Electromagnetic properties of Majorana neutrinos

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The electromagnetic properties of Majorana neutrinos are studied in general terms, with a careful discussion of the difference between the cases of Majorana and Dirac neutrinos. Some peculiarities associated with the Majorana character of the neutrinos are noted; for example, it is shown that for two Majorana neutrinos with the same *CP* parity their transition magnetic moment is of the type $\sigma_{\mu\nu}\gamma_5$ and not $\sigma_{\mu\nu}$ in contrast to the situation for the diagonal magnetic moment of Dirac neutrinos (or charged leptons). We also indicate how the electromagnetic form factors in the Majorana case can be obtained from those calculated as if the neutrinos were Dirac particles.

The advent of grand unified theories[1] that unify the weak, electromagnetic, and strong interactions, and the recent experimental reports[2] that neutrinos have finite masses in the cosmologically significant range, have motivated much interest in the cosmological implications of massive neutrinos. In particular, the effects of the radiative decay mode $\nu_1 \to \nu_2 + \gamma$ on a variety of astrophysical problems have been widely studied.[3] On the theoretical side, several calculations of the rates for $\nu_1 \to \nu_2 + \gamma$ in specific models exist in the literature.[4,5] In the simplest cases, these calculations are very similar to the calculations of the rates for $\mu \to e + \gamma$. In a recent paper by Pal and Wolfenstein[6] several models for the $\nu_1 \to \nu_2 + \gamma$ process are discussed, distinguishing the cases in which ν_1 and ν_2 are Dirac or Majorana neutrinos. Our purpose in the present work is to discuss the differences in these two cases without the adoption of specific models. Among our results we find that for Majorana neutrinos with the same *CP* parity, the transition moment is of the type $\sigma_{\mu\nu}\gamma_5$ and not $\sigma_{\mu\nu}$. This is opposite to the well known result for the magnetic moment of Dirac neutrinos and charged leptons and follows from general properties of the electromagnetic vertex. We also indicate how the mechanics of a specific calculation operate to yield this result, and show how the form factors in the Majorana case can be obtained from the form factors calculated as if the neutrinos were Dirac particles, for which general formulas exist in the literature.[5]

Model-independent considerations on the process $\mu \to e + \gamma$, which are also applicable to the case of Dirac neutrinos, but not to the case of Majorana neutrinos, have been previously discussed by Tung.[7] Although our interest is in the latter case, we first review the case of Dirac neutrinos. In standard notation the electromagnetic form factors of the neutrinos are defined by

$$\langle \nu_2(p_2) | J_\mu^{(\mathrm{EM})} | \nu_1(p_1) \rangle = \bar{u}(p_2)\Gamma_\mu(p_2,p_1)u(p_1) \ , \quad (1)$$

where, using gauge invariance and the Dirac equation for the spinors, the most general form of Γ_μ is

$$\Gamma_\mu(p_2,p_1) = (q^2\gamma_\mu - q_\mu \not{q})(F + f\gamma_5) + i\sigma_{\mu\nu}q^\nu(G + g\gamma_5) \ . \quad (2)$$

Here

$$q = p_1 - p_2$$

is the momentum of the photon. For a real photon only the terms involving G and g contribute. The Hermiticity of $J^{(\mathrm{EM})}$ and, if applicable, *CP* invariance imply certain reality conditions on the form factors which we now discuss. Let us consider the off-diagonal case $\nu_1 \neq \nu_2$ first. In this case it is easy to realize that Hermiticity by itself does not imply any restriction of the form factors. However, the assumption of *CP* invariance combined with Hermiticity imply some reality conditions. *CP* invariance implies that the fundamental Lagrangian is invariant under the transformation

$$\nu_{iL} \to \eta_i \nu_{iR}^c \ , \quad \nu_{iR} \to \eta_i \nu_{iL}^c \quad (i = 1,2) \ , \quad (3)$$

where, in the Dirac representation for the γ matrices,

$$\nu_i^c \equiv i\gamma_2 \nu_i^* \ . \quad (4)$$

In Eq. (3) it is understood that all other fields must transform appropriately; in particular for the photon $A_\mu \to -A_\mu$, which implies

$$J_\mu^{(\mathrm{EM})} \to -J_\mu^{(\mathrm{EM})} . \tag{5}$$

The transformation properties in Eqs. (3) and (5) can now be used to show that if the Lagrangian is CP-invariant, then F, f, G, and g must be relatively real; i.e.,

$$\left.\begin{matrix} F^* \\ f^* \\ G^* \\ g^* \end{matrix}\right\} = \eta_1 \eta_2^* \left\{\begin{matrix} F, \\ f, \\ G, \\ g, \end{matrix}\right. \tag{6}$$

where the η_i are defined in Eq. (3). In the diagonal case $\nu_1 = \nu_2$ there are more restrictions. In this case the most general form of Γ_μ is

$$\Gamma_\mu(p_2,p_1) = \gamma_\mu F + (q^2\gamma_\mu - q_\mu \not{q})\gamma_5 f + i\sigma_{\mu\nu}q^\nu(G+g\gamma_5) , \tag{7}$$

and the transformation properties in Eqs. (3) and (5) imply that

$$g = 0 \tag{8}$$

if CP invariance holds. In addition, Hermiticity by itself implies that F, f, and G are real:

$$\mathrm{Im}F = \mathrm{Im}f = \mathrm{Im}G = 0 . \tag{9}$$

The above discussion is merely intended as a brief review of well known results for Dirac neutrinos (and charged leptons). We now turn to our main purpose in the present work which is the extension of this discussion to the case of Majorana neutrinos. Before proceeding, it is appropriate to say a few words about our phase convention. In general a Majorana neutrino field satisfies

$$\nu^c = K\nu , \tag{10}$$

where ν^c is defined in Eq. (4) and K is a phase. For several neutrino flavors, ν in Eq. (10) can be interpreted as a column vector and K as a diagonal unitary matrix (in the basis of mass eigenstates). A redefinition of the phases of the left-handed components ν_L of the fields is equivalent to a redefinition of K. Therefore, without loss of generality, the phases of the ν_L can be chosen such that $K=1$, and the Majorana condition becomes

$$\nu^c = \nu \tag{11}$$

for all the neutrino flavors. Equation (11), together with a phase convention for the charged lepton fields, implies a convention for the Kobayashi-Maskawa matrix in the lepton sector which, in general, contains phases that break CP invariance.[8] Before deciding whether CP invariance is broken or not, we must discuss the transformation properties of Majorana neutrinos under CP. In the case of Majorana neutrinos the transformation in Eq. (3) for Dirac neutrinos must be replaced by

$$\begin{aligned} \nu_{iL} &\to \eta_i \nu_{iR} , \\ \nu_{iR} &\to \eta_i \nu_{iL} , \end{aligned} \tag{12}$$

since $\nu^c = \nu$ according to (11). Further, the Majorana condition $\nu^c = \nu$ restricts η_i to be real so that

$$\eta_i = \pm 1 . \tag{13}$$

Equation (13) can also be deduced by requiring the invariance of the mass terms $\bar{\nu}_i \nu_i = \nu_i^T (i\gamma_2\gamma_0)\nu_i$ under the transformation in Eq. (12). As with Eq. (3), Eq. (12) must be supplemented with the appropriate transformation of all the other fields. A given theory is CP invariant if the fundamental Lagrangian is unchanged under the transformation in Eq. (12) for some set of η_i. Finally we mention that the plane wave decomposition of the Majorana field is

$$\nu = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3p}{2E} \sum_s [a(p,s)u(p,s)e^{-ip\cdot x} + a^*(p,s)v(p,s)e^{ip\cdot x}] , \tag{14}$$

where the convention in Eq. (11) gives

$$v(p,s) = u^c(p,s) \equiv i\gamma_2 u^*(p,s) . \tag{15}$$

We are now in a position to extend the previous discussion to the case of Majorana neutrinos. We consider the off-diagonal case $\nu_1 \neq \nu_2$ first. The electromagnetic form factors are defined by equations identical to Eqs. (1) and (2):

$$\langle \nu_2(p_2) | J_\mu^{(\mathrm{EM})} | \nu_1(p_1) \rangle = \bar{u}(p_2)\Gamma_\mu(p_2,p_1)u(p_1) , \tag{16}$$

where

$$\Gamma_\mu(p_2,p_1) = (q^2\gamma_\mu - q_\mu \not{q})(F' + f'\gamma_5) + i\sigma_{\mu\nu}q^\nu(G'+g'\gamma_5) . \tag{17}$$

However, in contrast to the situation in the Dirac case, Hermiticity and CP invariance can be used independently to restrict the form factors. Hermiticity implies

$$\langle \nu_2(p_2) | J_\mu^{(\mathrm{EM})} | \nu_1(p_1) \rangle = \langle \nu_1(p_1) | J_\mu^{(\mathrm{EM})} | \nu_2(p_2) \rangle^* .$$

The left-hand side of this equation is given by Eqs. (14) and (15) while the right-hand side can be obtained using the substitution rule.[9] In this fashion we obtain

$$\bar{u}(p_2)\Gamma_\mu(p_2,p_1)u(p_1) = -(\bar{v}(p_2)\Gamma_\mu(-p_2,-p_1)v(p_1))^* ,$$

and using Eq. (15) this implies that F' and G' are imaginary while f' and g' are real:

$$\mathrm{Re}\, F' = \mathrm{Re}\, G' = \mathrm{Im}\, f' = \mathrm{Im}\, g' = 0 . \tag{18}$$

The assumption of CP invariance yields further conditions. Using the transformation properties in Eqs. (12), and (4), we can deduce that CP invariance implies

$$\bar{u}(p_2)\Gamma_\mu(p_2,p_1)u(p_1) = (-1)\eta_1\eta_2\bar{u}(p_2)\widetilde{\Gamma}_\mu(p_2,p_1)u(p_1) , \tag{19}$$

where $\widetilde{\Gamma}_\mu$ is obtained from Γ_μ by making the replacement $\gamma_5 \rightarrow -\gamma_5$. Equation (19) gives two possibilities:

(I) $\eta_1\eta_2 = +1$. In this case, Eq. (19) implies

$$F' = G' = 0 . \tag{20}$$

(II) $\eta_1\eta_2 = -1$. In this case, Eq. (19) implies

$$f' = g' = 0 . \tag{21}$$

It is interesting to notice the contrast between this and the situation in the Dirac case. In the present case, if the initial and final neutrinos have the same CP parity $\eta_1 = \eta_2$, then, as Eq. (20) indicates, the transition magnetic moment is of the type $\sigma_{\mu\nu}\gamma_5$ instead of $\sigma_{\mu\nu}$. The result is the opposite in the diagonal Dirac case, for which, in analogy with the off-diagonal Majorana case, CP invariance yields independent conditions on the form factors and the CP parity of the initial and final neutrino is the same. Also notice that in the off-diagonal Dirac case, CP invariance does not forbid the simultaneous presence of the $\sigma_{\mu\nu}$ and $\sigma_{\mu\nu}\gamma_5$ terms, in contrast to the situation in the Majorana case. In the off-diagonal Dirac case the simultaneous presence of the $\sigma_{\mu\nu}$ and $\sigma_{\mu\nu}\gamma_5$ terms is forbidden in a P-invariant theory, in which the P transformation property of neutrinos is precisely analogous to Eq. (12). However, in that case if the P parities are equal (opposite), the $\sigma_{\mu\nu}\gamma_5$ ($\sigma_{\mu\nu}$) term is absent, opposite to Eqs. (20) and (21). The difference in the two cases originates in the C transformation property of $J^{(\mathrm{EM})}$.

Let us now consider the diagonal Majorana case. In analogy with Eq. (7) we can define in this case

$$\langle \nu(p_2) | J_\mu^{(\mathrm{EM})} | \nu(p_1) \rangle = \bar{u}(p_2)\Gamma_\mu(p_2,p_1)u(p_1) , \tag{22}$$

where

$$\Gamma_\mu(p_2,p_1) = \gamma_\mu F' + (q^2\gamma_\mu - q_\mu \not{q})\gamma_5 f' + i\sigma_{\mu\nu}q^\nu(G' + g'\gamma_5) . \tag{23}$$

However, there is an alternative way to evaluate the matrix element. The Majorana condition $\nu^c = \nu$ [or equivalently, the expansion in Eq. (14)] and the substitution rule imply that the matrix element is also given by

$$\langle \nu(p_2) | J_\mu^{(\mathrm{EM})} | \nu(p_1) \rangle = -\bar{v}(p_1)\Gamma_\mu(-p_1,-p_2)v(p_2) . \tag{24}$$

The minus sign in Eq. (24) is just a reflection of the Pauli exclusion principle and follows from the Feynman rules when two external fermion lines are interchanged. Using Eq. (15), the consistency of Eqs. (22) and (24) then requires

$$F' = G' = g' = 0 . \tag{25}$$

Thus, in particular, a Majorana neutrino can have neither a magnetic moment nor an electric dipole moment. This statement has appeared repeatedly in the recent literature,[10] but it was not clear to us under what conditions it holds valid. In the above proof no assumption was made regarding CP invariance; thus CP-invariance violation does not invalidate the statement. However, we have used field theory concepts, such as the substitution rule and the anticommutativity of fermion fields, which imply CPT invariance. Although it does not constitute a general proof, this argument suggests that the existence of a magnetic moment for a Majorana neutrino is a signal of CPT nonconservation. A general proof is provided in the Appendix.

It is interesting to notice that Eqs. (25) and (21) allow us to recover the well-known results for a Dirac neutrino, Eqs. (8) and (9), when it is regarded as two degenerate Majorana neutrinos of opposite CP. In order to demonstrate this fact, let us consider the matrix element

$$\langle \nu | J_\mu^{(\mathrm{EM})} | \nu \rangle = \tfrac{1}{2}[\langle \nu_1 | J_\mu^{(\mathrm{EM})} | \nu_1 \rangle + \langle \nu_2 | J_\mu^{(\mathrm{EM})} | \nu_2 \rangle + i\langle \nu_1 | J_\mu^{(\mathrm{EM})} | \nu_2 \rangle - i\langle \nu_2 | J_\mu^{(\mathrm{EM})} | \nu_1 \rangle \,, \quad (26)$$

where ν is a Dirac neutrino, and ν_1 and ν_2 are two Majorana neutrinos of opposite CP defined by

$$\nu_1 = \frac{\nu + \nu^c}{\sqrt{2}} \,, \quad \nu_2 = \frac{\nu - \nu^c}{i\sqrt{2}} \,.$$

[We choose the phase of ν such that in Eq. (3), $\eta = 1$.] The $\nu_1 - \nu_2$ degeneracy implies that, regarded as a 2×2 matrix in $\nu_1 - \nu_2$ space, $J^{(\mathrm{EM})}$ must satisfy

$$R^{-1} J_\mu^{(\mathrm{EM})} R = J_\mu^{(\mathrm{EM})} \,, \quad (27)$$

where R is a 2×2 rotation matrix. This symmetry corresponds to the U(1) invariance associated with Dirac neutrinos. Equation (27) implies that $J^{(\mathrm{EM})}$ is of the form

$$J_\mu = a_\mu I + b_\mu \epsilon \,, \quad (28)$$

where I is the 2×2 unit matrix and

$$\epsilon = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} .$$

Equation (28) yields

$$\langle \nu_2 | J_\mu^{(\mathrm{EM})} | \nu_2 \rangle = \langle \nu_1 | J_\mu^{(\mathrm{EM})} | \nu_1 \rangle \,,$$
$$\langle \nu_1 | J_\mu^{(\mathrm{EM})} | \nu_2 \rangle = -\langle \nu_2 | J_\mu^{(\mathrm{EM})} | \nu_1 \rangle \,,$$

so that Eq. (26) becomes

$$\langle \nu | J_\mu^{(\mathrm{EM})} | \nu \rangle = \langle \nu_1 | J_\mu^{(\mathrm{EM})} | \nu_1 \rangle - i\langle \nu_2 | J_\mu^{(\mathrm{EM})} | \nu_1 \rangle$$

which, together with Eqs. (25) and (21), implies Eqs. (8) and (9).

It is useful to understand the mechanics that operate in a specific calculation to yield the results we have discussed. Let us consider the calculation of the form factors in

$$\langle \nu_2(p_2) | J_\mu^{(\mathrm{EM})} | \nu_1(p_1) \rangle \equiv \bar{u}(p_2)\Gamma_\mu^D(p_2,p_1)u(p_1)$$

for Dirac neutrinos (hence the superscript in Γ_μ^D) in the standard SU(2)$\times$U(1) model with the addition of right-handed single neutrinos. The most general form of Γ_μ^D is

$$\Gamma_\mu^D(p_2,p_1) = (q^2\gamma_\mu - q_\mu \not{q})(F_D + f_D\gamma_5) + i\sigma_{\mu\nu}q^\nu(G_D + g_D\gamma_5) \,. \quad (29)$$

To lowest order, the diagrams that enter the calculations are shown in Fig. 1. Let us now extend the model by giving the neutrinos Majorana masses. Thus, the mass matrix for the weak eigenstates neutrinos is the most general Dirac plus Majorana mass matrix. The mass eigenstates will be Majorana neutrinos, with an expansion given in Eq. (14). The form factors in

$$\langle \nu_2(p_2) | J_\mu^{(\mathrm{EM})} | \nu_1(p_1) \rangle \equiv \bar{u}(p_2) P \Gamma_\mu^M(p_2,p_1)u(p_1) \,,$$
$$\Gamma_\mu^M(p_2,p_1) = (q^2\gamma_\mu - q_\mu \not{q})(F_\mu + f_M\gamma_5) + i\sigma_{\mu\nu}q^\nu(G_M + g_M\gamma_5) \quad (30)$$

for Majorana neutrinos now receive two contributions. The first arises from a set of diagrams identical to those shown in Fig. 1, and we denote this contribution by

$$\bar{u}(p_2)\Gamma_\mu^D(p_2,p_1)u(p_1) \,.$$

However, the Majorana condition [or equivalently Eq. (14)] implies that for each diagram in Fig. 1 there is an additional diagram in which each vertex of a given diagram is replaced by its complex conjugate. The contribution from the set of diagrams thus obtained is given by the substitution rule and the complete matrix element is

$$\langle \nu_2(p_2) | J_\mu^{(\mathrm{EM})} | \nu_1(p_1) \rangle = \bar{u}(p_2)\Gamma_\mu^D(p_2,p_1)u(p_1) - \bar{v}(p_1)\Gamma_\mu^{\bar{D}}(-p_2,-p_1)v(p_2) \,, \quad (31)$$

where

$$\Gamma_\mu^{\bar{D}}(p_2,p_1) = \gamma_0[\Gamma_\mu^D(p_2,p_1)]^\dagger \gamma_0 \,.$$

The relative minus sign in Eq. (31) has the same origin as in Eq. (24). Using Eq. (15) and identifying the form factors according to Eqs. (29) and (30) we then obtain

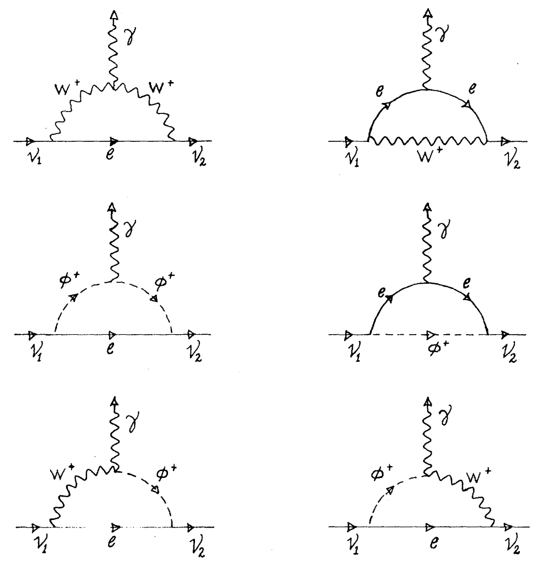

FIG. 1. Lowest-order diagrams for $\nu_1 \rightarrow \nu_2 + \gamma$ in the Dirac case, in the $SU(2)\times U(1)$ model with right-handed singlet neutrinos. ϕ is the standard Higgs doublet and e stands for a generic charged-lepton field.

$$F_M = F_D - F_D^* \ , \quad G_M = G_D - G_D^* \ ,$$
$$f_M = f_D + f_D^* \ , \quad g_M = g_D + g_D^* \ , \tag{32}$$

that ratifies Eq. (18). Further, substituting Eq. (6) in Eq. (32) we recover the results in Eqs. (20) and (21) for the CP-invariant case. Although we have derived the results in Eq. (32) by looking at the lowest-order diagrams in a particular model, a little thought reveals their general validity, as it must be according to the arguments leading to Eq. (18). The only subtle point is to recognize that for any diagram contributing to Γ_μ^D, the contribution of the charge conjugate diagram is given by the second term in Eq. (31). Equation (32) is also very useful from a practical point of view since it expresses the form factors for Majorana neutrinos in terms of the form factors calculated as if the neutrinos were Dirac particles, for which extensive calculations exist in the literature.

In summary, if the initial and final Majorana neutrinos in the process $\nu_1 \rightarrow \nu_2 + \gamma$ have the same (opposite) CP parity, then electric (magnetic) dipole radiation results. This result is against our intuition acquired by studying the Dirac case, but it is one of the somewhat peculiar properties of Majorana neutrinos that we have tried to understand in our work.

The present work was motivated in part by a recent paper by Pal and Wolfenstein. In the revised version of their paper[6] they report results in agreement with ours. We also learned of a paper by Schechter and Valle (Ref. 10) that discusses the electromagnetic properties of Majorana neutrinos, partially overlapping the present paper. We express our thanks to Palash B. Pal for bringing this reference to our attention and for a very useful discussion. After the revised version of the present paper was prepared we received a paper by Kayser[11] which treats subjects similar to ours. We express our thanks to Professor Kayser for sending us a copy of his work. Another reference on this subject has recently appeared: R. E. Shrock, Stony Brook Report No. ITP-SB-82-2, 1982 (unpublished).

APPENDIX

In this appendix we supply the proof that the existence of a magnetic moment for a Majorana neutrino is a signal of CPT nonconservation.

The basic ingredient in the proof is the appropriate definition of a Majorana neutrino in the absence of C and CP conservation. The physical definition of a Majorana neutrino is a neutrino for which the antiparticle is the particle itself. Therefore, the problem reduces to give a precise meaning to the antiparticle in the absence of C and CP conservation. In the $K^0 - \bar{K}^0$ system it is well known that, because of the lack of C and CP invariance in the $\Delta S = 2$ interactions, the $\bar{K}^0$ must be defined as the antiparticle of K^0 with respect to the CPT operation. A similar situation occurs in the present case. If C and CP are not conserved, the antiparticle must be defined through the CPT operation. Therefore, if $|\nu(\vec{p},s)\rangle$ represents a Majorana neutrino state of momentum $\vec{p}$ and spin s, the Majorana condition can be expressed in the form

$$\theta\,|\nu(\vec{p},s)\rangle = e^{i\phi}\,|\nu(\vec{p},-s)\rangle \ , \tag{A1}$$

where

$$\theta \equiv CPT \ . \tag{A2}$$

Using the antiunitary character of the θ operator, Eq. (A1) can be written in the form

$$\theta[e^{i\phi/2}\,|\nu(\vec{p},s)\rangle] = e^{i\phi/2}\,|\nu(\vec{p},-s)\rangle \ ,$$

so that with an appropriate phase convention we obtain

$$\theta | \nu(\vec{p},s)\rangle = | \nu(\vec{p},-s)\rangle \ . \tag{A3}$$

We adopt Eq. (A3) as the definition of a Majorana neutrino, valid in the absence of C and CP invariance. It represents the fact that the antiparticle of a Majorana neutrino, defined through CPT, is the particle itself.

The proof that a nonzero magnetic moment of a Majorana neutrino is a signal CPT nonconservation is now simple. In fact, we can use the well-known result that CPT invariance implies that particle and antiparticle, defined through CPT, have opposite magnetic moments. Since particle and antiparticle are the same for Majorana neutrinos, in the sense discussed above, their magnetic moment must be zero if CPT invariance holds.

We can make this argument more explicit. Under CPT the transformation of $J_\mu^{(\mathrm{EM})}$ is

$$\theta^{-1} J_\mu^{(\mathrm{EM})} \theta = -J_\mu^{(\mathrm{EM})} \ . \tag{A4}$$

If CPT invariance holds, Eqs. (A3) and (A4) imply

$$\langle \nu(\vec{p}\,',s') | J_\mu^{(\mathrm{EM})} | \nu(\vec{p},s)\rangle = -\langle \nu(\vec{p}\,',-s') | J_\mu^{(\mathrm{EM})} | \nu(\vec{p},-s)\rangle^* = -\langle \nu(\vec{p},-s) | J_\mu^{(\mathrm{EM})} | \nu(\vec{p}\,',-s')\rangle \ . \tag{A5}$$

For the left-hand side of Eq. (A5) we use the expression in Eq. (22) in the text. For the right-hand side we use the fact that the spinor representing the state $| \nu(\vec{p},s)\rangle$ is

$$u(\vec{p},-s) = \eta\gamma_5 u^c(\vec{p},s) = i\eta\gamma_5\gamma_2 u^*(\vec{p},s) \ , \tag{A6}$$

where η is an arbitrary phase, independent of $\vec{p}$ and s according to Eq. (A3). Then,

$$\langle \nu(\vec{p},-s) | J_\mu^{(\mathrm{EM})} | \nu(\vec{p}\,',-s')\rangle = u(\vec{p},-s)\Gamma_\mu(p,p')u(\vec{p}\,',-s') = -\bar{u}^c(\vec{p},s)\gamma_5\Gamma_\mu(p,p')\gamma_5 u^c(\vec{p}\,',s') \ ,$$

where Γ_μ is defined in Eq. (23) in the text. Equation (A5) then implies

$$\bar{u}(\vec{p}\,',s')\Gamma_\mu(p',p)u(\vec{p},s) = \bar{u}^c(\vec{p},s)\gamma_5\Gamma_\mu(p,p')\gamma_5 u^c(\vec{p}\,',s') \tag{A7}$$

while standard manipulations yield

$$\bar{u}^c(\vec{p},s)\gamma_5\Gamma_\mu(p,p')\gamma_5 u^c(\vec{p}\,',s') = \bar{u}(\vec{p}\,',s')[-\gamma_\mu F' + (q^2\gamma_\mu - q_\mu \not{q})\gamma_5 f' - i\sigma_{\mu\nu}q^\nu(G' + g'\gamma_5)]u(\vec{p},s) \ ,$$

so that only $f' \neq 0$.

We have presented a more formal version of the following, more intuitive, argument due to Wolfenstein.[12] Let us consider the nonrelativistic limit (ν rest frame) of the various operators:

$$\begin{aligned} &\sigma_{\mu\nu}F^{\mu\nu} \to \vec{\sigma}\cdot\vec{B} \ , \\ &\gamma_5\sigma_{\mu\nu}F^{\mu\nu} \to \vec{\sigma}\cdot\vec{E} \ , \\ &\gamma_\mu A^\mu \to \vec{p}\cdot\vec{A} \ , \\ &\gamma_\mu\gamma_5 A^\mu \to \vec{\sigma}\cdot\vec{A} \ . \end{aligned} \tag{A8}$$

TABLE I. Transformation properties of the various operators under C, P, and T.

	$\vec{\sigma}$	$\vec{p}$	$\vec{B}$	$\vec{E}$	$\vec{A}$
C	+	+	−	−	−
P	+	−	+	−	−
T	−	−	−	+	−

The transformation properties of these operators can be deduced from Table I. (The crucial point is that for a Majorana neutrino $\vec{p}$ and $\vec{\sigma}$ are even under C.) Of these four operators only $\vec{\sigma}\cdot\vec{A}$ is even under CPT so that, in particular, a magnetic moment is forbidden.

[1]See, e.g., the review by P. Langacker, Phys. Rep. 72, 185 (1981).

[2]V. A. Lyubimov *et al.*, Phys. Lett. 94B, 266 (1980).

[3]A. De Rújula and S. Glashow, Phys. Rev. Lett. 45, 942 (1980); F. W. Stecker, NASA Technical Memorandum No. 82158, 1981 (unpublished); R. Kimble, S. Bowyer, and P. Jakobsen, Phys. Rev. Lett. 46, 80 (1981); D. A. Dicus *et al.*, *ibid.* 39, 168 (1977); Phys. Rev. D 17, 1529 (1978), and references therein.

[4]See, e.g., T. Goldman and G. J. Stephenson, Phys. Rev. D 16, 2256 (1977).

[5]W. J. Marciano and A. I. Sanda, Phys. Lett. 67B, 303 (1977); B. W. Lee and R. E. Shrock, Phys. Rev. D 16, 1444 (1977).

[6]Palash B. Pal and Lincoln Wolfenstein, Phys. Rev. D 25, 766 (1982).

[7]W. K. Tung, Phys. Lett. 67B, 52 (1977).

[8]See, e.g., M. Doi *et al.*, Phys. Lett. 102B, 323 (1981).

[9]J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Springer, New York, 1976), p. 161.

[10]See, e.g., J. Schechter and J. W. F. Valle, Phys. Rev. D 24, 1883 (1981); 25, 283 (E) (1982).

[11]Boris Kayser, Report No. SLAC-PUB-2879, 1982 (unpublished).

[12]Lincoln Wolfenstein, private communication.