PHYSICAL REVIEW D VOLUME 26, NUMBER 11 1 DECEMBER 1982

Low-energy restoration of parity and maximal symmetry

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(Received 26 March 1982)

The maximal symmetry of fermions of one generation, SU(16), which includes the left-right-symmetric Pati-Salam group, $SU(4)_c \times SU(2)_L \times SU(2)_R$, as a subgroup, allows the possibility of a low-energy ($M_R \sim 100$ GeV) breaking of the left-right symmetry. It is known that such a low-energy restoration of parity can be consistent with weak-interaction phenomenology. We examine different chains of descent of SU(16) that admit a low value of M_R and determine the other intermediate symmetry-breaking mass scales associated with each of these chains. These additional mass scales provide an alternative to the "great desert" expected in some grand unifying models. The contributions of the Higgs fields in the renormalization-group equations are retained and are found to be important.

I. INTRODUCTION

In the recent literature a number of grand unified models have been considered ($[SU(4)]^4$, SO(10), SU(16), etc.) which include the Pati-Salam group $SU(4)_c \times SU(2)_L \times SU(2)_R$ as a subgroup and embody the twin features of left-right symmetry and quark-lepton unification. These models must of necessity include additional mass scales M_x [of $SU(4)_c$ quark-lepton symmetry breaking] and M_R [of $SU(2)_R$ left-right symmetry breaking] and hold the promise of providing an alternative to the "great desert" (between the grand unifying and electroweak mass scales) predicted in models with no intermediate symmetry breaking.

In this paper we study the SU(16) theory[1] with special emphasis towards the possibility of lowering M_R to phenomenologically interesting[2,3] values of 100 − 1000 GeV. SU(16) is the maximal symmetry of fermions belonging to one generation and includes the baryon number B, the lepton number L, and the fermion number F among its generators. In the SU(16) gauge theory these quantum numbers are exactly conserved in the symmetric limit and baryon- and lepton-number nonconservation is tied to the spontaneous symmetry breaking of the theory. SU(16) can descend to the low-energy $SU(3)_c \times U(1)_{EM}$ symmetry through various distinct chains, many of which have already been discussed.[1] The model allows novel decay modes of the proton ($p \to e^- \pi^+ \pi^+$, etc.) and, depending on the chain of descent, the different decay modes may coexist and N-$\bar{N}$ oscillations may be allowed.

The low-energy unbroken symmetry $SU(3)_c \times U(1)_{EM}$ is obtained after the breaking of the $SU(2)_L \times SU(2)_R$ left-right symmetry. The $SU(2)_L$ breaking follows the standard Glashow-Weinberg-Salam pattern and takes place at a scale M_L (~ 100 GeV). The observed parity violation in the weak interactions requires the right-handed weak bosons to be much heavier than M_L. Moreover, in grand unified models, the intermediate mass scales (e.g., M_R) affect the low-energy prediction of $\sin^2\theta_W$. A consistent solution in a grand unified theory with the usually accepted value of $\sin^2\theta_W \approx 0.23$ requires $M_R \geq 10^9$ GeV. However, it has recently been demonstrated by Rizzo and Senjanović[2] that if the neutrinos are Majorana particles, with the right-handed neutrino heavy ($\gtrsim 100$ GeV), then another scenario is also allowed by phenomenology. In this alternative situation M_R can be low (~ 100 GeV) and simultaneously $\sin^2\theta_W$ high (~ 0.27). In this paper we investigate the emergence of this latter possibility from an SU(16) grand unified theory.

The paper is structured as follows. In the next section we give some group-theoretic details of the SU(16) model. In Sec. III, we consider one chain of descent in detail and discuss the predictions of the model. In Sec. IV, we present the results for other chains of descent. We end with a concluding summary. The implications of the model for proton decay are discussed in another paper.[4]

II. SU(16) REPRESENTATIONS AND SUBGROUPS

The fermions of one generation transform as a fundamental representation (16) of SU(16) and all other irreducible representations may be generated by taking its Kronecker products. Some of the SU(16) Kronecker products are listed in Table I. If we denote the fundamental representation by ψ^α ($\equiv \underline{16}$) and its conjugate by ψ_α ($\equiv \underline{\overline{16}}$), then the other representations we find useful in this work are the adjoint $\psi^\alpha_\beta - \frac{1}{16}\delta^\alpha_\beta \psi^\gamma_\gamma$ ($\equiv \underline{255}$), $\psi^{\{\alpha,\beta\}}$ ($\equiv \underline{136}$), $\psi^{[\{\alpha,\beta\},\{\gamma,\delta\}]}$ ($\equiv \underline{5440}$), and the $\underline{18\,240}$ representation. The last representation is generated in the Kronecker product

$$136 \times \overline{136} = 1 + 255 + \underline{18\,240}\ .$$

We will denote just the $\underline{18\,240}$ piece of the above product by $\psi^{\{\alpha,\beta\}}_{\{\gamma,\delta\}}$. { } denotes symmetrization; [ ] denotes antisymmetrization.

Some of the important subgroups of SU(16) are $SU(8) \times SU(8) \times U(1)_F$, $SU(4)_L \times SU(4)_R \times SU(2)_L \times SU(2)_R \times U(1)_F$, and $SU(4)_c \times SU(2)_L \times SU(2)_R$. In Table II, we present the decomposition of SU(16) representations under $SU(8) \times SU(8) \times U(1)_F$, while in Table III we give the corresponding decompositions under $SU(4)_L \times SU(4)_R \times SU(2)_L \times SU(2)_R \times U(1)_F$. We can determine the decomposition of SU(16) representations under the Pati-Salam $SU(4)_c \times SU(2)_L \times SU(2)_R$ subgroup by using Table III in conjunction with Table IV, which lists the decomposition of $SU(4)_L \times SU(4)_R$ under $SU(4)_{L+R}$. Table V lists the decomposition of some useful SU(4) representations under $SU(3) \times U(1)$.

In the subsequent sections we make repeated use of these tables for two main purposes. To choose the Higgs scalars to break the symmetry at any stage ($G_n \to G_{n+1}$, $G_n \supset G_{n+1}$), we have to look for a Higgs field which is a singlet under the surviving symmetry (G_{n+1}) but is a nonsinglet under the higher symmetry (G_n). The tables provide a useful guide for this selection. The other important use of these tables is in the evaluation of the scalar-field contributions to the β functions. This point is discussed in detail in Sec. III and will not be elaborated here. Needless to say, these tables can be of use for other grand unifying groups which include $SU(8) \times SU(8)$ or $SU(4)_L \times SU(4)_R \times SU(2)_L \times SU(2)_R$ as subgroups.

TABLE I. Some SU(16) Kronecker products useful for this work.

$16 \times 16 = 120 + 136$
$16 \times \overline{16} = 1 + 255$
$136 \times \overline{136} = 1 + 255 + 18240$
$136 \times 136 = 3876 + 5440 + 9180$

III. A TYPICAL CHAIN OF DESCENT

In this section we discuss in detail the following chain of symmetry breaking (chain I):

$$\begin{aligned} SU(16) &\underset{M_u}{\longrightarrow} SU(4)_L \times SU(4)_R \times SU(2)_L \times SU(2)_R \times U(1)_F \\ &\underset{M_v}{\longrightarrow} SU(4)_c \times SU(2)_L \times SU(2)_R \\ &\underset{M_c}{\longrightarrow} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\underset{M_R}{\longrightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y \\ &\underset{M_L}{\longrightarrow} SU(3)_c \times U(1)_{EM}\ . \end{aligned}$$

Using Tables II, III, and IV, the Higgs fields required for the spontaneous symmetry breaking at any stage can be determined. In Table VI we have tabulated the different Higgs fields along with their transformation properties.

TABLE II. Reduction of some SU(16) representations under $SU(8)_L \times SU(8)_R \times U(1)_F$.

$16 = (8,1)_1 + (1,\overline{8})_{-1}$
$120 = (8,\overline{8})_0 + (28,1)_2 + (1,\overline{28})_{-2}$
$136 = (8,\overline{8})_0 + (36,1)_2 + (1,\overline{36})_{-2}$
$255 = (1,1)_0 + (63,1)_0 + (1,63)_0 + (8,8)_2 + (\overline{8},\overline{8})_{-2}$
$5440 = (36,\overline{36})_0 + (28,\overline{28})_0 + (168,\overline{8})_2 + (8,\overline{168})_{-2} + (336,1)_4 + (1,\overline{336})_{-4}$
$18\,240 = (63,63)_0 + (1,1)_0 + (1,63)_0 + (63,1)_0 + (1232,1)_0 + (1,1232)_0 + (280,8)_2 + (8,280)_2$ $+ (8,8)_2 + (\overline{8},\overline{280})_{-2} + (\overline{280},\overline{8})_{-2} + (\overline{8},\overline{8})_{-2} + (36,36)_4 + (\overline{36},\overline{36})_{-4}$

TABLE III. Reduction of some SU(16) representations under $SU(4)_L \times SU(4)_R \times SU(2)_L \times SU(2)_R \times U(1)_F$.

$16=(4,1,2,1)_1+(1,\bar{4},1,2)_{-1}$
$120=(4,\bar{4},2,2)_0+(10,1,1,1)_2+(6,1,3,1)_2+(1,\overline{10},1,1,)_{-2}+(1,6,1,3)_{-2}$
$136=(4,\bar{4},2,2)_0+(10,1,3,1)_2+(6,1,1,1)_2+(1,\overline{10},1,3)_{-2}+(1,6,1,1)_{-2}$
$255=(1,1,1,1)_0+(15,1,1,1)_0+(15,1,3,1)_0+(1,1,3,1)_0+(1,15,1,1)_0+(1,15,1,3)_0+(1,1,1,3)_0+(4,4,2,2)_2+(\bar{4},\bar{4},2,2)_{-2}$
$5440=(10,\overline{10},3,3)_0+(10,6,3,1)_0+(6,\overline{10},1,3)_0+(6,6,1,1)_0+(10,\overline{10},1,1)_0+(10,6,1,3)_0+(6,\overline{10},3,1)_0+(6,6,3,3)_0$ $+(20,\bar{4},4,2)_2+(20,\bar{4},2,2)_2+(20',\bar{4},2,2)_2+(\bar{4},\bar{4},2,2)_2+(4,\overline{20},2,4)_{-2}+(4,\overline{20},2,2)_{-2}+(4,\overline{20}',2,2)_{-2}+(4,\bar{4},2,2)_{-2}$ $+(45,1,3,1)_4+(35,1,1,1)_4+(20'',1,1,1)_4+(15,1,3,1)_4+(1,1,1,1)_4+(1,\overline{45},1,3)_{-4}+(1,\overline{35},1,1)_{-4}+(1,20'',1,5)_{-4}$ $+(1,20'',1,1)_{-4}+(1,15,1,3)_{-4}+(1,1,1,1)_{-4}+(20'',1,5,1)_4$
$18\,240=(15,15,1,1)_0+(15,15,1,3)_0+(15,1,1,3)_0+(15,15,3,1)_0+(15,15,3,3)_0+(15,1,3,3)_0+(1,15,3,1)_0+(1,15,3,3)_0$ $+(1,1,3,3)_0+3(1,1,1,1)_0+2(1,15,1,1)_0+3(1,15,1,3)_0+(1,1,1,3)_0+2(15,1,1,1)_0+3(15,1,3,1)_0+(1,1,3,1)_0$ $+(84,1,5,1)_0+(84,1,3,1)_0+(84,1,1,1)_0+(45,1,3,1)_0+(\overline{45},1,3,1)_0+(20'',1,1,1)_0+(15,1,5,1)_0+(1,1,5,1)_0$ $+(1,84,1,5)_0+(1,84,1,3)_0+(1,84,1,1)_0+(1,\overline{45},1,3)_0+(1,45,1,3)_0+(1,20'',1,1)_0+(1,15,1,5)_0+(1,1,1,5)_0$ $+(36,4,2,2)_2+(36,4,4,2)_2+(\overline{20},4,2,2)_2+(4,4,4,2)_2+3(4,4,2,2)_2+(4,36,2,2)_2+(4,36,2,4)_2+(4,\overline{20},2,2)_2$ $+(4,4,2,4)_2+(\bar{4},\overline{36},2,2)_{-2}+(\bar{4},\overline{36},2,4)_{-2}+(\bar{4},20,2,2)_{-2}+(\bar{4},\bar{4},2,4)_{-2}+3(\bar{4},\bar{4},2,2)_{-2}+(\overline{36},\bar{4},2,2)_{-2}$ $+(\overline{36},\bar{4},4,2)_{-2}+(20,\bar{4},2,2)_{-2}+(\bar{4},\bar{4},4,2)_{-2}+(10,10,3,3)_4+(10,6,3,1)_4+(6,10,1,3)_4+(6,6,1,1)_4+(\overline{10},\overline{10},3,3)_{-4}$ $+(\overline{10},6,3,1)_{-4}+(6,\overline{10},1,3)_{-4}+(6,6,1,1)_{-4}$

A. The renormalization-group equations

The different symmetry-breaking mass scales cannot be chosen arbitrarily. The predictions of $\sin^2\theta_W$ and α_s at low energies (calculated using the renormalization-group equations) are dependent on these intermediate mass scales. We use the standard renormalization-group equations[5]

$$\frac{1}{g^2(m)}=\frac{1}{g^2(M)}+2b\ln\frac{M}{m}\ , \tag{1}$$

where for the group SU(N) ($N=2,3,\ldots$)

$$b=-\frac{1}{16\pi^2}\left[\tfrac{11}{3}N-\tfrac{4}{3}\sum_f T(R_f)-\tfrac{1}{6}\sum_s T(R_s)\right]\ . \tag{2}$$

Here $T(R)$ is the quadratic Casimir invariant corresponding to the representation R. The three terms in the right-hand side of Eq. (2) represent the gauge boson, the fermion, and the Higgs scalar contributions, respectively. For the U(1) group there is no non-Abelian gauge-boson coupling and the first term is absent. The only other ingredient in the calculation is the boundary condition, i.e., the relationship among the coupling constants at

TABLE IV. Decomposition of $SU(4)_L \times SU(4)_R$ under $SU(4)_{L+R}$. The three 20-dimensional representations correspond to the Dynkin labels $20=(110)$, $20'=(300)$, and $20''=(020)$.

$4\times\bar{4}=1+15$
$4\times 4=10+6$
$4\times\overline{20}=6+\overline{10}+64$
$4\times\overline{20}'=\overline{10}+\overline{70}$
$4\times 36=10+64+70$
$6\times 6=1+15+20''$
$6\times 10=15+45$
$10\times 10=20''+35+45$
$10\times\overline{10}=1+15+84$
$15\times 15=1+15+15+20''$ $+45+\overline{45}+84$

TABLE V. Decomposition of some SU(4) representations under $SU(3)\times U(1)$.

$4=(3,\sqrt{1/24})+(1,-\sqrt{3/8})$
$6=(3,-\sqrt{1/6})+(\bar{3},\sqrt{1/6})$
$10=(1,-\sqrt{3/2})+(3,-\sqrt{1/6})$ $+(6,\sqrt{1/6})$
$15=(1,0)+(3,\sqrt{2/3})+(\bar{3},-\sqrt{2/3})$ $+(8,0)$

M_u. For the SU(16) theory these are[6]

$$g_{4L}(M_u)=g_{4R}(M_u)=g/\sqrt{2}\ ,$$
$$g_{2L}(M_u)=g_{2R}(M_u)=g/2\ . \qquad (3)$$

Using Eqs. (1), (2), and (3) it is straightforward to calculate the values of α_s ($\equiv g_3{}^2/4\pi$) and $\sin^2\theta_W$ ($\equiv e^2/g_2{}^2$) at low energies. It turns out that the fermionic contributions cancel out in the final expressions. In including the Higgs-field contributions we use the Appelquist-Carrazone decoupling theorem[7] and the extended survival hypothesis of del Aguila and Ibáñez.[8] Consider the chain

$$G_0 \underset{M_1}{\rightarrow} G_1 \underset{M_2}{\rightarrow} G_2 \rightarrow \cdots \underset{M_{n-1}}{\rightarrow} G_{n-1} \underset{M_n}{\rightarrow} G_n\ .$$

Let the Higgs scalar H_i ($\langle H_i \rangle \neq 0$) responsible for the symmetry breaking $G_i \rightarrow G_{i+1}$ be a member of the irreducible representation R_i of G_i. It is, of course, a singlet under G_{i+1}. R_i is contained in some irreducible representation, say R_{i0}, of the group G_0. Then

$$R_{i0} \supset R_{i1} \supset R_{i2} \cdots \supset R_{i(i-1)} \supset R_i\ ,$$

where R_{ij} ($i>j$) is some representation of G_j ($\supset G_i$) which contains the representation R_i of G_i. Now $R_{i(j-1)}$ contains, besides R_{ij}, other irreducible representations of G_j. According to the extended survival hypothesis, these fields which are contained in $R_{i(j-1)}$ but not in R_{ij} acquire masses of the order of M_j and do not contribute to the renormalization-group equations beyond this mass scale.

As a specific example, consider the Higgs scalar (Table VI) which is responsible for the symmetry breaking

$$\mathrm{SU}(4)_L \times \mathrm{SU}(4)_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times U(1)_F \underset{M_v}{\rightarrow} \mathrm{SU}(4)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R\ .$$

The vacuum expectation value, $\langle 0|\chi|0\rangle \neq 0$, is a singlet under $\mathrm{SU}(4)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. From Table III, it can be seen that χ is contained in the representation $(6,6,1,1)_4$ under $\mathrm{SU}(4)_L \times \mathrm{SU}(4)_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_F$, which itself is a member of the SU(16) multiplet $\psi\{^{\alpha,\beta}_{\gamma,\delta}\}$. According to the extended survival hypothesis, all members of the $(6,6,1,1)_4$ multiplet acquire masses of the order of M_v, while all the other fields in $\psi\{^{\alpha,\beta}_{\gamma,\delta}\}$ get masses of the order of M_u. According to the Appelquist-Carrazone decoupling theorem, these latter fields (mass M_u) do not contribute in the renormalization-group equations at all while the members of the $(6,6,1,1)_4$ multiplet contribute only between the energies M_u to M_v and decouple from then onwards. Using Table VI, it is simple to calculate all the Higgs-field contributions in the renormalization-group equations. For convenience, we have listed in Table VII the values of the quadratic Casimir operator for the groups and representations used in this paper.

The low-energy values of $\sin^2\theta_W$ and α_s are calculated in the standard manner. We find it convenient to introduce two variables σ and λ related to $\sin^2\theta_W$ and α_s through

$$\sigma = \frac{24\pi}{11\alpha \ln 10}\left(\tfrac{3}{8} - \sin^2\theta_W\right)\ , \qquad (4a)$$

$$\lambda = \frac{3\pi}{11\alpha \ln 10}\left[1 - \frac{8}{3}\frac{\alpha}{\alpha_s}\right]\ . \qquad (4b)$$

In Table VIII, we have listed the values of σ and λ for different values of $\sin^2\theta_W$ and α_s.

σ and λ are chosen so that they vanish at M_u. Their low-energy values obtained using the renormalization-group equations are functions of the intermediate mass scales. For any mass scale

TABLE VI. The Higgs fields that break the symmetry according to chain I. Note that a singlet under any group is also a singlet under all subgroups. In this table the U(1) quantum numbers are always written as subscripts. The SU(n) groups are written as n with a subscript to denote the chiral nature (if any), e.g., $\mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}$ is written as $3\,2_L 2_R 1_{B-L}$.

Symmetry-breaking mass scales	Higgs field	16	$4_L 4_R 2_L 2_R 1_F$	$4\,2_L 2_R$	$3\,2_L 2_R 1_{B-L}$	$3\,2_L 1_Y$	$3\,1_{EM}$
M_u	ρ	$\psi\{^{\alpha,\beta}_{\gamma,\delta}\}$	$(1,1,1,1)_0$				
M_v	χ	$\psi\{^{\alpha,\beta}_{\gamma,\delta}\}$	$(6,6,1,1)_4$	(1,1,1)			
M_x	ξ	$\underline{255}$	$(15,1,1,1)_0$ $(1,15,1,1)_0$	(15,1,1)	$(1,1,1)_0$		
M_R	Δ_R	$\underline{136}$	$(1,\overline{10},1,3)_{-2}$	$(\overline{10},1,3)$	$(1,1,3)_{\sqrt{3/2}}$	$(1,1)_0$	
M_L	Δ_L	$\overline{136}$	$(\overline{10},1,3,1)_{-2}$	$(\overline{10},3,1)$	$(1,3,1)_{\sqrt{3/2}}$	$(1,3)_{\sqrt{3/5}}$	$(1)_0$
	ϕ		$(\overline{4},4,2,2)_0$	(1,2,2)	$(1,2,2)_0$	$(1,2)_{\pm\sqrt{3/20}}$	$(1)_0$

TABLE VII. Values of the quadratic Casimir operator for different groups and representations found useful in this work. The representations are listed by their dimensionalities.

SU(4) representation	1	4	6	10	15
$T(R)$	0	$\frac{1}{2}$	1	3	4
SU(3) representation	1	3	6		
$T(R)$	0	$\frac{1}{2}$	$\frac{5}{2}$		
SU(2) representation	1	2	3		
$T(R)$	0	$\frac{1}{2}$	2		

M_i we introduce the variable n_i through $M_i=10^{n_i}$ GeV.

B. The results

In terms of the variables introduced in the previous subsection we find for the symmetry breaking of chain I,

$$\begin{aligned}\sigma=&-(6+\tfrac{2}{11}N_\Delta-\tfrac{12}{11}N_\chi-\tfrac{8}{11}N_\xi)n_u\\&+(4-\tfrac{1}{11}N_\Delta-\tfrac{12}{11}N_\chi)n_v\\&+(4+\tfrac{9}{11}N_\Delta-\tfrac{8}{11}N_\xi)n_x\\&+(3-\tfrac{15}{22}N_\Delta)n_R-(5-\tfrac{3}{22}N_\Delta)n_L\ ,\end{aligned}\tag{5}$$

$$\begin{aligned}\lambda=&(6+\tfrac{2}{11}N_\Delta-\tfrac{12}{11}N_\chi-\tfrac{8}{11}N_\xi)n_u\\&-(4-\tfrac{1}{11}N_\Delta-\tfrac{12}{11}N_\chi)n_v+(\tfrac{3}{11}N_\Delta+\tfrac{8}{11}N_\xi)n_x\\&+(1-\tfrac{5}{22}N_\Delta)n_R-(3+\tfrac{7}{22}N_\Delta)n_L\ .\end{aligned}\tag{6}$$

Here N_ξ and N_χ are the number of ξ-type and χ-type fields. Δ_L and ϕ (Δ_R) come from the $\underline{136}$ $(\overline{\underline{136}})$ representation of SU(16). We choose the number of such representations to be equal to each other (N_Δ). In the following calculation we set $N_\xi=N_\chi=1$. We also choose $N_\Delta=1$. This corresponds to one $\underline{136}$ and one $\overline{\underline{136}}$ multiplet of Higgs fields.[9] It has recently been shown[10] that with two such multiplets it is possible to give a natural explanation of the expected masses of the mirror fermions [which have to be introduced in an SU(16) theory to cancel triangle anomalies]. We have taken all Higgs fields to be complex; this introduces an additional factor of 2 in the Higgs-field contributions in Eq. (2).

In our analysis n_L has been set to 2 (i.e., $M_L\sim100$ GeV). Low-energy restoration of parity ($M_R\sim100-1000$ GeV) requires n_R to be between 2 and 3. Adding Eqs. (5) and (6) ($N_\xi=N_\chi=N_\Delta=1$, $n_L=2$),

$$(\sigma+\lambda)=\tfrac{56}{11}n_x+\tfrac{34}{11}n_R-\tfrac{180}{11}\ .\tag{7}$$

From Table VIII it can be seen that for the range of variation of α_s usually entertained, it is a good approximation to choose $\lambda=40$. The increase of σ with decreasing $\sin^2\theta_W$ is more rapid. Using Eq. (7), we can get the following bounds:

$$\begin{aligned}n_R=2:&\ \sin^2\theta_W\le 0.23,0.25,0.27\\&\rightarrow n_x\ge 20.8,19.3,17.7\ ,\\n_R=3:&\ \sin^2\theta_W\le 0.23,0.25,0.27\\&\rightarrow n_x\ge 20.2,18.7,17.1\ .\end{aligned}\tag{8}$$

Note that $n_R=2$ and $\sin^2\theta_W\le0.25$ or $n_R=3$ and $\sin^2\theta_W\le0.23$ requires $M_x>M_{\text{Planck}}$ ($\simeq10^{19}$ GeV) and are hence inadmissible.

Furthermore, since $n_u\ge n_v\ge n_x$, Eq. (6) implies

$$\tfrac{28}{11}n_x+\tfrac{17}{22}n_R-\tfrac{73}{11}\le\lambda\ (\simeq40)\ .\tag{9}$$

The two sides are equal if $n_u=n_v=n_x$. The bounds on n_x that follow from the above relation are

$$\begin{aligned}n_R=2:&\ n_x\le 17.8\ ,\\n_R=3:&\ n_x\le 17.4\ .\end{aligned}\tag{10}$$

Equations (8) and (10) together show that only a small range of M_x is allowed for $\sin^2\theta_W=0.27$ and

TABLE VIII. σ and λ for different values of $\sin^2\theta_W$ and α_s. Note $\alpha(M_W)\simeq\frac{1}{129}$.

$\sin^2\theta_W$	0.23	0.24	0.25	0.26	0.27	0.28
σ	55.7	51.8	48.0	44.2	40.3	36.5
α_s	0.10	0.11	0.12	0.13	0.14	0.15
λ	38.1	39.0	39.7	40.4	41.0	41.4

$\sin^2\theta_W \le 0.25$ ruled out even for $n_R = 3$.

It is clear from the above discussion that in the allowed situations low-energy restoration of parity requires the scale of quark-lepton unification (M_x) to be high. A similar analysis of Eq. (7) with low M_x (i.e., $n_x \sim 4-5$) indicates that the opposite is also true, i.e., low-mass quark-lepton unification goes together with high values of M_R. In passing, we note that in view of the above results, N-$\bar{N}$ oscillations will be suppressed in this chain of descent of SU(16).

In Table IX, we have listed a number of solutions that are allowed by Eqs. (5) and (6). All the solutions, except the last two, have $M_R = 10^2$ or 10^3 GeV. The last two solutions with high M_R are included for the sake of comparison with the low-M_R results.

It is easy to convince oneself that if Higgs-field contributions are neglected, then no consistent solution of Eqs. (5) and (6) can be found with low M_R. In that situation, in place of Eq. (7) one has

$$\sigma + \lambda = 4n_x + 4n_R - 8n_L$$

and

$$n_R = 2: \ \sin^2\theta_W \le 0.23, 0.25, 0.27 \rightarrow n_x \ge 26, 24, 22\ ,$$

$$n_R = 3: \ \sin^2\theta_W \le 0.23, 0.25, 0.27 \rightarrow n_x \ge 25, 23, 21\ .$$

Thus for this chain of descent of SU(16), only when the contributions from the Higgs scalars are retained does one have the possibility of low-mass parity restoration.

IV. OTHER CHAINS OF DESCENT

In the previous section, we have discussed in detail one chain of symmetry breaking of the SU(16) group to the low-energy unbroken $SU(3) \times U(1)_{EM}$ symmetry. In this section we consider a few alternative scenarios.

A. Chain II: The possibility of chiral color

$$SU(16) \underset{M_u}{\rightarrow} SU(4)_L \times SU(4)_R \times SU(2)_L \times SU(2)_R$$

$$\underset{M_x}{\rightarrow} SU(3)_L \times SU(3)_R \times U(1)_L^{B-L}$$

$$\times U(1)_R^{B-L} \times SU(2)_L \times SU(2)_R$$

$$\underset{M_v}{\rightarrow} SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$$

$$\underset{M_R}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\underset{M_L}{\rightarrow} SU(3)_c \times U(1)_{EM}\ .$$

If SU(16) breaking follows the above pattern of descent, then the chiral SU(4) groups are broken first (at M_x) to chiral SU(3) (color) and U(1) ($B-L$) groups. At a lower mass scale M_v, the symmetry is further broken; the axial-vector gauge bosons become massive and only the vectorial $SU(3)_c$ and $U(1)_{B-L}$ pieces survive.

The Higgs fields that break the symmetry in this fashion are exactly the same as in chain I (see Table VI) except that in this case the scalar ξ picks up a vacuum expectation value at an energy scale M_x which is now *higher* than the scale M_v at which the field χ gets its vacuum expectation value. To include the effect of the Higgs scalars in the renormalization-group equations, it is now necessary to know the decomposition of $SU(4)_L \times SU(4)_R$ representations under $SU(3)_L \times SU(3)_R$

TABLE IX. The results for chain I. Higgs-field contributions are included. All masses are in GeV.

Mass M_R	Mass M_x	Mass M_v	Mass M_u	$\sin^2\theta_W$
10^2	5.13×10^{17}	5.13×10^{17}	5.13×10^{17}	0.271
10^3	2.57×10^{17}	2.57×10^{17}	2.57×10^{17}	0.267
10^2	10^{17}	1.50×10^{18}	1.50×10^{18}	0.28
10^2	2.75×10^{17}	2.75×10^{17}	3.98×10^{17}	0.275
10^2	2×10^{17}	5×10^{17}	6.31×10^{17}	0.276
10^3	6.31×10^{16}	6.40×10^{17}	6.40×10^{17}	0.275
10^3	2.66×10^{16}	2.66×10^{16}	10^{17}	0.28
10^3	3.59×10^{16}	2.51×10^{17}	3.98×10^{17}	0.278
10^{10}	2×10^{15}	2×10^{15}	2×10^{15}	0.238
1.12×10^{14}	1.12×10^{14}	1.12×10^{14}	1.12×10^{14}	0.223

$\times U(1)_L^{B-L} \times U(1)_R^{B-L}$. These decompositions can be read off from Table V which lists the SU(3) $\times$U(1) content of the relevant SU(4) representations.

The low-energy values of $\sin^2\theta_W$ and α_s can be calculated in the standard manner. In terms of the variables σ and λ and in the notation introduced in the previous section, we get

$$\begin{aligned}\sigma = & -(6+\tfrac{2}{11}N_\triangle - \tfrac{8}{11}N_\xi - \tfrac{12}{11}N_\chi)n_u \\ & +(8+N_\triangle - \tfrac{8}{11}N_\xi - \tfrac{6}{11}N_\chi)n_x \\ & -\tfrac{3}{11}(N_\triangle + 2N_\chi)n_v + (3-\tfrac{15}{22}N_\triangle)n_R \\ & -(5-\tfrac{3}{22}N_\triangle)n_L \,, \end{aligned} \tag{11}$$

$$\begin{aligned}\lambda = & (6+\tfrac{2}{11}N_\triangle - \tfrac{8}{11}N_\xi - \tfrac{12}{11}N_\chi)n_u \\ & +(\tfrac{5}{11}N_\triangle + \tfrac{6}{11}N_\chi + \tfrac{8}{11}N_\xi)n_x \\ & -(4+\tfrac{1}{11}N_\triangle - \tfrac{6}{11}N_x)n_v \\ & +(1-\tfrac{5}{22}N_\triangle)n_R - (3+\tfrac{7}{22}N_\triangle)n_L \,. \end{aligned} \tag{12}$$

As a consistency check we note that chain I and chain II become identical if one chooses $M_x = M_v$ and Eqs. (11) and (12) become identical to Eqs. (5) and (6), respectively, in this limit.

As discussed already, we are interested in the case where $N_\xi = N_\chi = N_\triangle = 1$ and $N_L = 2$. Combining (11) and (12) we find that in this situation

$$\sigma + \lambda = \tfrac{104}{11}n_x - \tfrac{48}{11}n_v + \tfrac{34}{11}n_R - \tfrac{180}{11} \,.$$

Since for this chain $n_x \geq n_v$, we note that the lower bounds on n_x that are obtained by setting $n_x = n_v$ are exactly the same as those obtained for chain I [using Eq. (7)]. In particular, N-$\bar{N}$ oscillations are also suppressed in this chain. The upper bounds on n_x obtained by setting $n_u = n_v = n_x$ in chain I [see Eq. (10)] become bounds on n_v for this chain.

In Table X we have presented various solutions that are allowed in this chain of descent of SU(16). The first two and the last two solutions of Table IX (chain I) which have $n_u = n_v = n_x$ are also solutions for this chain, since in that limit chains I and II are identical. We have included one example of low-energy parity restoration that is allowed in this chain when Higgs-field contributions are ignored. Note that in this case $M_u = M_v$ is very high as is $\sin^2\theta_W$.

B. Chain III: The possibility of $M_{W_R^\pm} \gg M_{Z_2}$

$$\begin{aligned} SU(16) & \underset{M_u}{\rightarrow} SU(4)_L \times SU(4)_R \times SU(2)_L \times SU(2)_R \\ & \underset{M_v}{\rightarrow} SU(4)_c \times SU(2)_L \times SU(2)_R \\ & \underset{M_x}{\rightarrow} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ & \underset{M_{R^+}}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\ & \underset{M_{R^0}}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y \\ & \underset{M_L}{\rightarrow} SU(3)_c \times U(1)_{EM} \,. \end{aligned}$$

This chain of symmetry breaking is somewhat similar to chain I discussed in Sec. III. The only difference is that the $SU(2)_R$ symmetry breaking takes place in two stages. At M_{R^+}, $SU(2)_R$ is broken to $U(1)_R$; the charged right-handed vector bosons $W_R^\pm$ pick up masses of this order. $U(1)_R$ is broken at a lower mass scale M_{R^0} (100–1000 GeV) which is the mass scale of the second neutral-vector boson Z_2. $M_{R^+} \gg M_L$ also implies $g_R > g_L$. (In the context of SO(10) grand unification such a situation has been investigated by Rajpoot.[11]) This chain of descent thus leads to a left-

TABLE X. The results for chain II. The last entry is a solution where Higgs-field contributions have been dropped. Masses in GeV.

Mass M_R	Mass M_v	Mass M_x	Mass M_u	$\sin^2\theta_W$
10^2	10^{15}	1.36×10^{16}	1.36×10^{16}	0.279
10^2	2.75×10^{17}	2.75×10^{17}	3.98×10^{17}	0.275
10^2	10^{15}	1.26×10^{16}	1.41×10^{16}	0.28
10^3	2.66×10^{16}	2.66×10^{16}	10^{17}	0.28
10^3	10^{14}	2.66×10^{15}	2.66×10^{15}	0.278
10^3	10^{15}	7.94×10^{15}	1.12×10^{16}	0.278
10^3	5.62×10^{17}	10^{19}	10^{19}	0.278

right asymmetric ($g_L \neq g_R$) theory with the charged right-handed vector bosons much heavier than the corresponding left-handed ones and the two neutral-vector bosons (which have masses of the same order). It has been found[3] that such a theory is also in agreement with all neutral-current data with $\sin^2\theta_W$ in the range 0.23–0.28.

The Higgs scalars required for this symmetry-breaking chain is the same as in chain I, except that one needs an additional scalar field for the symmetry breaking at M_{R^+}. This breaking can take place through the Higgs field σ_R ($\subset \underline{255}$) which transforms as $(1,1,1,3)_0$ under $\mathrm{SU}(4)_L \times \mathrm{SU}(4)_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_F$. To maintain left-right symmetry in the unbroken Lagrangian, we also include a field σ_L ($\subset \underline{255}$) which transforms as $(1,1,3,1)_0$.

Using the renormalization-group equations we get

$$\begin{aligned}\sigma = &-(6-\tfrac{12}{11}N_\chi-\tfrac{8}{11}N_\xi+\tfrac{2}{11}N_\triangle+\tfrac{2}{11}N_\sigma)n_u\\&+(4-\tfrac{12}{11}N_\chi-\tfrac{1}{11}N_\triangle)n_v+(4+\tfrac{9}{11}N_\triangle-\tfrac{8}{11}N_\xi)n_x\\&+(3-\tfrac{9}{22}N_\triangle-\tfrac{3}{11}N_\sigma)n_{R^+}-\tfrac{3}{11}N_\triangle n_{R^0}\\&-(5-\tfrac{3}{22}N_\triangle-\tfrac{5}{11}N_\sigma)n_L\ ,\end{aligned} \tag{13}$$

$$\begin{aligned}\lambda = &(6-\tfrac{12}{11}N_\chi-\tfrac{8}{11}N_\xi+\tfrac{2}{11}N_\triangle+\tfrac{2}{11}N_\sigma)n_u\\&-(4-\tfrac{12}{11}N_\chi-\tfrac{1}{11}N_\triangle)n_v+(\tfrac{3}{11}N_\triangle+\tfrac{8}{11}N_\xi)n_x\\&+(1-\tfrac{3}{22}N_\triangle-\tfrac{1}{11}N_\sigma)n_{R^+}-\tfrac{1}{11}N_\triangle n_{R^0}\\&-(3+\tfrac{7}{22}N_\triangle+\tfrac{1}{11}N_\sigma)n_L\ .\end{aligned} \tag{14}$$

In the case that we examine in detail, $N_\chi = N_\xi = N_\triangle = N_\sigma = 1$ and $n_L = 2$ (i.e., $M_L \sim 100$ GeV), we get from Eqs. (13) and (14)

$$\sigma+\lambda=\tfrac{56}{11}n_x+\tfrac{34}{11}n_{R^+}-\tfrac{4}{11}n_{R^0}-\tfrac{172}{11}\ , \tag{15}$$

$$\lambda=\tfrac{50}{11}n_u-\tfrac{31}{11}n_v+\tfrac{17}{22}n_{R^+}+n_x-\tfrac{1}{11}n_{R^0}-\tfrac{75}{11}\ . \tag{16}$$

Using $n_x \geq n_{R^+}$, we can set the following lower bounds on n_x from Eq. (15):

$$n_{R^0}=2,\ \sin^2\theta_W \leq 0.23, 0.25, 0.27 \rightarrow n_x \geq 13.8, 12.8, 11.8\ . \tag{17}$$

On the other hand, from Eq. (16), using $n_u \geq n_v \geq n_x \geq n_{R^+}$ and setting $n_{R^0}=2$, we get

$$n_{R^0}=2,\ \sin^2\theta_W \leq 0.23, 0.25, 0.27 \rightarrow n_x \leq 13.29, 13.69, 14.08\ . \tag{18}$$

Inequalities (17) and (18) cannot be simultaneously satisfied for $\sin^2\theta_W \leq 0.23$. In the allowed cases, M_x turns out to be very large ($\sim 10^{13}$ GeV) so that N-$\bar{N}$ oscillations are again suppressed.

In Table XI, some of the solutions allowed within this chain of symmetry breaking are exhibited. Note that the additional flexibility in this chain makes it possible to accommodate as low value of $\sin^2\theta_W$ as 0.235. The unification mass M_u is also comparatively lower in this case.

We do not present any results with the Higgs contribution ignored. This is because in that situation the additional mass scale M_{R^0} drops out from Eqs. (13) and (14) (its coefficients are proportional to $N_\triangle$). The equations for this chain are then identical to those of chain I and, as discussed in Sec. III, do not allow any solution with low M_R.

An alternative situation with $M_{R^+} > M_x$ can be envisaged. We do not discuss this case except to point out that exactly the same bounds set by (17) and (18) apply in that case to the mass scale M_{R^+}.

A variant of chain II with $M_{R^+} \neq M_{R^0}$ can also be considered. The results are qualitatively the same and we do not present them here.

V. SUMMARY AND CONCLUSIONS

We have examined the possibility of a low-mass restoration of parity in the framework of the

TABLE XI. Results for chain III. Masses in GeV.

Mass M_{R^0}	Mass M_{R^+}	Mass M_x	Mass M_v	Mass M_u	$\sin^2\theta_W$
10^2	2.68×10^{13}	2.68×10^{13}	2.68×10^{13}	2.68×10^{13}	0.235
10^3	2.85×10^{13}	2.85×10^{13}	2.85×10^{13}	2.85×10^{13}	0.235
10^2	10^{10}	2.51×10^{14}	2.51×10^{14}	2.51×10^{14}	0.25
10^2	10^3	2.42×10^{16}	2.42×10^{16}	2.42×10^{16}	0.28
10^2	10^{12}	5×10^{12}	10^{13}	3.68×10^{13}	0.257
10^2	10^{12}	10^{12}	7.85×10^{14}	7.85×10^{14}	0.267

SU(16) model of grand unification. It has been noted recently that such a low-energy breaking of left-right symmetry is consistent with present data.[2,3] If in the upcoming experiments at LEP, ISABELLE, or the $\bar{p}p$ collider, any evidence of a low-mass W_R is seen, then that would rule out models of grand unification which cannot accommodate left-right symmetry, e.g., SU(5). One would still have to distinguish between the different models which *can* allow such a situation, e.g., SO(10), SU(16), and larger groups. In this paper we have presented the detailed predictions of SU(16) for several chains of symmetry breaking. The implications of these results for baryon-number violation will be discussed in a companion paper.

We find many different allowed scenarios including the possibility of a two-step breaking of the right-handed symmetry. In this latter situation the charged right-handed vector bosons are superheavy while there are two light neutral-vector bosons. Low-energy parity restoration in this case would thus be in the neutral-current sector only.

As a special case of our results we can obtain the predictions of the SO(10) model when it is first broken to the Pati-Salam $SU(4)_c \times SU(2)_L \times SU(2)_R$. These correspond to the results obtained by setting $M_u = M_v$ in our calculations.

In our calculations we have retained the contributions in the renormalization-group equations coming from the Higgs fields. We find that these contributions are significant and important.

Note added. During the completion of this work we received a paper by R. N. Mohapatra and M. Popović [Phys. Rev. D 25, 3012 (1982)] in which related ideas have been examined. Similar investigations have also been carried out by A. Mohanty and J. C. Pati at the University of Maryland (private communication from J. C. Pati).

ACKNOWLEDGMENTS

This work was begun in collaboration with J. C. Pati and S. Rajpoot and we are grateful for their help. A. R. thanks Abhijit Majumdar for encouragement. This research was supported by the University Grants Commission, India.

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