PHYSICAL REVIEW D VOLUME 26, NUMBER 11 1 DECEMBER 1982

Tensor-meson dominance in the Υ system

H. Genz
Center for Theoretical Studies, University of Miami, Coral Gables, Florida 33124
and Institut für Theoretische Kernphysik der Universität Karlsruhe, D7500 Karlsruhe, Federal Republic of Germany
(Received 1 March 1982)

Tensor-meson dominance is extended to the $\bar{b}b$ system and a sum rule to test the validity of this extension is derived. The sum rule connects the widths $\Gamma(\Upsilon''\to\gamma f_b')$, $\Gamma(\Upsilon''\to\gamma f_b)$, $\Gamma(f_b'\to\gamma\Upsilon')$, $\Gamma(f_b'\to\gamma\Upsilon)$, $\Gamma(\Upsilon'\to\gamma f_b)$, and $\Gamma(f_b\to\gamma\Upsilon)$ to each other (with f_b and f_b' the narrow $\bar{b}b$ tensor mesons predicted by potential models). The corresponding result for the $\bar{c}c$ system is known to predict $\Gamma(f_c(3.55)\to\gamma\psi)=700$ keV proportional to the experimental $\Gamma(\psi'\to\gamma f_c(3.55))$.

I. INTRODUCTION

It is the main assumption of tensor-meson dominance (TMD) that Zweig-rule-allowed matrix elements of the symmetric energy-momentum tensor $T_{\mu\nu}$ are dominated by tensor-meson poles. For a given quark content $(\bar{u}u+\bar{d}d)/\sqrt{2}$, $\bar{s}s$, or $\bar{c}c$ only the ground-state tensor mesons $f(1.27)$, $f'(1.52)$, and $f_c(3.55)$ are known at present. Using these to saturate the corresponding matrix elements, various predictions on amplitudes involving them have been made.[1–6] These are collected, discussed, and compared to experiment in Table I of Ref. 6. With a few exceptions, the predictions of TMD are in reasonable agreement with experiment.

In this paper, TMD is extended to the Υ system. Potential models imply[7] that there are two narrow $\bar{b}b$ tensor mesons f_b and f_b' below the threshold for explicit b flavor. Since obviously both might contribute to Zweig-rule-allowed $\bar{b}b$ matrix elements of $T_{\mu\nu}$, the results of TMD on the ψ system with only one narrow $\bar{c}c$ tensor meson—the $f_c(3.55)$—are modified in a nontrivial way. In particular, TMD predicts the width $\Gamma(f_c(3.55)\to\gamma-\psi)$ in terms of $\Gamma(\psi'\to\gamma f_c(3.55))$ if one saturates the t dependences $[t=(p-q)^2]$ of the matrix elements

$$\langle\psi(p)\,|\,T_{\mu\nu}\,|\,\psi(q)\rangle$$

and

$$\langle\psi'(p)\,|\,T_{\mu\nu}\,|\,\psi'(q)\rangle\ ,$$

with the only narrow $\bar{c}c$ tensor meson, the $f_c(3.55)$. As a generalization of this result to the Υ system, we will obtain the sum rule in Eq. (19) which connects the partial widths of the various γ transitions between the three vector mesons $V_a\equiv\Upsilon,\Upsilon',\Upsilon''$ and narrow tensor mesons $T_\xi\equiv f_b,f_b'$ of types $V_a\to\gamma T_\xi$ and $T_\xi\to\gamma V_a$ to each other. Further possible tests of the scheme are also pointed out. For the sake of generality, we derive our results for an arbitrary number N of narrow tensor mesons and $N+1$ vector mesons.

The spectrum is assumed to be similar to the $\bar{b}b$ spectrum expected in potential models.[7] Namely, denoting by m_ξ^T (with $\xi=1,\dots,N$) the masses of the N narrow tensor mesons and by m_a^V (with $a=1,\dots,N+1$) the masses of the $N+1$ vector mesons, we assume

$$m_1^V<m_1^T<\cdots<m_N^T<m_{N+1}^V\ .$$

For the Υ system, $N=2$ and the threshold for explicit b flavor lies above $m_{\Upsilon''}$ according to experiment[8] and below the mass of a possible f_b'' according to theoretical models.[7] We will consider the transitions $V_a\to\gamma T_\xi$ for $\xi\le a-1\le N$ and $T_\xi\to\gamma V_a$ for $a\le\xi\le N$. The assumptions to be used are TMD and vector-meson dominance (VMD).

II. DERIVATION OF THE SUM RULE AND CONCLUSIONS

We consider the matrix elements of $\epsilon_\xi^{\mu\nu}T_{\mu\nu}$ (with $\epsilon_\xi^{\mu\nu}$ the spin-2 polarization tensor of the tensor meson T_ξ) between states of a $\bar{b}b$ vector meson V_a (the ϵ_p,ϵ_q are the spin-1 polarization tensors):

$$\langle V_a(p)\,|\,\epsilon_\xi^{\mu\nu}T_{\mu\nu}\,|\,V_a(q)\rangle = [F_1^a(\epsilon_p^*\cdot\epsilon_q)+F_2^a(\epsilon_p^*\cdot q)(\epsilon_q\cdot p)][(p+q)\cdot\epsilon_\xi\cdot(p+q)]$$
$$+2F_3^a\{(\epsilon_p^*\cdot q)[\epsilon_q\cdot\epsilon_\xi\cdot(p+q)]+(\epsilon_q\cdot p)[\epsilon_p^*\cdot\epsilon_\xi\cdot(p+q)]\}-2F_4^a\epsilon_p^*\cdot\epsilon_\xi\cdot\epsilon_q$$
$$+2F_7^a\{(\epsilon_p^*\cdot q)[\epsilon_q\cdot\epsilon_\xi\cdot(p+q)]-(\epsilon_q\cdot p)[\epsilon_p^*\cdot\epsilon_\xi\cdot(p+q)]\}\ . \qquad (1)$$

The form factors F_1^a, F_2^a, F_3^a, F_4^a, and F_7^a are functions of p^2, q^2, and t. Obviously, F_7^a is antisymmetric under $p^2 \leftrightarrow q^2$ whereas the other four form factors are symmetric. Thus

$$F_7^a(p^2,q^2=p^2,t)=0\ , \qquad (2)$$

which implies that F_7^a vanishes if both vector mesons are on their mass shell, i.e., $p^2=q^2=(m_a^V)^2$.

The hadronic Hamiltonian H is

$$H=\int d^3x\ T_{00}(x) \qquad (3a)$$

and the third component of the total angular momentum M_3 is

$$M_3=\int d^3x[x_1T_{02}(x)-x_2T_{01}(x)]\ . \qquad (3b)$$

From this, one easily derives[1]

$$F_1^a(p^2=(m_a^V)^2,q^2=(m_a^V)^2,t=0)=-\tfrac{1}{2} \qquad (4a)$$

and

$$F_3^a=\tfrac{1}{2} \qquad (4b)$$

at the same kinematical point as in Eq. (4a).

Since one-particle states $|z\rangle$ diagonalize the Hamiltonian, $\langle z'|H|z\rangle$ vanishes if the particles z and z' differ. Accordingly, in any effective canonical theory the part of $T_{\mu\nu}$ which is bilinear in the physical particle fields is at the same time diagonal in these fields. Thus one assumes in TMD that matrix elements of the type $\langle z'|T_{\mu\nu}|z\rangle$ are negligible as compared to $\langle z|T_{\mu\nu}|z\rangle$ if the particles z and z' differ from each other. Using this property, we will argue that F_7^a approximately vanishes identically.

For $p^2=(m_a^V)^2$ and arbitrary q^2 and t we define functions $G_{1,2,3,4,7}^{a,\xi}$ as

$$G_{1,2,3,4,7}^{a,\xi}=[(m_\xi^T)^2-t]\frac{e}{2\gamma_a g_\xi}F_{1,2,3,4,7}^a \qquad (5)$$

with

$$\Gamma(V_a\to\text{electron,positron})=\pi m_a^V\alpha^2/3\gamma_a^2\ ,$$

where $\alpha=e^3/4\pi=\frac{1}{137}$ and g_ξ is defined by ($|0\rangle$ is the hadronic vacuum)

$$\langle 0|T^{\mu\nu}|T_\xi\rangle=g_\xi\epsilon_\xi^{\mu\nu}\ . \qquad (6)$$

To define the p^2 and q^2 dependence of the functions $F_{1,2,3,4,7}^a$, we use the electromagnetic current as an interpolating field to take the vector mesons off their mass shells. Therefore, the amplitudes

$$G_{1,2,3,4,7}^{a,\xi}(p^2=(m_a^V)^2,\ q^2=0,\ t=(m_\xi^T)^2)$$

for $\xi\le a-1$ describe the transitions $V_a\to\gamma T_\xi$ and for $a\le\xi$ describe the transitions $T_\xi\to\gamma V_a$. If we replace at this kinematical point ϵ_q^μ by q^μ, the total matrix element in Eq. (1) must vanish due to gauge invariance. This requires

$$(p\cdot q)G_2^{a,\xi}=-G_1^{a,\xi}-G_3^{a,\xi}+G_7^{a,\xi} \qquad (7a)$$

and

$$G_4^{a,\xi}=2(p\cdot q)(G_3^{a,\xi}+G_7^{a,\xi}) \qquad (7b)$$

at the decay points. We therefore may write the decay widths as

$$\Gamma(V_a\to\gamma T_\xi)=\frac{(m_a^V)^2-(m_\xi^T)^2}{24\pi(m_a^V)^3}\sum_{\sigma=0}^{2}(B_\sigma^{a,\xi})^2 \qquad (8a)$$

and

$$\Gamma(T_\xi\to\gamma V_a)=\frac{(m_\xi^T)^2-(m_a^V)^2}{40\pi(m_\xi^T)^3}\sum_{\sigma=0}^{2}(B_\sigma^{a,\xi})^2 \qquad (8b)$$

with the helicity amplitudes $B_{\sigma=0,1,2}^{a,\xi}$ involving the tensor meson T_ξ with helicities σ given by

$$B_0^{a,\xi}=\frac{2}{\sqrt{6}}\,|(m_a^V)^2-(m_\xi^T)^2|\left[G_3^{a,\xi}+G_7^{a,\xi}-\frac{(m_a^V)^2-(m_\xi^T)^2}{(m_\xi^T)^2}G_1^{a,\xi}\right],$$

$$B_1^{a,\xi}=\sqrt{2}\,|(m_a^V)^2-(m_\xi^T)^2|\,\frac{m_a^V}{m_\xi^T}\left[G_3^{a,\xi}+\left(\frac{m_\xi^T}{m_a^V}\right)^2G_7^{a,\xi}\right],\quad B_2^{a,\xi}=2\,|(m_a^V)^2-(m_\xi^T)^2|\,(G_3^{a,\xi}+G_7^{a,\xi}) \qquad (9)$$

at $p^2=(m_a^V)^2$, $q^2=0$, and $t=(m_\xi^T)^2$.

We argue that $G_7^{a,\xi}$ approximately vanishes. If p^2 is fixed at $(m_a^V)^2$, the VMD saturation of F_7^a with vector-meson poles in q^2 can [due to Eq. (2)] only be nonvanishing if $T_{\mu\nu}$ has nonvanishing matrix elements $\langle V_a | T_{\mu\nu} | V_{a'}\rangle$ between states of *different* vector mesons V_a and $V_{a'}$. This is, however, excluded by the assumptions of TMD. The width formulas, therefore, become

$$\Gamma(V_a\to\gamma T_\xi)=\frac{[(m_a^V)^2-(m_\xi^T)^2]^3}{12\pi(m_a^V)^3}\tilde{\Gamma}^{a,\xi} \qquad (10a)$$

and

$$\Gamma(T_\xi\to\gamma V_a)=\frac{[(m_\xi^T)^2-(m_a^V)^2]^3}{20\pi(m_\xi^T)^3}\tilde{\Gamma}^{a,\xi} \qquad (10b)$$

with

$$\tilde{\Gamma}^{a,\xi}=\left[\frac{10}{3}+\frac{(m_a^V)^2-(m_\xi^T)^2}{(m_\xi^T)^2}\right](G_3^{a,\xi})^2+\frac{2}{3}\frac{(m_a^V)^2-(m_\xi^T)^2}{(m_\xi^T)^2}(-G_1^{a,\xi})G_3^{a,\xi}+\frac{1}{2}\frac{(m_a^V)^2-(m_\xi^T)^2}{(m_\xi^T)^2}(G_1^{a,\xi})^2 . \qquad (10c)$$

In the present application, this expression can be simplified further. Namely, since

$$\epsilon\equiv\left|\frac{(m_a^V)^2-(m_\xi^T)^2}{(m_\xi^T)^2}\right|\lesssim\frac{2(m_{\Upsilon''}-m_\Upsilon)}{m_{\Upsilon'}}\approx 0.2 ,$$

first, we may neglect ϵ as compared to $\frac{10}{3}$ in the main term. Second, since $G_3^{a,\xi}$ and $G_1^{a,\xi}$ presumably are of the same order of magnitude [Eqs. (4)], we may neglect all terms proportional to ϵ and arrive at

$$\tilde{\Gamma}^{a,\xi}=\tfrac{10}{3}(G_3^{a,\xi})^2 . \qquad (11)$$

Saturating the t dependence of F_3^a by the tensor-meson poles at $t=(m_\xi^T)^2$ with residues $(m_\xi^T)^2\tilde{F}_3^{a,\xi}$, i.e., writing

$$F_3^a(p^2=(m_a^V)^2,q^2,t)=\sum_{\xi=1}^{N}\frac{(m_\xi^T)^2\tilde{F}_3^{a,\xi}}{(m_\xi^T)^2-t} \qquad (12)$$

we arrive at the sum rule

$$\sum_{\xi=1}^{N}\tilde{F}_3^{a,\xi}=\tfrac{1}{2} \qquad (13)$$

for the physical $V_aT_\xi\gamma$ coupling.[9] (F_3^a is independent of q^2 according to VMD.) Except for their signs, the $\tilde{F}_3^{a,\xi}/g_\xi$ are given by measurable quantities as

$$Z^{a,\xi}\equiv\pm\frac{6}{(m_\xi^T)^2}\left[\frac{(m_a^V)^3\gamma_a{}^2\Gamma(V_a\to\gamma T_\xi)}{10\alpha[(m_a^V)^2-(m_\xi^T)^2]^3}\right]^{1/2}=\frac{\tilde{F}_3^{a,\xi}}{g_\xi} \qquad (14a)$$

for $a>\xi$ and

$$Z^{a,\xi}\equiv\pm\left[\frac{6\gamma_a{}^2\Gamma(T_\xi\to\gamma V_a)}{\alpha[(m_\xi^T)^2-(m_a^V)^2]^3m_\xi^T}\right]^{1/2}=\frac{\tilde{F}_3^{a,\xi}}{g_\xi} \qquad (14b)$$

for $\xi\le a$.

In order to obtain our sum rule, we substitute the $N(N+1)$ observables $Z^{a,\xi}$ for the $\tilde{F}_3^{a,\xi}$ in Eq. (13) and obtain

$$\sum_{\xi=1}^{N}g_\xi Z^{a,\xi}=\tfrac{1}{2} . \qquad (15)$$

These are $N+1$ linear inhomogeneous equations for the N unknowns g_ξ. Then we expect one relation between the $Z^{a,\xi}$. To derive it, we substract Eq. (15) for $a+1$ from Eq. (15) for a (with $a=1,\ldots,N$) and find the N homogeneous relations

$$\sum_{\xi=1}^{N}(Z^{a,\xi}-Z^{a+1,\xi})g_\xi=0,\quad a=1,\ldots,N . \qquad (16)$$

For these to have a nontrivial solution we must have

$$\det_{\substack{a=1,\ldots,N\\ \xi=1,\ldots,N}}(Z^{a,\xi}-Z^{a+1,\xi})=0 . \qquad (17)$$

This is the desired sum rule for an arbitrary $\bar{Q}Q$ system which fulfills our assumptions concerning the mass spectrum.

For $N=1$, the $\bar{c}c$ system, we have explicitly

$$\Gamma(f_c(3.55)\to\gamma\psi)=\frac{3}{5}\left[\frac{m_{f_c}{}^2-m_\psi{}^2}{m_{\psi'}{}^2-m_{f_c}{}^2}\right]^3\left[\frac{m_{\psi'}}{m_{f_c}}\right]^3\times\left[\frac{\gamma_{\psi'}}{\gamma_\psi}\right]^2\Gamma(\psi'\to\gamma f_c) . \qquad (18)$$

The exact formula[3] contains an additional factor

$$(m_{\psi}{}^4+3m_{\psi}{}^2m_{f_c}{}^2+6m_{f_c}{}^4)\times(m_{\psi'}{}^4+3m_{\psi'}{}^2m_{f_c}{}^2+6m_{f_c}{}^4)^{-1}=0.85\ .$$

Turning to the $\bar{b}b$ system, the signs of the $Z^{a,\xi}$ are obviously not determined by Eqs. (14). It is most plausible, however, that they all agree. Namely, the contributions of vector mesons to the $T_{\mu\nu}^{\text{canonical}}$ of any effective canonical theory all have the same sign. In such a theory, the sources $(\Box+m_{\xi}^{T})\theta_{\mu\nu}^{\xi}$ of the tensor meson fields $\theta_{\mu\nu}^{\xi}$ are taken to be proportional to $T_{\mu\nu}^{\text{canonical}}$ and therefore the signs of the $\tilde{F}_3^{a,\xi}$ do not depend on a for any given ξ. Assuming that renormalization effects do not change these signs, we can choose the signs of the g_{ξ} by a phase convention for the states $|T_{\xi}\rangle$ in such a way that all the $Z^{a,\xi}$ are positive. Thus we obtain as our main result the sum rule

$$\begin{aligned}
&\left[\frac{10\gamma_{\Upsilon}{}^2\gamma_{\Upsilon'}{}^2\Gamma(f_b\to\gamma\Upsilon)\Gamma(f_b'\to\gamma\Upsilon')}{6(m_{f_b}{}^2-m_{\Upsilon}{}^2)^3(m_{f_b'}{}^2-m_{\Upsilon'}{}^2)^3}\right]^{1/2}+\left[\frac{6\gamma_{\Upsilon'}{}^2\gamma_{\Upsilon''}{}^2m_{\Upsilon'}{}^3m_{\Upsilon''}{}^3\Gamma(\Upsilon'\to\gamma f_b)\Gamma(\Upsilon''\to\gamma f_b')}{10m_{f_b}{}^3m_{f_b'}{}^3(m_{\Upsilon'}{}^2-m_{f_b}{}^2)^3(m_{\Upsilon''}{}^2-m_{f_b'}{}^2)^3}\right]^{1/2}\\
&\quad+\left[\frac{\gamma_{\Upsilon}{}^2\gamma_{\Upsilon''}{}^2m_{\Upsilon''}{}^3\Gamma(\Upsilon''\to\gamma f_b)\Gamma(f_b'\to\gamma\Upsilon)}{m_{f_b}{}^3(m_{\Upsilon''}{}^2-m_{f_b}{}^2)^3(m_{f_b'}{}^2-m_{\Upsilon}{}^2)^3}\right]^{1/2}=\left[\frac{\gamma_{\Upsilon'}{}^2\gamma_{\Upsilon}{}^2m_{\Upsilon'}{}^3\Gamma(\Upsilon'\to\gamma f_b)\Gamma(f_b'\to\gamma\Upsilon)}{m_{f_b}{}^3(m_{\Upsilon'}{}^2-m_{f_b}{}^2)^3(m_{f_b'}{}^2-m_{\Upsilon}{}^2)^3}\right]^{1/2}\\
&\qquad+\left[\frac{\gamma_{\Upsilon''}{}^2\gamma_{\Upsilon'}{}^2m_{\Upsilon''}{}^3\Gamma(\Upsilon''\to\gamma f_b)\Gamma(f_b'\to\gamma\Upsilon')}{m_{f_b}{}^3(m_{\Upsilon''}{}^2-m_{f_b}{}^2)^3(m_{f_b'}{}^2-m_{\Upsilon'}{}^2)^3}\right]^{1/2}\\
&\qquad+\left[\frac{\gamma_{\Upsilon}{}^2\gamma_{\Upsilon''}{}^2m_{\Upsilon''}{}^3\Gamma(\Upsilon''\to\gamma f_b')\Gamma(f_b\to\gamma\Upsilon)}{m_{f_b'}{}^3(m_{\Upsilon''}{}^2-m_{f_b'}{}^2)^3(m_{f_b}{}^2-m_{\Upsilon}{}^2)^3}\right]^{1/2}
\end{aligned}\tag{19}$$

for the $V_a\to\gamma T_{\xi}$ and $T_{\xi}\to\gamma V_a$ widths. The sum rule contains only measurable quantities.

We conclude with a remark. Just as for the $\bar{c}c$ system,[3] the matrix elements of $T_{\mu\nu}$ in Eq. (6), i.e., the g_{ξ}, can be computed in analogy to the conventional computation of the γ_V in potential models. Namely, in QCD the $\bar{q}q$ part of $T_{\mu\nu}$ is

$$T_{\mu\nu}^{\text{QCD},\bar{q}q}=\frac{i}{4}:\bar{q}(\gamma_{\mu}\overleftrightarrow{\partial}_{\nu}+\gamma_{\nu}\overleftrightarrow{\partial}_{\mu})q\ :$$

and inserting this into Eq. (6) one finds

$$g_{\xi}{}^2=\frac{9m_{\xi}^{T}}{\pi}\,|\,\phi_{\xi}'(0)\,|^2$$

with $\phi_{\xi}'(0)$ the derivative of the T_{ξ} wave function at the origin. It is therefore obvious that reliable potential models for the $\bar{b}b$ system would imply further relations between the widths in Eqs. (14). The analogous calculation for the $\bar{c}c$ system,[3] using the $|\,\phi_{f_c(3.55)}'(0)\,|$ of Ref. 10, predicts $\Gamma(\psi'\to\gamma f_c(3.55))=16$ keV, in agreement with the experimental[8] 15 ± 5 keV.

ACKNOWLEDGMENTS

The author would like to thank Professor B. Kursunoglu and Professor A. Perlmutter for the kind hospitality extended to him at the Center for Theoretical Studies. This work was supported in part by Stiftung Volkswagenwerk.

[1]B. Renner, Phys. Lett. 33B, 599 (1970); B. Renner, Nucl. Phys. B30, 634 (1971).
[2]W. Gampp and H. Genz, Phys. Lett. 76B, 319 (1978).
[3]W. Gampp and H. Genz, Phys. Lett. 79B, 267 (1978).
[4]Kl. Bongardt, W. Gampp, and H. Genz, Z. Phys. C 3, 233 (1980); W. Gampp and H. Genz, *ibid.* 1, 199 (1979).
[5]For reviews compare H. Genz, in *Quarks and Leptons as Fundamental Particles,* proceedings of the XVIII Internationale Universitatswochen für Kernphysik, Schladming, Austria, 1979, edited by P. Urban (Springer, Vienna, 1979) [Acta Phys. Austriaca, Suppl.

21, 559 (1979)]; W. Gampp, Thesis, Karlsruhe, 1978 (unpublished).
[6]H. Genz and S. Mallik, Fortschr. Phys. (to be published).
[7]E. Eichten and K. Gottfried, Phys. Lett. 66B, 286 (1979).
[8]Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).
[9]The sum rule is derived from Eq. (4b). An analogous sum rule for the contributions to F_1^a obviously follows from Eq. (4a).
[10]R. Barbieri, R. Gatto, and R. Kögerler, Phys. Lett. 60B, 183 (1976).