Leading $q\bar{q}$ Regge singularities in perturbative quantum chromodynamics

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The QCD-motivated integral equation for the $q\bar{q}$ pair is derived taking into account Reggeization of quarks and asymptotic freedom. This equation is shown to generate an infinite number of Regge poles accumulating at $j=0$. It is expected that this short-range effect constraining the intercept of leading Regge singularities to be at least near zero independently of the flavors involved should not be affected by confinement. The phenomenological relevance of this constraint for charm production is pointed out.

The high-energy behavior of scattering amplitudes in non-Abelian gauge theories corresponds to the Reggeon calculus of Reggeized gauge bosons.[1–3] In the leading $\ln s$ approximation this calculus gives multiperipheral equations with Reggeized gauge-boson exchange in the ladder. For amplitudes corresponding to singlet of the underlying gauge group those equations are free from infrared divergences and are meaningful for genuine QCD with massless gluons.[2,3]

Multiple-gluon exchange, although asymptotically dominant, only contributes to the flavor-singlet channel and the purpose of this note is to discuss leading Regge singularities corresponding to the flavor-nonsinglet channel. In the leading $\ln s$ approximation which we adopt, this reduces to a multiperipheral equation with Reggeized quarks in the ladder.

The quark Regge trajectories $\frac{1}{2}+\alpha(q_\perp)$ and effective double-Regge vertices Γ_μ^D with two Reggeized fermions (see Fig. 1) were calculated in Ref. 4 for the group SU(N). For QCD the result is

$$\alpha(q_\perp)=(M-q_\perp)\eta(q_\perp)=(M-q_\perp)(-\tfrac{4}{3})\frac{g^2}{(2\pi)^3}\int\frac{d^2\vec{q}\,'}{(M-q'_\perp)[(\vec{q}\,'_\perp-\vec{q}_\perp)^2+\mu^2]}\,, \tag{1}$$

$$\Gamma_\mu^D=-g\frac{\lambda^D}{2}\left[\gamma^\mu-(M-q_{1\perp})\frac{p_A^\mu}{q_2\cdot p_A}-(M-q_{2\perp})\frac{p_B^\mu}{q_1\cdot p_B}\right], \tag{2}$$

where M and μ are the quark and gluon masses, respectively, the latter to be put equal to zero at the end, and g is the coupling constant of the QCD Lagrangian. (Apart from trivial factors the same results also hold for massive QED.[5,6] A difference appears however in the negative-signature fermion channel due to presence of "direct" forces coming from gluon exchange in addition to fermion exchange.[4] In our case, however, exchange of the negative-signature partner of a quark can be neglected in the leading $\ln s$ approximation.) The momenta q_i and $p_{A,B}$ are defined in Fig. 1. Using the expression (2) for the vertex Γ_μ^D and taking into account the identity

$$\frac{p_A\cdot p_B}{(q_2\cdot p_A)(q_1\cdot p_B)}=-\frac{2}{(\vec{q}_{1\perp}-\vec{q}_{2\perp})^2+\mu^2}\,, \tag{3}$$

which is valid in the double-Regge region, we can obtain the kernel of the equation we are looking for. For color singlets in the t channel this equation has the following form (see Fig. 2):

$$\begin{aligned}
jF_j(\vec{q}_\perp)&-F_j(\vec{q}_\perp)\eta(q_\perp)S^{-1}(q_\perp)-S^{-1}(q_\perp)\eta(q_\perp)F_j(\vec{q}_\perp)\\
&=F^{(0)}+\frac{4}{3}\frac{g^2}{(2\pi)^3}\int d^2\vec{q}\,'_\perp\{\tfrac{1}{2}\gamma_\perp^\mu S(q'_\perp)F_j(\vec{q}\,'_\perp)S(q')\gamma_\perp^\mu[\theta(\vec{q}_\perp^{\,2}-\vec{q}\,'^{2}_\perp)+(\vec{q}_\perp^{\,2}/\vec{q}\,'^{2}_\perp)^j\theta(\vec{q}\,'^{2}_\perp-\vec{q}_\perp^{\,2})]\\
&\qquad+[(\vec{q}_\perp-\vec{q}\,'_\perp)^2+\mu^2]^{-1}[S^{-1}(q_\perp)S(q')F_j(\vec{q}\,'_\perp)+F_j(\vec{q}\,'_\perp)S(q'_\perp)S^{-1}(q_\perp)]\}
\end{aligned} \tag{4}$$

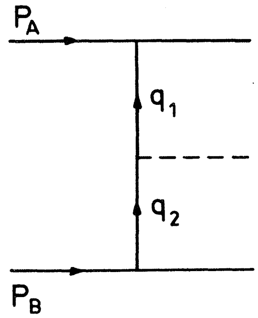

FIG. 1. The double-Regge vertex with Reggeized quarks.

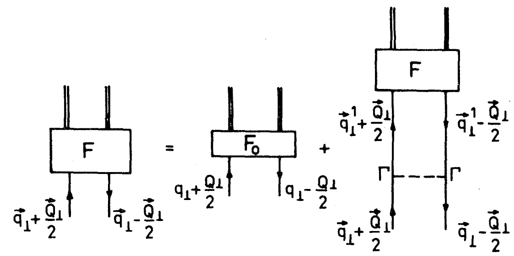

FIG. 2. Multiperipheral integral equation with Reggeized quark and antiquark exchange.

and for simplicity we consider the forward configuration, i.e., $\vec{Q}_\perp=0$. In Eq. (4) j is the t-channel angular momentum, $F_j(\vec{q}_\perp)$ is the partial-wave matrix with external Reggeized quark-antiquark pairs and $F^{(0)}$ is the appropriate inhomogeneous term. $S(q\!\!\!/_\perp)$ is the fermion propagator, i.e., $S(q\!\!\!/_\perp)=(M-q\!\!\!/_\perp)^{-1}$. The part of the kernel containing the product of γ matrices gives double ln$s$ terms (i.e., powers of g^2/j^2) which are typical for fermion exchange.[7,8] The j-dependent factor is the reflection of appropriate limits in the longitudinal-momentum integration.[7,8]

The following properties of Eq. (4) should be emphasized:

(i) There is a cancellation of infrared divergences in the limit $\mu^2\to 0$ between the divergent trajectory function $\alpha(q\!\!\!/_\perp)$ and the singular part of the kernel which is proportional to $[(\vec{q}_\perp-\vec{q}\,'_\perp)^2+\mu^2]^{-1}$. Thanks to this cancellation, which is similar to that for gluon amplitudes,[1-3] this equation is well defined for genuine QCD with massless gluons.

(ii) For large $\vec{q}_\perp^{\,2}$ the leading $\ln\vec{q}_\perp^{\,2}$ terms come from the first part of the kernel. Logarithmic terms coming from the quark trajectory $[\alpha(q\!\!\!/_\perp)\sim\ln\vec{q}_\perp^{\,2}]$ cancel with logarithmic terms generated by the singular part of the kernel. To achieve this cancellation it is essential that the logarithmic behavior of the fermion trajectory for large values of $\vec{q}_\perp^{\,2}$ comes only from the gluon propagator in the loop [i.e., from the region $(\vec{q}\,'-\vec{q})^2<<q^2$] while the fermion propagator (i.e., the region $M^2<<q'^2<<q^2$) does not contribute to this behavior. Notice the difference of a factor of 2 in comparison with the asymptotic behavior of the gluon trajectory which is defined by the two-gluon loop.[1-3] After taking into account asymptotic-freedom corrections Eq. (4) reduces in the leading $\ln\vec{q}_\perp^{\,2}$ approximation to the evolution equation[9] with anomalous dimension approximated by its pole at $j=0$.

(iii) The double ln$s$ terms (i.e., powers of g^2/j^2) are entirely generated by the first part of the kernel. Including reggeization of quarks together with additional terms in the kernel neither cancels nor alters the logarithmic divergence of the transverse-momentum integrals responsible for this double ln$s$ behavior. Similar equations with gluons[1-3] give the single-logarithmic terms only.

In the leading double ln$s$ approximation Eq. (4) simplifies and apart from trivial factors becomes equivalent to the equation derived in Ref. 7 for QED. It takes the following form:

$$jf_j(\delta)=f^{(0)}+\frac{2}{3}\frac{g^2}{(2\pi)^2}\left[\int_0^\delta d\delta' f_j(\delta')+\int_\delta^\infty d\delta' \exp j(\delta-\delta')f_j(\delta')\right], \tag{5}$$

where

$$\delta=\ln\vec{q}_\perp^{\,2}/q_0^{\,2} \tag{6}$$

and

$$F_j=\gamma_\perp^\mu f_j\gamma_\perp^\mu . \tag{7}$$

The solution of this equation has a fixed branch point at $j=j_0$:

$$j_0=2\left[\frac{2}{3}\frac{g^2}{(2\pi)^2}\right]^{1/2} . \tag{8}$$

This fixed branch point comes entirely from the ultraviolet part. It can therefore be expected that asymptotic-freedom corrections should convert this singularity into Regge poles.[10] These corrections amount to replacing g^2 by the running coupling constant $g^2(\delta')$.[9] Equation (5) takes then the following form:

$$f_j(\delta)=\frac{f^{(0)}}{j}+\frac{c}{j}\left[\int_0^{\delta}\frac{d\delta'}{\delta'+1/h}f_j(\delta')+\int_{\delta}^{\infty}\frac{d\delta'}{\delta'+1/h}\exp j(\delta-\delta')f_j(\delta')\right], \tag{9}$$

where $c=8/(33-2n_F)$ and $h=[(33-2n_f)/48\pi^2]g^2(\delta'=0)$ (n_F denotes the number of flavors).

Solution of this equation can easily be obtained by reducing it to Kummer's differential equation[11] and taking into account boundary conditions implied by the integral equation itself. For a constant inhomogeneous term $f^{(0)}$ the solution is

$$f_j(\delta)=\frac{f^{(0)}}{j}\left[U(-\frac{c}{j},1;j/h)\right]^{-1}U(-\frac{c}{j},0;j(\delta+1/h)), \tag{10}$$

where $U(a,b;z)$ is the Kummer function.[11] This solution contains an infinite number of Regge poles j_n which correspond to zeros of the function $U(-c/j,1;j/h)$ appearing in the denominator in formula (11). They accumulate at $j=0$ according to the following rule:

$$j_n=c/n+O(1/n^2). \tag{11}$$

Let us notice that the leading term c/n in the formula (11) is independent of h but depends only upon the parameter c controlling behavior of the running coupling constant for large momenta.

The existence of an infinite number of flavor nonsinglet Regge poles accumulating at $j=0$ was assumed in Ref. 12 and Eqs. (9)–(11) indicate the possible origin of those poles. This singularity structure can however be modified by absorptive corrections (i.e., Pomeron-Regge pole cuts) but this type of correction goes beyond the leading $\ln s$ approximation adopted in our paper.

Physically, Regge poles j_n correspond to the short-range part of the interaction alone. One can, however, expect that the implicit bound on the intercept of the leading Regge singularities implied by (11) (i.e., $\bar{j}(0)\gtrsim 0$) should not be affected by long-range confining forces. Although this constraint is of little importance for Regge trajectories of mesons "built up" from light quarks, it becomes relevant when heavy quarks are involved. In particular, it should invalidate straightforward linear extrapolations of D^* or ψ trajectories which give very low, negative intercepts.

This possibility that the intercept of Regge singularities cannot be arbitrarily low independently of the flavors involved should be important for the phenomenology of high-energy reactions proceeding through charm exchange. The high intercept (i.e.,$\simeq$0) of the D^* trajectory could, in particular, be relevant for the explanation of the presence of leading Λ_c particles produced at pp collisions at the CERN ISR.[13]

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