10% uncertainty in Λ_m gives rise to an approximately equal uncertainty in $\sigma_{p\text{-air}}$ (see Fig. 2).

Even without the problem of heavy primaries, measurement of x_m or $x_{1/4}$ alone cannot determine an arbitrarily large proton cross section because of intrinsic fluctuations in shower development. The results of our calculations bear this out, as shown in Fig. 2. Here we show Λ_m for proton showers only, as

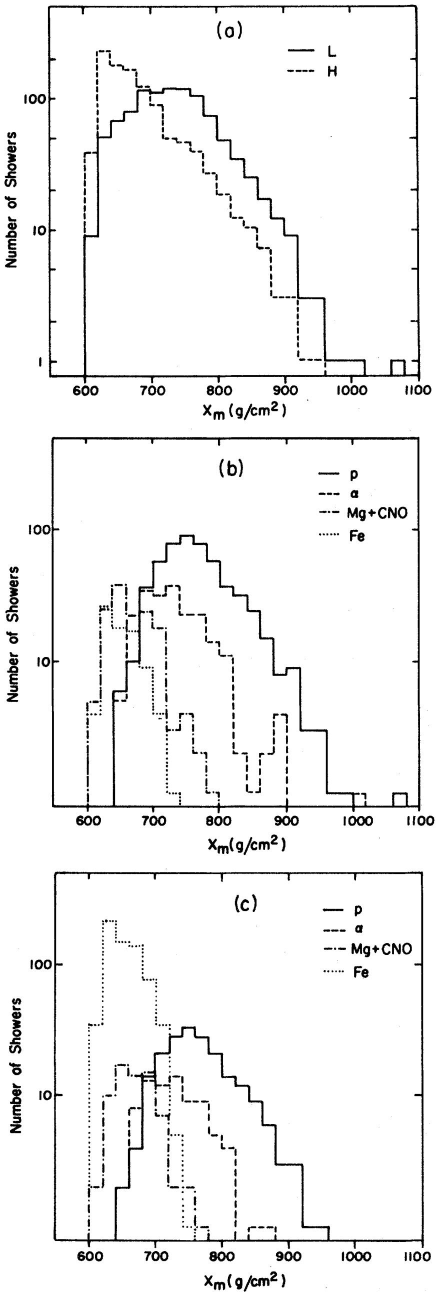

FIG. 1. (a) Distribution of depth of maximum for showers of energy $\geq 3 \times 10^{17}$ eV per nucleus for two compositions: $p:\alpha:\text{CNO}+\text{Mg}:\text{Fe} = 0.55:0.21:0.16:0.08$, as at low energy ($L$); and $p:\alpha:\text{CNO}+\text{Mg}:\text{Fe} = 0.2:0.08:0.07:0.65$, denoted H. (b) and (c) show the components separately for the low-energy and the heavy composition, respectively.

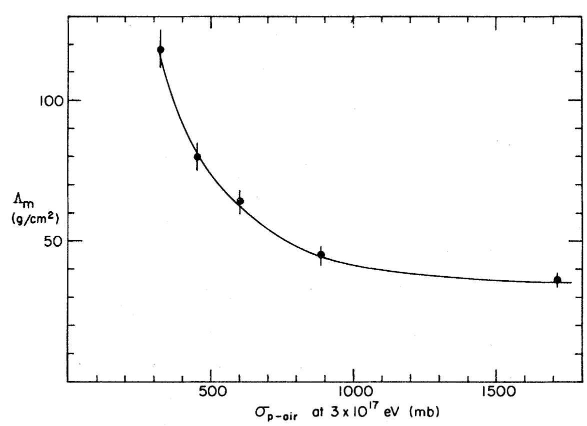

FIG. 2. Λ_m vs $\sigma_{p\text{-air}}$ for proton showers chosen from a power-law energy spectrum (differential index = 2) with $E_0 > 3 \times 10^{17}$ eV. Error bars show statistical uncertainty from the simulation result. Since the figure shows proton showers only, it cannot be used for an accurate determination of σ. See text.

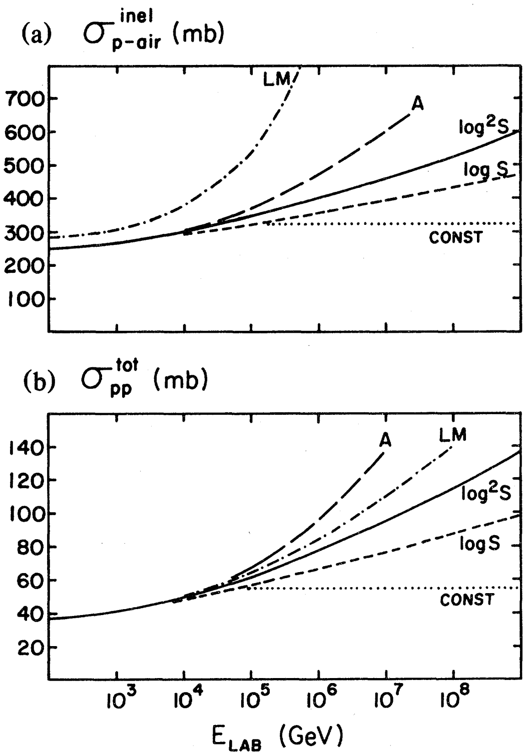

FIG. 3. (a) Energy-dependent cross sections for inelastic p-air collisions used for the calculations shown in Fig. 2. The curves are labeled to correspond to the values of $\sigma_{pp}^{\rm tot}$ shown in part (b). (b) Energy-dependent pp total cross sections. The curves labeled $\log s$ and $\log^2 s$ are extrapolations of fits to the cross section up to CERN ISR energies. The curve labeled A is an extrapolation of the estimate of Afek *et al.* (Ref. 15) and LM stands for Leader and Maor (Ref. 16).

a function of $\sigma_{p\text{-air}}$ at 3×10^{17} eV. For the atmosphere,

$$\lambda_{p\text{-air}}\ (\mathrm{g/cm^2}) = \frac{2.4\times10^4}{\sigma_{p\text{-air}}\ (\mathrm{mb})}\quad .$$

We emphasize that Fig. 2 cannot at present be used for an accurate determination of $\sigma_{p\text{-air}}$ from Λ_m because of the dependence on composition mentioned above. In addition, possible effects of uncertainties in the interaction model and of instrumental fluctuations need to be understood.

Primary energies of the protons were chosen from a power-law spectrum with $E_0 > 3\times10^{17}$ eV. To approximate Fly's Eye conditions we used a differential spectral index of 2. The true index is about 3 but the acceptance is proportional to E. Cascade development depends on the hadronic cross section at all energies up to the primary energy, though the overall profile is dominated by the high-energy behavior. Figure 3(a) shows the energy dependences of $\sigma_{p\text{-air}}$ used to construct Fig. 2. [Each point in Fig. 2 corresponds to one curve in Fig. 3(a).] The p-air and nucleus-air cross sections were obtained from $\sigma_{pp}^{\rm tot}$ (and the pp slope parameter) using Glauber theory as described in Ref. 13. The corresponding values of $\sigma_{pp}^{\rm tot}$ are shown in Fig. 3(b).[14]

TABLE I. Characteristic lengths for various cross sections. Errors are statistical uncertainty in simulation results.

Cross section	Input nucleon interaction length in air at 3×10^{17} eV	Simulation results $\Lambda_{1/4}$ (g/cm^2)	Λ_m (g/cm^2)
const	75	91 ± 6	118 ± 7
$\log s$	53	67 ± 4	80 ± 5
$\log^2 s$	40	53 ± 3	64 ± 4
A	28	35 ± 2	45 ± 3
LM	15	27 ± 2	36 ± 2

Inspection of Fig. 2 suggests that experiments that can measure shower profiles should be able to estimate the proton cross section at $\sqrt{s}\sim50$ TeV if it is not too large (say $\sigma_{p\text{-air}} < 600$ mb or $\sigma_{pp}^{\rm tot} \lesssim 120$ mb) and to place a lower bound otherwise. If the abundance of α primaries were much larger than we have assumed or if heavy primaries were very abundant, the dividing line between measurement and lower bound would be somewhat lower. Perhaps surprisingly, Λ_m is as sensitive to cross section as $\Lambda_{1/4}$, though for a given cross section $\Lambda_{1/4}$ is numerically closer to $\lambda_{\rm int}$ than Λ_m (i.e., $\Lambda_m > \Lambda_{1/4} > \lambda_{\rm int}$—see Table I). This may be of practical importance since x_m appears easier to measure than $x_{1/4}$. Determination of relative abundance of heavy nuclei, as well as cross section, will require unbiased measurements of x_m (and/or $x_{1/4}$) over the full range of depths (see Fig. 1).

Complete results for shower profiles calculated with various models (including scale breaking) over a range of energies and description of details of the calculation will be published elsewhere.

The work of R.W.E. and G.B.Y. was supported in part by the U.S. National Science Foundation under Grant No. PHY-7909272. The work of T.K.G. was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-78ER05007. The work of T.S. was supported in part by the U.S. National Science Foundation under Grant No. PHY-8008901.

*Permanent address: Institute for Nuclear Research and Nuclear Energy, Sofia, Bulgaria.

[1]G. B. Khristiansen, in *Sixteenth International Cosmic Ray Conference, Kyoto, 1979, Conference Papers* (Institute of Cosmic Ray Research, University of Tokyo, Tokyo, 1979), Vol. 14, p. 360, and references therein; N. N. Kalmykov *et al.*, in *Seventeenth International Cosmic Ray Conference, Paris, 1981, Conference Papers* (Centre d' Études Nucleaires, Saclay, 1981), Vol. 6, p. 114.

[2]K. Orford and K. E. Turver, Nature (London) 264, 727 (1976); M. P. Chantler *et al.*, in *Seventeenth International Cosmic Ray Conference, Paris, 1981, Conference Papers* (Ref. 1), Vol. 6, p. 121; A.A. Andam *et al.*, *ibid.*, Vol. 6, pp. 57, 125, 129; Phys. Rev. D 26, 23 (1982).

[3]G. L. Cassiday *et al.*, in *Proton-Antiproton Collider Physics—1981,* proceedings of the Workshop on Forward Collider Physics, Madison, Wisconsin, edited by V. Barger, D. Cline, and F. Halzen (AIP, New York, to be published), and references therein. See also J. W. Elbert, in *Cosmology and Particles,* proceedings of the First Moriond Astrophysics Meeting, edited by J. Audouze *et al.* (Editions Frontieres, Dreux, 1981), p. 69.

[4]The Durham group have compared their experimental results (Ref. 2) to Monte Carlo simulations that include generation of the signals they actually measure. See T. J. L. McComb and K. E. Turver, in *Seventeenth International Cosmic Ray Conference, Paris, 1981, Conference Papers* (Ref. 1), Vol. 6, p. 130, and references therein. In order to be more general, we calculate the longitudinal shower profiles here.

[5]S. Fukui *et al.*, Prog. Theor. Phys. Suppl. 16, 1 (1960); and J. Linsley, in *Proceedings of the Eighth International Conference on Cosmic Rays, Jaipur, India, 1963,* edited by R. Daniels *et al.* (Commercial Printing Press, Ltd., Bombay, India, 1964–5), Vol. 4, p. 295.

[6]T. K. Gaisser, T. Stanev, P. Freier, and C. Jake Waddington, Phys. Rev. D 25, 2341 (1982).

[7]E. H. Loh (private communication); G. Cassiday, in *Proton-Antiproton Collider Physics—1981* (Ref. 3).

[8]R. W. Ellsworth *et al.*, Phys. Rev. D 23, 771 (1981).

[9]This type of Feynman scaling for $F(x) = (1/\sigma) d\sigma/dx$ produces a central plateau which does not rise with energy. Recent data from the $\bar{p}p$ collider at CERN SPS show, however, that the plateau continues to rise at least to 540 GeV [G. Arnison *et al.* (UA1 Collaboration), Phys. Lett. 107B, 320 (1981); K. Alpgard *et al.* (UA5 Collaboration), Phys. Lett. 107B, 310, 315 (1981)]. Presumably this rise continues asymptotically. Unless it is accompanied by a significant change for $F(x)$ in the fragmentation region, however, it will not affect high-energy cascade calculations, which depend primarily on the fragmentation region (say $x \gtrsim 0.05$). Effects of possible fragmentation scaling violations [J. Wdowczyk and A. W. Wolfendale, in *Proton-Antiproton Collider Physics—1981* (Ref. 3); Nuovo Cimento 54A, 433 (1979)] on this type of calculation will be investigated in a future paper.

[10]We have simulated 500 proton showers for each of five trial cross sections (see Fig. 3). In addition, we calculated 200 α showers, 100 nitrogen showers, 100 Mg showers, and 120 Fe showers for the $\log^2 s$ cross section. These represent five major groups of primary cosmic rays: p, α, CNO, $(9 \leq Z \leq 14)$, and $(15 \leq Z \leq 26)$, respectively. For clarity, Mg and CNO have been plotted as a single group in Fig. 1.

[11]This estimate for $\lambda_{\alpha\text{-air}}$ is based on a very simple multiple-scattering calculation in which it was assumed that the shape of the nucleon profile is independent of energy. In fact, in most theories with a rising nucleon-nucleon cross section the proton becomes increasingly transparent as energy increases. Thus the shadowing will decrease in collisions between nuclei. So if $\lambda_{p\text{-air}}$ has decreased from 80 to 40 g/cm^2 we expect the corresponding high-energy value of $\lambda_{\alpha\text{-air}}$ to be significantly less than 35 g/cm^2. (It is 40 g/cm^2 at low energy.) We are grateful to J. Ball for pointing out this fact, which will tend to alleviate the possible problem of contamination by α's.

[12]If there is a rigidity-dependent steepening in the primary spectrum in which the spectral index increases by 0.5 an α/p ratio of 0.4 below, the break would tend to 0.6 above. Such a bend may occur around 10^{15}–10^{16} eV. The energy range of interest here is much higher, and there are independent suggestions in the recent literature that the primary spectrum may contain a significant fraction of protons around 10^{18} eV. See, e.g., J. Linsley and A. A. Watson, Phys. Rev. Lett. 46, 459 (1981), and references therein.

[13]V. Barger *et al.*, Phys. Rev. Lett. 33, 1051 (1974); T. K. Gaisser *et al.*, in *Proceedings of the Fourteenth International Conference on Cosmic Rays, Munich, 1975* (Max-Planck-Institut für Extraterrestrische Physik, Garching, West Germany, 1975), Vol. 7, p. 2161; T. K. Gaisser, C. J. Noble, and G. B. Yodh (unpublished).

[14]For all except the curve labeled LM we assumed $b = 0.296$ GeV$^{-2} \times [\sigma_{pp}^{\text{tot}}$ (mb)].

[15]Y. Afek *et al.*, Phys. Rev. Lett. 45, 85 (1980).

[16]E. Leader and U. Maor, Phys. Lett. 43B, 505 (1973); U. Maor and S. Nussinov, *ibid.* 46B, 99 (1973).