Exclusive decays of charmonium: The ratio $\Gamma(^3S_1 \to \pi^+\pi^-)/\Gamma(^3P_0 \to \pi^+\pi^-)$

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The ratio $\Gamma(^3S_1 \to \pi^+\pi^-)/\Gamma(^3P_0 \to \pi^+\pi^-)$ is analyzed for exclusive decays of the charmonium. In leading order, the result is independent of the soft contribution, thus providing an interesting test of the method. The calculated value is compared with recent experimental results.

The recent advance in the analysis of exclusive processes at large momentum transfer[1,2] has opened up an important field of applications: the study of exclusive decays of quarkonia,[3–5] characterized by the large constituent mass M. Here we consider two particular decay modes of the charmonium, both having the same final state, a pair of charged pions. One of these, the decay of the $^3S_1$ state, $\psi(3100) \to \pi^+\pi^-$, is mediated by electroweak interactions (G parity is violated), and its analysis is similar to the analysis of the pion form factor.[6] Even the other decay mode, $\chi(3415) \to \pi^+\pi^-$, mediated by two gluons, has already been thoroughly investigated.[3] However, we want to point out in this paper that the ratio of decay rates for both processes is, in leading order, independent of the soft dynamics. Thus, the leading-order result, entirely controlled by the renormalization group, might provide an interesting test of the method. With the present experimental accuracy, the leading result should be a good approximation. However, any improvement in the experimental technique combined with the inclusion of (calculable) subleading terms would make an even more ambitious project possible, namely, the differentiation between various potential models for charmonium states. Our method and notation follow Refs. 1 and 3.

The decay of the $^3S_1$ state in lowest order is described by diagrams in Fig. 1. The amplitude of the hard subprocess is given by[7]

$$T_S = \frac{1}{\sqrt{4\pi M}}\phi_0(0)\,\mathrm{Tr}[M(1+\gamma_0)\not{\epsilon}\mathcal{M}_S] \ , \tag{1}$$

where ϵ^μ is the polarization vector of the initial state, while $\phi_0(0)$ is the nonrelativistic radial wave function of the S state at the origin. $\mathcal{M}$ is the sum of contributions 1(a)–1(d). Neglecting corrections of order $1/M$, we obtain

$$\mathcal{M}_S = \frac{8g^2e^2}{M^5}\frac{1-x_1x_2+(x_1-x_2)/3}{(1-x_1^2)(1-x_2^2)}\frac{\not{p}_1-\not{p}_2}{M}+O(K) \ . \tag{2}$$

From (1) and (2) we find

$$T_S = \frac{32}{\sqrt{4\pi M}}\frac{g^2e^2}{M^5}(p_1\epsilon - p_2\epsilon)\phi_0(0)\left[\frac{1-x_1x_2+(x_1-x_2)/3}{(1-x_1^2)(1-x_2^2)}\right] \ . \tag{3}$$

In the decay of the $^3P_0$ state, one must keep even the terms linear in the relative momenta K. If $\mathcal{M}_P$ is written as an expansion, $\mathcal{M}_p = \overline{\mathcal{M}}_p + K_\mu \mathcal{M}_P^\mu + \cdots$, then the relevant amplitude[7]

$$T_P = \frac{1}{\sqrt{4\pi M}}\phi_1'(0)\,\mathrm{Tr}\left[3\overline{\mathcal{M}}_P + M\left(\gamma_\mu - \frac{(\not{p}_1+\not{p}_2)(p_1+p_2)_\mu}{M^2}\right)\mathcal{M}_P^\mu(1+\gamma_0)\right] \tag{4}$$

is proportional to the derivative $\phi_1'(0)$ of the radial wave function of the P state at the origin. The diagrams shown in Fig. 2 give

$$\overline{\mathcal{M}}_P = \frac{16g^4}{M^5}\frac{1}{(1-x_1^2)(1-x_2^2)(1+x_1x_2)} \ ,$$
$$\mathcal{M}_P^\mu = \overline{\mathcal{M}}_P\left[\frac{(-2)}{M^3}\frac{1}{1+x_1x_2}(\not{p}_1 p_2^\mu + \not{p}_2 p_1^\mu + x_1^2\not{p}_1 p_1^\mu + x_2^2\not{p}_2 p_2^\mu)\right] \ , \tag{5}$$

and one finally obtains

$$T_P = \frac{128}{\sqrt{4\pi M}}\frac{g^4}{M^5}\phi_1'(0)\left[\frac{1+x_1x_2+(x_1+x_2)^2/4}{(1-x_1^2)(1-x_2^2)(1+x_1x_2)}\right] \ . \tag{6}$$

It is well known[1,2] that T_S and T_P satisfy the renormalization-group equation which can be diagonalized by pro-

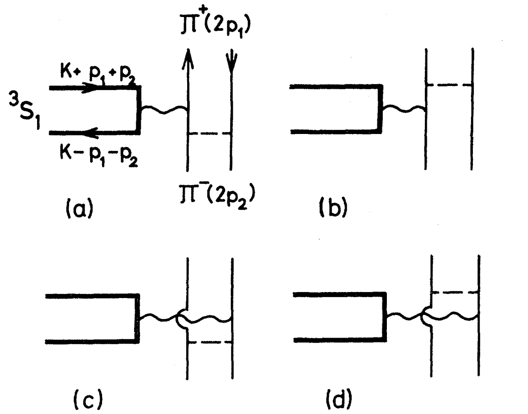

FIG. 1. Lowest-order contributions to the $^3S_1$-state decay into a pair of pions. The double, wavy, and dashed lines denote heavy quark, photon, and gluon, respectively.

jecting (3) and (6) onto the Gegenbauer polynomials:

$$T \equiv T(x_1,x_2) = \sum_{N_1,N_2} C_{N_1}^{3/2}(x_1)\,T_{N_1N_2}\,C_{N_2}^{3/2}(x_2)\ . \tag{7}$$

The asymptotic solution for $T_{N_1N_2}$ is then

$$T_{N_1N_2}(M^2) \sim (\ln M^2/\mu^2)^{-A_{N_1}-A_{N_2}}\,T_{N_1N_2}^{\text{asymp}}(\mu^2, g^2(M^2/\mu^2))\ , \tag{8}$$

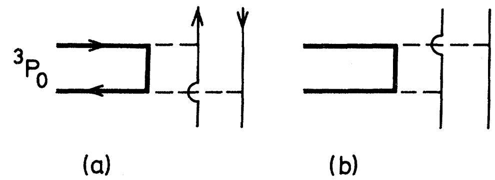

FIG. 2. Lowest-order contributions to the decay of the $^3P_0$ state. The notation is the same as in Fig. 1.

with A denoting anomalous-dimension factors. $T_{N_1N_2}^{\text{asymp}}$ are determined by the asymptotic form of the relevant Born graphs, and their evaluation requires knowledge of the integrals of the form

$$h_{N_1N_2} = \frac{(2N_1+3)!!(2N_2+3)!!}{4(N_1+2)!(N_2+2)!} \times \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2\, C_{N_1}^{3/2}(x_1) H(x_1,x_2) C_{N_2}^{3/2}(x_2)\ . \tag{9}$$

H in (9) stands for the functions inside the square brackets of expressions (3) and (6).

One may now go to the pion pole on each of the final-meson legs, write the complete decay amplitude, and finally obtain the required ratio of partial widths, G. Assuming that the $^3S_1$ state is unpolarized, one obtains

$$G = \frac{\Gamma(^3S_1 \to \pi^+\pi^-)}{\Gamma(^3P_0 \to \pi^+\pi^-)} = \frac{1}{3}\left[\frac{e^2}{4g_{\text{eff}}^2}\frac{M\phi_0(0)}{\phi_1'(0)}\right]^2 \left[\frac{\displaystyle\sum_{n_1n_2N_1N_2} (\ln M^2/\mu^2)^{-A_{N_1}^S - A_{N_2}^S} f_\pi^{(n_1)} e_{n_1}^{(N_1)} h_{N_1N_2}^S e_{n_2}^{(N_2)} f_\pi^{(n_2)}}{\displaystyle\sum_{m_1m_2M_1M_2} (\ln M^2/\mu^2)^{-A_{M_1}^P - A_{M_2}^P} f_\pi^{(m_1)} e_{m_1}^{(M_1)} h_{M_1M_2}^P e_{m_2}^{(M_2)} f_\pi^{(m_2)}}\right]^2 . \tag{10}$$

For the definitions of $f_\pi^{(n)}$ and e_n^N, see, e.g., Ref. 1.

Equation (10) reduces to a simple expression in the leading-order approximation, in which only the dominant fractional powers of $\ln M^2$ are kept. The remaining pion-decay constants f_π cancel, and the value for G becomes

$$G = \frac{1}{48}\left[\frac{\alpha}{\alpha_{\text{eff}}(2M)}\right]^2 \left[\frac{M\phi_0(0)}{\phi_1'(0)}\right]^2 w\ , \tag{11}$$

where

$$w = \left[\frac{h_{00}^S}{h_{00}^P}\right]^2 = \left[\frac{16}{3\pi^2-8}\right]^2 .$$

The factor

$$r = [\phi_1'(0)/M\phi_0(0)]^2 \tag{12}$$

changes from model to model. For the typical values, $r = 0.06$ (Ref. 8) and $\alpha_{\text{eff}}(3.1\text{ GeV}) = \frac{1}{3}$, one obtains

$$G_{\text{theor}} \simeq 0.9 \times 10^{-4}\ . \tag{13}$$

The experimental value for G may be found by combining the results for $a_1 = B(^3S_1 \to \pi^+\pi^-)$, $a_2 = \Gamma(^3S_1 \to \text{all})$, $b_1 = B(^3P_0 \to \pi^+\pi^-)$, and $b_2 = \Gamma(^3P_0 \to \text{all})$. The most accurate values available at present are

$$a_1 = (0.13 \pm 0.06) \times 10^{-3} \quad (\text{Ref. 9}) \ ,$$

$$a_2 = (69 \pm 9) \times 10^{-3}\ \text{MeV} \quad (\text{Ref. 10}) \ ,$$

$$b_1 = (9 \pm 2) \times 10^{-3} \quad (\text{Ref. 10}) \ ,$$

$$b_2 = (16 \pm 4)\ \text{MeV} \quad (\text{Ref. 11}) \ ,$$

which give

$$G_{\text{exp}} = 0.6(1 \pm 0.6) \times 10^{-4} \ . \tag{14}$$

Most of the uncertainty in the result (14) comes from the value for a_1. However, a more precise result for this branching ratio may also be expected in the near future.

What can one learn from this analysis? The leading-order result (for values of α_{eff} which are not too small) is already in fair agreement with the experimental value. This is certainly an encouraging piece of information. Remember that the data for most of other analyzed exclusive processes (e.g., pion form factor, Υ decays, etc.) are still not available. Another conclusion is that one cannot decide at present which r [Eq. (12)] is preferred by experiment. However, as soon as more accurate measurements stimulate the calculation of subdominant terms in (10), a more direct test of potential models will become available.

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