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Mean-field approximation for chiral quark bag models

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A chirally invariant quark model of the nucleon is studied by the use of a Tomonaga type of approximation. Static nucleon properties are calculated as a function of the bag radius and compared against corresponding quantities obtained perturbatively, as indeed has been done invariably in existing chiral bag calculations. It is found that for some observables (such as the energy) the perturbation treatment becomes unacceptably inaccurate for $R \lesssim 0.9$ fm. However, electromagnetic properties may be accurately calculated for much smaller bag radii, provided that a coupling constant (f_0) is appropriately adjusted to give the correct asymptotic pion field strength.

INTRODUCTION

Chirally symmetric bag models,[1–6] an extension of the original MIT bag model,[7] have attracted a great deal of interest and attention in recent years. It is well known that the original MIT bag model violates chiral invariance. This is remedied in the chiral bag models by coupling to an explicit pion field in such a manner as to make the axial-vector current continuous everywhere, including at the bag boundary. The (spherical) bag surface acts as a fixed source, absorbing and emitting pions with a strength which increases rapidly with decreasing radius. Different versions of the chiral bag now exist, distinguished by whether pions are allowed to penetrate the bag volume,[2–5] or whether they are strictly confined to the outside region.[6,8,9] Earlier treatments[8] of the pion field had been classical, using a "hedgehog" ansatz. Later, a quantum-mechanical solution was given by Thomas, Théberge, and Miller[3–5] (the "cloudy bag model," CBM) and by DeTar,[2] based on a linearized Lagrangian and a one-intermediate-meson approximation. Good fits to low-energy π-N scattering and nucleon charge radii, magnetic moments, etc. were obtained. The success of the model for nucleons has stimulated work on the properties of the strange baryons[9,10] and on the P_{11} nucleon resonance.[11] Chiral bags have been particularly interesting for nuclear physicists since the Yukawa pion tail is present in the theory from the very outset.

Although the success of chiral models appears encouraging, one must keep in mind that the extreme complexity of the full chiral Lagrangian[1] describing quarks and pions has necessitated a number of approximations and assumptions in all quantum treatments for the sake of tractability. Therefore, before one can accept the chiral bag as a quantitatively useful model of the nucleon, each one of these assumptions needs to be checked. In this work, we examine a key assumption used in all the current literature to date, namely, that the pion field is sufficiently weak for a single pion in intermediate states to adequately describe the solution. While this is certainly true for large bag radii (i.e., the weak-coupling limit), the single-meson assumption cannot be adequate for small R.

In this work, we shall consider the chiral bag model of Refs. 3–5 in a Hartree-type approximation. In the weak-coupling (large-R) limit and equal nucleon and Δ masses (no gluon splitting), the CBM results are recovered exactly. For smaller radii, it is shown that the single-meson approximation very rapidly breaks down for some observables.

As a starting point, we shall use the CBM Hamiltonian as in Refs. 3–5. However, one should remember that this Hamiltonian is the result of linearizing in the pion field the original (highly complicated) nonlinear chiral Lagrangian. For smaller radii this can no longer be expected to be a valid procedure and one would expect both a sizable contribution from pionic self-interactions as well as multipion coupling to the surface. Although consistency requires that these terms be treated to the same order as the others, the complication is substantial and we shall restrict the present treatment to the linear Hamiltonian only. The effect of including the nonlinear parts is currently under study and will be reported separately. However, at the very minimum, the present work allows an assessment of the smallest bag radius which may safely be used in a calculation.

NOTATION AND THEORY

Since several papers on the subject of chiral bags exist in the literature, we shall recall below only such details as are needed to establish continuity and the necessary notation.

A bag theory for baryons which respects PCAC (partial conservation of axial-vector current) is provided by the following Lagrangian density:

$$\mathscr{L}(x)=\left[\frac{i}{2}\bar{q}_a\overleftrightarrow{\partial}q_a-B\right]\theta(R-r)$$
$$+\tfrac{1}{2}(D_\mu\vec{\phi})^2-\tfrac{1}{2}m_0{}^2\phi^2$$
$$-\tfrac{1}{2}\bar{q}_a e^{i\tau\cdot\vec{\phi}\gamma_5/f_0}q_a\delta(r-R)\ . \quad (1)$$

In Eq. (1), q_a is the quark field operator with a being the color index ($a=1,2,3$), the θ function serving to confine the quarks within a radius R. $\vec{\phi}$ denotes the pion field with bare mass m and B is the energy density of the vacuum. For definiteness, the pion field will be allowed to exist both inside and outside the bag. However, the procedure developed in this paper may readily be used for the case of pions strictly outside the bag. For the present, we follow the customary route of approximating the highly complicated covariant derivative $D_\mu\phi$ by $\partial_\mu\phi$, and expanding the exponential term to first order in ϕ. The Hamiltonian corresponding to the linearized version of Eq. (1), after projection onto the space of bare nucleons and Δ bags, is[3-5]

$$H=H_{\rm MIT}+H_\pi+H_{\rm int}\ , \quad (2)$$

where

$$H_{\rm MIT}=m_{0N}N^\dagger N+m_{0\Delta}\Delta^\dagger\Delta\ , \quad (3)$$

$$H_\pi=\tfrac{1}{2}\int d^3r(\dot{\vec{\phi}}\cdot\dot{\vec{\phi}}+\nabla\vec{\phi}\cdot\nabla\vec{\phi}+m^2\vec{\phi}\cdot\vec{\phi})\ , \quad (4)$$

and

$$H_{\rm int}=\frac{i}{2f_0}\int d^2S\,\bar{q}_a\gamma_5\tau_\alpha q_a\phi_\alpha\ . \quad (5)$$

In the above, m_{0N} and $m_{0\Delta}$ are the bare masses of the nucleon and Δ (i.e., the masses in the absence of pions) which are defined by the mass formula[12] below,

$$m_0=\frac{3\omega}{R}+\frac{4}{3}\pi BR^3$$
$$+\frac{0.24\alpha_c}{R}[-9+S(S+1)+3I(I+1)]\ , \quad (6)$$

where α_c is the color coupling constant, S is the total spin, and I is the total isospin of the specific baryon. The last term in Eq. (6) generates a mass splitting between bare nucleon and Δ bags. The subscript α in Eq. (5) is used there and, hereafter, for the three charge states of the pion. Note that γ_5 and τ_α are quark operators. The integration in Eq. (5) is over the bag surface since we have only surface coupling between pions and quarks. In $H_{\rm int}$, the parameter f_0, whose value will turn out to be in the vicinity of the pion decay constant $f_\pi\approx 93$ MeV, will be taken to be a free parameter which, for a given value of bag radius R, is required to yield the measured strength of the asymptotic pion field. f_0 is found to be a slowly increasing function of R.

The analysis of the CBM Hamiltonian [Eq. (2)] proceeds in Refs. 2–5 by expansion of the pion field operator $\vec{\phi}(\vec{r},t)$ in momentum eigenstates. Since plane waves are completely delocalized, they do not constitute a good basis set for describing the localized pion cloud and spherical symmetry of the bag. A more suitable expansion, at least for the bound-state problem, is provided by the following unitarily equivalent expansion [a sum over repeated indices is implied]:

$$\phi_\alpha(\vec{r})=\frac{1}{\sqrt{\pi}}\int_0^\infty\frac{dk}{\sqrt{\omega}}kU_l(kr)Y_{lm}(\hat{r})a_{lm\alpha}(k)$$
$$+\text{H.c.}\ , \quad (7)$$

where $U_l(kr)$ is a complete set of radial functions obeying the free-pion-field equation

$$(\nabla^2+k^2)U=0\ . \quad (8)$$

[Models requiring the pion field to vanish for $r<R$ may be handled by requiring that Eq. (8) be satisfied for $r>R$ and that $r\cdot\nabla\phi=0$ at the surface. It may be readily verified that U is then a linear combination of Hankel functions.] The completeness relations are

$$\frac{2}{\pi}\int_0^\infty dk\,k^2U_l(kr)U_l(kr')=\frac{1}{r^2}\delta(r-r') \quad (9)$$

and

$$\frac{2}{\pi}\int_0^\infty dr\, r^2 U_l(kr)U_l(kr')=\frac{1}{k^2}\delta(k-k') . \tag{10}$$

It is advantageous to make a further unitary transformation on the pion creation and destruction operators,

$$a_{lm\alpha}(k)=\sum_{n=0}^{\infty} F_n(k)a_{nlm\alpha} , \tag{11}$$

where the $F_n(k)$ constitute a complete, orthonormal set of momentum-space wave functions. (It is also possible, but less convenient, to work directly with coordinate-space wave functions since one must then deal with differential equations rather than algebraic ones.) Hence,

$$\int_0^\infty dk\, F_n^*(k)F_{n'}(k)=\delta_{nn'} \tag{12a}$$

and

$$\sum_{n=0}^{\infty} F_n^*(k)F_n(k')=\delta(k-k') . \tag{12b}$$

The pion field operator is, then,

$$\phi_\alpha(\vec{r})=\sum_{n=0}^{\infty}\frac{1}{\sqrt{\pi}}\int_0^\infty \frac{dk}{\sqrt{\omega}}kU_l(kr)F_n(k) \times Y_{lm}(\hat{r})a_{nlm\alpha}+\text{H.c.} \tag{13}$$

Imagine now that one were to take as a trial wave function the most general state consisting of three quarks populating the levels of the bag in an arbitrary way, and consisting of any number of pions populating the pion levels labeled by n. Then, clearly, one would be solving the CBM Hamiltonian exactly—obviously an intractable problem. However, guided by earlier developments in the treatment of intermediate coupling between nucleons and pions, we shall make the so-called Tomonaga approximation[13–16] by restricting the sum over n in Eq. (13) to only one term and, hence, make the following variational ansatz for the physical baryon state $|\bar{\alpha}\rangle$,

$$|\bar{\alpha}\rangle=(\,|\text{quarks}\rangle\,|\text{pions in mode } n=0\rangle)^{J,T} , \tag{14}$$

where the quark wave function is given by

$$q_a(r)=\left[\frac{\omega^2}{4\pi R^3[1-j_0{}^2(\omega)]}\right]^{1/2} \times \begin{bmatrix} j_0(\omega r/R) \\ i\vec{\sigma}\cdot\hat{r}j_1(\omega r/R)\end{bmatrix} v_a . \tag{15}$$

In the above, ω is the usual quark frequency ($\omega=2.04$) for the lowest-lying state and v_a is the spin-isospin wave function. The superscripts on the right denote coupling to the quantum numbers of the baryon, J and T.

The motivation for the ansatz in Eq. (14) is straightforward. In the weak-coupling limit, the motion of pions and quarks will be dynamically decoupled. Furthermore, it will be seen later that the pion wave function collapses to that of first-order perturbation theory in this limit. The present ansatz is just a form of Hartree approximation in which any number of bosons are allowed to populate a single state, that state being determined variationally. The natural tendency for bosons to cluster into the same state motivates the present ansatz, in much the same way as the exclusion principle justifies the nuclear shell model for fermions.

Within a space spanned by the set of states of the form Eq. (14), a reduced Hamiltonian can be defined such that

$$\langle\bar{N}\,|\,H_R\,|\,\bar{N}\rangle=\langle\bar{N}\,|\,H\,|\,\bar{N}\rangle , \tag{16}$$

which is satisfied by

$$H_R=H_{\text{MIT}}+\int_0^\infty dk\,\omega F^2(k)A^\dagger_{m\alpha}A_{m\alpha} +\gamma_0\int_0^\infty dk\,\frac{k}{\sqrt{\omega}}j_1(kR)F(k) \times[\sigma_m(a)\tau_\alpha(a)A_{m\alpha}+\text{H.c.}], \tag{17}$$

where

$$\gamma_0=-\frac{\omega}{(\omega-1)}\frac{1}{4\sqrt{3}\pi f_0 R} . \tag{18}$$

Since only p-wave pions can couple to the quarks in the no-recoil approximation, we have dropped all $l\neq 1$ terms and put $a_{1m\alpha}\equiv A_{m\alpha}$. $F(k)$ is the normalized but otherwise arbitrary wave function of the single state. The quark operators $\sigma_m(a)$ and $\tau_\alpha(a)$ in the interaction part of the Hamiltonian Eq. (17) can cause transitions between Δ and nucleon states, as well as cause diagonal transitions.

To determine $F(k)$, we demand that

$$\frac{\delta}{\delta F(k)}\langle\bar{N}\,|\,H_R\,|\,\bar{N}\rangle=0 , \tag{19}$$

subject to the normalization constraint $\langle\bar{N}\,|\,\bar{N}\rangle=1$. This readily yields a familiar form[14–16]

$$F(k)=\frac{\mathcal{N}kj_1(kR)}{\omega^{1/2}(\omega+\lambda)} , \tag{20}$$

where λ is a free parameter to be varied, being essentially a Lagrange multiplier. $\mathcal{N}$ is a normalization factor determined from $\int dk\, F^2(k)=1$.

Using the above expression for $F(k)$, the reduced Hamiltonian takes the form

$$H_R(\lambda)=H_{\rm MIT}+\left[\frac{L_1(\lambda)}{L_2(\lambda)}-\lambda\right]A^\dagger_{m\alpha}A_{m\alpha}+G_0(\lambda)[\sigma_m(a)\tau_\alpha(a)A_{m\alpha}+\text{H.c.}]\ , \tag{21}$$

where

$$G_0(\lambda)=\gamma_0 L_1(\lambda)/L_2{}^{1/2}(\lambda) \tag{22}$$

and

$$L_n(\lambda)=\int_0^\infty dk\frac{k^2 j_1{}^2(kR)}{\omega(\omega+\lambda)^n}\ . \tag{23}$$

In the reduced space, the physical state of the nucleon or Δ obeys the Schrödinger equation

$$H_R(\lambda)\,|\,\bar\alpha\,\rangle=m_\alpha\,|\,\bar\alpha\,\rangle,\ \ \bar\alpha=\bar N,\bar\Delta\ . \tag{24}$$

The relation between the physical masses m_α and bare masses $m_{0\alpha}$ is given by

$$m_\alpha=m_{0\alpha}-\frac{\langle\,\bar\alpha\,|\,H_{\rm int}\,|\,\alpha\,\rangle}{\langle\,\bar\alpha\,|\,\alpha\,\rangle}\ . \tag{25}$$

Equations (24) and (25) are solved simultaneously by iteration. m_N and m_Δ have been taken to be 938 and 1232 MeV, respectively.

Consider, for the moment, the perturbative solution of Eq. (25) to order $1/f_0{}^2$. Minimizing $\langle\,\bar N\,|\,H_R(\lambda)\,|\,\bar N\,\rangle$, it may be easily shown that λ is zero and, hence, all static quantities (probabilities, self-energy, magnetic moments, etc.) agree exactly to this order with the corresponding quantities calculated perturbatively in the usual way, provided that the N-Δ mass splitting is ignored. This makes the ansatz Eq. (14) plausible. The present scheme in effect treats the spin-isospin degrees of freedom correctly but takes account of the pion wave function in an average way.

Although Eq. (24) is a much simplified reduction from the full theory, it still cannot be solved exactly. In fact, much effort has been made in the past to solve a simpler form of the same equation. We follow Harlow and Jacobsohn[16] by recognizing that H conserves angular momentum and isospin, and therefore expand the physical nucleon state $|\bar N\rangle$ in a quasimeson basis with the correct symmetries,

$$|\,\bar\alpha\,\rangle=\sum_{n=0}^{N}\sum_{L,T}\sum_{s,t}\sum_{\beta}C_\alpha(n,L,T,s,t,\beta)[\,|\,LT(n)\rangle\,|\,st\,\rangle]^{J,\tau}\ . \tag{26}$$

In the above, L and T denote the total orbital angular momentum and isospin of n quasipions, $s=t=\frac{1}{2}$ for the nucleon and $s=t=\frac{3}{2}$ for the Δ, β distinguishes between pionic states with the same n, L, and T, and the superscripts to the right indicate the total spin-isospin (J,τ) of the baryon. N is the maximum number of quasipions, and an exact solution of Eq. (25) is attained for $N=\infty$. The complexity of the algebra for SU(2)$\times$SU(2) limits the practically feasible values of N to be rather small.

With the expansion of the wave function Eq. (26), the problem of solving the Schrödinger equation becomes a matrix eigenvalue problem of small dimension with the off-diagonal matrix elements proportional to

$$\begin{aligned}\langle(n+1)(L's')^{1/2}(T't')^{1/2}\,|\,\sigma_m(a)\tau_\alpha(a)A^\dagger_{m\alpha}\,|\,(Ls)^{1/2}(Tt)^{1/2}(n)\rangle\\ =(-1)^{L'+T'-s-t}[s']^{1/2}[t']^{1/2}[L']^{1/2}[T']^{1/2}W(ss'LL';1J)W(tt'TT';1\tau)\\ \times\langle s't'||\sigma(a)\tau(a)||st\rangle\langle(n+1)L'T'||A^\dagger||LT(n)\rangle\ .\end{aligned} \tag{27}$$

In the above, $[s]=2s+1$, etc. The reduced matrix element of quark operators in the above equation is calculated by writing down the explicit form of the quark wave functions for the nucleons and Δ.[17] The second reduced matrix element can be recognized as a fractional parentage coefficient. Bolsterli's[18] scheme of coherent meson pair states could be usefully introduced here as well. This would enable an extension of the variational subspace with relatively little effort. However, we find the straightforward expansion adequate since, for very small radii, the product ansatz Eq. (14) would not be valid anyway.

All calculated observables in the present scheme can be expressed in terms of a renormalized coupling constant γ related to γ_0 by the relation

$$\gamma_0\langle\,\bar N\,|\,\sigma_m(a)\tau_\alpha(a)\,|\,\bar N\,\rangle=\gamma\langle\,N\,|\,\sigma_m(a)\tau_\alpha(a)\,|\,N\,\rangle\ . \tag{28}$$

On inserting the expression for $|\bar{N}\rangle$ and manipulating suitably, the above gives

$$Z \equiv \frac{\gamma}{\gamma_0} = \frac{2}{5} \sum C^*(nLTst\alpha)C(nLTs't'\beta)[s']^{1/2}[t']^{1/2}W(1s\tfrac{1}{2}L,s'\tfrac{1}{2})W(1t\tfrac{1}{2}T,t'\tfrac{1}{2})\langle s't'||\sigma(a)\tau(a)||st\rangle \ . \tag{29}$$

The coefficients C are the amplitudes for states with the indicated numbers, and are the output of the matrix eigenvalue problem discussed above.

ELECTROMAGNETIC PROPERTIES

The interaction of photons with the chiral quark bag proceeds through coupling with the nucleon current j_N^μ,

$$j_N^\mu = j_Q^\mu + j_\pi^\mu \ , \tag{30}$$

where j_Q^μ is the quark current as calculated from Eq. (15) and j_π^μ the pion current. The latter can be expressed in terms of $F(k)$ and the pion operators $A_{m\alpha}^\dagger$ and $A_{m\alpha}$.

Because the static model is valid only for low momentum transfer, the only electromagnetic properties that have been calculated here are the nucleon magnetic moments and charge radii. In Refs. 4 and 5, the formation for calculating electromagnetic properties in the CBM has been given in ample detail, and so only the results in the mean-field approximation will be given below.

The calculation of all electromagnetic properties requires the evaluation of matrix elements of the current operator Eq. (30) between physical nucleon states. From the Hamiltonian $H(\lambda)$, one may establish the following useful identities:

$$A_{z3}|\bar{N}\rangle = -G_0(\lambda)\left[\left[\frac{L_1(\lambda)}{L_2(\lambda)} - \lambda\right] + H\right]^{-1} \times \sigma_z(a)\tau_3(a)|\bar{N}\rangle \tag{31}$$

and

$$A_{z3}^\dagger|\bar{N}\rangle = -G_0(\lambda)\left[\left[\frac{L_1(\lambda)}{L_2(\lambda)} - \lambda\right] - H\right]^{-1} \times \sigma_z(a)\tau_3(a)|\bar{N}\rangle \ . \tag{32}$$

A rather lengthy calculation, using the above identities, yields the following result for the contribution of the pion charge to the nucleon rms charge radii:

$$\int d^3r\, r^2 j_\pi^0(\vec{r}) = e\langle N|\tau_3|N\rangle H_\lambda(R)G^2(\lambda) \times \left[\frac{25}{(L_1/L_2-\lambda)^2} - \frac{8}{(L_1/L_2-\lambda+\omega_\Delta)^2} - \frac{8}{(L_1/L_2-\lambda)^2-\omega_\Delta{}^2}\right] , \tag{33}$$

where

$$H_\lambda(R) = \frac{4}{3}\int_0^\infty \frac{dk}{k^2}\frac{d}{dk}\left[\frac{kF(k)}{\sqrt{\omega}}\right]\frac{d}{dk}[k\sqrt{\omega}F(k)] \ . \tag{34}$$

$G(\lambda)$ is the renormalized counterpart of $G_0(\lambda)$ [Eq. (22)], defined through

$$\frac{G(\lambda)}{G_0(\lambda)} = Z \tag{35}$$

and

$$\omega_\Delta = m_\Delta - m_N \ . \tag{36}$$

The contribution of the quark charges to the nucleon rms charge radii, calculated from Eq. (15), is

$$\int d^3r\, r^2 j_Q^0(\vec{r}) = eC_N \int d^3r\, r^2[j_0{}^2(\omega r/R) + j_1{}^2(\omega r/R)]\theta(R-r) \ , \tag{37}$$

where C_N is to be chosen such that the total (pion + quark) charge is e for the proton and zero for the neutron.

Turning next to the calculation of nucleon magnetic moments, we first evaluate the pion current contribution by calculating the expectation value of the operator μ_N^π between physical nucleon states, where

$$\mu_N^\pi = -i \lim_{q\to 0} \frac{1}{2} \frac{\vec{\sigma}\times\vec{q}}{\vec{q}\cdot\vec{q}} \int d^3r\, e^{i\vec{q}\cdot\vec{r}} j_\pi(\vec{r}) \ . \tag{38}$$

After expressing the pion current in terms of $F(k)$ and creation/annihilation operators and integrating over angles, the following expression results:

$$\mu_N^\pi = \left[\frac{1}{(1-\lambda L_2/L_1)^2} + \frac{4/25}{[(1-\lambda L_2/L_1)+\omega_\Delta L_2/L_1]^2} + \frac{4/25}{(1-\lambda L_2/L_1)^2-(\omega_\Delta L_2/L_1)^2} \right] \times \frac{100}{9} \frac{\gamma^2}{R^2} \int_0^\infty dk \frac{k^2 j_1{}^2(kR)}{\omega^2(\omega+\lambda)^2} \ . \tag{39}$$

The contribution to the magnetic moments from the quark currents is calculated from

$$\begin{aligned} \mu_N^Q &= \mu_0 \langle \bar{N} \,|\, \bar{q}(a)\sigma_z(a)Q(a)q(a) \,|\, \bar{N} \rangle \\ &= 3\mu \langle \bar{N} \,|\, \bar{q}(3)\sigma_z(3)Q(3)q(3) \,|\, \bar{N} \rangle \ , \end{aligned} \tag{40}$$

where $Q(a)$ is the charge of the ath quark ($a=1,2,3$) and μ_0 is calculated from the quark wave function to be

$$\mu_0 = \frac{R(4\omega-3)}{12\omega(\omega-1)} \ . \tag{41}$$

Inserting the form for $|\bar{N}\rangle$, μ_N^Q may be written as

$$\mu_N^Q = \mu_0 \sum C^*(nLTst\alpha)C(nLTs't'\alpha')M(s't',st,LT) \tag{42}$$

and the only nonzero elements of the matrix elements M are

$$\begin{aligned}
&(1)\ s=t=\tfrac{1}{2},\ s'=t'=\tfrac{1}{2},\ L=0,\ T=0,\ M=\begin{bmatrix} 1 \\ -2/3 \end{bmatrix} ,\\
&(2)\ s=t=\tfrac{1}{2},\ s'=t'=\tfrac{1}{2},\ L=1,\ T=1,\ M=\tfrac{1}{27}\begin{bmatrix} 1 \\ -4 \end{bmatrix} ,\\
&(3)\ s=t=\tfrac{3}{2},\ s'=t'=\tfrac{3}{2},\ L=1,\ T=1,\ M=\tfrac{1}{27}\begin{bmatrix} 20 \\ -5 \end{bmatrix} ,\\
&(4)\ s=t=\tfrac{3}{2},\ s'=t'=\tfrac{3}{2},\ L=2,\ T=1,\ M=\tfrac{1}{9}\begin{bmatrix} -4 \\ 1 \end{bmatrix} ,\\
&(5)\ s=t=\tfrac{3}{2},\ s'=t'=\tfrac{3}{2},\ L=1,\ T=2,\ M=\tfrac{1}{9}\begin{bmatrix} 0 \\ 5 \end{bmatrix} ,\\
&(6)\ s=t=\tfrac{3}{2},\ s'=t'=\tfrac{3}{2},\ L=2,\ T=2,\ M=\tfrac{1}{3}\begin{bmatrix} 0 \\ 1 \end{bmatrix} ,\\
&(7)\ s=t=\tfrac{1}{2},\ s'=t'=\tfrac{3}{2},\ L=1,\ T=1,\ M=\frac{8\sqrt{2}}{27}\begin{bmatrix} 1 \\ -1 \end{bmatrix} ,\\
&(8)\ s=t=\tfrac{3}{2},\ s'=t'=\tfrac{1}{2},\ L=1,\ T=1,\ M=\frac{8\sqrt{2}}{27}\begin{bmatrix} 1 \\ -1 \end{bmatrix} .
\end{aligned} \tag{43}$$

The upper (lower) entry in the above applies to the proton (neutron), respectively. The additional terms, relative to Eq. (5.23) of Ref. 5, arise from the fact that when two or more pions are allowed in intermediate states, they may couple to total allowed orbital angular momentum and isospin equal to 0,1,2.

RESULTS

The model discussed in this paper has all its parameters defined when f_0 and the bag radius R are specified. However, f_0 must be related to the strength of the asymptotic pion field and, therefore, we consider the expectation value of the pion field at a distance r from the center of the bag,

$$\bar{\phi}_\infty(\vec{r}) \equiv \langle \bar{N} \,|\, \phi_\infty(\vec{r}) \,|\, \bar{N} \rangle = \frac{\omega}{\omega-1} \frac{5}{12\pi^2 f R (1-\lambda L_2/L_1)} \langle N \,|\, \tau_\alpha \vec{\sigma}\cdot\hat{r} \,|\, N \rangle \int_0^\infty dk \frac{k^2 j_1(kR) j_1(kr)}{\omega(\omega+\lambda)} . \tag{44}$$

Unfortunately, for $\lambda \neq 0$, the asymptotic behavior of the above integral is not of the form $e^{-m_0 r}$. This can be understood in simple physical terms. Considered as a coordinate-space wave function, Eq. (20) contains a piece which goes as $e^{-m_0 r}$, together with another piece with a longer range behavior if $\lambda \neq 0$. The latter piece serves to "mock up" the effect of continuum states and provides a means for lowering the total energy by including a small admixture of these states. Hence, the nonexponential behavior is purely an artifact arising from the single-mode restriction. This dilemma has also been noted by Bolsterli,[19] who argues that the choice $\lambda = 0$ is necessary in order to cancel the lowest-order effects coming from states orthogonal to the single internal mode. Of course, this choice of λ will not, in general, also minimize the energy. Fortunately, as will be discussed below, the energy is fairly insensitive in the range $0 \leq \lambda \leq \lambda_{\min}$, where $\lambda_{\min}$ minimizes the energy. Therefore, consider for the moment the case $\lambda = 0$. From Eq. (14) it readily follows that the asymptotic strength of the pion field is given by the coupling constant

$$f_{\pi NN}^2 = \frac{\omega}{\omega-1} \frac{5(m_0 R \cosh m_0 R + \sinh m_0 R)}{24\pi (f/m_0) m_0^3 R^3} . \tag{45}$$

The requirement that $f_{\pi NN}^2 \sim 0.081$ then gives f (or f_0) as a function of the bag radius R.

We turn now to a discussion of numerical results.

The self-energy, defined as $\Sigma = m_N - m_{0N}$, is shown in Fig. 1 as a function of the bag radius R. The parameter λ has been set to zero in Fig. 1 and the asymptotic strength of the pion field is given by $f_{\pi NN}^2 \sim 0.081$. The perturbation solution is shown, together with the matrix eigenvalue solution of $H(0) \,|\, \bar{N} \rangle = m_{0N} \,|\, \bar{N} \rangle$ for the maximum number of quasimesons equal to 1, 2, and 3. It can be seen that for moderately large values of R, the convergence of the solutions is rapid. Furthermore, the perturbation solution is quite inadequate for R even as large as 0.9 fm. This has important implications for several calculations existing in the literature. For example, the extraction of α_c in Refs. 4 and 5, and the mass corrections and $B(R)$ in Ref. 9 can be considered adequate only for $r \gtrsim 1$ fm.

At this point, the reader is reminded that the perturbation calculation, which involves a maximum of one meson in intermediate states, is not equivalent to the $n=1$ calculation, even though they agree numerically for very weak coupling. The latter is known as a Tamm-Dancoff approximation[15] and is a diagonalization of the

TABLE I. The nucleon self-energy calculated for two different bag radii and for both $\lambda=0$ as well as $\lambda=\lambda_{\min}$. λ is measured in units of pion mass and the energy in MeV.

	Perturbation	$n=1$		$n=2$		$n=3$	
	$\lambda=0$	$\lambda=0$	$\lambda=2.4$	$\lambda=0$	$\lambda=1.4$	$\lambda=0$	$\lambda=1.1$
$R=1.0$ fm	-186	-202	-210	-228	-232	-234	-237
	$\lambda=0$	$\lambda=0$	$\lambda=4.8$	$\lambda=0$	$\lambda=2.4$	$\lambda=0$	$\lambda=1.5$
$R=0.75$ fm	-386	-487	-518	-585	-602	-623	-633

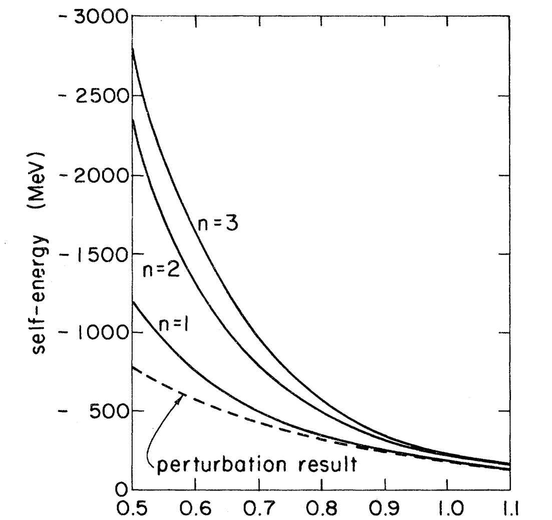

FIG. 1. The nucleon self-energy arising from pion effects as a function of bag radius, as calculated using lowest-order perturbation theory and with $n=1$, 2, and 3 quasipions.

Schrödinger equation with a normalized wave function, whereas the perturbed wave function to second order is well known not to be normalized to unity. As can be seen, the energy of the $n=1$ state is indeed lower than the energy calculated perturbatively to second order in G.

In Table I, we show the results for λ with a fixed value of f_0, both for the case where $\lambda=0$ and when it is allowed to assume the value which minimizes the energy. As can be seen, the energy

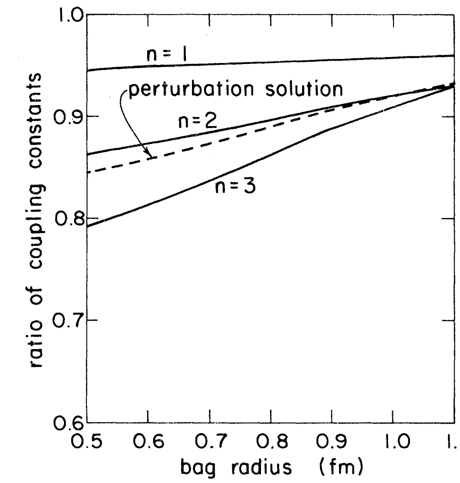

FIG. 3. Z, the ratio of coupling constants as defined in Eq. (29) of the text, calculated in different approximations. For $R\rightarrow\infty$, there is no vertex dressing by pions and Z approaches unity.

shifts are reasonably small in going from $\lambda=0$ to $\lambda=\lambda_{\min}$. Furthermore, $\lambda_{\min}$ decreases with increasing n. We shall, therefore, accept the prescription of Bolsterli[19] and debate this point no more. Thus, in all subsequent figures λ has been set to zero, and f_0 adjusted to give the correct asymptotic pion field—irrespective of the order of approximation.

The physical nucleon is a bare bag only part of the time. Figure 2 shows the probability of finding a bare nucleon as a function of R both as com-

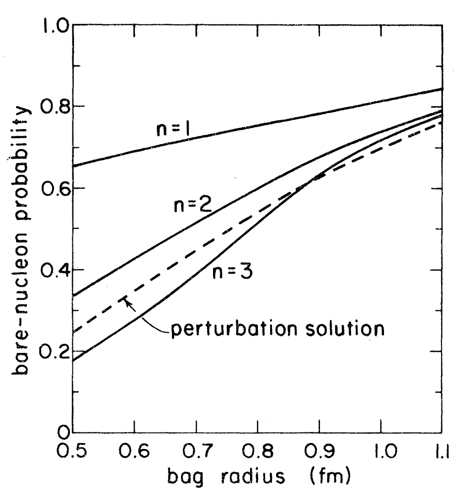

FIG. 2. The probability of a bare-nucleon (MIT) bag as a function of bag radius calculated in different approximations. For $R\rightarrow\infty$, the probability approaches unity and pionic effects vanish.

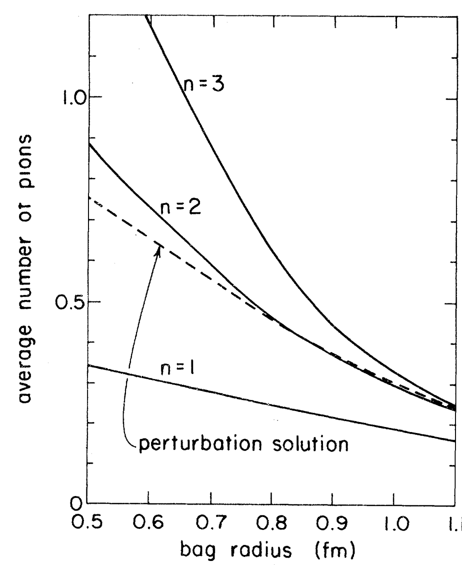

FIG. 4. The average number of pions surrounding the nucleon bag as a function of bag radius, calculated in different approximations. These numbers are well within the bounds given by Dodd *et al.* (Ref. 20).

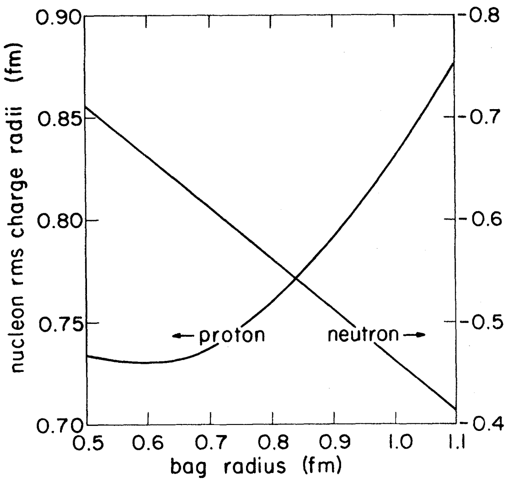

FIG. 5. The proton and neutron rms charge radii as a function of bag radius. The different approximations considered in previous figures all yield the same result as a consequence of renormalizing to the correct asymptotic pion field. (For an explanation, see text.) Center-of-mass corrections are not included. The scale on the left (right) corresponds to the proton (neutron).

puted perturbatively and with $n=1,2,3$ quasi-mesons, respectively. As can be seen, the perturbation solution lies closer to the exact solution than the $n=1$ solution, even though the latter has a lower energy. This feature persists for all observables. In Fig. 3 is shown $Z(R)$ the ratio of coupling constants defined as in Eqs. (28) and (39). For $R\to\infty$, $Z\to 1$, even though the convergence is rather slow and may not be obvious from the

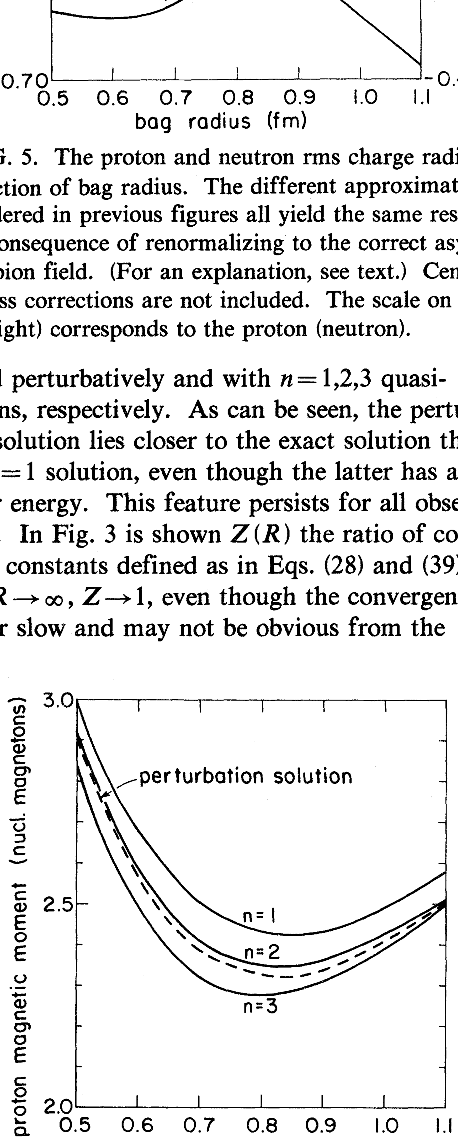

FIG. 6. The proton magnetic moment plotted as a function of bag radius, calculated in different approximations. The differences between the different approximations essentially result from the different quark contributions.

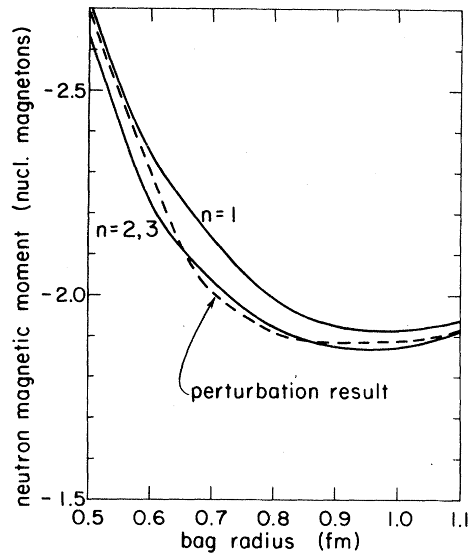

FIG. 7. The neutron magnetic moment plotted as a function of bag radius. Since the $n=2$ and $n=3$ approximations cannot be usefully separated on this scale, they have been drawn together.

graph.

Although the total number of pions surrounding the nucleon fluctuates, the average number $\langle n \rangle$ is well defined and provides an indication of the importance of pionic effects in the chiral bag. This number is illustrated in Fig. 4 as a function of bag radius and the different approximations considered. In every case, the value of $\langle n \rangle$ is well within the bounds given by Dodd *et al.*[20]

The proton and neutron rms charge radii are shown as functions of R in Fig. 5. Because we have renormalized the asymptotic field strength to the measured value for all cases, the perturbation and $n=1,2,3$ results agree identically. *This indicates that, when renormalized, the lowest-order perturbative treatment yields answers for the rms radius well beyond the normally expected range of validity for R.* We have made no corrections for center-of-mass effects, since there is considerable ambiguity as to the proper procedure. Nevertheless, the results in Fig. 5 are in the vicinity of experimental numbers $\langle r^2\rangle_p^{1/2}=0.83$ fm and $|\langle r^2\rangle_N^{1/2}|=0.34$ fm for $R\sim 1$ fm.

Magnetic moments of the proton and neutron are shown in Figs. 6 and 7, respectively. Here, the different approximations are all rather close to each other but are not identical as for the charge radii. The distinction lies in that for the magnetic-moment calculation the core contributes differently in each approximation, whereas for the charge radius calculation the core contribution is fixed from the requirement of definite nucleon charge [see Eq. (37)]. Again, no center-of-mass effects are included, but the calculated numbers are in the vicinity of the experimental values $\mu_p = 2.79$ nuclear magnetons and $\mu_n = -1.91$ nuclear magnetons for $R \sim 1$ fm.

CONCLUSION

When examined in an intermediate-coupling approximation, rather than in lowest-order perturbation theory, the cloudy bag model shows significant deviations for quantities such as the self-energy, probabilities for specific pion states, average number of pions, and the color coupling constant α_c, even for rather large bag radii. However, the charge radii and magnetic moments are essentially the same when calculated either perturbatively or in the mean-field approximation used here, provided that the coupling constant f_0 is adjusted to give the correct asymptotic pion field strength.

For small bag radii, the linearized Hamiltonian used in the chiral bag calculations cannot be a correct reduction of the full Hamiltonian. The mean-field approximation developed in this paper can be extended to the nonlinear terms in the theory to calculate their importance for physical properties. This study is currently in progress.

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