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Pion polarizabilities from backward and fixed-u sum rules

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Various contributions to the pion polarizabilities are estimated using sum rules obtained from backward and fixed-$u=\mu^2$ ($\mu=$ pion mass) dispersion relations for the relevant pion Compton-scattering amplitude. While the s-channel part can be quite reliably computed in terms of the strong and radiative widths of known meson resonances, the evaluation of the t-channel piece remains highly model dependent despite important clarifications provided recently by measurements of the $\gamma\gamma\rightarrow\pi\pi$ reaction.

The possibility of investigating the pion Compton effect and the pion polarizabilities in the radiative scattering of high-energy pions in nuclear Coulomb fields[1] or in radiative single-pion photoproduction on protons[2] has been recently stressed. At the same time, information on the process $\gamma\gamma\rightarrow\pi\pi$ may be obtained by studying the colliding-beam reaction $e^+e^-\rightarrow e^+e^-\pi\pi$ and indeed interesting results have already been found (see, for instance, Ref. 3). A natural theoretical framework which simultaneously involves quantities relevant to both the $\gamma\pi\rightarrow\gamma\pi$ and $\gamma\gamma\rightarrow\pi\pi$ channels is provided by sum rules for the pion polarizabilities derived from backward or fixed-u dispersion relations. Such sum rules have been first put forward[4] and used[4-7] in connection with the difference $\alpha-\beta$ between the electric (α) and magnetic (β) polarizabilities of the proton. Their analogs in the pion case are particularly appealing since for pions, unlike for nucleons, $\alpha-\beta$ is the most important combination [on quite general grounds one expects[8] $(\alpha+\beta)_\pi << |(\alpha-\beta)_\pi|$].

For a review on previous calculations of the pion polarizabilities within various approaches (quantum field theoretical, quark models, forward dispersion relations), we refer the reader to Ref. 9. Here we recall only that predictions based on forward finite-energy sum rules (FFESR's) are strongly model dependent because of difficulties in evaluating reliably the high-energy asymptotic contributions.

In this note we shall present some simple numerical estimates of the pion polarizabilities using backward and fixed-$u=\mu^2$ ($\mu=$pion mass, $u=$the usual Mandelstam variable) sum rules. In some sense our approach looks complementary to that of FFESR's, since the annihilation-channel exchanges are now taken into account directly, mainly through $\gamma\gamma\rightarrow\pi\pi$ amplitudes, rather than indirectly by means of Regge parameters describing the asymptotics. While the s-channel contributions expressed by integrals over cross sections for photoabsorption on the pion can be more or less reliably computed using the known radiative and strong widths of vector, axial-vector, and tensor meson resonances, the evaluation of the annihilation-channel contributions still remains largely affected by model dependence, despite important clarifications brought recently by the experimental study of pion pair production in photon-photon collisions. Although in our procedure the model-dependence problem then appears merely shifted rather than much mitigated, one has at least the advantage of starting with a convergent (subtraction-free) dispersion representation for $(\alpha-\beta)_\pi$ and there is also a realistic hope that further better knowledge of ($\gamma\gamma\rightarrow$hadrons)-processes shall help reduce the existing ambiguities in the determination of the pion polarizabilities.

We shall deal with the following two slightly different sum rules for $(\alpha-\beta)_\pi$ (Ref. 10):

(1) Fixed-angle $\theta=180°$ sum rule:

$$(\alpha-\beta)_\pi=(\alpha-\beta)_\pi^{(s)}+(\alpha-\beta)_\pi^{(t)} , \tag{1}$$

$$(\alpha-\beta)_\pi^{(s)}=-\frac{1}{8\pi^2\mu}\int_{4\mu^2}^{\infty}ds\frac{s+\mu^2}{s(s-\mu^2)}M^{(s)}\left[s,t=-\frac{(s-\mu^2)^2}{s}\right] , \tag{2}$$

$$(\alpha-\beta)_\pi^{(t)}=-\frac{1}{8\pi^2\mu}\int_{4\mu^2}^{\infty}\frac{dt}{t}M^{(t)}\left[t,u=\mu^2-\frac{t}{2}\pm\frac{1}{2}[t(t-4\mu^2)]^{1/2}\right] . \tag{3}$$

(2) Fixed-$u=\mu^2$ sum rules:

$$(\alpha-\beta)_\pi=(\alpha-\beta)_\pi^{[s]}+(\alpha-\beta)_\pi^{[t]} , \tag{1'}$$

$$(\alpha-\beta)_\pi^{[s]}=-\frac{1}{8\pi^2\mu}\int_{4\mu^2}^{\infty}\frac{ds}{s-\mu^2}M^{(s)}(s,u=\mu^2) , \tag{2'}$$

$$(\alpha-\beta)_\pi^{[t]}=-\frac{1}{8\pi^2\mu}\int_{4\mu^2}^{\infty}\frac{dt}{t}M^{(t)}(t,u=\mu^2) . \tag{3'}$$

$M^{(s)}$ and $M^{(t)}$ are the s- and t-channel absorptive parts of the amplitude

$$M=2A+\left[\frac{t}{4}-\mu^2\right]B\equiv\frac{4f_{++}}{t} , \tag{4}$$

where A,B are the invariant amplitudes (free of kinematical problems[11]) specifying the pion Compton scattering S-matrix element and f_{++} is the helicity amplitude describing transitions with photon helicities $+1$, $+1$ in the $\gamma\gamma\to\pi\pi$ channel:

$$\langle\gamma(k'),\pi(p')\,|\,\gamma(k),\pi(p)\rangle=\delta_{f,i}+i(2\pi)^{-2}(16k_0'k_0p_0'p_0)^{-1/2}\epsilon_\mu^\dagger(k')T_{\mu\nu}(p',k';p,k)\epsilon_\nu(k)\delta^4(\cdots) , \tag{5}$$

$$T_{\mu\nu}(p',k';p,k)=A(s,t,u)(k\cdot k'g_{\mu\nu}-k_\mu k_\nu')$$
$$\qquad -B(s,t,u)[k\cdot k'P_\mu P_\nu-(P\cdot K)(P_\mu k_\nu'+P_\nu k_\mu)+g_{\mu\nu}(P\cdot K)^2] ,$$

$$P=\frac{p+p'}{2} ,\quad K=\frac{k+k'}{2} ,\quad s=(p+k)^2 ,\quad t=(k-k')^2 ,\quad s+t+u=2\mu^2 ,$$

where $s=\mu^2+2\mu\omega$ (ω=the incident photon energy in the laboratory system).

As shown in Ref. 10, one can put the above sum rules in the form

$$(\alpha-\beta)_\pi^{(s)}=\frac{1}{2\pi^2}\int_{\omega_0=3\mu/2}^{\infty}\frac{d\omega}{\omega^2}\left[1+\frac{\omega}{\mu}\right][\sigma(\text{yes})-\sigma(\text{no})] , \tag{2a}$$

$$(\alpha-\beta)_\pi^{(t)}=-\frac{1}{2\pi^2\mu}\int_{4\mu^2}^{16\mu^2}\frac{dt}{t^2}\sum_{J=\text{even}}(2J+1)\left[\frac{t(t-4\mu^2)}{16}\right]^{J/2}g_+^J(t)h_J^*(t)$$
$$+\text{higher-annihilation-channel contributions} , \tag{3a}$$

$$(\alpha-\beta)_\pi^{[s]}=\frac{1}{2\pi^2\mu}\int_{\omega_0=3\mu/2}^{\infty}\frac{d\omega}{\omega^2}\sum_{l=1}^{\infty}d_{1,-1}^l\left[x=-\frac{s+\mu^2}{s-\mu^2}\right][\sigma_{El}(\omega)-\sigma_{Ml}(\omega)] , \tag{2'a}$$

$$(\alpha-\beta)_\pi^{[t]}=-\frac{1}{2\pi^2\mu}\int_{4\mu^2}^{16\mu^2}\frac{dt}{t^2}\sum_{J=\text{even}}(2J+1)\left[\frac{t(t-4\mu^2)}{16}\right]^{J/2}g_+^J(t)h_J^*(t)P_J(\cos\psi)$$
$$+\text{higher-annihilation-channel contributions} . \tag{3'a}$$

σ(yes) and σ(no) stand for the sum of the photoabsorption cross sections containing, respectively, the parity-flip and -nonflip multipoles:

$$\sigma(\text{yes}) = \sum_{l=\text{odd}} \sigma_{El}(\omega) + \sum_{l=\text{even}} \sigma_{Ml}(\omega) , \qquad (6)$$

$$\sigma(\text{no}) = \sum_{l=\text{even}} \sigma_{El}(\omega) + \sum_{l=\text{odd}} \sigma_{Ml}(\omega) ; \qquad (7)$$

$h_J(t) = \exp[i\delta_J(t)]\sin\delta_J(t)$ and $g_+^J(t)$ denote, respectively, the $\pi\pi \to \pi\pi$ and $\gamma\gamma \to \pi\pi$ partial waves [for t in the elastic-unitarity region $4\mu^2 \le t \le 16\mu^2$ they have (modulo π) the same phase $\delta_J(t)$]; $d^l_{1,-1}(x)$ and $P_J(\cos\psi)$ are the usual rotation group functions and Legendre polynomials (ψ=the t-channel center-of-mass (c.m.) angle, $\cos\psi = (u-s)/[t(t-4\mu^2)]^{1/2}$; x =the cosine of the c.m. angle for $\gamma\pi \to \gamma\pi$).

We start now discussing the evaluation of the (more reliable) s-channel contributions $(\alpha-\beta)_\pi^{(s),[s]}$. Only photoabsorption channels with two and three pions are retained; the process $\gamma\pi \to \pi\pi$ is considered in the ρ-resonance approximation, while for $\gamma\pi \to \pi\pi\pi$ we take only ω, ϕ, A_1, A_2 resonance contributions to $\gamma\pi \to \pi\rho$. So we retain only $E1$, $M1$, and $M2$ transitions [corresponding, respectively, to the A_1, (ρ,ω,ϕ), and A_2 resonances] and integrate the Breit-Wigner forms

$$\sigma_J(s) = \frac{2\pi s}{(s-\mu^2)^2}(2J+1)\frac{\Gamma_i \Gamma_f}{(\sqrt{s}-M_R)^2 + \frac{1}{4}{\Gamma_{\text{total}}}^2} \qquad (8)$$

with corresponding angular momentum factors included in Γ_f to ensure correct threshold behavior, etc. Masses and strong and total widths are taken from Ref. 12. The following radiative widths are used (see Refs. 9 and 12): $\Gamma(\rho \to \pi\gamma) = 0.063$, $\Gamma(\omega \to \pi\gamma) = 0.88$, $\Gamma(\phi \to \pi\gamma) = 0.57 \times 10^{-2}$, $\Gamma(A_1 \to \pi\gamma) = 0.60$, $\Gamma(A_2 \to \pi\gamma) = 0.45$ (all values in MeV). Numerical integration leads then to the results (for polarizabilities the units of 10^{-4} fm^3 are employed throughout this paper):

$$(\alpha-\beta)_{\pi^\pm}^{(s)} = \underset{(\rho)}{-0.98} + \underset{(A_1)}{2.13} + \underset{(A_2)}{1.37} \simeq 2.5 , \qquad (9)$$

$$(\alpha-\beta)_{\pi^0}^{(s)} = \underset{(\rho)}{-0.98} \underset{(\omega)}{-14.37} \underset{(\phi)}{-0.06} \simeq -15.4 . \qquad (10)$$

Analogously one finds for the s-channel contribution in the fixed $u=\mu^2$ sum rule

$$(\alpha-\beta)_{\pi^\pm}^{[s]} = \underset{(\rho)}{-0.94} + \underset{(A_1)}{2.10} + \underset{(A_2)}{1.41} \simeq 2.6 , \qquad (9')$$

$$(\alpha-\beta)_{\pi^0}^{[s]} = \underset{(\rho)}{-0.94} \underset{(\omega)}{-13.91} \underset{(\phi)}{-0.06} \simeq -14.9 . \qquad (10')$$

Below we display for comparison the results of the evaluation of $(\alpha-\beta)_\pi^{(s),[s]}$ in a narrow-width resonance approximation:

$$(\alpha-\beta)_{\pi^\pm}^{(s)} = \frac{2}{\pi\mu}\left[-\frac{{g_\rho}^2({M_\rho}^2+\mu^2)}{{M_\rho}^2-\mu^2)} + \frac{{g_{A_1}}^2({M_{A_1}}^2+\mu^2)}{{M_{A_1}}^2-\mu^2} + \frac{5}{3}\frac{{g_{A_2}}^2({M_{A_2}}^2+\mu^2)}{{M_{A_2}}^2-\mu^2}\right]$$

$$\simeq(-1.0+3.2+2.3) = 4.5 ,$$

$$(\alpha-\beta)_{\pi^0}^{(s)} = -\frac{2}{\pi\mu}\sum_{R=\rho,\omega,\phi}\frac{{g_R}^2({M_R}^2+\mu^2)}{{M_R}^2-\mu^2} \simeq (-1.0-14.2-0.03) = -15.2 , \qquad (11)$$

$$(\alpha-\beta)_{\pi^\pm}^{[s]} = \frac{2}{\pi\mu}\left\{-\frac{{g_\rho}^2{M_\rho}^2}{{M_\rho}^2-\mu^2} + \frac{{g_{A_1}}^2{M_{A_1}}^2}{{M_{A_1}}^2-\mu^2} + \frac{5}{3}\frac{{g_{A_2}}^2{M_{A_2}}^2}{{M_{A_2}}^2-\mu^2}\frac{{M_{A_2}}^2+3\mu^2}{{M_{A_2}}^2-\mu^2}\right\} \simeq 4.7 ,$$

$$(\alpha-\beta)_{\pi^0}^{[s]} = -\frac{2}{\pi\mu}\sum_{R=\rho,\omega,\phi}\frac{{g_R}^2{M_R}^2}{{M_R}^2-\mu^2} \simeq -15.1 , \qquad (11')$$

$$g_R=\left[\frac{6\pi M_R{}^3\Gamma(R\to\pi\gamma)}{(M_R{}^2-\mu^2)^3}\right]^{1/2}, \quad R=\rho,\omega,\phi,A_1,A_2 .$$

The A_1 contribution has been computed using for the vertex $A_1\pi\gamma$ the expression[13] $2g_{A_1}F_{\mu\nu}\mathcal{F}^{\mu\nu}\phi_\pi$, where ϕ_π, $F_{\mu\nu}$, and $\mathcal{F}^{\mu\nu}$ stand for the pion, the electromagnetic, and the A_1 fields.

Before discussing the t-channel pieces $(\alpha-\beta)_\pi^{(t),[t]}$, we mention that saturation of the known sum rule

$$\alpha+\beta=\frac{1}{2\pi^2}\int_{\omega_0=3\mu/2}\frac{\sigma_T(\omega)d\omega}{\omega^2} \tag{12}$$

(σ_T=the total cross section for photoabsorption on pions) by retaining only $E1$, $M1$, and $M2$ transitions and proceeding as we did above in connection with $(\alpha-\beta)_\pi^{(s),[s]}$, yields the (almost certainly underestimated) values

$$(\alpha+\beta)_{\pi^\pm}\simeq 0.2\ , \quad (\alpha+\beta)_{\pi^0}\simeq 0.5\ . \tag{13}$$

Saturation of Eq. (12) in narrow-width approximation (with the same set of intermediate states) leads to

$$(\alpha+\beta)_{\pi^\pm}\simeq 0.2\ , \quad (\alpha+\beta)_{\pi^0}\simeq 0.9\ . \tag{13'}$$

The evaluation of the t-channel contributions $(\alpha-\beta)_\pi^{(t),[t]}$ is a much more delicate task and at least reasonable knowledge of the appearing $\gamma\gamma\to\pi\pi$ amplitudes is needed. The $\gamma\gamma\to\pi\pi$ process has been investigated by several groups from measurements of the colliding-beam reaction $e^+e^-\to e^+e^-\pi\pi$ (see the review[3]). The dominant feature observed is a strong signal from the f(1270 MeV) $[I^G(J^P)C_n=0^+(2^+)_+]$ meson; so far no trace of the ϵ($\simeq$700 MeV) meson seems to appear; the TASSO group provides the limit $\Gamma(\epsilon\to\gamma\gamma)<1.5$ keV for the radiative width of the ϵ meson.[14] The Crystal Ball group at SPEAR has found, by studying the angular distribution of the $f\to\pi^0\pi^0$ decay, that the production of the f(1270 MeV) meson in $\gamma\gamma$ scattering is strongly dominated by photon pairs with opposite helicity,

$$\Gamma(f\to\gamma\gamma)\simeq 3\ \text{keV}\ ,$$
$$\Gamma(f\to\gamma(+)\gamma(+))<<\Gamma(f\to\gamma(+)\gamma(-))\ ,$$

thus confirming previous theoretical expectations.[15] Since the helicity channel of interest to us is that with both photons of helicity $+1$ [see Eq. (4)], the general scheme which seems to be required for our purposes is to consider $(\alpha-\beta)_\pi^{(t),[t]}$ as dominated by $I=J=0$ contributions and employ a model for the $(I=J=0)\gamma\gamma\to\pi\pi$ amplitude which does not deviate too much from its corresponding Born (quantum electrodynamical) expression. The simplest way of satisfying these demands would be to use for the absorptive part $M^{(t)}$ in Eqs. (3) and (3′) an effective $\epsilon(0^+)$ Breit-Wigner model assuming a large total width for this resonance[16]:

$$M^{(t)}(t)^{(I=J=0)}\simeq\tfrac{3}{2}g_{\epsilon\gamma\gamma}g_{\epsilon\pi\pi}\left[-\frac{4}{M_\epsilon{}^2}\right]\frac{\left[\dfrac{t-4\mu^2}{M_\epsilon{}^2-4\mu^2}\right]^{1/2}M_\epsilon\Gamma}{(M_\epsilon{}^2-t)^2+M_\epsilon{}^2\Gamma^2\left[\dfrac{t-4\mu^2}{M_\epsilon{}^2-4\mu^2}\right]}\ , \tag{14}$$

$$g_{\epsilon\gamma\gamma}=4[\pi M_\epsilon\Gamma(\epsilon\to\gamma\gamma)]^{1/2}\ , \quad g_{\epsilon\pi\pi}=4M_\epsilon\left[\frac{2\pi}{3}\frac{\Gamma(\epsilon\to\pi\pi)}{(M_\epsilon{}^2-4\mu^2)^{1/2}}\right]^{1/2}, \quad \Gamma\simeq\Gamma(\epsilon\to\pi\pi)\ .$$

Recalling the relationship between charge and isospin labels,

$$M^{(\pi^\pm)}=\tfrac{2}{3}[M^{(I=0)}+\tfrac{1}{2}M^{(I=2)}]\ ,$$
$$M^{\pi^0}=\tfrac{2}{3}[M^{(I=0)}-M^{(I=2)}]\ , \tag{15}$$

taking $M_\epsilon\simeq 660$ MeV, $\Gamma_{\text{total}}\simeq\Gamma(\epsilon\to\pi\pi)\simeq 640$ MeV, $\Gamma(\epsilon\to\gamma\gamma)\simeq 1.3$ keV, and integrating over t in Eqs. (3) and (3′) from $4\mu^2$ to ∞, one finds

$$(\alpha-\beta)_{\pi^\pm,\pi^0}^{(t),[t];(\epsilon)}\simeq 8.3\ . \tag{16}$$

To what extent this value is representative of the actual $(\alpha-\beta)_\pi^{(t),[t]}$ is hard to say. In the following we shall confine ourselves to the more modest task of computing the $(I=J=0)$ contribution to $(\alpha-\beta)_\pi^{(t),[t]}$ coming only from the elastic-unitarity region $4\mu^2\le t\le 16\mu^2$ of the t-channel cut, leaving open the question of higher waves and higher than $\pi\pi$ state contributions to the unitarity sum. The partial waves $g_+^{(I=J=0)}(t)$ and $h^{(I=J=0)}(t)$ entering Eqs. (3a) and (3′a) are taken as given by the resonance model devised in the last work of Ref. 6 (for details see Ref. 17). We recall that although very crude, this model incorporates (approximately) the experimental knowledge of the $\pi\pi\to\pi\pi(I=J=0)$ phase shift and worked apparently well in connection with the proton polarizabilities and proton Compton scattering. We have found

$$\begin{aligned}(\alpha-\beta)_{\pi}^{(t),\pi\pi(I=J=0)} &= (\alpha-\beta)_{\pi}^{[t],\pi\pi(I=J=0)} \\ &= -\frac{1}{2\pi^2\mu}\int_{4\mu^2}^{16\mu^2}\frac{dt}{t^2}g_+^{(I=J=0)}(t)h^{*(I=J=0)}(t) \\ &\simeq 3.7 \end{aligned} \quad (17)$$

and hence, taking into account Eqs. (15),

$$(\alpha-\beta)_{\pi^\pm,\pi^0}^{(t),[t]}(\pi\pi;s\text{ wave};4\mu^2\le t\le 16\mu^2)\simeq 2.5\ . \quad (18)$$

Although it has to be regarded with some caution, this value for the s-wave contribution of the elastic-unitarity portion of the t-channel cut should be typical for situations in which the amplitude $g_+^{(I=J=0)}(t)$ does not differ too much from its Born approximation and does not have a zero in the immediate vicinity of the threshold. The amplitude g_+ actually employed by us develops a zero at $t\simeq 21\mu^2$; in order to keep the whole amplitude $g_+(t)$ close to its Born approximation, an arbitrary subtraction constant, appearing in the N/D equations which determine it, has been fixed by demanding that at threshold ($t=4\mu^2$), $g_+\simeq g_+^{\text{Born}}$.

The authors of Ref. 18 remove an analogous ambiguity by relating the subtraction constant to $(\alpha-\beta)_{\text{proton}}$ in the context of a backward sum rule for the latter. Their resulting partial wave $g_+(t)$ in the region $4\mu^2\le t\le 16\mu^2$ does not seem to differ too much from ours.

Strictly speaking, since the $(I=J=0)$ t-channel piece is the same in both the fixed $\theta=180°$ and fixed $u=\mu^2$ sum rules [Eqs. (3a) and (3′a)], other waves should also be included in $(\alpha-\beta)^{(t)}$ and $(\alpha-\beta)^{[t]}$ to avoid inconsistencies between the two sum rules [as seen from Eqs. (9), (9′), (10), and (10′), $(\alpha-\beta)^{(s)}$ and $(\alpha-\beta)^{[s]}$ although practically equal for $\pi^\pm$ differ somewhat in the π^0 case]. If the large uncertainties affecting the $I=J=0$ contribution could be removed, one may try to use simultaneously the two sum rules in order to constrain less reliable contributions from higher waves.

The large model dependence of the t-channel contributions in the sum rules discussed here has as its correspondence in the FFESR approach presented in Ref. 9 the equally large uncertainties affecting the high-energy asymptotic contributions which account for the same annihilation-channel effects by means of Regge-pole exchanges. It is worth noting that for the combination (of t-channel isospin $I=2$) $(\alpha-\beta)_{\pi^\pm}-(\alpha-\beta)_{\pi^0}$, almost entirely dependent only upon s-channel effects, both our results and that of Ref. 9 agree remarkably well with each other. Indeed, one finds $(\alpha-\beta)_{\pi^\pm}-(\alpha-\beta)_{\pi^0}\simeq 18$ [from Eqs. (9) and (10)]; $\simeq 17.5$ [from Eqs. (9′) and (10′)]; $\simeq 20$ [from Eqs. (11) and (11′)] in this work, while Table VI of Ref. 9 gives the value $\simeq 20.5$.

We conclude with the remark that for a reliable calculation of the annihilation-channel contributions to the pion polarizabilities which would permit a good prediction not only for $\alpha_{\pi^\pm}-\alpha_{\pi^0}$, but for $\alpha_{\pi^\pm}$ and α_{π^0} separately, further more detailed experimental investigation of the $\gamma\gamma\to\pi\pi$, $\gamma\gamma\to\pi\pi\pi\pi$, $\gamma\gamma\to K\bar{K}$, and $\gamma\gamma\to p\bar{p}$ reactions is needed in order to obtain the necessary information on the helicity channel $\lambda=0$ (both photons with equal helicity) of interest in this context.

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