Nucleon-antinucleon potential due to the annihilation of one gluon

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The short-range part of the interaction between nucleon and antinucleon due to one-gluon exchange is explicitly calculated in the framework of the quark model. This is a first step towards treating anihilation channels for $N\bar{N}$ systems. It is found that it gives a spin- and isospin-dependent repulsion. The relativistic effect due to a large momentum of quarks is also considered and found to enhance the strength of the potential. It is argued in connection with the nucleon-nucleon potential that the one-gluon-exchange potential would provide almost none of the characteristics of the nucleon-antinucleon interaction at short distances expected in the meson-exchange theory.

Several attempts[1–6] have been made to understand the short-range part of the nucleon-nucleon (NN) interaction from a theory based on the quark model and quantum chromodynamics. Since the nucleon-antinucleon ($N\bar{N}$) interaction is closely related to the nucleon-nucleon interaction the meson-exchange model by the G-parity transformation, it will be interesting to investigate these connections in the quark model. In this paper we wish to examine the one-gluon-exchange contribution to the nucleon-antinucleon interaction in the same framework as is used to study the short-range part of the nucleon-nucleon interaction.

It is well known[7] that in the meson-exchange model of the nucleon-nucleon interaction the short-range repulsive core is due to ω-meson ($J^P=1^-$, $T=0$) exchange. This model can also tell the nucleon-antinucleon interaction by the G-parity transformation given by

$$G\begin{bmatrix} p \\ n \end{bmatrix} = \begin{bmatrix} \bar{n} \\ -\bar{p} \end{bmatrix} . \tag{1}$$

Since the ω meson has negative parity under the G transformation, the interaction between nucleon and antinucleon has the opposite sign to the nucleon-nucleon potential. Thus it is expected by the meson-exchange model that the nucleon-antinucleon potential has a very strong attraction at a short distance. There have been some attempts[1–6] to understand the short-range part of the nucleon-nucleon interaction using the one-gluon-exchange potential.[8] Therefore, it will be very interesting to see what kind of potential will come out for nucleon-antinucleon interaction using a quark-quark interaction represented by the exchange and annihilation diagrams, Figs. 1 and 2. Since the exchange diagram of Fig. 1 vanishes between a $N\bar{N}$ state due to the color-singlet nature of the nucleon and antinucleon, we calculate here the $N\bar{N}$ interaction using the annihilation diagram of Fig. 2 as the force between a quark and antiquark.

Using the quark-gluon vertex,

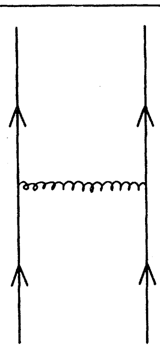

FIG. 1. Quark-antiquark scattering due to one-gluon exchange.

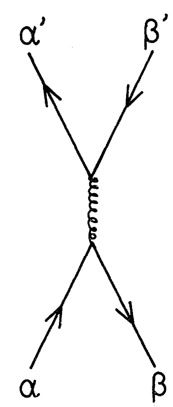

FIG. 2. Quark-antiquark annihilation diagram.

$$i(4\pi\alpha_s)^{1/2}\sum_{\mu=1}^{4}\sum_{c=1}^{8}\bar{\psi}\gamma_\mu\frac{\lambda^{(c)}}{2}\psi G_\mu{}^{(c)}\ , \qquad (2)$$

we evaluate the contribution from the term in Fig. 2. Here ψ is a quark spinor and G_μ is a gluon field. The $\lambda^{(c)}$ are the color SU(3) generators and γ_μ are the usual Dirac γ matrices. The gluon propagator in Fig. 2 is written as

$$D_{\mu\nu}^{cc'}=\frac{-i}{q_0{}^2-\vec{q}^{\,2}+i\epsilon}\delta_{cc'}\delta_{\mu\nu}$$
$$\cong\frac{-i}{4m^2-\vec{Q}^{\,2}+i\epsilon}\delta_{cc'}\delta_{\mu\nu}\ , \qquad (3)$$

where the gluon momentum $\vec{q}$ is replaced by the c.m. momentum $\vec{Q}$ of quark and antiquark. The quark is bound in the nucleon and thus it is off-shell. Its total energy is a little less than the effective mass m plus $\frac{1}{3}$ the kinetic energy of a nucleon. Since we do not know the off-shell energy q_0 which is the sum of the quark and antiquark energy we replace it by $q_0 \approx 2m_q$. This uncertainty affects only slightly the real part but might give a "yes" or "no" answer for the imaginary part. This point will be discussed below. We keep only central terms up to $O(p^2/m^2)$. If we restrict ourselves to S-wave interaction between $N\bar{N}$, tensor and two-body spin-orbit terms do not contribute:

$$M=4\pi\alpha_s\frac{1}{4m^2-\vec{Q}^{\,2}+i\epsilon}\frac{(\xi_{\bar{\beta}}^\dagger\lambda\xi_\alpha)(\xi_{\alpha'}^\dagger\xi_{\bar{\beta}'})}{4}(\chi_{\bar{\beta}}^\dagger\vec{\sigma}\chi_\alpha)$$
$$\times(\chi_{\alpha'}^\dagger\vec{\sigma}\chi_{\bar{\beta}'})\left[1+\frac{1}{12m^2}(\vec{q}^{\,2}+\vec{q}^{\,\prime 2}-\tfrac{3}{2}\vec{Q}^{\,2})\right]\ . \qquad (4)$$

Here ξ and χ are color and spin wave functions, respectively. The $\vec{q}$ and $\vec{q}\,'$ are relative momenta between quarks and antiquarks in the initial and final states. The color, spin, and isospin parts are rewritten in a more familiar form in the following way. We explain the derivation only for the color part. Using the completeness of the color-SU(3) generators $\lambda^{(c)}$ $(c=1,8)$ together with the unit matrix, we obtain the following rearrangement formula:

$$\sum_{c=1}^{8}\lambda_{\alpha'\beta'}^{(c)}\lambda_{\beta\alpha}^{(c)}=-\tfrac{1}{3}\sum_{c=1}^{(c)}\lambda_{\alpha'\alpha}^{(c)}\lambda_{\beta\beta'}^{(c)}+\tfrac{16}{9}\delta_{\alpha'\alpha}\delta_{\beta'\beta}\ . \qquad (5)$$

Then the color part is written as

$$-\tfrac{1}{12}(\xi_{\alpha'}^\dagger\lambda\xi_\alpha)(\xi_{\bar{\beta}}^\dagger\lambda\xi_{\bar{\beta}'})+\tfrac{4}{9}\delta_{\alpha'\alpha}\delta_{\beta'\beta}\ . \qquad (6)$$

Keeping in mind that λ_1 and $-\lambda_2^*$ are color-SU(3) generators for quarks and antiquarks, Eq. (6) is written in the following form:

$$\tfrac{1}{24}(\lambda_1-\lambda_2^*)^2\ . \qquad (7)$$

Similarly, we obtain for spin and isospin parts as follows:

$$\frac{(\vec{\sigma}_1+\vec{\sigma}_2)^2}{4}\frac{1-\vec{\tau}_1\cdot\vec{\tau}_2}{2}\ . \qquad (8)$$

Finally the potential in momentum space between quark and antiquark coming from the pair annihilation due to one-gluon exchange in Fig. 2 becomes

$$V_{12}^{(\mathrm{ann})}=4\pi\alpha_s\frac{(\lambda_1-\lambda_2^*)^2}{24}\frac{(\vec{\sigma}_1+\vec{\sigma}_2)^2}{4}\frac{1-\vec{\tau}_1\cdot\vec{\tau}_2}{2}$$
$$\times\frac{1}{4m^2-\vec{Q}^{\,2}+i\epsilon}$$
$$\times\left[1+\frac{1}{12m^2}(\vec{q}^{\,2}+\vec{q}^{\,\prime 2}-\tfrac{3}{2}\vec{Q}^{\,2})\right]\ . \qquad (9)$$

It must be noted that each factor of color, spin, or isospin says that the potential has a nonvanishing value only for color octet 8, spin=1, and isospin=0 state of a quark and antiquark pair, which corresponds to the gluon quantum numbers. From Eq. (9), we immediately see that the nucleon-antinucleon potential due to one-gluon exchange in Fig. 2 is repulsive when the momenta of quark and antiquark are small.

We now evaluate the nucleon-anticnucleon potential assuming that the nucleon and the antinucleon consist of three quarks and antiquarks confined in the lowest configuration. The wave function of the nucleon-antinucleon system is given by

$$|\Phi,\vec{r}\rangle$$
$$=\Psi(\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4\vec{r}_5\vec{r}_6,\vec{r})\,|\,q^3(\tfrac{1}{2}\ \tfrac{1}{2}),\bar{q}^{\,3}(\tfrac{1}{2}\ \tfrac{1}{2});ST\rangle\ , \qquad (10)$$

where S and T are the total spin and isospin. The orbital wave function $\Psi(\vec{r}_1\cdots\vec{r}_6,\vec{r})$ is given by

$$\Psi(\vec{r}_1\cdots\vec{r}_6,\vec{r})=\prod_{\substack{i=1,2,3\\ j=4,5,6}}\phi_{0s}\left[r_i-\frac{\vec{r}}{2}\right]\phi_{0s}\left[\vec{r}_j+\frac{\vec{r}}{2}\right]\ , \qquad (11)$$

where $\vec{r}$ is the relative distance between the center of mass of the nucleon and antinucleon. The single-particle wave function ϕ_{0s} is taken to be a Gaussian form,

$$\phi_{0s}(\vec{X}) = \left[\frac{1}{\sqrt{\pi} b}\right]^{3/2} e^{-\vec{X}^2/2b^2}, \tag{12}$$

where b is a size parameter. Thus we neglect polarization effects.

First we consider the nonrelativistic case where all the momentum dependence in Eq. (9) is ignored. In this case the quark and antiquark potential in orbital space becomes local and of zero range,

$$\frac{1}{(2\pi)^3}\int 4\pi\alpha_s \frac{1}{4m^2} e^{-i\vec{p}\cdot\vec{r}_{12}} d^3\vec{p} = \frac{\alpha_s \pi}{m^2}\delta(\vec{r}_{12}) . \tag{13}$$

Then the evaluation of the matrix element of the interaction between nucleon and antinucleon is straightforward:

$$V_{N\bar{N}}(r) = \langle \Phi, \vec{r} \,|\, \sum_{\substack{i=1,3\\ j=4,6}} V_{ij}^{(\mathrm{ann})} \,|\, \Phi, \vec{r}\rangle = \frac{\alpha_s}{81}(243 + 9\vec{\sigma}_N\cdot\vec{\sigma}_{\bar{N}} - 27\vec{\tau}_N\cdot\vec{\tau}_{\bar{N}} - 25\vec{\sigma}_N\cdot\vec{\sigma}_{\bar{N}}\vec{\tau}_N\cdot\tau_{\bar{N}})\frac{1}{4m^2b^3}\left[\frac{2}{\pi}\right]^{1/2} e^{-r^2/2b^2}$$

$$= V_{ST} e^{-r^2/2b^2} , \tag{14}$$

where $\vec{\sigma}_N$ and $\vec{\sigma}_{\bar{N}}$ are nucleon and antinucleon spins, and $\vec{\tau}_N$ and $\vec{\tau}_{\bar{N}}$ are their isospins. The magnitude of the potential is estimated using the parameters used by Oka and Yazaki.[4] They are $m = 300$ MeV/c^2, $\alpha_S = 1.39$, $b = 0.6$ fm. Then the coefficients V_{ST} are $V_{00} = 97$, $V_{01} = 357$, $V_{10} = 552$, $V_{11} = 271$ MeV.

Now we consider the modification of the potential in Eq. (15) due to the relativistic effects by keeping the momentum dependence in Eq. (10). In this case, the evaluation is easily done in momentum space by transforming the wave function in Eq. (11) into momentum space. The result is

$$\mathrm{Re}V_{N\bar{N}}(r) = V_{ST}\left[\left[1 + \frac{1}{4b^2m^2} - \frac{r^2}{24m^2b^4}\right] f(2m^2b^2) - \frac{3}{8m^2b^2} g(2m^2b^2)\right] e^{-r^2/2b^2} , \tag{15}$$

where f and g are given by

$$f(\alpha^2) = \frac{4\alpha^2}{\sqrt{\pi}} \mathrm{P}\int_0^\infty \frac{e^{-\chi^2}\chi^2}{\alpha^2 - \chi^2} d\chi ,$$

$$g(\alpha^2) = \frac{8\alpha^2}{3\sqrt{\pi}} \mathrm{P}\int_0^\infty \frac{e^{-\chi^2}\chi^4}{\alpha^2 - \chi^2} d\chi . \tag{16}$$

These functions are normalized so that $f(\infty) = g(\infty) = 1$, which corresponds to the nonrelativistic limit for the gluon propagator. The corrections which remain even when $f = g = 1$ are due to the vertex corrections corresponding to $O(p^2/m^2)$ terms in Eq. (4).

The functions f and g are given in Fig. 3. When we take $\alpha^2 = 2m^2b^2 \approx 1.66$, we have $f \approx 0.84$. This means that the inclusion of the momentum depen-

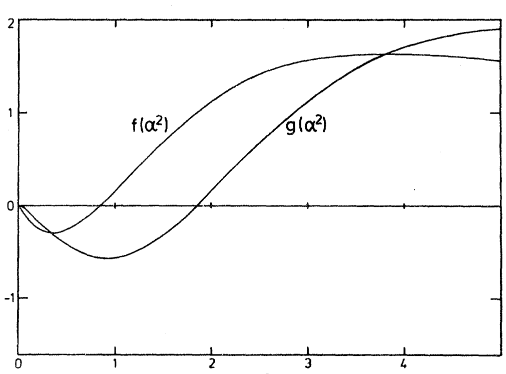

FIG. 3. Functions $f(\alpha^2)$ and $g(\alpha^2)$ defined in Eq. (17) vs $\alpha^2 \equiv 2m^2b^2$.

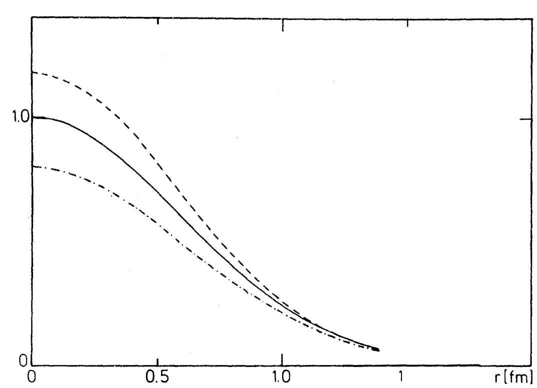

FIG. 4. Nucleon-antinucleon potential without V_{ST} vs the relative distance r. The solid line represents the real part for the nonrelativistic case, and the dotted line for the case where the momentum dependence up to $O(p^2/m^2)$ is taken into account. The dashed-dotted line gives the imaginary part (17) with a factor $-V_{ST}$ removed.

dence in the gluon propagator reduces the strength of the potential. As seen in Eq. (15), the vertex correction enhances the strength of the potential. Thus the repulsion is increased by roughly 20% by including the relativistic effect for $r=0$. In Fig. 4 we show the potential without a factor V_{ST}, together with the nonrelativistic result.

The quark-antiquark potential in Eq. (9) also yields in our one-gluon approximation an imaginary part of $V_{NN}(r)$. The existence of an imaginary part depends on the fact that the propagator of the gluon (3) has a pole. Since q_0 is off-shell $[<(m^2+p_1{}^2)^{1/2}+(m^2+p_2{}^2)^{1/2}$; with p_1 and p_2 the momenta of the annihilated quark-antiquark pair], it cannot be calculated reliably. But since the decay products of the intermediate state $2p\bar{p}+$ gluon have a smaller rest mass than a nucleon-antinucleon pair, we assume the same for our "doorway state" ($2q\bar{q}+$ gluon). This assumption was made also by Freedman *et al.*[9] who calculated the imaginary part in the bag model using similar approximations. We obtain for the imaginary part

$$\operatorname{Im} V_{N\bar{N}}(r) = -4\sqrt{2}\pi m^3 b^3 e^{-2m^2b^2}\left[1+\frac{1}{4b^2m^2}-\frac{r^2}{24m^2b^4}-\frac{1}{2}\right]V_{ST}e^{-r^2/2b^2} \simeq -0.8V_{ST}e^{-r^2/2b^2}\,. \qquad (17)$$

If one specializes to the proton-antiproton interaction, our result for Im $V_{p\bar{p}}^{\text{central}}R/\alpha_s$ with the bag radius $R=1$ fm is at $r=0$ a factor 10 and at $r=0.6$ fm a factor 2.6 larger than the result of Freedman *et al.*[9] If the bag radius R is decreased, the result of Freedman *et al.*[9] increases to the same magnitude for $R=0.4$ fm. The width $\Gamma_{1s}(p\bar{p})$ using (17) is only of the order of 1 keV,

$$\Gamma_{1s}(p\bar{p}) \approx -860e^{2(b/a)^2}\left[\frac{b}{a}\right]^3 \text{ MeV} \qquad (18)$$

(the Bohr radius is $a=57$ fm). If one scales the $p\bar{p}$ to the size of a deuteron ($a\approx 2$ fm), one obtains a width of the order of 50 MeV in good agreement with what one expects qualitatively.

At the end of this work we want to point out that although the approximations used are still crude (annihilation in only one gluon), the G-parity argument for deriving the $N\bar{N}$ potential can only be correct if the two bags are not overlapping. At this more distant range meson exchange should be prevailing and G parity is a valuable concept for deriving the $N\bar{N}$ interaction. At short distances the quark picture takes over. Our work indicates at these short distances a repulsion. At the moment there seems to be no experimental evidence against such a view. In order to draw some reliable conclusions, calculations within a more reliable model for the nucleons and antinucleons with the annihilation into more than one gluon and the exchange of two gluons are needed. In addition more precise experiments with low-energy antiprotons are needed. One hopes that such data can be obtained with LEAR projected at CERN.

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