Unity of quark and lepton interactions with symplectic gauge symmetry

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Properties of symplectic groups are reviewed and the gauge structure of Sp(2n) derived. The electroweak unification of leptons within Sp(8) gauge symmetry and grand unification of quarks and leptons within Sp(10) gauge symmetry are discussed.

I. INTRODUCTION

The hypothesis that the observed elementary-particle interactions are low-energy manifestations of a single fundamental force is central to current thought in high-energy theory. The disparity in the strengths of these interactions is attributed to the different symmetries associated with these interactions at present energies. Empirical evidence, which is in abundance for the $SU(2)_L \times U(1)$ structure[1] of electroweak interactions and which seems adequate for the SU(3) structure of strong interactions, supports $G = SU(2)_L \times U(1) \times SU(3)_c$ as the basic underlying symmetry of these interactions. It is possible that the present electroweak interactions have probed only the G part of the full symmetry $\tilde{G} = SU(2)_L \times SU(2)_R \times U(1) \times SU(3)_c$ that is required if right-handed currents of ordinary fermions exist and parity violation in weak interactions is spontaneous.[2]

Unfortunately, there are few clues at present as to what the grand symmetry of the fundamental force is. Extrapolation of either the $SU(2)_L \times U(1) \times SU(3)_c$ or the $SU(2)_L \times SU(2)_R \times U(1) \times SU(3)_c$ symmetries can lead to the parent symmetries[3] SU(5) or[4] SO(10). These symmetries embed G or $\tilde{G}$ in a most economical way, where "economy" is taken to mean minimal rank and simplicity in the choice of fermion representations. Fundamental issues such as the inclusion of fermion families, neutral bosons lighter than the Z^0 of the Weinberg-Salam theory, and an acceptable spectrum of neutrino masses has led to the discussion of "nonminimal" grand unified theories based on the parent symmetries such as[5] $SU(5+n)$, $SO(10+2n)$, E_6, and E_8.

In this note unity of the fundamental interactions with symplectic gauge symmetry for nonminimal unified theories is shown to be an equally viable alternative. For the sake of completeness, properties of symplectic groups are first reviewed. Then the structure of the gauge-boson matrix is derived and is used to discuss the properties of fermion representation. Finally electroweak unification of leptons with Sp(8) gauge symmetry and grand unification of quarks and leptons with Sp(10) gauge symmetry are discussed.

II. SYMPLECTIC GROUPS

The set of transformations S that leave a skew-symmetric bilinear form

$$x^T y = x_i y_{n+i} - x_{n+i} y_i \quad (i=1 \text{ to } n) \tag{2.1}$$

invariant constitute a symplectic group. In the above notation x (y) is a column vector of dimension $2n$ and the metric of the bilinear form is

$$G = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{2.2}$$

where 1 is the $n \times n$ identity matrix. Skew symmetric metrics of any other form can be brought into the standard form of Eq. (2.2) by suitable rotation of the bases.

Classical mechanics offers adequate examples where metrics of the form (2.2) arise in classical mechanics. For example, Hamilton's equations of motion for a conservative dynamical system with $2n$ degrees of freedom (p_i, q_i, $i=1$ to n)

$$-\frac{\partial}{\partial t}\begin{pmatrix} p_i \\ q_i \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \dfrac{\partial H}{\partial p_i} \\ \dfrac{\partial H}{\partial q_i} \end{pmatrix} \tag{2.3}$$

and the Poisson bracket of two differentiable functions $f(p_i, q_i)$ and $g(p_i, q_i)$

$$\{f,g\} = \begin{pmatrix} \dfrac{\partial f}{\partial p_i} & \dfrac{\partial f}{\partial q_i} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \dfrac{\partial g}{\partial p_i} \\ \dfrac{\partial g}{\partial q_i} \end{pmatrix} \tag{2.4}$$

exhibit the metric G.

Invariance of Eqs. (2.1), (2.3), and (2.4) under the set of linear transformations S amounts to the condition

$$SGS^T = G \tag{2.5}$$

(T indicates transpose). All $2n \times 2n$ matrices S satisfying Eq. (2.5) form a symplectic group Sp($2n$) of rank n and $n(2n+1)$ generators. Unlike orthogonal groups symplectic groups are unimodular, i.e., Det$G = +1$, and hence defined for only even-dimensional spaces. This follows from Det$G = (-1)^n$ from Eq. (2.2).

Let the infinitesimal generators of Sp($2n$) effecting global transformations in the neighborhood of the identity be $\sigma_{\alpha\beta}$ ($\alpha, \beta = 1$ to $2n$). The generator $\sigma_{\alpha\beta}$ is represented by unity at the αth row and βth column of the $2n \times 2n$ matrix. Using condition (2.5), the $n(2n+1)$ generators $\sigma_{\alpha\beta}$ are redefined in terms of the diagonal and off-diagonal generators

$$A_{ij} = \sigma_{ij} - \sigma_{n+j,n+i}\,, \tag{2.6}$$

$$B_{ij} = \sigma_{i,n+j} + \sigma_{j,n+i} = B_{ji}\,, \tag{2.7}$$

$$C_{ij} = \sigma_{n+i,j} + \sigma_{n+j,i} = C_{ji}\,. \tag{2.8}$$

These generators satisfy the commutation relations

$$[A_{ij}, A_{kl}] = g_{ik}A_{il} - g_{li}A_{kj}\,, \tag{2.9}$$

$$[A_{ij}, B_{kl}] = g_{jk}B_{il} + g_{jl}B_{ik}\,, \tag{2.10}$$

$$[A_{ij}, C_{kl}] = -g_{ik}C_{jl} - g_{il}C_{jk}\,, \tag{2.11}$$

$$[B_{ij}, C_{kl}] = g_{jk}A_{il} + g_{ik}A_{jl} + g_{jl}A_{ik} + g_{il}A_{jk}\,, \tag{2.12}$$

$$[B_{ij}, B_{kl}] = [C_{ij}, C_{kl}] = 0\,. \tag{2.13}$$

For elementary-particle physics it is further required that the generators be Hermitian. This implies $C = B^\dagger$ and relation (2.11) is redundant. The structure of the gauge-boson matrix entering the covariant derivative

$$D_\mu \psi_\alpha = \left(\partial_\mu \delta_{\alpha\beta} + \frac{i}{2} g [\sigma_{ij} W^{ij}]_{\alpha\beta}\right)\psi^\beta \tag{2.14}$$

is given by

$$\sigma \cdot W = \frac{1}{2}\begin{bmatrix} U & X \\ X^\dagger & -U^T \end{bmatrix}\,. \tag{2.15}$$

The form of the matrix of Eq. (2.15) exhibits two things explicitly.

(i) The diagonal gauge fields U represent a unitary matrix with n^2 parameters. Thus the maximal subgroup of Sp($2n$) is U(n), a result that can also be derived by other group-theoretic considerations.

(ii) The gauge-field matrix $-U^T$ gives particle interactions that are conjugate to the ones given by the gauge fields U. Hence the $2n$-dimensional representation ψ contains one set of n fermions that is "conjugate" to the other set. (ψ is not reducible.) A consequence of this property is that a nonminimal grand unified theory based on the symplectic gauge symmetry Sp(16) containing the particles and antiparticles of any one of the e, μ, and τ families will not have weak interactions since all currents are vectorlike. The reality of the representations also guarantees anomaly cancellation with the exception[6] of $n = 3$.

III. UNIFICATION WITH SYMPLECTIC SYMMETRY

A. Electroweak unification of leptons

Several factors make it worthwhile to consider unification of just the leptonic interactions. Leptons—at least of the first two generations—have a pointlike structure. They are free from the mysteries of strong interactions and do not face the electric charge assignment dilemma encountered in the case of quarks. In the following sections, it will be assumed that neutrinos are massive Dirac particles. The fundamental representation that contains the leptonic fields of the electron family is that of Sp(8). These fields are assigned in $\psi_L^{(e)}$ $[=\frac{1}{2}(1+\gamma_5)\psi]$, where

$$\psi_L^{(e)} = \begin{pmatrix} \nu_e \\ e^- \\ e^+ \\ -\nu_e^c \\ E^- \\ E^0 \\ E^{0c} \\ E^+ \end{pmatrix}_L\,. \tag{3.1}$$

The conjugate fields $(E^+, E^-, E^0, E^{0c})_L$ also form a family which is "twin" to the electron family. One noteworthy feature of this twin electron family is that it has right-handed currents with strength G_F.

The diagonal generators of Sp(8) are

$$T_L^3 = \text{diag}\,\tfrac{1}{2}(1, -1, 0, 0, 0, 0, -1, 1)\,, \tag{3.2}$$

$$T_R^3 = \text{diag}\,\tfrac{1}{2}(0, 0, 1, -1, -1, -1, 0, 0)\,, \tag{3.3}$$

$$T_0 = \text{diag}\,\frac{1}{2\sqrt{2}}(-1, -1, 1, 1, -1, -1, 1, 1)\,, \tag{3.4}$$

$$V_0 = \text{diag}\,\frac{1}{2\sqrt{2}}(1, 1, 1, 1, -1, -1, -1, -1)\,. \tag{3.5}$$

T_L^3, T_R^3, and T_0 are some linear combinations of the conventional SU(4) generators[7] V_3, V_8, and

V_{15}. The electric charge of the leptons in $\psi_L^{(e)}$ in Eq. (3.1) is given by

$$Q_{EM} = T_L^3 + T_R^3 + \sqrt{2}\, T_0 . \tag{3.6}$$

Let g_0 be the bare coupling of Sp(8) and $[g_{2L}, g_{2R}, g_{L+R}]$ be the renormalized couplings of the subgroups $[SU(2)_L, SU(2)_R, U(1)_{L+R}]$ in the hierarchy

$$Sp(8) \underset{M_1}{\longrightarrow} SU(4)\times U(1) \underset{M_2}{\longrightarrow} SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times U(1) \underset{M_H}{\longrightarrow} SU(2)_L \times SU(2)_R \times U(1)_{L+R} \underset{M_R}{\longrightarrow} SU(2)_L \times U(1) \underset{M_L}{\longrightarrow} U(1)_{EM} . \tag{3.7}$$

The above symmetry breaking can be implemented by suitable choice of the vacuum expectation values of two adjoint, one symmetric, and one (two if mixing between the left- and right-handed weak gauge bosons is to be avoided) fundamental Higgs scalar fields.

In terms of the renormalized couplings the photon field A and the electric charge e are given by

$$\frac{A}{e} = \frac{W_L^0}{g_{2L}} + \frac{W_R^0}{g_{2R}} + \frac{U}{g_{L+R}} , \tag{3.8}$$

$$e^{-2}(M_L) = g_{2L}{}^{-2}(M_L) + g_{2R}{}^{-2}(M_L) + g_{L+R}{}^{-2}(M_L) , \tag{3.9}$$

where the renormalization point μ is taken to be equal to the charged-weak-boson mass M_L in the usual way. From the definition of the weak angle $\sin^2\theta_W$ it follows that the bare value of the weak angle $\sin^2\theta_W$ is $\frac{1}{4}$ at the electroweak unifying mass M_1. The renormalized and the bare weak angles are related by the expression

$$\sin^2\theta_W = \tfrac{1}{4} - \frac{11 e^2(M_L)}{24\pi^2}\left(\ln\frac{M_2}{M_L} + \tfrac{1}{2}\ln\frac{M_R}{M_L}\right) . \tag{3.10}$$

From the experimental value of the weak angle $\sin^2\theta_W = 0.23 \pm 0.015$, $e^2/4\pi = \frac{1}{13}$, and Eq. (3.10) the masses M_2, M_L, M_R satisfy the constraint

$$M_2{}^2 M_R \sim 10^4 M_L{}^3 \tag{3.11}$$

or $M_2 \sim 10^{1.8} M_L$ for the currently favored value $M_R \simeq 300$ GeV. The mass scale M_1 is not constrained.

The neutral-current sector of Sp(8) is rich. There are three massive neutral bosons: one is the familiar neutral boson of the $SU(2)_L \times U(1)$ theory with mass 94 GeV; the second one has a mass of order 300 GeV to get the right amount of parity violation in polarized-electron–deuteron inelastic scattering; the third neutral boson does not affect the observed neutrino neutral-current interactions. Its mass can be lighter than the mass of the standard $SU(2)_L \times U(1)$ neutral boson. The precise mass will be determined by measurements on the forward-backward charge asymmetry in the leptonic reaction[8] $e^+e^- \to \mu^+\mu^-$.

Note that if neutrinos have no right-handed counterparts the relevant symplectic gauge symmetry is Sp(6). The bare value of $\sin^2\theta_W$ in a gauge theory of leptons based on Sp(6) is equal to one quarter. Such a theory will also have a light neutral boson in addition to the one of the standard $SU(2)_L \times U(1)$ theory.

B. Grand unification of quarks and leptons

The relevant symplectic gauge symmetry that contains the known $SU(3)_c \times SU(2)_L \times U(1)$ gauge interactions is Sp(10). The fermions of the theory are split between the fundamental and the antisymmetric representations of Sp(10), as in the simple SU(5) model of Georgi and Glashow (Ref. 3). The contents of the fundamental 10 are

$$\psi_{10} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ \nu_e \\ e^- \\ E^+ \\ E^{0c} \\ D_3 \\ D_2 \\ D_1 \end{pmatrix}_L \tag{3.12}$$

while the antisymmetric 45-dimensional representation is reducible into a singlet and a 44-dimensional representation.[9] The rest of the fermionic fields of the electron family not in (3.12) are assigned to the irreducible 44 of Sp(10) as follows:

$$\psi_{44}=\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & u_3 & -u_2 & -d_1^c & -u_1^c & F_{11} & F_1 & F_2 & F_3 & F_4 \\ -u_3 & 0 & u_1 & -d_2^c & -u_2^c & F_1^c & F_{22} & F_5 & F_6 & F_7 \\ u_2 & -u_1 & 0 & -d_3^c & -u_3^c & F_2^c & F_5^c & F_{33} & F_8 & F_9 \\ d_1 & d_2 & d_3 & 0 & -e^- & F_3^c & F_6^c & F_8^c & F_{44} & F_{10} \\ u_1 & u_2 & u_3 & e^- & 0 & F_4^c & F_7^c & F_9^c & F_{10}^c & F_{55} \\ -F_{11} & -F_1^c & -F_2^c & -F_3^c & -F_4^c & 0 & E^+ & U_3 & U_2 & U_1 \\ -F_1 & -F_{22} & -F_3^c & -F_6^c & -F_4^c & -E^+ & 0 & D_3 & D_2 & D_1 \\ -F_2 & -F_5 & -F_{33} & -F_8^c & -F_9^c & -U_3 & -D_3 & 0 & -U_1^c & U_2^c \\ -F_3 & -F_6 & -F_8 & -F_{44} & -F_{10}^c & -U_2 & -D_2 & U_1^c & 0 & -U_3^c \\ -F_4 & -F_7 & -F_9 & -F_{10} & -F_{55} & -U_1 & -D_1 & -U_2^c & U_3^c & 0 \end{bmatrix}_R \tag{3.13}$$

with

$$F_{11}+F_{22}+F_{33}+F_{44}+F_{55}=0\,.$$

The representations ψ_{10} and ψ_{44} are anomaly free.

The five diagonal generators of Sp(10) are chosen to be

$$\begin{aligned} T_L^3 &= \mathrm{diag}\,\tfrac{1}{2}(0,0,0,1,-1,-1,1,0,0,0)\,, \\ U_0 &= \mathrm{diag}\frac{1}{2\sqrt{15}}(2,2,2,-3,-3,3,3,3,-2,-2,-2)\,, \\ V_3 &= \mathrm{diag}\,\tfrac{1}{2}(1,-1,0,0,0,0,0,0,1,-1)\,, \\ V_8 &= \mathrm{diag}\frac{1}{2\sqrt{3}}(1,1,-2,0,0,0,0,2,-1,-1)\,, \\ T &= \mathrm{diag}\frac{1}{\sqrt{10}}(1,1,1,1,1,-1,-1,-1,-1,-1)\,. \end{aligned} \tag{3.14}$$

The electric-charge operator is identical to that of the standard SU(5) theory [i.e., $Q=T_L^3+(\frac{5}{3})^{1/2}U_0$] if fermion assignments are given by ψ_{10} and ψ_{44}. Hence, the predictions of the renormalized and the unrenormalized weak angle are identical in the Sp(10) and SU(5) grand unified theories. Although the extra U(1) generator T_0 does not contribute to the electric charge, the fermions in Eqs. (3.12) and (3.13) do carry the quantum numbers of T_0. The top and bottom halves of ψ_{10} have T_0 quantum numbers 1 and -1 (modulo the normalization). The fermions in the top and bottom diagonal blocks in ψ_{44} carry $+2$ and -2 units of T_0 quantum numbers while the off-diagonal F fermions carry $+4$ units of T_0 quantum numbers.

The Sp(10) gauge symmetry can descend to the low energy symmetry $SU(3)_c\times U(1)_{EM}$ through different intermediate symmetries. The route followed here involves SU(5) as one intermediate stage.[10] The hierarchy

$$\mathrm{Sp}(10)\xrightarrow[M_G]{}SU(5)\times U(1)_H\xrightarrow[M]{}SU(3)_c\times SU(2)_L\times U(1)\times U(1)_T\xrightarrow[M_H]{}SU(3)_c\times SU(2)_L\times U(1)\xrightarrow[M_W]{}SU(3)_c\times U(1)_{EM} \tag{3.15}$$

is implemented by two scalar multiplets (ϕ_G,ϕ) in the adjoint, one antisymmetric tensor scalar field $\phi_{\alpha\beta\gamma\delta\sigma}$ and one fundamental ϕ_{10} of Sp(10). The masses (M_G, M, M_H, M_W) are proportional to the vacuum expectation values of the scalar fields $(\phi_G,\phi,\phi_{\alpha\beta\gamma\delta\sigma},\phi_{10})$ which are

$$\begin{aligned} &\langle\phi_G\rangle=\mathrm{diag}(a,a,a,a,a,-a,-a,-a,-a,-a)\,,\quad \langle\phi\rangle=\mathrm{diag}(2b,2b,2b,-3b,-3b,3b,3b,-2b,-2b,-2b)\,, \\ &\langle\phi_{\alpha\beta\gamma\delta\sigma}\rangle=\phi_{12345}\,,\quad \langle\phi_{10}\rangle=(0,0,0,c,0,0,0,0,0,0)\,. \end{aligned} \tag{3.16}$$

The spontaneous symmetry breaking depicted in Eq. (3.15) leads to the neutral-current Lagrangian

$$L_N=\tfrac{1}{4}c^2(g_{2L}W_L^0-g_1B^0+g_HH^0)^2+\tfrac{1}{4}\phi_{12345}{}^2g_H{}^2H^{0\,2}\,, \tag{3.17}$$

where $[g_{2L},g_1,g_H]$ are the renormalized couplings of the subgroups $[SU(2)_L, U(1), U(1)_H]$ and (B^0,H^0) are the gauge fields corresponding to the U(1) generators U_0 and T_0. The diagonalization of L_N is trivial; it leads to the massless photon A_μ and two neutral eigenstates $(N_{1\mu},N_{2\mu})$ with masses (M_{N_1},M_{N_2}), where

$$A=W_L^3\cos\theta_W+B^0\sin\theta_W\,, \tag{3.18}$$

$$N_1=Z^0\cos\alpha+H^0\sin\alpha\,, \tag{3.19}$$

$$N_2=H^0\cos\alpha-Z^0\sin\alpha\,, \tag{3.20}$$

$$M_{N_{1(2)}}{}^2 = \frac{M_{W_L}{}^2}{2\cos^2\theta_W}\left\{1+\left(\frac{g_H}{g_1}\cos\theta_W\right)^2 - (+)\left[\left(\frac{g_H}{g_1}\cos\theta_W\right)^2\left(1+\frac{\phi_{12345}{}^2}{c^2}\right)+4\frac{g_H{}^2}{g_1{}^2}\cos^2\theta_W\right]^{1/2}\right\}, \tag{3.21}$$

$$\tan 2\alpha = \frac{2(g_H/g_1)\cos\theta_W}{1-\left(\frac{g_H}{g_1}\cos\theta_W\right)^2\left(1+\frac{\phi_{12345}{}^2}{c^2}\right)}. \tag{3.22}$$

Z^0 and θ_W are the neutral boson and weak angle of the $SU(2)_L \times U(1)$ theory. The bare values of the couplings g_1 and g_H are $(\frac{3}{5})^{1/2}g_{10}$ and $(\frac{5}{2})^{1/2}g_{10}$ where $g_{10} \equiv g$ (Sp(10)). M_{W_L} is the mass of the charged weak boson W_L. The neutrino neutral-current phenomenology of N_1 and N_2 is identical to that of the standard $SU(2)_L \times U(1)$ theory and imposes no restriction on the masses[11] of N_1 and N_2. However the electron neutral-current phenomenology serves as a probe of the masses of N_1 and N_2. The forward-backward charge-asymmetry measurement[12] in polarized-electron–deuterium deep inelastic scattering constrains the mass of N_1 to be almost that of Z^0 of the standard $SU(2)_L \times U(1)$ theory and that of N_2 to be of order 350 GeV at the weak-angle value $\sin^2\theta_W \simeq 0.23$.

IV. DISCUSSION

Unitary symplectic gauge groups are shown to be viable candidates for nonminimal unification of the quark and lepton interactions. Two noteworthy features of the unified models discussed in the previous section are as follows.

(i) They have a rich neutral-current sector that is heavily constrained by the existing neutral-current data involving neutrinos and electrons.

(ii) The fermionic representations contain exotic fermions that lead to right-handed interactions with strength G_F. These mirror fermions form twin families to the ordinary e, μ, τ families and can carry their own fermion number, a quantum number possibly violated through mixing of terms in the fermion mass matrix. Unfortunately mixing angles and masses of these mirror fermions are as yet unpredictable, because of the large number of Yukawa parameters producing the mass matrix.

Sp($2n$) can be broken down to SU(n)$\times$U(1) with vacuum expectation values of Higgs fields in the adjoint representation. This mode of symmetry breaking satisfies the Goddard-Olive criterion[13] of generating monopoles with charges identical to the gauge bosons. It is interesting to note that the U(1) field in the SU(n)$\times$U(1) subsymmetry and a linear combination of the diagonal generators in the off-diagonal blocks (X, X^+) constitute an SU(2) symmetry with correctly normalized SU(2) generators. This hidden SU(2) leads to gauge interactions between the fermions of the ordinary and the twin families. Thus the maximal subgroup of Sp($2n$) is SU(n)$\times$SU(2). Finally, the smallest symplectic group that contains the left-right-symmetric group $SU(2)_L \times SU(2)_R \times SU(4)^c_{L+R}$ and has the sixteen fermions and antifermions of any one family of the ordinary fermions assigned to its fundamental representation is Sp(32). This contains the subgroup SU(16) discussed in the literature as the maximal symmetry[14] underlying the interactions of any one of the e, μ, or τ families.

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