π^0 and η decays into lepton pairs

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A model for the rare decays of the neutral pseudoscalar mesons into lepton pairs is presented. By using a once-subtracted dispersion relation for the amplitude, we are able to obtain form-factor-insensitive results. The calculated branching ratios are $\Gamma(\pi^0\to e^+e^-)/\Gamma(\pi^0\to \text{all}) = 2.0\times10^{-7}$ and $\Gamma(\eta\to\mu^+\mu^-)/\Gamma(\eta\to\text{all}) = 6.1\times10^{-7}$, in excellent agreement with experiment. We also predict the branching ratio for $\eta\to e^+e^-$ to be 9.1×10^{-9}. We suggest that this decay may be accessible to experiment.

Recently, better experimental data have become available for the rare decays of the neutral pseudoscalar mesons into lepton pairs, which we denote by $P\to\bar{l}\,l$; for the decay $\pi^0\to e^+e^-$, Mischke *et al.*[1] have reported the branching ratio

$$\Gamma(\pi^0\to e^+e^-)/\Gamma(\pi^0\to\text{all}) = (1.8\pm0.6)\times10^{-7} \ ,$$

which is compatible with the earlier observation by Fischer *et al.*,[2]

$$\Gamma(\pi^0\to e^+e^-)/\Gamma(\pi^0\to\text{all}) = (2.23^{+2.4}_{-1.1})\times10^{-7} \ .$$

In the case of the decay $\eta\to\mu^+\mu^-$, a new measurement by Dzhelyadin *et al.*[3] gives

$$\Gamma(\eta\to\mu^+\mu^-)/\Gamma(\eta\to\text{all}) = (6.5\pm2.1)\times10^{-6} \ ,$$

which is two standard deviations below the older result by Hyams *et al.*,[4]

$$\Gamma(\eta\to\mu^+\mu^-)/\Gamma(\eta\to\text{all}) = (2.2\pm0.8)\times10^{-5} \ .$$

These results are of considerable interest since the decay $P\to\bar{l}\,l$ is thought to be dominated by a two-photon intermediate state [Fig. 1(a)] which probes the pseudoscalar electromagnetic structure at large virtual-photon mass.[5] Indeed, this configuration saturates the absorptive part of the $\pi^0\to e^+e^-$ amplitude and dominates the absorptive part for $\eta\to\mu^+\mu^-$, from which the model-independent unitarity bounds[6]

$$\Gamma(\pi^0\to e^+e^-)/\Gamma(\pi^0\to\text{all}) \geq 4.7\times10^{-8}$$

and[7]

$$\Gamma(\eta\to\mu^+\mu^-)/\Gamma(\eta\to\text{all}) \geq 4.1\times10^{-6}$$

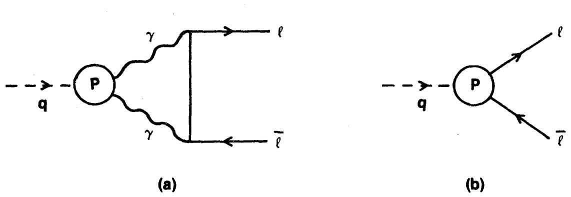

FIG. 1. Feynman diagrams for the decay $P\to\bar{l}\,l$.

are determined.

In the limit of a pointlike $P(q)\to\gamma(k_1)+\gamma(k_2)$ interaction, the dispersive part of the decay amplitude is logarithmically divergent when expressed as an unsubstracted dispersion relation[5] or Feynman integral,[6] so that the sensitivity of the branching ratio to the pseudoscalar-meson structure is expressed by the introduction of a form factor $f(k_1{}^2,k_2{}^2,q^2)$ at this vertex. A constraint upon the parametrization of f is provided by the form-factor slope a which is defined as

$$a \equiv q^2\frac{\partial}{\partial k^2}f(k^2,0,q^2)\bigg|_{k^2=0} \ . \tag{1}$$

This quantity is measured in the decay $P\to\bar{l}\,l\gamma$; Dzhelyadin *et al.*[8] find both a_η and their $\eta\to\mu^+\mu^-$ branching ratio[3] to be in fair agreement with the vector-meson-dominance model for f advocated by Quigg and Jackson.[7] In contrast, this same model seriously underestimates both a_π, as measured by Fischer *et al.*,[9] and the $\pi^0\to e^+e^-$ branching ratio quoted above. Similarly, the single-particle-propagator model for f, first proposed by Berman and Geffen[6] and, more recently, derived from QCD by Bergström,[10] gives an uncomfortably small $\pi^0\to e^+e^-$ branching ratio when a_π is taken as an input. Pratap and Smith[11] have calculated f in a nucleon-loop model; this effort was motivated by the earlier $\eta\to\mu^+\mu^-$ results, and their branching ratio is a factor of 2 larger than the current experimental value. Although this model does give a reasonable $\pi^0\to e^+e^-$ branching ratio, it also predicts values for a_π and a_η which are 1–2 orders of magnitude smaller than recent observations allow.[12]

Since the contributions of neutral currents[13] and massive Higgs bosons[14] are expected to be small, the disagreement between theory and experiment for the $\pi^0\to e^+e^-$ decay constitutes a serious problem. Bergström[10] has advanced this discrepancy as a possible indication of new interactions; however, we feel that before invoking anomalous couplings it is important to consider a less radical departure from existing theoretical ideas.

The $P \to \bar{l}\, l$ partial width can be expressed as[5]

$$\frac{\Gamma(P\to \bar{l}\, l)}{\Gamma(P\to\gamma\gamma)} = \frac{1}{2}\left(\frac{\alpha}{\pi}\frac{m_l}{m_P}\right)^2 |K(m_P{}^2)|^2 \left[1-4\left(\frac{m_l}{m_P}\right)^2\right]^{1/2} , \tag{2}$$

so that the two-photon contribution to the absorptive part of $K(q^2)$ is given by

$$\mathrm{Im}K_{\gamma\gamma}(q^2) = \frac{\pi}{\beta}\ln\left(\frac{1-\beta}{1+\beta}\right) f(0,0,q^2) \quad , \tag{3}$$

where $\beta = (1-4m_l{}^2/q^2)^{1/2}$ and the form factor is normalized to $f(0,0,m_P{}^2)=1$. Drell[5] has advocated an unsubtracted dispersion relation for $K(q^2)$ with $\mathrm{Re}K(q^2)$ completely calculable if the imaginary part is known. This assumption, which is implicit in the models discussed above, becomes suspect if we note that the effective current-current weak Lagrangian contains direct P-$\bar{l}$-l coupling terms [Fig. 1(b)]. We therefore postulate a once-subtracted dispersion relation

$$\mathrm{Re}K(q^2) = K_P(0) + \frac{q^2}{\pi}\int_0^\infty \frac{\mathrm{Im}K(t)}{t(t-q^2)}\,dt \quad , \tag{4}$$

with the subtraction constant $K_P(0)$ fixed by current algebra[15] and the PCAC (partial conservation of axial-vector current) relations for $P\to\gamma\gamma$ (Ref. 16):

$$K_\pi(0) = (1-\tfrac{1}{2}\sin^2\theta_C)\left(\frac{2\pi f_\pi}{\alpha}\right)^2 G_F \quad , \tag{5a}$$

$$K_\eta(0) = \tfrac{1}{2}\sin^2\theta_C\left(\frac{2\pi f_\pi}{\alpha}\right)^2 G_F \quad , \tag{5b}$$

where G_F is the Fermi coupling, f_π is the charged-pion decay constant, and θ_C is the Cabbibo angle. The dispersion integral is sufficiently convergent that we may take $f(0,0,q^2)=1$ over the entire range of q^2 and thereby obtain

$$\mathrm{Re}K_{\gamma\gamma}(q^2) = \frac{1}{\beta}\left[\frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right) - 2\Phi\left(\frac{1-\beta}{1+\beta}\right) + \frac{\pi^2}{6}\right] , \tag{6}$$

where $\Phi(x) = \int_0^x (dt/t)\ln(1+t)$ is the Spence function. It should be noted that the leading logarithm-squared behavior is typical[6,10]; what distinguishes this model from previous ones is the absence of logarithmic cutoff terms, which always reduce the value of $\mathrm{Re}K_{\gamma\gamma}(q^2)$.

Although by definition $\mathrm{Re}K_{\gamma\gamma}(0)=0$, for the physical masses $\mathrm{Re}K_{\gamma\gamma}(q^2) >> K_P(0)$. Our model predicts the branching ratios

$$\Gamma(\pi^0\to e^+e^-)/\Gamma(\pi^0\to \mathrm{all}) = 2.0\times 10^{-7} \tag{7a}$$

and

$$\Gamma(\eta\to\mu^+\mu^-)/\Gamma(\eta\to\mathrm{all}) = 6.1\times 10^{-6} \quad , \tag{7b}$$

which are in excellent agreement with the latest experimental values cited above. These results are, moreover, relatively insensitive to higher-mass intermediate states since their contributions are suppressed by q^2/Λ^2, where Λ is of the order of the ρ mass. As a concrete example of this behavior we have considered the vector-meson-dominance model for the electromagnetic structure of the pseudoscalar meson, which gives additional contributions from γV and VV intermediate states:

$$\mathrm{Re}K_V(q^2) = 2\sum_V g_{\gamma V}\left(\frac{\epsilon}{3}\right)(\ln\xi - \tfrac{11}{6}) + O(\epsilon^2) + O(\xi) \quad , \tag{8a}$$

$$\mathrm{Re}K_{VV}(q^2) = -\tfrac{2}{3}\sum_V g_{VV}\epsilon + O(\epsilon^2) + O(\xi) \quad . \tag{8b}$$

Here, $\epsilon = (q^2/M_V{}^2)$, $\xi = (m_l/m_V)^2$, M_V is the vector-meson mass, and $g_{\gamma V} = (4\pi\alpha/F)(f_{P\gamma V}/f_V)$, where $f_{P\gamma V}$, f_V, and F are, respectively, the P-γ-V, V-γ, and P-γ-γ coupling constants. Using the known coupling constants, and approximating $g_{VV}\simeq g_{\gamma V}$, we find $\mathrm{Re}K_{\gamma V}(m_\pi{}^2) = -0.37$ and $\mathrm{Re}K_{VV}(m_\pi{}^2) = -0.02$, which are negligible compared to $\mathrm{Re}K_{\gamma\gamma}(m_\pi{}^2) = 63.8$. A more substantial effect is found for $\eta\to\mu^+\mu^-$, where Dzhelyadin *et al.*[8] find ρ dominance with $g_{\gamma V}=1.14$; in this case, including the ϵ^2 and ϵ^3 terms, we obtain $\mathrm{Re}K_{\gamma\rho}(m_\eta{}^2) = -2.53$ and $\mathrm{Re}K_{\rho\rho}(m_\eta{}^2) = -0.40$. Our prediction for $\eta\to\mu^+\mu^-$ is thereby reduced by approximately 20%, to

$$\Gamma(\eta\to\mu^+\mu^-)/\Gamma(\eta\to\mathrm{all}) = 4.9\times 10^{-6} \quad , \tag{9}$$

which is still well within experimental uncertainty.

Only a large upper limit[17]

$$\Gamma(\eta\to e^+e^-)/\Gamma(\eta\to\mathrm{all}) < 3\times 10^{-4}$$

presently exists for the decay $\eta\to e^+e^-$. The unitarity bound is

$$\Gamma(\eta\to e^+e^-)/\Gamma(\eta\to\mathrm{all}) \gtrsim 1.7\times 10^{-9} \quad ,$$

and, including vector-meson effects, our model predicts

$$\Gamma(\eta\to e^+e^-)/\Gamma(\eta\to\mathrm{all}) = 9.1\times 10^{-9} \quad . \tag{10}$$

Although small, it is not inconceivable for such a branching ratio to be observed given a sufficiently good source of η mesons.

In summary, it appears that the existing $P\to\bar{l}\, l$ data can be accommodated by a model with a two-photon intermediate state dominating if a subtraction is made at $q^2=0$. It should finally be noted that, since theories with direct P-γ-γ couplings are nonrenormalizable,[5] a detailed understanding of the subtraction remains an open question.

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