Poincaré and de Sitter gauge theories of gravity with propagating torsion

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We consider a gauge approach to the gravitational theory based on the local Poincaré P_{10} or de Sitter S_{10} groups. The P_{10} gauge rotations and translations take place in the tangent spaces to the space-time manifold. We interpret the independence of matter fields from the tangent vectors as the necessity to use a nonlinear realization of the P_{10} or S_{10} groups thus effectively breaking the full symmetry to the Lorentz group. The Lagrangian we choose is the S_{10} Yang-Mills invariant with the space-time metric expressed in terms of the translational part of the S_{10} nonlinear gauge field. Various consequences of the theory are discussed, including the correspondence with general relativity, the propagating spin-connection interactions, the analogy with the chiral Higgs mechanism, instantonlike solutions, a possibility of gravitational repulsion due to the noncompactness of the Lorentz group, etc. We also analyze the quantization of the theories with torsion with special emphasis on the presence of the nonlinear realization. We stress the possibility of obtaining a renormalizable theory if the metric is not quantized but is expressed in terms of a mean value of the quantized S_{10} nonlinear gauge field.

I. INTRODUCTION

Recently, there has been considerable interest in attempts to extend the Einstein theory of gravity or general relativity (GR) using the gauge approach (for reviews see Refs. 1–8). In the most natural of these modifications [generalizing in that or another way the Einstein-Cartan theory[8] (ECT) one considers the localization of the Poincaré group $P_{10}=T_4\times)L_6$ [or the semidirect product of the translational $T_4=R^{1,3}$ and Lorentz $L_6=\mathrm{SO}(1,3)$ subgroups] thus obtaining the tetrad e^a_μ and the Lorentz connection $\omega^{ab}{}_\mu$ as the gauge potentials and the torsion $\Theta^a{}_{\mu\nu}=\mathscr{D}_\mu e^a_\nu-\mathscr{D}_\nu e^a_\mu$ and the curvature $R^a{}_{b\mu\nu}=[\partial_\mu\omega_\nu+\omega_\mu\omega_\nu-(\mu\leftrightarrow\nu)]^a_b$ as the gauge strengths. As a consequence, the Einstein interaction of e^a_μ (or the metric) with the energy-momentum tensor t^μ_a is supplemented by the interaction of $\omega^{ab}{}_\mu$ and the spin-density tensor $S_{ab}{}^\mu$ ("ω-S interaction"). The Lagrangian of the theory contains the "translational" part (linear in R and quadratic in Θ), providing the correspondence with GR, as well as the "rotational" part (quadratic in R) giving a dynamical (or noncontact, cf. with ECT) ω-S interaction. The possibility of two independent coupling constants (dimensional and dimensionless) follows from the structure of the P_{10} itself.

The consideration of these theories is based on the following reasons: (i) the Poincaré group and the concept of spin are fundamental in particle physics (cf. Refs. 1–3, and 8); (ii) the gauge principle is very successful as a guide in constructing theories of fundamental interactions; (iii) one may hope that a gauge theory of gravity with torsion (or its supersymmetric generalization) will improve the quantum behavior of GR.[9–12] At the same time, there are a number of problems in the formulation of the "kinematics" as well the dynamics of the P_{10} gauge theory. The proposed approaches (see, e.g., Refs. 8, 13, and 14 and references in Refs. 1, 3, and 7) use in fact as a gauge group not P_{10} but the direct product $T_4\times L_6$, or some deformed algebra with different commutation relations from those for P_{10}.[8,1] As a result, e^a_μ cannot be consistently identified with an ordinary gauge field and the Lagrangian is invariant under the independent T_4 and L_6 groups. In this paper (Sec. 1) we consider the T_4 translations acting in the tangent affine spaces (and not as infinitesimal coordinate transformations) and show that one must use the nonlinear realization of the T_4 symmetry in order to obtain a gravitational theory and not the "internal" P_{10} theory (the spontaneous breakdown of T_4 in our approach corresponds to the trivial invariance of the ordinary matter fields under the "internal" T_4 transformations). Gravity is thus described by the *nonlinear* P_{10} *gauge field* (for the definition of nonlinear gauge fields see Ref. 15) with $k_0{}^{-1}e^a_\mu([k_0]=\mathrm{cm})$ and $\omega^a{}_{b\mu}$ being its T_4 and L_6 parts (and hence it is the L_6 part of gravity that is the "usual" or linear gauge field).

Another problem is the ambiguity in choosing the P_{10} Lagrangian: the independent T_4 and L_6 symmetries do not sufficiently restrict the Lagrangian. Therefore one can use various combinations of the Θ^2 and R^2 terms with arbitrary coefficients[1–3,16] and thus needs some *additional* assumptions for selecting a particular one.[17] Our interpretation of this ambiguity is the following: the use of the (e,ω) variables corresponds to the use of the nonlinear realization of P_{10} and so the fixation of the dynamics is just the fixation of a "nonlinear Lagrangian".[15] The only distinguished Lagrangian is the Yang-Mills invariant of the *linear* gauge theory. However, the latter choice is not a satisfactory one for the Poincaré group, P_{10} being not semisimple (Sec. III). That is why we consider in Sec. IV the nonlinear gauge theory of the De Sitter group $S_{10}=\mathrm{SO}(1,4)$ (or the "nearest" semisimple extension of P_{10}) and choose the S_{10} Yang-Mills Lagrangian as a basic one. This fixes a particular nonlinear P_{10} Lagrangian, possessing a number of attractive properties, presented in Sec. V. Here we discuss the correspondence with GR, the spin-spin interactions, the instanton solutions, a possibility of indefiniteness of the energy for ω (due to the noncompactness of L_6), etc. In Sec. VI we analyze the problems of quantization of the (e,ω) theories with torsion and show that the use of the nonlinear realization and the Yang-Mills structure of the S_{10} Lagrangian provides the possibility to obtain a renormalizable theory, where the metric is not quantized but is equal to a mean value of a quantized field.

Some of the results of this paper were already summarized in Ref. 18.

II. NONLINEAR GAUGE THEORY OF THE POINCARÉ GROUP

Let us consider first the construction of the linear P_{10} gauge theory following the analogy with "internal" gauge theories. In order to realize P_{10} on the linear vector space and not on the affine space we use the map $P_{10}\to\mathrm{GL}(5,R)$

$$P_{10}=\left\{\mathscr{A}=\begin{bmatrix}L & b\\ 0 & 1\end{bmatrix},\ L\in\mathrm{SO}(1,3),\ b\in R^{1,3}\right\},$$
$$\mathscr{A}\begin{bmatrix}a\\ 1\end{bmatrix}=\begin{bmatrix}La+b\\ 1\end{bmatrix},\ a\in R^{1,3}\,. \tag{2.1}$$

Defining the $\mathrm{GL}(5,R)$ connection on the vector bundle over space-time M^4 with the fiber R^5 and restricting it on P_{10}, we get the following P_{10} connection (one-form on M^4 with values in the Lie algebra of P_{10}):

$$\hat\Omega=\begin{bmatrix}\hat\omega & \hat\theta\\ 0 & 0\end{bmatrix},\ \hat\omega\in\mathscr{S}\mathrm{O}(1,3),\ \hat\theta\in R^{1,3}\,,$$
$$\hat\omega'=L\hat\omega L^{-1}+L\,dL^{-1},\ \hat\theta'=L\hat\theta-\hat{\mathscr{D}}'b\,, \tag{2.2}$$
$$\hat{\mathscr{D}}'=d+\hat\omega'\,.$$

The corresponding curvature two-form is

$$\hat{\mathscr{R}}=d\hat\Omega+\hat\Omega\wedge\hat\Omega=\begin{bmatrix}\hat R & \hat\Theta\\ 0 & 0\end{bmatrix},$$
$$\hat R=d\hat\omega+\hat\omega\wedge\hat\omega,\ \ \hat\Theta=\hat{\mathscr{D}}\hat\theta\,. \tag{2.3}$$

It is evident from (2.2) that the splitting of $\hat\Omega$ on $\hat\omega$ and $\hat\theta$ depends on the splitting of P_{10} on L_6 and T_4. This freedom can be naturally separated by the introduction of the vector field $\xi(x)\in R_x^{1,3}$, fixing in each tangent space $T_x(M)$ a stability point for L_6 [here

$$\left\{\begin{bmatrix}a\\ 1\end{bmatrix}\right\}\subset R_x{}^5$$

is identified with $T_x(M)$ so that the corresponding bundle is soldered to M^4 with $\hat\theta$ playing the role of the generalized soldering form[19–21]]. Under the action of P_{10}: $\xi'=L\xi+b$, $\bar\theta'=L\theta$, where $\bar\theta=\hat\theta+\hat{\mathscr{D}}\xi$. As a result, one can identify $\hat\omega$ with the Lorentz connection ω on $T(M)$, $k_0\bar\theta$ ($[k_0]=\mathrm{cm}$) with the canonical one-form θ ($=e^a_\mu dx^\mu$) and $\hat\Omega$—with the generalized affine connection on the tangent affine bundle over $M^{4,19}$, i.e.,

$$\hat\Omega=\begin{bmatrix}\omega & k_0{}^{-1}\theta-\mathscr{D}\xi\\ 0 & 0\end{bmatrix},\ \ \hat{\mathscr{R}}=\begin{bmatrix}R & k_0{}^{-1}\Theta-R\xi\\ 0 & 0\end{bmatrix}, \tag{2.4}$$

where R and $\Theta=\mathscr{D}\theta$ are the curvature and torsion of ω and θ.

The above construction of the P_{10} kinematics is natural and gives all necessary results by the restriction of various GL_5 formulas to the P_{10} case. For example, from the Bianchi identity $\hat D\hat{\mathscr{R}}\equiv d\hat{\mathscr{R}}+\hat\Omega\wedge\hat{\mathscr{R}}-\hat{\mathscr{R}}\wedge\hat\Omega=0$ we have

$$\mathscr{D}R=0,\ \ \mathscr{D}\Theta-R\wedge\theta=0\,, \tag{2.5}$$

or the well-known Bianchi and Ricci identities.[19]

The introduction of ξ in (2.2) can also be treated in the following remarkable way: ξ is the "chiral" field providing the passage from the linear P_{10} gauge field $\hat{\Omega}$ to the nonlinear P_{10} gauge field

$$\Omega=\begin{bmatrix}\omega & k_0{}^{-1}\theta\\ 0 & 0\end{bmatrix}.$$

The notion of a nonlinear gauge field[15] was used in theories with "phenomenological Lagrangians" for the description of interactions of gauge and adjoint chiral fields and formally coincides with that of the connection on the bundle with a homogeneous space as a fiber. Let $P(M,G)$ be a principal bundle and $\mathcal{F}(M,F,G,P)$ be the associated bundle with the fiber $F=G/K$, i.e., the homogeneous space (the space of a nonlinear realization of G). Here K is a subgroup of G, i.e., for $S\in G$ there is a unique decomposition

$$S=S_F S_K,\ S_F\in F,\ S_K\in K$$

For the Lie algebra g of G we have

$$g=k\oplus f,\ [k,k]\subseteq k,\ [k,f]\subseteq f\ , \qquad (2.6)$$

where k is the Lie algebra of K,f is the vector space, corresponding to F, and F is assumed to be a weakly reductive homogeneous space. Let $\hat{A}\in g$ be the one-form on M^4 corresponding to the connection on $P(M,g)$ for the fixed section of the bundle, i.e., the linear gauge field. Under the change of the section $\hat{A}'=S\hat{A}S^{-1}+S\,dS^{-1}$, $S\in G$. Now let us define the connection A on the bundle $\mathcal{F}$ or the nonlinear gauge field

$$A=\phi^{-1}\hat{A}\phi+\phi^{-1}d\phi,\ A\in g,\ \phi\in F\ . \qquad (2.7)$$

Here A is a one-form on M^4 depending on $\hat{A}$ and on the fixed point $\phi(x)\equiv S_{0F}\in F_x$ in each copy of the homogeneous space, i.e., on the adjoint chiral field. Under the action of G,

$$S\phi=(S_F S_K)S_{0F}=S'_{0F}S'_K,\ S'_{0F}=\phi'\ . \qquad (2.8)$$

Decomposing A according to (2.6),

$$A=A_k+A_f,\ A_k\in k,\ A_f\in f\ , \qquad (2.9)$$

we get the following transformation laws for the parts of A under (2.8):

$$A'_k=S'_K A_k S'^{-1}_K+S'_K dS'^{-1}_K,\quad A'_f=S'_K A_f S'^{-1}_K\ . \qquad (2.10)$$

As a result, A_k transforms as an ordinary connection (or a linear gauge field) but A_f transforms homogeneously (this is a characteristic feature of a nonlinear gauge field). The nontriviality of the passage from $\hat{A}$ to A is due to the fact that S'_K is in general a complicated function of S_K, S_F, and ϕ and so the transformed fields in (2.10) depend on S in a nonlinear way.

The Higgs mechanism is a remarkable example of the use of the nonlinear gauge fields.[22] Consider a multiplet of scalar fields φ linearly transforming under the action of G. If there is a spontaneous symmetry breaking $G\to K$, then

$$\rho=\phi^{-1}\varphi,\ \phi=\exp(\alpha^a t_a)\in F,\ \{t_a\}\in f\ , \qquad (2.11)$$

where ϕ corresponds to a set of Goldstone fields and transforms as a chiral field according to (2.8) (as a consequence, ρ transforms as a nonlinear multiplet, $\rho'=S'_K\rho$). Introducing the vacuum ρ_0 (invariant under K) we get the Higgs field $\chi=\rho-\rho_0$, transforming according to the nonlinear realization of G. As a result of (2.11), $\langle\hat{\mathscr{D}}\varphi\hat{\mathscr{D}}\varphi\rangle=\langle\mathscr{D}\varphi\mathscr{D}\varphi\rangle$, $\mathscr{D}\rho=\mathscr{D}\chi+A_f\rho_0$, i.e., $\hat{A}$ transforms into A by absorbing ϕ and $\rho_0\neq 0$ provides a nonzero mass for A_f. The final Higgs-field Lagrangian is invariant under the nonlinear realization of G and so the mass term for A_f is admissible under the transformation law (2.10). The field strength of A is

$$F=\phi^{-1}\hat{F}\phi=dA+A\wedge A\ , \qquad (2.12)$$

so for the Lagrangian four-form we have

$$L=\tfrac{1}{4}\operatorname{tr}(\hat{F}\wedge *\hat{F})=\tfrac{1}{4}\operatorname{tr}(F\wedge *F)\ . \qquad (2.13)$$

Now let us turn to the case of our interest: $G=P_{10}$, $K=L_6$, and $G/K=T_4$ being the homogeneous affine Minkowski space $\mathscr{M}$. It is the fact that $\mathscr{M}$ is the affine and not the linear space that suggests here the use of nonlinear gauge fields. Taking

$$\phi=\begin{bmatrix}1 & \xi\\ 0 & 1\end{bmatrix}\in T_4\ ,\qquad S=S_L S_T=\begin{bmatrix}L & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & b\\ 0 & 1\end{bmatrix}, \qquad (2.14)$$

from (2.8) we get $L'=L$, $\xi'=L\xi+b$. As a consequence of (2.2), (2.7), and (2.14) we obtain the nonlinear P_{10} gauge field in the form

$$\Omega=\begin{bmatrix}\omega & \bar{\theta}\\ 0 & 0\end{bmatrix},\quad \mathscr{R}=\begin{bmatrix}R & \bar{\Theta}\\ 0 & 0\end{bmatrix}. \qquad (2.15)$$

Thus the identification $\bar{\theta}=\hat{\theta}+\mathscr{D}\xi$, $\hat{\omega}=\omega$ in (2.4) follows directly from the definition (2.7). Because of the correct transformation law $\bar{\theta}'=L\bar{\theta}$ [following from (2.10)] we can now identify $k_0\bar{\theta}$ with the canonical one-form on M^4 and introduce the metric tensor on M^4 ($g_{\mu\nu}=e^a_\mu e^a_\nu=k_0{}^2\bar\theta^a_\mu\bar\theta^b_\nu\eta_{ab}$) in terms of the translational part of the P_{10} nonlinear gauge field. We see that the P_{10} case is in some sense a trivial example of a nonlinear realization (though a nontrivial redefinition is $\hat\theta\to\bar\theta$): $L'=L$, i.e., L' is linear in L and independent of the translational parameter (and so ω' and $\bar\theta'$ "do not remember" about b).

We stress that in our approach translations take place in the tangent spaces [for example, as a shift of the origin of the affine basis, $\eta'=\eta+b$, $\eta \in T_x(M)$] and so T_4 has no relation to coordinate transformations (see also Refs. 20 and 21, cf. Refs. 4 and 5). The ordinary physical fields $\varphi(x)$ depending only on the point $x\in M_4$ of the base space are trivially invariant under T_4. Thus P_{10} can be realized (as a semidirect product) only on the ring of smooth functions on the tangent bundle $C^\infty(T(M))=\{\psi(x,\eta)\}$ and not on $C^\infty(M) =\{\varphi(x)\}$, $\psi'(x,\eta)=\mathscr{O}(L)\psi(x,L^{-1}(\eta-b))$ [$\mathscr{O}(L)$ defines a representation of L_6], i.e., on the functions depending not only on x, but also on the tangent vector $\eta(x)\in T_x(M)$ (similar fields are discussed in Refs. 20, 21, and 23). One can consider this fact as a manifestation of a spontaneous T_4-symmetry breaking ($P_{10}\to L_6$) by the physical vacuum (for the fields depending only on x but not on η). In analogy with (2.11) we can define [here φ stands for ρ and ψ for φ in (2.11)]

$$\varphi(x)=\exp(-\xi^a t_a)\psi(x,\eta),\quad t_a=\frac{\partial}{\partial\eta^a} \tag{2.16}$$

and choosing $\xi=\eta$ (as a gauge) we get $\varphi(x) =\psi(x,0)$. Thus P_{10} is realized nonlinearly on $\{\varphi(x)\}$ [in the mentioned trivial sense, cf. (2.14)] and we come to the necessity of using the nonlinear gauge field (2.15). The "chiral field" ξ plays the role of (four) Goldstone fields, all physical fields are of the "Higgs type" for gravity and gravity itself is described by the nonlinear P_{10} gauge field. In this final interpretation our approach differs considerably from the earlier suggestions of treating gravity as a nonlinear gauge field.[24–27] It is interesting to note that translations play a rather peculiar role (cf. Ref. 28): they do not act on the physical fields (and thus the P_{10} covariant derivative coincides with the L_6 one) but their localization is necessary for the introduction of the tetrad one-form (and thus metric) as the part of the nonlinear gauge field (note that this interpretation of θ is unrelated to the attempts at considering the metric as a Goldstone field[29].

We come to the conclusion that the L_6 symmetry is unbroken (and so is realized linearly) and the corresponding L_6 gauge field ω is a part of the gravitational field which is really a (linear) gauge field. The tetrad field (or metric), i.e., the Einstein gravitational field, is the "nonlinear" T_4 part of the nonlinear P_{10} gauge field, which (due to the dimensional character of the T_4 coupling constant) provides a clear explanation of a possibility of nonpolynomial (in e^a_μ or $g_{\mu\nu}$) interactions (present in the Einstein theory).

III. THE CHOICE OF THE DYNAMICS FOR THE POINCARÉ GAUGE THEORY

Now let us consider two possible ways of constructing the dynamics of the P_{10} theory. First, one can start with the nonlinear gauge field (2.15) and use its parts ω and θ separately in a Lagrangian, invariant under the nonlinear realization of P_{10} (or actually under L_6) and of course under the general coordinate transformations. Hence a Lagrangian is not fixed uniquely [note that the possibility of the linear in curvature $R(\omega)$ term, providing the correspondence with GR, is due to the homogeneous transformation law for θ, cf. Ref. 30]. Second, one can try to construct the dynamics in analogy with internal gauge theories, i.e., consider the Lagrangian depending on Ω (or $\mathscr{R}$) as a whole object. The simplest nontrivial choice is the Yang-Mills Lagrangian (2.13),

$$L_P=\frac{1}{4\lambda}\,\mathrm{tr}(\mathscr{R}\wedge *\mathscr{R})\ . \tag{3.1}$$

However, here it is necessary to assume that $g_{\mu\nu}$ in the star operation in (2.13) is expressed in terms of θ. That is why the use of the Ω as a whole is not sufficient, and this is a well-known obstacle to describing gravity by the Lagrangian, polynomial in the linear gauge field (cf. Refs. 5 and 30–32). In view of the nonsemisimple nature of the P_{10}, the Killing-Cartan form "tr" in (3.1) is degenerate, leading to the lack of equivalence of two procedures: (i) varying (3.1) as a GL_5 Yang-Mills Lagrangian and going to P_{10} only on the level of the field equations; (ii) treating (3.1) as the P_{10} Lagrangian from the very beginning, substituting (2.15) and varying the independent P_{10} potentials. The latter procedure gives the unsatisfactory result $L_P=L_L=(1/4\lambda)\,\mathrm{tr}(R\wedge *R)$ or only the L_6 Lagrangian without a T_4 part (necessary to establish the correct Einstein limit). The first procedure gives the following field equations:

$$D*\mathscr{R}=-\lambda*J,\quad J=\begin{bmatrix} S & -k_0t \\ 0 & 0 \end{bmatrix}, \tag{3.2}$$

or

$$\begin{aligned} &\mathscr{D}*R=-\lambda*S\ ,\\ &\mathscr{D}*\Theta-*R\wedge\theta=\frac{k^2}{2}*t\quad (k^2=2\lambda k_0{}^2)\ . \end{aligned} \tag{3.3}$$

These equations provide the dynamical ω-S interaction and a correspondence with GR [when $\Theta=0$ and $J=0$ (3.3) is equivalent to the vacuum Einstein equations, see also Ref. 34]. However, Eqs. (3.3) are nonvariational in a two-fold sense. First, they cannot be obtained from some action by varying ω and θ. Second, the dependence of $g_{\mu\nu}$ [in "$*$" in (3.1)] on the θ part of the dynamical variable Ω is not reflected in (3.2) [otherwise t in (3.3) would be supplemented by the energy-momentum tensor of $\mathscr{R}$]. As a result, one cannot identify t_a^μ with the energy-momentum tensor of matter $[=e^{-1}\partial(e\mathscr{L}_m)/\partial e^a_\mu,\ e=\det e^a_\mu]$ though it is possible to identity $S_{ab}{}^\mu$ with the spin density $(=\partial\mathscr{L}_m/\partial\omega^{ab}{}_\mu)$.

We conclude that the approach based on (3.1) does not give an acceptable theory in the case of the P_{10} as a gauge group. It seems natural now to pass to the De Sitter group, or the "nearest" semisimple extension of P_{10} (with the nondegenerate trace tr) and to consider again a unique Yang-Mills invariant (3.1). This gives the theory with the variational field equations, analogous in their structure to Eqs. (3.3).

IV. NONLINEAR GAUGE THEORY OF THE DE SITTER GROUP

The following parametrization of the De Sitter group $S_{10}=\mathrm{SO}(1,4)$ is useful in separating the $L_6=\mathrm{SO}(1,3)$ subgroup:

$$S=\begin{bmatrix} (1-\sigma b\otimes b^T)L & b \\ b^TL & \gamma \end{bmatrix}=S_bS_L\ ,$$
$$L\in L_6, b\in R^{1,3}\ , \tag{4.1}$$

$$S_L=\begin{bmatrix} L & 0 \\ 0 & 1 \end{bmatrix},\quad S_b=\exp\begin{bmatrix} 0 & \beta \\ \beta^T & 0 \end{bmatrix},$$
$$b=\frac{\sin\mu}{\mu}\beta,\ \mu^2=-b^2\ ,$$
$$\gamma=(1+b^2)^{1/2},\ (\beta^a)^T=\beta_a\ . \tag{4.2}$$

Let $\mathscr{E}(M^4,R^{1,4},S_{10},P)$ be the vector bundle, associated with the principal bundle $P(M^4,S_{10})$ and $\hat\Omega$ be the connection on $\mathscr{E}$ or the linear S_{10} gauge field [one-form on M^4 with values in the $\mathscr{S}\mathrm{O}(1,4)$ algebra]

$$\hat\Omega=\begin{bmatrix} \hat\omega & \hat\theta \\ \hat\theta^T & 0 \end{bmatrix},\quad \hat\omega\in\mathscr{S}\mathrm{O}(1,3),\ \hat\theta\in R^{1,3}\ . \tag{4.3}$$

The corresponding curvature is given by

$$\hat{\mathscr{R}}=\begin{bmatrix} \hat R+\hat\pi & \hat\Theta \\ \hat\Theta^T & 0 \end{bmatrix},\quad \hat R=d\hat\omega+\hat\omega\wedge\hat\omega\ ,$$
$$\hat\pi=\hat\theta\wedge\hat\theta,\ \hat\Theta=\hat{\mathscr{D}}\hat\theta\ . \tag{4.4}$$

The splitting of $\hat\Omega$ (4.3) is invariant only under the L_6 transformations ($b=0$). In order to have a possibility to establish the relation of $\hat\omega$ and $\hat\theta$ to the Lorentz connection and the canonical one-form on $T(M)$ it is necessary to consider the nonlinear realization of S_{10}. This effectively provides us with the space-time (noninternal) gauge theory and corresponds to the "breaking" of the symmetry $S_{10}\rightarrow L_6$. We can now construct the nonlinear S_{10} gauge field by considering the connection on the associated bundle $\mathscr{F}(M^4,\Sigma^4,S_{10},P)$ with the De Sitter spaces $\Sigma^4=S_{10}/(L_6)_\mathscr{O}$ as fibers ($\mathscr{O}$ is a fixed point in $R^{1,4}\supset\Sigma^4$). Using the prescription (2.7) with ϕ equal to S_b (4.2) we get [compare with (2.14) and (2.15) and note that the correspondence with the P_{10} theory can be obtained by the Inönu-Wigner contraction $S_{10}\rightarrow P_{10}$, i.e., by formally putting all quantities with index T equal to zero].

$$\Omega=\begin{bmatrix} \omega & \bar\theta \\ \bar\theta^T & 0 \end{bmatrix},\quad \omega\in\mathscr{S}\mathrm{O}(1,3),\ \bar\theta\in R^{1,3}\ , \tag{4.5}$$

$$\begin{aligned} &\omega=\hat\omega-\sigma(b\otimes b^T\hat\omega+\hat\omega b\otimes b^T)\ ,\\ &\bar\theta=\gamma\bar\theta+d\beta+\hat\omega b+\sigma(\hat\theta b^T)b\ . \end{aligned} \tag{4.6}$$

Under the action of S_{10},

$$\omega'=L'\omega L'^{-1}+L'dL'^{-1},\quad \bar\theta'=L'\theta\ , \tag{4.7}$$

where L' is defined from $SS_b=S_{b'}S_{L'}$, $S\equiv S_aS_L$.

Finally we are able to establish the necessary identifications of the ω and $\theta=k_0\bar\theta$ (coinciding with those of P_{10} case). Two remarks are now in order. First, one can observe that the Goldstone fields here are represented by the five-vector

$$\begin{bmatrix} b(x) \\ \gamma(x) \end{bmatrix}\in\Sigma_x{}^4$$

or the section of the bundle $\mathscr{F}$ (see also Refs. 27 and 35). The second remark is that the bundle $\mathscr{F}$

is soldered to M^4 with $\bar{\theta}$ being the soldering form (for detailed discussion of the De Sitter bundle see Ref. 20). The curvature of (4.5) can be written in the form

$$\mathscr{R}=\begin{bmatrix} R+\bar{\pi} & \bar{\Theta} \\ \bar{\Theta}^T & 0 \end{bmatrix}, \quad \bar{\pi}=k_0{}^{-2}\pi,\ \pi=\theta\wedge\theta^T, \tag{4.8}$$

where R and Θ coincide with the same quantities in (2.15).

Having defined the metric on M^4 in terms of $e^a_\mu=k_0\bar{\theta}^a_\mu$ we choose as a Lagrangian the S_{10} Yang-Mills term [cf. (2.13) and (3.1)]

$$\begin{aligned} L_S&=\frac{1}{4\lambda}\operatorname{tr}(\hat{\mathscr{R}}\wedge *\hat{\mathscr{R}}) \\ &=\frac{1}{4\lambda}\operatorname{tr}(\mathscr{R}\wedge *\mathscr{R}), \end{aligned} \tag{4.9}$$

invariant under the nonlinear realization of S_{10} (or even under the linear realization of S_{10} if $g_{\mu\nu}$ is considered as an invariant object).

It is important to recognize that only the L_6 symmetry is physical for the ordinary matter fields [this fact is a manifestation of a nonlinear realization of S_{10} (or P_{10})]. Thus various earlier attempts[32,35–39] at treating gravity directly as the De Sitter gauge field seem to be inappropriate. The correct attitude toward the problem (i.e., basing on a nonlinear realization) has been presented in Refs. 25–27, but from a different point of view regarding the reason for the nonlinear realization (see also Sec. II).

As far as the dynamics of the S_{10} theory is concerned, there have been proposed different variants of the S_{10} Lagrangian, but only (4.9) corresponds to the natural physical requirements [(4.9) was also discussed in Refs. 36 and 38 but with a different emphasis and in a different context]. For example, the Lagrangians of Refs. 27, 32, 37, and 40 are simply the reparametrizations of the ECT Lagrangian.[8] The approach of Ref. 39 (though similar to ours in the requirement of noncontactness of the ω-S interaction) is an unsatisfactory one: the Lagrangian here is a total derivative while all physical effects are obtained from the gauge-fixing term.

Returning to our Lagrangian (4.9) we can rewrite it [using (4.8)] as follows [$L_S=\mathscr{L}_S\sqrt{g}\,d^4x$ ($g=-\det g_{\mu\nu}$)]:

$$\begin{aligned} \mathscr{L}_S=&-\frac{1}{k^2}[R(\omega)-2\Lambda]+\frac{1}{2k^2}\Theta^a_{\mu\nu}\Theta_a{}^{\mu\nu} \\ &+\frac{1}{8\lambda}R^a{}_{b\mu\nu}R^b_a{}^{\mu\nu},\quad \Lambda=-\frac{6\lambda}{k^2},\quad k^2=2k_0{}^2\lambda\,. \end{aligned} \tag{4.10}$$

Varying (4.9) plus the matter term

$$\begin{aligned} &L_m=-\tfrac{1}{2}\operatorname{tr}(*J\wedge\Omega),\quad J=\begin{bmatrix} S & -k_0t \\ -k_0t & 0 \end{bmatrix}, \\ &L_m=\tfrac{1}{2}\omega^{ab}{}_\mu S_{ab}{}^\mu-e^a_\mu t^\mu_a\,, \end{aligned} \tag{4.11}$$

with respect to ω and e, we get the following equations:

$$\mathscr{D}*R+2\lambda k^{-2}(\mathscr{D}*\pi+\theta\wedge*\Theta^T-*\Theta\wedge\theta^T)=-\lambda*S\,, \tag{4.12}$$

$$\mathscr{D}*\Theta-*R\wedge\theta-2\lambda k^{-2}*\pi\wedge\theta=\frac{k^2}{2}(*t+\lambda^{-1}*T_0)\,. \tag{4.13a}$$

They differ from the S_{10} Yang-Mills equations only by the presence of the energy-momentum tensor T_0 of the Ω itself [arising due to the dependence of $g_{\mu\nu}$ on θ in (4.10)]:

$$\begin{aligned} &T^a_{0\mu}=e^a_\nu T^\nu_{0\mu}\,, \\ &T_{0\mu\nu}=-\frac{4\lambda}{k^2}(e^\lambda_b e_{a(\mu}R^{ab}_{\ \ \nu)\lambda}-\tfrac{1}{4}g_{\mu\nu}e^\lambda_b e^\rho_a R^{ab}_{\rho\lambda})+\frac{2\lambda}{k^2}(\Theta^a_{\mu\lambda}\Theta^\lambda_{a\nu}-\tfrac{1}{4}g_{\mu\nu}\Theta^a_{\lambda\rho}\Theta^{\lambda\rho}_a)+\tfrac{1}{2}(R^{ab}_{\mu\lambda}R_{ba\nu}{}^\lambda-\tfrac{1}{4}g_{\mu\nu}R^{ab}_{\lambda\rho}R_{ba}{}^{\lambda\rho})\,. \end{aligned} \tag{4.13b}$$

The Lagrangian (4.10) can be given the following useful interpretation. A spontaneous symmetry breaking $S_{10}\rightarrow L_6$ occurs in the free S_{10} gauge theory itself, supplying the L_6 gauge field ω with the "mass term." The difference from the case of the internal nonlinear gauge theory lies in the fact that the "exterior" metric in (4.9) is constructed of the $\bar{\theta}$ part of the gauge potential and thus can "absorb" (by the contraction of indices) the $\bar{\theta}$ multipliers in various terms of (4.10). As a simple analog of this situation let us consider the chiral Higgs mechanism,

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}\mathscr{D}^\mu\varphi\mathscr{D}^\mu\varphi\,, \tag{4.14}$$

where A_μ is the SO_3 gauge field and $\varphi \in S^2 = SO_3/SO_2$ is the adjoint chiral field ($\varphi^2 = m^2, \{\varphi^i\} \in R^3$). In view of the local SO_3 invariance of (4.14) we can simply put $\varphi = \varphi_0 = (0,0,m)$, obtaining the Lagrangian

$$\mathscr{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} [(A^1_\mu)^2 + (A^2_\mu)^2] \tag{4.15}$$

with the reduced symmetry SO_2. The only drawback of this "Higgs" mechanism is its perturbative nonrenormalizability (see, however Ref. 41). The correspondence with our theory is obvious:

$$\begin{aligned} &\varphi \to e^a_\mu, \quad \varphi^2 = m^2 \to e^a_\mu e^b_\nu \eta_{ab} = g_{\mu\nu} , \\ &\mathscr{D}_\mu \varphi \to \Theta^a_{\mu\nu} = \mathscr{D}_\mu e^a_\nu - \mathscr{D}_\nu e^a_\mu . \end{aligned} \tag{4.16}$$

As a result, e^a_μ being, from the geometrical point of view, the "covariant" part of the nonlinear S_{10} gauge field plays the role of the vector chiral Higgs field in the Lagrangian (4.10). Let us note that possible interpretation of θ as a vector Higgs field was also mentioned in Ref. 42. Quantizing the connection on the flat background $e^a_\mu \simeq \delta^a_\mu$ we really do obtain the mass term for ω (see Secs. V B and VI).

V. SOME CONSEQUENCES OF THE S_{10} THEORY

A. Correspondence with general relativity

There are two possible approaches to the problem of correspondence with GR in the context of the $(R + \Theta^2 + R^2)$ gauge theories of gravity. In the first, one naively assumes that torsion must be approximately zero in the Einstein limit and thus needs the linear in curvature term in the Lagrangian in order to obtain the Einstein equations. It should be understood that this approach is in no way necessary and compulsory and suffers from possible drawbacks when matter is present. The second approach (see, e.g., Ref. 43 and references therein) is based on the remark that the Einstein limit corresponds to the case of the spinless matter and thus to the theory, where only the translational subgroup of the Poincaré group is localized.[28] As a result we are led to a teleparallelism theory with $\Theta^a_{\mu\nu} \neq 0$, $R^a_{b\mu\nu} = 0$. If the Θ^2 part of the Lagrangian contains the appropriate combinations of the invariants, one can in principle establish the correct correspondence with GR. Though this prescription is rather appealing and natural in the context of the Hehl *et al.* Lagrangian,[17,43] it seems not to be well suited for our case (4.10), where there is the explicit R term (while the Θ^2 term does not give a realistic teleparallelism theory). Keeping in mind that this inapplicability of the second approach may be considered as a possible shortcoming of the choice of (4.10), we shall treat here the question of correspondence with GR assuming that torsion is zero.

Putting S, t, and Θ in (4.12) and (4.13) equal to zero we get

$$\begin{aligned} &\tilde{R}_{\mu\nu} - \tfrac{1}{2}(\tilde{R} - 2\Lambda) g_{\mu\nu} = \frac{k^2}{2\lambda} \tilde{\tau}_{\mu\nu} , \\ &\tilde{\tau}_{\mu\nu} = \tfrac{1}{2}(\tilde{R}^a{}_{b\mu\lambda} \tilde{R}^b{}_{a\nu}{}^\lambda - \tfrac{1}{4} g_{\mu\nu} \tilde{R}^a{}_{b\lambda\rho} \tilde{R}^b{}_a{}^{\lambda\rho}) , \end{aligned} \tag{5.1}$$

$$\tilde{R}^a{}_{b\nu\rho}{}^{;\nu} = 0, \text{ or } \tilde{R}_{\mu[\nu;\lambda]} = 0 , \tag{5.2}$$

where the wave denotes the objects constructed with the help of the Riemannian connection. Equation (5.1) can be rewritten as

$$\frac{2\lambda}{k^2} E_{\mu\nu} = C_{\mu\lambda\nu\rho} E^{\lambda\rho}, \quad \tilde{R} = 4\Lambda , \tag{5.3}$$

where $E_{\mu\nu} = \tilde{R}_{\mu\nu} - \frac{1}{4} g_{\mu\nu} \tilde{R}$ and $C_{\lambda\mu\nu\rho}$ is the Weyl tensor. Equations (5.2) and (5.3) are obviously satisfied if $\tilde{R}_{\mu\nu} = \Lambda g_{\mu\nu}$. It seems very probable that the Einstein spaces are the only vacuum solutions with zero torsion of the system (4.12) and (4.13). Though we failed to prove this statement for $\Lambda \neq 0$, it seems sufficient to assume that $\Lambda \simeq 0$ (the cosmological constant seems to be very small in our real world) and then to use the result of Ref. 44 that the system (5.1) and (5.2) with $\Lambda = 0$ is equivalent to $\tilde{R}_{\mu\nu} = 0$.

One can also try to prove for (4.12) and (4.13) some form of the Birkhoff theorem: the unique solution of (4.12) and (4.13) with $S = t = 0$ for the SO_3 spherically symmetric ansatz for e^a_μ and ω^{ab}_μ is $\Theta = 0$ and $g_{\mu\nu} =$ the Schwarzschild-DeSitter metric. This is really possible for some classes of the $(R + \Theta^2 + R^2)$ Lagrangians.[45,46] At the same time for the $(\Theta^2 + R^2)$ Lagrangian of Hehl *et al.*[17,2] there exists the SO_3 symmetric solution of Baekler[47] with $\Theta \neq 0$. It is probable that in our case [(4.10)] torsion will be nonzero but will swiftly fall down away from the origin with $r \to \infty$ (thus representing a "torsion monopole", cf. Refs. 9, 45, and 48).

The apparent problem is the relation of magnitudes of Λ and the ω-S interaction constant λ (note that $\Lambda \sim \lambda/k^2$ also for the Baekler solution[47]). It is possible to solve the problem of the great Λ (with $\lambda \neq 0$) merely subtracting the Λ term from (4.10). However, one must keep in mind that the great Λ term arises also in a number of other cases[49] and may also take into account the possibility of some kind of "confinement" on the microlevel due to the ω-S interaction (hadrons as Schwarzschild-De Sitter "universes"). Finally, let us note that even for a spinless matter ($S=0$) the condition of $\Theta=0$ is too stringent, because according to Ref. 50 nearly all solutions of (5.2) are the vacuum Einstein spaces (cf. Ref. 2; see also the discussion of the Newtonian limit for the $(R+R^2)$ theories in Ref. 46).

B. Spin-Spin interactions

Taking $e^a_\mu=\delta^a_\mu,\Lambda=0$ one can write the $(R^2+\Theta^2)$ part of (4.10) in the form

$$\mathcal{L}=\frac{1}{k^2}(\check{\omega}_a\check{\omega}^a+18\hat{\omega}_a\hat{\omega}^a)\ ,$$
$$\check{\omega}_a=\omega^b{}_{ab},\quad \hat{\omega}_d=\frac{1}{3!}\epsilon_{abcd}\omega^{abc}\ . \tag{5.4}$$

Making use of the relation $S_{abc}=\epsilon_{abcd}\hat{S}^d$, $\hat{S}^d=-\frac{1}{2}\bar{\psi}\gamma^d\gamma_5\psi$ for the spin $-\frac{1}{2}$ fermions and assuming that $\omega_{abc}=\epsilon_{abcd}\hat{\omega}^d$, we get the Lagrangian for the ω-S interaction,

$$\mathcal{L}=\tfrac{3}{2}[-(\partial_a\hat{\omega}_b)^2-\tfrac{1}{2}(\partial_a\hat{\omega}_a)^2+\mu^2\hat{\omega}_a{}^2-3(\hat{\omega}_a\hat{\omega}^a)^2-2\lambda\hat{\omega}_a\hat{S}^a]\ ,$$
$$\mu^2\equiv 12k^{-2}\lambda\ . \tag{5.5}$$

It leads to the following static nonrelativistic potential[51] for the interaction of two fermions with spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$ in the linear approximation and massless limit:

$$V_P\sim\lambda^2\left[A\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{r}+B\frac{(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^3}\right]+[\delta(r)\text{ terms}]\ . \tag{5.6}$$

Another approximation is obtained if we neglect the transferred momentum in the $\hat{\omega}$ propagator ($\mu\neq 0$), giving the contact potential as the ECT case.[8] Thus the L_6 Yang-Mills term R^2 in (4.10) modifies the ECT just like the theory with intermediate vector bosons do with the Fermi theory of weak interactions. The potential (5.6) will lead to some new physical effects, depending on the value of the L_6 dimensionless constant λ. In Ref. 39 λ was taken to be 10^{-38} (or $\mu\sim$the mass of the proton), but the absence of contradiction with known atomic effects allows it to be sufficiently greater[52] [e.g., 10^{-17} (Ref. 10)]. One can observe that the spin-spin interaction is already present in the Einstein theory ($V_E\sim -k^2(\vec{\sigma}_1\cdot\vec{\sigma}_2)/r^3$ is the analog of the Breit potential[51]). The point, however, is that the gaugelike ω-S interaction provides the $1/r$ spin-spin potential analogous to the fundamental $1/r$ potentials in electrodynamics and gravitation.

C. Instanton solutions

The field equations (4.12) and (4.13) in vacuum possess the remarkable solutions, intimately connected with the Yang-Mills structure of (4.9), i.e., the self-dual solutions [usually considered in the Euclidean case SO(1,4)$\rightarrow$SO(5), $\eta_{ab}\rightarrow-\delta_{ab}$ or $k_0{}^2\rightarrow -k_0{}^2$ in (4.10)]

$$\mathcal{R}=*\mathcal{R}\ \text{ or }\ *(R-\overline{\pi})=R-\overline{\pi},\quad \Theta=*\Theta\ . \tag{5.7}$$

The solutions of (5.7) realize the local minima of the Euclidean action

$$I_S=\frac{1}{8\lambda}\int d^4x\sqrt{g}\,[(R-\overline{\pi})^2+2\overline{\Theta}^2]$$

bounded from below by the topological invariant

$$P_{1S_{10}}=-\frac{1}{8\pi^2}\int \mathrm{tr}(\mathcal{R}\wedge\mathcal{R})=P_{1L_6}\ .$$

It is important to stress that here we do not confront the difficulty of the indefiniteness of the Euclidean action, present in the Einstein theory[53] (and also in ECT and in supergravity). Note that though the action of Ref. 32 is also non-negative, it does not lead to the convergence of the path integral (as compared to I_S). The absolute minimum $I_S=0$ is realized on the De Sitter space $S^4,\mathcal{R}=0$, or $\Theta=0$, $R^{ab}{}_{\mu\nu}=(2/k_0{}^2)e^{[a}_{\underline{\mu}}e^{b]}_{\nu}$. Then the minima with $\Theta=0$ and self-dual $(R-\overline{\pi})$, which coincide with the solutions of the Einstein equations with self-dual Weyl tensor, are realized (e.g., CP^2 and its generalizations considered in Ref. 54). Instantons with $\Theta\neq 0$ have greater action (or topological number).

One can *a priori* hope that the theory (4.10) may provide the realization of the idea of Hanson and Regge[9] about the analogy of gravity with torsion $\Theta\neq 0$ and the theory of superconductivity (with the Meissner effect corresponding to the phase of zero torsion, thus dynamically explaining why $\Theta=0$ in GR, and localized regions with $\Theta\neq 0$ being the analogs of the Abrikosov vortices). In Ref. 9 this analogy was proposed to hold in the Euclidean case with e and ω being spherically symmetric in the four-dimensional sense ("torsion instantons").

However, (5.7) does not admit nontrivial solutions of this type. All solutions for the most general SO_4 ansatz ($\rho = x^a x_a$),

$$\omega^{ab}{}_\mu = 2Ax^{[a}\delta^{b]}_\mu + B\epsilon^{ab}{}_{\mu\nu}x^\nu, \quad e^a_\mu = f\delta^a_\mu + gx^a x_\mu \tag{5.8}$$

are exhausted by the trivial S^4 instanton:

$$A = \frac{2}{\rho^2 + a^2}, \quad f = \frac{\sqrt{2}ak_0}{\rho^2 + a^2},$$
$$B = 0, \quad g = 0$$

[ω corresponds to the SO_4 connection for the SU_2 one-instanton solution[55]]. This result has a simple explanation. Looking for solutions with $\Theta \neq 0$ we are looking for "multiinstanton" configurations, which are not spherically symmetric. The correct point of view probably is that the Hanson-Regge analogy takes place in the Minkowski-signature space-time and what we really need is the SO_3-symmetric static solution, corresponding to the torsion "monopole" (similar solutions were already found in a number of theories[45–48]. This solution is analogous to the SU_2 't Hooft-Polyakov monopole with e^a_μ playing the role of the Higgs field and $\omega^{ab}{}_\mu$ of the Yang-Mills potential. In the case of M^4 being the Minkowski space one is able to prove the topological stability of such solutions. Putting $e^0_0 = 1$, $e^0_i = e^i_0 = 0$, $e^i_j = \partial y^i / \partial x_j \in SO_3$ and choosing

$$e^a_\mu \underset{r\to\infty}{\longrightarrow} \delta^a_\mu$$

we get the map e^a_μ: $R^3 \cup \{\infty\} \to SO_3$ classified by $\pi^3(SO_3) = Z$. The necessary degeneracy of the e^a_μ vacuum is provided by the condition $e^a_\mu e^b_\nu \eta_{ab} = \eta_{\mu\nu}$ cf. the discussion of the analogy with the chiral Higgs mechanism in Sec. IV).

D. Model Lagrangian and physical consequences of noncompactness of the Lorentz group

Dropping the torsion "mass terms" (and Λ term) in (4.10) [in $(R + \Theta^2)$] we obtain the following approximation for (4.10),

$$\mathscr{L}_0 = -\frac{1}{k^2}\tilde{R}(g) + \frac{1}{8\lambda}R^a{}_{b\mu\nu}(\omega)R^b{}_a{}^{\mu\nu}(\omega) . \tag{5.9}$$

The $\mathscr{L}_0$ theory may be considered as a model of the P_{10} dynamics, corresponding in some sense to the $T_4 \times L_6$ gauge theory (the analogous Lagrangians were discussed in Refs. 56–58 and 7). One can treat $\mathscr{L}_0$ also as the Lagrangian for the Einstein gravity, interacting with the SO(1,3) gauge field. (Note that ω interacts only with spinors.) The field equations have the form [we use matrix notations in $a,b,\ldots,$ e.g., $R_{\mu\nu} = (R^a{}_{b\mu\nu})$, $\mathrm{tr}(R_{\mu\nu}R_{\alpha\beta}) = R^a{}_{b\mu\nu}R^b{}_{a\alpha\beta}$],

$$\tilde{R}_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\tilde{R} = \frac{k^2}{2}(t_{\mu\nu} + \lambda^{-1}\tau_{\mu\nu}), \quad t_{[\mu\nu]} = 0 ,$$
$$\tau_{\mu\nu} = \tfrac{1}{2}\,\mathrm{tr}(R_{\mu\lambda}R_\nu{}^\lambda - \tfrac{1}{4}g_{\mu\nu}R_{\lambda\rho}R^{\lambda\rho}) , \tag{5.10}$$

$$\overline{\mathscr{D}}_\mu R^{\mu\nu} = \lambda S^\nu, \quad \overline{\mathscr{D}} \equiv \mathscr{D} + \{\ \} . \tag{5.11}$$

Introducing $E_i = R_{0i}$, $H_i = \frac{1}{2}\epsilon_{ijk}R^{jk}$, $\vec{\mathscr{E}}^i_E = \vec{\mathrm{E}}^{i0}$, $\vec{\mathscr{E}}^i_H = \vec{\mathrm{H}}^{i0}$, $\vec{\mathscr{H}}^i_E = \frac{1}{2}\epsilon^{ikj}\vec{\mathrm{E}}_{kj}$, $\vec{\mathscr{H}}^i_H = \frac{1}{2}\epsilon^{ikj}\vec{\mathrm{H}}_{kj}$, we get $\tau_{00} = \frac{1}{2}(\mathscr{H}_E{}^2 + \mathscr{H}_H{}^2 - \mathscr{E}_E{}^2 - \mathscr{E}_H{}^2)$, $\mathscr{L}_L = \frac{1}{2}(\mathscr{E}_H{}^2 + \mathscr{H}_E{}^2 - \mathscr{E}_E{}^2 - \mathscr{H}_H{}^2)$, i.e., the noncompactness of $L_6 = SO(1,3)$ (the indefiniteness of the group metric) imply the indefiniteness of the energy for ω. The lack of the positive definiteness of the energy is the necessary consequence of the localization of L_6, drastically influencing physical predictions of the theory (e.g., violating the conditions for singularities, providing the possibility for the "gravitational repulsion," etc.). We conclude that the Poincaré gauge theory, taking into account the ω-S interaction of the L_6 Yang-Mills type, may provide a possible answer to the question of Wu and Yang[59] about the physical applications of gauge theories with noncompact gauge groups.

There exists the situation when the system of spins creates the field ω with negative energy (the following example is also interesting from the point of view of possible experimental observation of the ω-S interaction). Namely, let us consider the cylindric ferromagnetic specimen with all spins polarized in the z direction so that inside the specimen

$$\psi = \sqrt{n}\,e^{-imt}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \quad \hat{S}^a = \{0,0,0,\tfrac{1}{2}n\}$$

(cf. Sec. V B) (n is the concentration of the electrons). Supposing that $g_{\mu\nu} = \eta_{\mu\nu}$ we have from (5.11) (in the linear approximation) the following interior solution:

$$R^{i0} = \frac{\lambda}{3}x^i S^0, \quad R^{ik} = \lambda x^{[i}S^{k]} ,$$
$$\Delta M = \int \tau_{00}d^3x = -\lambda^4 n^2 \pi r^2 l\left[\frac{A}{3}l^2 + \frac{B}{4}r^2\right] , \tag{5.12}$$
$$A = 14/288, \quad B = 5/288$$

(r is the radius and l is the length of the specimen). As a result, the spins produce the L_6 gauge field with the negative energy and thus decrease the total mass of the system of "sources + field" and hence, effectively, of the specimen. For example, when $l\sim 10$ cm, $r\sim 1$ cm, $n\sim 10^{24}$ cm^{-3}, then $\Delta M\sim\lambda^4 10^{12}g$ ($\hbar=c=1$) and therefore in the world with $\lambda\sim 10^{-5}$ one will witness the "antigravity" after the instantaneous polarization of the specimen (cf. the recent proposal of antigravity in supergravity[60]).

Another consequence of the noncompactness of L_6 is the possibility of static regular solutions of the L_6 Yang-Mills equations in vacuum, absent for compact gauge groups. For example, let us consider the following complex solution for the SU_2 gauge theory found in Ref. 59 in flat space:

$$A_k^i=\epsilon_{ikj}f(r)n^j,\ A_0^k=n^k g(r),\ \vec{n}=\frac{\vec{x}}{r}\ \ (i,j,k,..=1,2,3)\ ,$$
$$1+rf=\beta r/\sinh(\beta r),\ 1+rg=\beta r\coth(\beta r),\ \beta=a+ib\in C,\ a>0\ . \tag{5.13}$$

The corresponding SL(2,C) or SO(1,3) solution is obtained according to the rule $A_\mu^k=W_\mu^k+iV_\mu^k$, $\omega_\mu^{ok}=V_\mu^k$, $\omega^{ij}{}_\mu=\epsilon^{ijk}W_\mu^k$. The field strength for (5.13) is regular in $r=0$ and goes as r^{-2} when $r\to\infty$. The energy of the solution is zero, i.e., it is possible to generalize it to the approximate solution of (5.10) and (5.11) for conformally flat metrics. We remark that the existence of regular solutions like (5.13) is probably a manifestation of a long-range character of "noncompact" gauge fields (analogous to that of the Abelian fields). Thus the $L_6(\omega)$ gravity has a long-range nature just like the Einstein $T_4(e)$ gravity (so the idea of "confinement" of ω proposed in Refs. 1–3 seems to be inappropriate and based on an incorrect choice of variables).

Treating the vacuum $\mathscr{L}_0$ theory as GR + SO(1,3) gauge field it is easy to obtain the general spherically symmetric solution of (5.10) and (5.11), cf. Ref. 61 (for an analogous solution for an arbitrary gauge group see Ref. 62),

$$\theta^0=a\,dt,\ \theta^1=b\,dr,\ \theta^2=r\,d\theta,\ \theta^3=r\,\sin\theta\,d\varphi,\ \omega=\frac{u}{r}\,dt-\zeta\cos\theta\,d\varphi\ ,$$
$$R=E\theta^0\wedge\theta^1+H\theta^2\wedge\theta^3, E=\frac{u}{r^2},\ H=\frac{\zeta}{r^2},\ a^2=b^2=1-\frac{2GM}{r}+\frac{4\pi G}{r^2}q^2, q^2=\tfrac{1}{2}\,\mathrm{tr}(u^2+\zeta^2) \tag{5.14}$$

(u and ζ are commuting matrices from the algebra of L_6). Note that $q^2\gtrless 0$ due to the noncompactness of L_6. Here u and ζ are connected with the "charge" (i.e., with the spin) of the central body and hence there is a possibility of preventing the formation of a horizon by a nonzero total spin $\vec{\Sigma}$ of the star [$q^2=(\lambda^2/6\pi^2)\vec{\Sigma}^2$]. It is interesting to note that torsion is nonzero for (5.14) [and thus the Birkhoff theorem of Ref. 45 is not valid for (5.9)] but goes down too slowly ($\sim 1/r$) to provide the interpretation of a "torsion monopole" to the Abelian solution (5.14).

As far as cosmological solutions of (5.10) and (5.11) are concerned ω can be considered as some background field (due to a probable lack of its sources). Here, in general, $\Theta\neq 0$. It is important to realize, however, that torsion is not a directly observable quantity. Namely, from (5.10) and (5.11) one gets the following conservation laws (or equations of motion):

$$\overline{\mathscr{D}}_\mu S^\mu=0,\ \ \tau^{\mu\nu}{}_{;\nu}=\tfrac{1}{2}\,\mathrm{tr}(R^{\mu\nu}S_\nu) \tag{5.15}$$

and so the spinless particles move along the Riemannian geodesics, while in the presence of spin they are influenced by ω via the generalized Lorentz force (see also Ref. 8). As a result, one can in principle detect the existence of the ω-S interaction observing the differences in the trajectories of particles with and without spin near a probable source of ω (i.e., a ferromagnetic specimen or a rotating neutron star).

VI. QUANTIZATION OF GAUGE THEORIES OF GRAVITY WITH TORSION

A. General remarks and propagator

The noncompactness of L_6 (or the absence of positive definiteness of the energy) presents problems in quantization of the theory, including the $\mathscr{L}_L$ part in the Lagrangian [as in (5.9)]: the Hamiltonian $H \gtrless 0$ is not bounded from below, there is no ground state and unitarity, etc. The correct quantization scheme is absent for the noncompact gauge fields. From the point of view of the path-integral quantization, the indefiniteness of H apparently implies the divergence of the Euclidean path integral. However, our case of L_6 is a remarkable exception. Here the Wick rotation takes place not only on the base space ($g_{\mu\nu} \rightarrow g_{\mu\nu}^{(E)}$) but also "in the fibers" [SO(1,3)$\rightarrow$SO(4)], because the fiber here is the tangent space to the base. As a result, the Euclidean action is positive [compare again with GR (Ref. 53)], signifying that in spite of the ghosts in the Minkowski sector the corresponding quantum theory may be meaningful.

Let us now consider the propagator in the S_{10} theory, i.e., the bilinear approximation in (4.10) in terms of ω and B, $e_\mu^a \simeq \delta_\mu^a + kB_\mu^a$ (we disregard the Λ term). The analysis of the spectrum of particles in the theory allows the following ambiguity: (i) one can treat $(e_\mu^a, \omega_\mu^{ab})$ as a multiplet of ten vector gauge fields, i.e., $40(e,\omega) = 10 \times (1^- \oplus 0^-)$; (ii) in the bilinear approximation there is formally no difference between the latin and greek indices and so one can consider B_{ab} as the basic variables

$$B_{ab} = \bar{B}_{ab} + \tfrac{1}{4}\delta_{ab}B + B_{[ab]} = \{2^+, 1^-, 0^+; 0^+; 1^-, 1^+\};$$

$$\omega_{abc} = \bar{\omega}_{abc} + \tfrac{2}{3}\delta_{c[a}\hat{\omega}_{b]} + \epsilon_{abcd}\hat{\omega}^d$$
$$= \{2^+, 2^-, 1^-, 1^+; 0^+, 1^-; 0^-, 1^+\} .$$

The second approach is preferable when dealing with the propagator and is in clear correspondence with GR (graviton as 2^+), while the first one is essential in the exact theory. Let us note that the appearence of the 2^+ particles from the ω_{abc} (besides the gravitaton from the B) provides the interpretation of the (e,ω) theory as a variant of "strong gravity."[2,57,10] The propagators in (e,ω) theories with quadratic Lagrangians recently were discussed in detail in Refs. 10, 11, and 16 so we shall only emphasize the essential point of our approach. From the above dicussion it is evident that there is $(e-\omega)$ mixing in the $2^+, 1^-, 1^+, 0^+$ sectors of the propagators. However, for the aim of revealing the pole structure it is advantageous to analyze the propagator in terms of the SO(4) and not of the SO(3) spin. Here again the mixing occurs and the propagator P has the following sectors: "trace" $\check{P}$, "pseudotrace" $\hat{P}$ and "irreducible" $\bar{P}$. $\bar{P}$ turns out to have no massive poles, while $\check{P}$ and $\hat{P}$ have the following tachyonic poles: $\check{\mu}^2 = -12k^{-2}\lambda$, $\hat{\mu}^2 = -6k^{-2}\lambda$, $\lambda > 0$ [cf. (5.5)]. The passage to the Euclidean theory implies $S_{10} \rightarrow SO_5$ or $k^2 \rightarrow -k^2$ and the change of the square of momentum sign [our signature is $(+ - - -)$]. As a result, the Euclidean propagator has no real poles just as in the case of a "respectable" Euclidean theory (we hope that this fact makes the objection of the absence of unitary raised against (4.10) in Ref. 11 harmless). Thus one may suspect (as one does in the case of the Einstein theory[53]) that it is the Euclidean approach that possibly provides the correct quantization of the theory.

Analyzing the propagator one can formally act in terms of the nonlinear S_{10} potential Ω (4.5). However, it is necessary to take into consideration the spontaneous symmetry breaking $\langle e_\mu^a \rangle = \delta_\mu^a$. It is remarkable that in our approach this condition (extremely essential also in GR) *is the direct consequence of the use of the nonlinear realization* of S_{10} (or P_{10}). Assuming that for the linear gauge field $\hat{\Omega}$ (4.3) $\langle \hat{\Omega} \rangle = 0$, or $\langle \hat{\omega} \rangle = 0$, $\langle \hat{\theta} \rangle = 0$, we get from the definitions (4.5) and (4.6) that $\langle \omega \rangle = 0$, $\langle \bar{\theta} \rangle = d\beta \neq 0$ [the Goldstone parameter β in (4.6) is considered as a classical field]. Dealing, for simplicity, with the case of the P_{10} theory (2.4) and (2.15) we have $\langle \theta_\mu^a \rangle = \langle \partial_\mu \xi^a \rangle = \delta_\mu^a$ (where $\xi^a = x^a$ when M^4 is flat, i.e., coincides with the tangent space). Thus the problem is reduced to the question about the values of Goldstone parameters in the vacuum state and clearly corresponds *to the spontaneous T_4 symmetry breaking.*

One may notice that the nonzero $\langle \theta \rangle$ had led to the mass terms for ω [more accurately, to the poles of the combined (e,ω) propagator]. This fact prompts the possibility of obtaining the spontaneous symmetry breaking in a sourceless internal (e.g., SO_n) gauge theory by the use of a nonlinear realization ($SO_n \rightarrow SO_{n-1}$) under some natural assumptions about the vacuum values of Goldstone parameters. As a consequence, the SO_{n-1} gauge field will get some mass and so this mechanism may be an interesting alternative to the ordinary Higgs mechanism, being based only the use of gauge fields.

An additional remark about the Λ term in (4.10) is in order. Even if $2\Lambda k^{-2}$ is subtracted from (4.10), the minimum $(=-2\Lambda k^{-2})$ of the Euclidean action is realized on the De Sitter space. Thus the value of Λ is a serious problem (note, however, that in GR even the notion of Euclidean vacuum is senseless in view of the indefiniteness of the action and hence the choice of the flat space as the vacuum state is an additional assumption).

B. The problem of renormalizability

We shall consider sucessively a number of examples of the (e,ω) theories to illustrate the main difficulties confronted in the case of the S_{10} theory (4.10). Let us start with the $\mathscr{L}_0$ theory (5.9). In the Euclidean variant it is unitary but nonrenormalizable (as GR plus SO_4 gauge theory). Taking into account the result of Ref. 63 one can get the one-loop counterterms in the form [$\epsilon=8\pi^2(n-4)$, $r_L=6$, $C_{2L}=C_2(SO_4)$]

$$\Delta\mathscr{L}_0=\frac{1}{\epsilon}\left[\left[\frac{137}{60}+\frac{r_L-1}{10}\right]\widetilde{R}_{\mu\nu}{}^2+C_{2L}\tfrac{11}{12}(-\tfrac{1}{2})\mathrm{tr}[R_{\mu\nu}(\omega)R^{\mu\nu}(\omega)]\right] . \tag{6.1}$$

Using the field equations (5.10) and (5.11) we have $\widetilde{R}=0$, $\widetilde{R}_{\mu\nu}{}^2=\frac{1}{4}\widetilde{R}_{\lambda\mu\nu\rho}{}^2+\mathrm{div}$, or $\Delta L_0=a_1\widetilde{R}_{\lambda\mu\nu\rho}{}^2+a_2(R^a{}_{b\mu\nu})^2$. Extraction for the torsion part from ω yields $(R_{ab\mu\nu})^2=(\widetilde{R}_{\lambda\mu\nu\rho}+T_{\lambda\mu\nu\rho})^2$, $T=\partial\widetilde{T}+\widetilde{T}\widetilde{T}$, $\widetilde{T}=\omega-\omega_0(e)$, and thus the renormalizability takes place after the additional assumption that the background torsion is zero (this condition may have physical sense if the idea about the Meissner effect for torsion is true[9]). This "conditional renormalizability" may be compared with the finiteness of supergravity beyond the one-loop level when the background fields satisfy certain self-duality relations.[64] If one puts also the quantum torsion equal to zero, the $\mathscr{L}_0$ theory is reduced to the $\widetilde{R}+\widetilde{R}^2$ theory, which is renormalizable[65] but apparently nonunitary (see also Ref. 66 and references therein). We conclude that torsion plays the part of the field, restoring the unitarity (see also Ref. 11) but in general breaking the renormalizability of the theory with a quadratic Lagrangian.

Our next example is the theory of Fairchild,[17] $\mathscr{L}_F\sim R(\omega)+R(\omega)R(\omega)$ [coinciding with (4.10) up to the Λ and Θ^2 terms]. Here the kinetic term for e^a_μ is absent making the quantization of e^a_μ (or metric) somewhat senseless. If e^a_μ is not quantized, $\mathscr{L}_F$ is simply the Lagrangian of the SO_4 gauge field in curved space-time, interacting in a gauge-invariant manner with the external source $\pi^{ac}_{\mu\nu}=e^{[a}_\mu e^{c]}_\nu$ by means of the Pauli-like term $R=R^{ab}{}_{\mu\nu}\pi^{\mu\nu}_{ab}$. This theory is unitary (cf. the opposite claim in Ref. 11 in the case of Minkowski signa re and quantized e^a_μ. Using the algorithm of Rt. 67, one can obtain the one-loop counterterms in the form

$$\Delta\mathscr{L}_F=\frac{1}{\epsilon}\left\{\frac{r_L}{20}C_{\lambda\mu\nu\rho}{}^2+C_{2L}(-\tfrac{1}{2})\mathrm{tr}\left[\left[R_{\mu\nu}+\frac{2\lambda}{k^2}\pi_{\mu\nu}\right]^2-\tfrac{1}{12}R_{\mu\nu}{}^2\right]\right\} \tag{6.2}$$

where the Weyl tensor $C_{\lambda\mu\nu\rho}$ and π depend only on the external field and so do not disturb the renormalizability.[68] However, if e^a_μ is quantized, the additional $\widetilde{R}^2$ terms appear in (6.2) thus breaking renormalizability if the background torsion is nonzero.

Turning now to the case of the S_{10} theory (4.10) we conclude, that it is unitary (in the Euclidean region) but in general is nonrenormalizable in view of unavoidable $\widetilde{R}^2$ counterterms [arising from the coupling of metric to the kinetic $(\partial\omega)^2$ and $(\partial e)^2$ terms in (4.10)]. Thus we have a strong feeling that Lagrangians like (4.10) *lead to nonrenormalizable theories* (this is especially obvious when a nonsupersymmetric matter is present, giving additional counterterms absent in the initial Lagrangian). As a result, the S_{10} theory has the quantum behavior analogous to GR (excluding the remarkable fact of the positivity of the Euclidean action and an interesting possibility of obtaining the renormalizable theory when the classical torsion is equal to zero). There is, however, a chance to improve the situation by treating the metric as a classical object and quantizing only the linear S_{10} gauge field $\hat{\Omega}$ (4.3). Let us assume (in the context of the background field method[69]) that

$$\hat{\Omega}=\hat{\Omega}_{\mathrm{cl}}+\hat{\Omega}_{\mathrm{g}},\quad \hat{\Omega}_{\mathrm{cl}}=\Omega ,$$

or

$$\langle\hat{\omega}\rangle=\omega ,\quad \langle\hat{\theta}^a_\mu\rangle=k_0e^a_\mu , \tag{6.3}$$

i.e., that the breaking of the symmetry $S_{10}\to L_6$ is established by the condition that the classical part of the quantized linear gauge field (4.3) is equal to the nonlinear gauge field (4.5). If the condition (6.3) is absent and $g_{\mu\nu}$ is some classical metric we have, for (4.9),

$$\Delta\mathscr{L}_S=\frac{1}{\epsilon}\left[\frac{r_S}{20}(C_{\lambda\mu\nu\rho})^2+\tfrac{11}{12}C_{2S}(-\tfrac{1}{2})\mathrm{tr}(\hat{\mathscr{R}}_{\mu\nu}\hat{\mathscr{R}}^{\mu\nu})\right] , \tag{6.4}$$

implying the renormalizability.[68] Taking now into consideration (6.3) and the equality (4.9) one can observe that renormalizability is still present (up to a subtle point of establishing the background gauge invariance in view of the dependence of $g_{\mu\nu}$ on the part of the background S_{10} gauge field). The latter problem does not arise in the proof of the renormalizability (in the flat space-time) of the SO_n gauge theory with the spontaneous symmetry breaking $SO_n \to SO_{n-1}$ of the type discussed in Sec. VI B. We emphasize that the obtained result is based essentially on (a) the Yang-Mills (renormalizable) structure of the Lagrangian (4.9), and (b) on the use of the nonlinear realization [necessary to establish the interpretation of (4.9) as a gravity theory]. The approach based on (6.3) is sort of a compromise: though the metric is not quantized, it is not purely classical being expressed in terms of the mean value of the quantum field $\hat{\theta}$. The apparent drawback of this suggestion lies in the fact that matter interacts naturally with e and ω but not with $\hat{\theta}$ and $\hat{\omega}$. The radical point of view is that the quantized part of the gravitational field is the connection ω (or the L_6 gauge field[58]), the tetrad or metric being purely classical.

Our final comment is about the interaction of ω with the spin-$\frac{1}{2}$ fermions. As is well known (see, e.g., Ref. 8), the corresponding term in the Lagrangian is $\mathscr{L}_{\rm int} \sim (\bar{\psi}\gamma^{\mu}\gamma_5\psi)\hat{\omega}_{\mu}$, $\hat{\omega}_{\mu} = -(1/3!)\epsilon_{\mu\lambda\rho\sigma}\omega^{\lambda\rho\sigma}$, i.e., only the axial part of the connection (or torsion) contributes to it. As a result, in order to establish the renormalizability (even if the metric is classical) one needs the following bare term in the (gravity + matter) Lagrangian:

$$\mathscr{L} = \frac{1}{4\lambda'} W_{\mu\nu} W^{\mu\nu}, \quad W_{\mu\nu} = \partial_{\mu}\hat{\omega}_{\nu} - \partial_{\nu}\hat{\omega}_{\mu} \,. \tag{6.5}$$

It is this term [and not $R^2(\omega)$ in (5.9)] that should be added to the Einstein Lagrangian for the renormalizability of the $\omega - S$ interaction. This fact seems to be overlooked in earlier investigations (i.e., various Lagrangians of Refs. [1–3, 8, 16, 17, and 36–40 are nonrenormalizable in the presence of fermions).

VII. FURTHER PROBLEMS

Let us recollect here a number of questions left in the theory: (1) What is the value of λ in (4.10) and thus the magnitude of physical effects due to the interaction of spin and connection? (2) Are there any monopolelike solutions of Eqs. (4.12) and (4.13) with the localized torsion and what is their impact on the quantum theory (cf. Refs. 9 and 48)? (3) Does there exist a supersymmetric extension of the Lagrangian (4.10), partially improving the quantum behavior of the theory (see in this connection Ref. 70)? (4) Is it possible to develop a sensible quantum theory of (4.10) by continuing the Euclidean theory back to the Lorentzian sector? (5) What is the exact form of the one-loop counterterms for (4.1) when both e^a_μ and ω^{ab}_μ are quantized? (Let us point out that the one-loop counterterms in the theories with nonzero torsion can be evaluated using the well-known algorithm of Refs. 67 and 71 without the modification proposed in Ref. 72).

The questions related to our theory in internal gauge theories are (i) how to quantize gauge theories with noncompact gauge groups, and (ii) whether it is possible to establish a viable alternative to the Higgs mechanism by working only in terms of the pure gauge theory under the assumption of a nonlinear realization and choosing appropriate conditions on the "broken" part of gauge potentials [cf. (6.3)], (see also Ref. 73 about "noncompact" gauge theories).

The main question is about the existence in nature of the interaction of the Lorentz connection and spin. If the answer is negative, the Lorentz group should not be gauged and one must give up the attempts to interpret gravity as a gauge theory (analogous to the internal gauge theories). However, if the answer is positive, we believe that the $\omega - S$ interaction will be of the Yang-Mills type and thus the total Lagrangian will look like (4.10) [probably plus (6.5)].

After the completion of this work appear several references useful to our view: similar approaches to the kinematics of the Poincaré gauge theory were proposed in Ref. 74; the Lagrangians analogous to (4.10) were discussed in Ref. 75; for instantons in the R^2 theory see Ref. 76.

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