Symmetry breaking and higher representations in the Cabibbo theory

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The most recent data for semileptonic hyperon decays are compared with the Cabibbo theory. Two significant deviations from the Cabibbo-theory predictions have been observed. The deviation for Γ in $\Sigma^- \to \Lambda e\nu$ can be the signal of the symmetry breaking. The discrepancy for α_e in $\Sigma^- \to ne\nu$ cannot be explained by the first-order symmetry breaking or by admixture of higher representations in the axial-vector current.

I. INTRODUCTION

With the advent of new experiments in hyperon semileptonic decays (HSD) it has been possible to test the standard Cabibbo theory[1] (CT) much more rigorously than before.[2] A very important consequence of this recent test is that the predictive power of the CT is clearly established. For example, if only the available transition rates are used the CT has limited predictive power: the three parameters involved (F, D, and θ) still have enough freedom left to change dramatically.[2] However when the electron-neutrino angular-correlation coefficients are used, then F, D, and θ acquire stable values, which do not change when other data are added to the fit of the CT. The other pieces of data then provide a test of the CT. Precisely because of the above features it is possible to establish deviations from the CT predictions. In particular, two major deviations[3,4] have been established in the currently available data.[5–7] Although both derivations are statistically speaking highly significant they may have quite different implications for the CT.

The deviation from the experimental transition rate in $\Sigma^- \to \Lambda e\nu$ is big because the error bars are quite small while the central value is not too far from the theoretical prediction. In contrast the deviation in the electron-spin asymmetry in $\Sigma^- \to ne\nu$ comes from the central value being far away from the CT prediction with ample error bars.

The first deviation may be a signal of first-order SU(3)-symmetry-breaking corrections to the CT, which were expected to occur eventually since the CT was never intended to be exact. The second deviation may be challenging[8] the SU(3)-octet hypothesis for the axial-vector current A_μ. It could mean either that other (10, $\overline{10}$, and 27) representations are present in A_μ or that the octet hypothesis is masked by strong symmetry breaking.

After updating the status of the CT by incorporating the most recent data (Sec. II), we shall study the effect of the first-order symmetry breaking in CT, which is what actually puts the CT in its best predictive position (Sec. III). Next we shall study a modification of the CT by incorporating higher representations to A_μ (Sec. IV). Finally, we shall discuss our results and draw conclusions (Sec. V).

II. CURRENT STATUS OF THE CABIBBO THEORY

We have seen before[2] that in order to compare the CT with HSD data it is important to include the radiative corrections and the q^2 dependence[9] of the leading vector and axial-vector form factors. New data have been published after our earlier analysis; we now include them in our comparison of the CT to experiment, including all the corrections discussed in Ref. 2. In addition to updating the present status of the CT we also set a point of reference for our later discussion. The results can be found in Table I.

We have taken into account all of the new data of the WA2 collaboration[3] and the recently published[6] new world average for α_e in $\Sigma^- \to ne\nu$.

The new data have a negative effect upon the

TABLE I. Comparison of the data for HSD with the standard Cabibbo model. The experimental data are from Refs. 5–7. The α asymmetries (Ref. 10) A and B in $\Sigma^- \to \Lambda e\nu$ and $\Xi^- \to \Lambda e\nu$ were obtained using the same assumptions as in Ref. 5 (see also Ref. 11). Decay rates are in 10^6 sec^{-1} except for neutron decay which is in 10^{-3} sec^{-1}.

Process	Experimental value	Predicted value	Contribution to χ^2
$n \to pe\nu$ (rate)	1.091 ± 0.017	1.068	1.78
$\Sigma^+ \to \Lambda e\nu$ (rate)	0.253 ± 0.059	0.270	0.08
$\Sigma^- \to \Lambda e\nu$ (rate)	0.378 ± 0.018	0.448	15.18
$\Lambda \to pe\nu$ (rate)	3.165 ± 0.053	3.222	1.16
$\Sigma^- \to ne\nu$ (rate)	7.085 ± 0.194	6.839	1.60
$\Xi^- \to \Lambda e\nu$ (rate)	3.244 ± 0.218	2.893	2.57
$\Xi^- \to \Sigma^0 e\nu$ (rate)	0.524 ± 0.122	0.515	0.00
$\Lambda \to p\mu\nu$ (rate)	0.597 ± 0.133	0.601	0.00
$\Sigma^- \to n\mu\nu$ (rate)	3.086 ± 0.271	3.134	0.03
$\Xi^- \to \Lambda\mu\nu$ (rate)	1.580 ± 1.580	0.876	0.20
$n \to pe\nu$ ($\alpha_{e\nu}$)	-0.074 ± 0.004	-0.074	0.03
$n \to pe\nu$ (α_e)	-0.084 ± 0.003	-0.081	0.87
$n \to pe\nu$ (α_ν)	1.001 ± 0.038	0.989	0.10
$\Sigma^+ \to \Lambda e\nu$ ($\alpha_{e\nu}$)	-0.400 ± 0.18	-0.404	0.08
$\Sigma^- \to \Lambda e\nu$ ($\alpha_{e\nu}$)	-0.412 ± 0.062	-0.412	0.00
$\Sigma^- \to \Lambda e\nu$ (A)	0.065 ± 0.072	0.050	0.05
$\Sigma^- \to \Lambda e\nu$ (B)	0.853 ± 0.07	0.897	0.36
$\Lambda \to pe\nu$ ($\alpha_{e\nu}$)	-0.009 ± 0.019	-0.017	0.18
$\Lambda \to pe\nu$ (α_e)	0.125 ± 0.066	0.010	3.00
$\Lambda \to pe\nu$ (α_ν)	0.821 ± 0.06	0.976	6.71
$\Lambda \to pe\nu$ (α_p)	-0.508 ± 0.065	-0.578	1.16
$\Sigma^- \to ne\nu$ ($\alpha_{e\nu}$)	0.279 ± 0.032	0.330	2.58
$\Sigma^- \to ne\nu$ (α_e)	0.26 ± 0.19	-0.620	21.48
$\Xi^- \to \Lambda e\nu$ (A)	0.604 ± 0.12	0.454	1.55
Total value of χ^2			60.75

$F=1.098 \quad D=-1.458 \quad \sin\theta=0.228$

agreement between experiment and the CT, with the χ^2 increasing to 60.7. In addition to the deviation reported before in the $\Lambda \to pe\nu$ spin asymmetries, the contributions of the $\Sigma^- \to \Lambda e\nu$ rate and of the α_e in $\Sigma^- \to ne\nu$ build up to almost $\frac{2}{3}$ of such a high χ^2.

As an illustration of what we meant in the Introduction about establishing the predictive power of the CT, let us compare the new values of F, D, and θ namely,

$$F=1.098, \quad D=-1.458, \quad \sin\theta=0.228 ,$$

with those obtained before,[2]

$$F=1.069 \pm 0.023 ,$$

$$D=-1.490 \pm 0.014 ,$$

$$\sin\theta=0.225 \pm 0.015 ,$$

when the earlier values of rates and electron-neutrino angular coefficients were used. The three new values remain remarkably close to the older ones, despite the addition of the two strongly deviating pieces of data.

III. FIRST-ORDER SYMMETRY BREAKING

Certainly one of the most attractive features of HSD is that they may provide clean experimental evidence on SU(3) symmetry breaking (SB) other than that coming from hyperon mass differences. As we remarked before, the CT was never intended to be exact and deviations from experiment are expected to appear. Therefore, one must first incorporate SB corrections to the CT before one may draw conclusions about its detailed success. Hence, it is most interesting to see how the predictions of the standard CT are changed once first-order SB is taken into account.

The straightforward way to proceed would be to obtain model calculations of SB, add them to the CT, and then compare it with experiment. Unfortunately, this kind of calculation is difficult to perform and only a few are available.[12] Because of the difficulties involved, the model calculations contain very particular assumptions and this kind of approach may not be as general as would be desirable. We shall take a different point of view.

We shall attempt to extract from experiment the SB corrections in a form which is as general as possible. This procedure may allow us to obtain the improved predictions of the CT. Assuming that SB comes from the eighth component of an octet in the strong-interaction Hamiltonian, one can obtain the most general first-order corrections to the axial-vector form factors g_1 and g_2. The SB contributions to g_1 would come from[13]

$$\begin{aligned}
&A_1 \mathrm{Tr}(\{\lambda_i,\lambda_8\}\bar{B}B),\quad B_1\mathrm{Tr}(\bar{B}\{\lambda_i,\lambda_8\}B)\,,\\
&C_1[\mathrm{Tr}(\bar{B}\lambda_i B\lambda_8)-\mathrm{Tr}(\bar{B}\lambda_8 B\lambda_i)]\,,\\
&D_1[\mathrm{Tr}(\bar{B}\lambda_i)\mathrm{Tr}(B\lambda_8)+\mathrm{Tr}(\bar{B}\lambda_8)\mathrm{Tr}(B\lambda_i)]\,,\\
&E_1\mathrm{Tr}(\bar{B}B)\mathrm{Tr}(\lambda_i\lambda_8)\,,
\end{aligned} \tag{1}$$

and to g_2 would come from

$$\begin{aligned}
&A_2\mathrm{Tr}([\lambda_i,\lambda_8]\bar{B}B),\quad B_2\mathrm{Tr}(\bar{B}[\lambda_i,\lambda_8]B)\,,\\
&C_2[\mathrm{Tr}(\bar{B}\lambda_i)\mathrm{Tr}(B\lambda_8)-\mathrm{Tr}(\bar{B}\lambda_8)\mathrm{Tr}(B\lambda_i)]\,.
\end{aligned} \tag{2}$$

This will lead to the following expressions for the axial-vector form factors g_1 and g_2 in terms of F, D, and the new reduced form factors $A_1,\ldots,D_1,A_2,\ldots,C_2$ for the SB contributions:

$$\begin{aligned}
g_1(n\to p)&=\frac{1}{\sqrt{6}}F-\left[\frac{3}{10}\right]^{1/2}D+\frac{2}{\sqrt{3}}(B_1-C_1)\,,\\
g_1(\Sigma^\pm\to\Lambda)&=-\frac{1}{\sqrt{5}}D+\frac{\sqrt{2}}{3}(A_1+B_1+3D_1)\,,\\
g_1(\Lambda\to p)&=-\tfrac{1}{2}F+\frac{1}{2\sqrt{5}}D\\
&\quad+\frac{\sqrt{2}}{6}(-A_1+2B_1+3C_1+6D_1)\,,\\
g_1(\Sigma^-\to n)&=-\frac{1}{\sqrt{6}}F-\left[\frac{3}{10}\right]^{1/2}D\\
&\quad-\frac{1}{\sqrt{3}}(A_1+C_1)\,,\\
g_1(\Xi^-\to\Lambda)&=\tfrac{1}{2}F+\frac{1}{2\sqrt{5}}D\\
&\quad+\frac{\sqrt{2}}{6}(2A_1-B_1-3C_1+6D_1)\,,\\
g_1(\Xi^-\to\Sigma^0)&=\frac{1}{2\sqrt{3}}F-\frac{1}{2}\left[\frac{3}{5}\right]^{1/2}D\\
&\quad+\frac{1}{\sqrt{6}}(-B_1+C_1)
\end{aligned} \tag{3}$$

and

$$\begin{aligned}
g_2(n\to p)&=0\,,\\
g_2(\Sigma^\pm\to\Lambda)&=-\sqrt{2}C_2\,,\\
g_2(\Lambda\to p)&=\sqrt{2}(-\tfrac{1}{2}A_2+B_2+C_2)\,,\\
g_2(\Sigma^-\to n)&=-\sqrt{3}A_2\,,\\
g_2(\Xi^-\to\Lambda)&=\sqrt{2}(A_2-\tfrac{1}{2}B_2-C_2)\,,\\
g_2(\Xi^-\to\Sigma^0)&=-(\tfrac{3}{2})^{1/2}B_2\,.
\end{aligned} \tag{4}$$

The form factors are defined as usual by the hadronic part of the transition matrix element[9]:

$$\bar{u}_B[f_1(q^2)\gamma_\mu+f_2(q^2)i\sigma_{\mu\nu}q^\nu+f_3(q^2)q_\mu+g_1(q^2)\gamma_\mu\gamma_5+g_2(q^2)i\sigma_{\mu\nu}q^\nu\gamma_5+g_3(q^2)q_\mu\gamma_5]u_A\,. \tag{5}$$

Although there should be in all eight new contributions coming from Eqs. (1) and (2), in practice there are only seven. The reason is that the term $\mathrm{Tr}(\bar{B}B)\mathrm{Tr}(\lambda_i\lambda_8)$ is diagonal and cannot contribute to these matrix elements. The new quantities A_1, B_1, C_1, D_1, A_2, B_2, and C_2 in Eqs. (3) and (4) can be used to parametrize the SB in the experimental data, provided none of them becomes too large.

Before going into more detail it is important to discuss further what is meant by first-order SB in the CT. Although one customarily says that the CT assumes that the symmetry limit is a good approximation, in reality SB is introduced to all orders into CT by keeping the physical masses of hyperons, since otherwise the available phase space would be zero and the decays would not take place at all. So what is really meant in the CT by the symmetry limit is that only the form factors are

TABLE II. Comparison of the experimental data for HSD with the Cabibbo model with first-order symmetry breaking. (A) incorporates the corrections in g_1 only, (B) incorporates the corrections in g_2 only, and (C) incorporates the corrections in g_1 and g_2. The parameters $A_1, \ldots, D_1$ and $A_2, \ldots, C_2$ come from Eqs. (3) and (4).

	(A)		(B)		(C)	
Process	Predicted value	Contribution to χ^2	Predicted value	Contribution to χ^2	Predicted value	Contribution to χ^2
$n \to pe\nu$ (rate)	1.083	0.20	1.079	0.54	1.079	0.46
$\Sigma^+ \to \Lambda e\nu$ (rate)	0.228	0.17	0.236	0.08	0.228	0.17
$\Sigma^- \to \Lambda e\nu$ (rate)	0.379	0.00	0.390	0.45	0.379	0.00
$\Lambda \to pe\nu$ (rate)	3.163	0.00	3.223	1.21	3.145	0.13
$\Sigma^- \to ne\nu$ (rate)	7.088	0.00	6.826	1.78	7.173	0.21
$\Xi^- \to \Lambda e\nu$ (rate)	3.242	0.00	2.856	3.17	3.250	0.00
$\Xi^- \to \Sigma^0 e\nu$ (rate)	0.524	0.00	0.534	0.00	0.524	0.00
$\Lambda \to p\mu\nu$ (rate)	0.601	0.00	0.600	0.00	0.601	0.00
$\Sigma^- \to n\mu\nu$ (rate)	3.253	0.38	3.196	0.16	3.410	1.43
$\Xi^- \to \Lambda\mu\nu$ (rate)	0.985	0.14	0.869	0.20	0.921	0.17
$n \to pe\nu$ ($\alpha_{e\nu}$)	−0.076	0.22	−0.075	0.06	−0.077	0.32
$n \to pe\nu$ (α_e)	−0.084	0.01	−0.083	0.06	−0.084	0.07
$n \to pe\nu$ (α_ν)	0.988	0.11	0.989	0.11	0.988	0.11
$\Sigma^+ \to \Lambda e\nu$ ($\alpha_{e\nu}$)	−0.404	0.00	−0.370	0.03	−0.403	0.00
$\Sigma^- \to \Lambda e\nu$ ($\alpha_{e\nu}$)	−0.412	0.00	−0.374	0.37	−0.411	0.00
$\Sigma^- \to \Lambda e\nu$ (A)	0.054	0.02	0.052	0.03	0.054	0.02
$\Sigma^- \to \Lambda e\nu$ (B)	0.898	0.41	0.890	0.28	0.898	0.41
$\Lambda \to pe\nu$ ($\alpha_{e\nu}$)	−0.002	0.12	0.007	0.75	−0.015	0.10
$\Lambda \to pe\nu$ (α_e)	0.018	2.60	−0.006	3.96	0.073	0.61
$\Lambda \to pe\nu$ (α_ν)	0.972	6.38	0.980	7.06	0.939	3.87
$\Lambda \to pe\nu$ (α_p)	−0.580	1.25	−0.592	1.66	−0.543	0.29
$\Sigma^- \to ne\nu$ ($\alpha_{e\nu}$)	0.306	0.73	0.262	0.28	0.294	0.22
$\Sigma^- \to ne\nu$ (α_e)	−0.646	22.73	−0.592	20.09	0.000	1.87
$\Xi^- \to \Lambda e\nu$ (A)	0.609	0.00	0.504	0.69	0.603	0.00
Total value of χ^2		35.47		43.02		10.46
	$F=1.220$		$F=1.195$		$F=1.453$	
	$D=-1.381$		$D=-1.399$		$D=-0.870$	
	$\sin\theta=0.229$		$\sin\theta=0.227$		$\sin\theta=0.241$	
	$A_1=-0.152$		$A_2=0.250$		$A_1=0.143$ $A_2=1.660$	
	$B_1=-0.033$		$B_2=0.165$		$B_1=0.329$ $B_2=1.338$	
	$C_1=-0.036$		$C_2=-0.229$		$C_1=0.164$ $C_2=-0.00$	
	$D_1=0.049$				$D_1=-0.006$	

kept at their symmetry-limit values, while the difference ΔM between the hyperon masses in kept to all orders. Therefore, incorporating first-order SB into HSD in the spirit of the CT means that first-order corrections should be incorporated into $f_1(0)$ and $g_1(0)$ only, while f_2, g_2, and the slopes of the q^2 dependence of f_1 and g_1 should be kept at their symmetry-limit values. Strictly speaking one should add all SB corrections in such a way that the order is well kept. For example, if the q^2 dependence of f_1 and g_1 is introduced, then second-order SB corrections to $f_1(0)$ and $g_1(0)$ should be included and first-order corrections to $f_2(0)$ and $g_2(0)$ should also be included and powers of $(\Delta M)^3$ and higher should be dropped; this would be a rigorous accounting of second-order SB in CT. We shall not pursue this approach in this paper. We shall instead stay close to the original spirit of the CT. Therefore, by first-order SB we mean that we shall keep f_2 at its conserved-vector-current value $g_2=0$ because of the absence of second-class currents[14] and f_1 at its symmetry-limit value because of the Behrends-Sirlin[15] and Ademollo-Gatto[13] theorems. Thus, only the changes in g_1 introduced in Eq. (3) should be considered.

Fitting A_1, B_1, C_1, and D_1 along with F, D, and θ we obtain the results (A) in Table II. They are quite interesting. Comparing the values of these F, D, and θ with those of Table I, we see that they changed very little, meaning that SB is indeed small. Also the values of A_1, B_1, C_1, and D_1 are small enough as to be acceptable as first-order SB. Looking at the new predictions we can observe that this pattern of SB leads to fine readjustments of some quantities. Indeed the rates for $\Sigma^- \to \Lambda e\nu$ and $\Sigma^- \to \Lambda e\nu$ are reduced by $\sim 17\%$ and the rates for $\Xi^- \to \Lambda e\nu$ and $\Xi^- \to \Lambda\mu\nu$ are increased by $\sim 10\%$. $\alpha_{e\nu}$ in $\Sigma^- \to ne\nu$ is decreased by $\sim 9\%$ and α_e in $\Sigma^- \to ne\nu$ and A in $\Xi^- \to \Lambda e\nu$ are increased by $\sim 9\%$ and $\sim 25\%$, respectively. All other changes are fairly small. The most dramatic change is in the rate for $\Sigma^- \to \Lambda e\nu$. It is changed by 3.8 standard deviations and its experimental value is very well reproduced. But, the new value of α_e in $\Sigma^- \to ne\nu$ still remains totally off, as was to be expected.

Because of this we shall consider the incorporations of the g_2 form factors. Although it may not be easy to give an example, it is conceivable that for some reason SB may be stronger in the g_2 terms than elsewhere and thus g_2 could be incorporated while keeping all other form factors as before. Besides it is interesting to see how big a g_2 is required by present data. In Table II we give two more cases (B) and (C). In (B) A_1, B_1, C_1, and D_1 are kept at zero and only A_2, B_2, and C_2 are allowed to vary and in (C) we take the combined effect of the corrections to g_1 and g_2. Except for minor details, fit (B) is very similar to fit (A). In contrast, fit (C) has a very much reduced χ^2 but this requires very large A_1, B_1, and D_1, too large to be considered a manifestation of first-order SB.

Our main conclusion in this section is that small symmetry breaking through the g_1 leads to very good agreement with present HSD data except for the value of α_e in $\Sigma^- \to ne\nu$. Nevertheless, we can only claim that the incorporation of first-order SB into the CT is just consistent with present data. The approach of this section leads to noticeable modifications in the predictions of the original CT, which cannot yet be rigorously tested at present because of the laxity of some pieces of data. For example, the substantial reduction predicted in the $\Sigma^+ \to \Lambda e\nu$ rate requires a much more precise measurement of such a rate. Clearly, with a substantial improvement of the precision of the data it will be possible to extract from the data in a rather general way important information on SB that would be of great use in guiding the theoretical work in this area.

IV. HIGHER REPRESENTATIONS

As we mentioned in the Introduction the new world average of α_e in $\Sigma^- \to ne\nu$ is such that it may challenge[8] the octet hypothesis for the axial-vector current A_μ. As we have just seen in Sec. II, first-order SB contributions to the CT can do practically nothing to bring agreement with the experimental value of α_e. It is conceivable that it is the octet assumption for A_μ that needs revision and not the assumption on the validity of the symmetry limit in the CT. This is the issue we shall study in this section, namely, we shall assume that A_μ may be given by an admixture of higher SU_3 representations, while the symmetry limit is still valid. Therefore, in addition to the octet there may be 10, $\overline{10}$, and 27 in A_μ; i.e., A_μ is given by

$$A_\mu = A_\mu^{(8)} + A_\mu^{(10)} + A_\mu^{(\overline{10})} + A_\mu^{(27)} . \tag{6}$$

The Cabibbo universality assumes that the full weak current is obtained from the $\Delta S = 0$ current through a rotation around the 7th axis in the SU(3) space.[16] We assume that this construction is also valid for higher representations. We thus have

$$A_\mu = e^{-2i\theta_C F_7} \tilde{A}_\mu e^{2i\theta_C F_7} , \tag{7}$$

where

$$\tilde{A}_\mu = A_\mu^{(8)}(0,1,1) + A_\mu^{(10)}(0,1,1) + A_\mu^{(\overline{10})}(0,1,1) + A_\mu^{(27)}(0,1,1) . \tag{8}$$

In Eq. (8) $A_\mu^{(n)}(Y,I,I_3)$ denotes the current that transforms according to the n representation of the SU(3) group and is the (Y,I,I_3) member of this representation. Equation (8) indicates that in addition to the SU(3)-invariant form factors F and D there will be three more form factors F_{10}, $F_{\overline{10}}$, and F_{27}. After performing the rotation (7) of the current (8) we obtain the following form of the weak axial-vector current:

$$A_\mu=-\left[\frac{5}{2}\right]^{1/2}\frac{1+\cos2\theta_C}{2}\sin\theta_C A_\mu^{(27)}(-1,\tfrac{3}{2},\tfrac{3}{2})+\frac{\sqrt{5}}{4}(1-\cos2\theta_C)\cos\theta_C A_\mu^{(27)}(0,2,1)$$
$$+\tfrac{1}{4}(5\cos2\theta_C-1)\cos\theta_C A_\mu^{(27)}(0,1,1)-\left[\frac{5}{6}\right]^{1/2}\sin\theta_C\cos2\theta_C A_\mu^{(27)}(1,\tfrac{3}{2},\tfrac{1}{2})$$
$$+\frac{1}{2\sqrt{6}}\sin\theta_C(3+5\cos2\theta_C)A_\mu^{(27)}(1,\tfrac{1}{2},\tfrac{1}{2})-\left[\frac{5}{2}\right]^{1/2}\frac{1-\cos2\theta_C}{2}\cos\theta_C A_\mu^{(27)}(2,1,0)$$
$$-\sqrt{3}\frac{1+\cos2_C}{2}\sin\theta_C A_\mu^{(\overline{10})}(-1,\tfrac{3}{2},\tfrac{3}{2})+\frac{3\cos2\theta_C-1}{2}\cos\theta_C A_\mu^{(\overline{10})}(0,1,1)$$
$$+\frac{3\cos2\theta_C+1}{2}\sin\theta_C A_\mu^{(\overline{10})}(1,\tfrac{1}{2},\tfrac{1}{2})+\sqrt{3}\frac{1-\cos2\theta_C}{2}\cos\theta_C A_\mu^{(\overline{10})}(2,0,0)$$
$$+\cos\theta_C A_\mu^{(10)}(0,1,1)-\sin\theta_C A_\mu^{(10)}(1,\tfrac{3}{2},\tfrac{1}{2})+\cos\theta_C A_\mu^{(8)}(0,1,1)+\sin\theta_C A_\mu^{(8)}(1,\tfrac{1}{2},\tfrac{1}{2})\ . \tag{9}$$

The matrix elements of the current (9) can be calculated with the help of the Wigner-Eckart theorem. The corresponding Clebsch-Gordan coefficients[17] are compiled in Table III.

In the case of higher representations we have performed several fits. We have first tried separately each of the higher representations, then combinations of each with $A_\mu^{(8)}$, and finally all of them together. In neither one of all these options was there any improvement with respect to α_e in $\Sigma^-\to ne\nu$ found. Therefore we shall only display the results for the last case, they are given in Table IV. Table IV shows that the discrepancy for $\Gamma(\Sigma^-\to\Lambda e\nu)$ disappears, while the deviation for α_e in $\Sigma^-\to ne\nu$ is not improved at all. The values of form factors F_{10}, $F_{\overline{10}}$, and F_{27} are small in comparison with the values of F and D and these latter two do not differ significantly from the values in

TABLE III. SU(3) Clebsch-Gordan coefficients for HSD. The normalizations are as in Ref. 17.

Process	8^F	8^D	10	10*	27 (I)	27 (I)
$n\to p$	$\frac{1}{\sqrt{6}}$	$-(\frac{3}{10})^{1/2}$	$(\frac{2}{15})^{1/2}$	$(\frac{2}{15})^{1/2}$	$\frac{2}{3}(\frac{2}{15})^{1/2}$ (1)	0 (2)
$\Sigma^\pm\to\Lambda$	0	$-\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$-\frac{2}{3\sqrt{5}}$ (1)	0 (2)
$\Lambda\to p$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{5}}$	0	$-\frac{1}{\sqrt{5}}$	$-(\frac{2}{15})^{1/2}$ $(\frac{1}{2})$	0 $(\frac{3}{2})$
$\Sigma^-\to n$	$-\frac{1}{\sqrt{6}}$	$-(\frac{3}{10})^{1/2}$	$(\frac{2}{15})^{1/2}$	$(\frac{2}{15})^{1/2}$	$-\frac{2}{9\sqrt{5}}$ $(\frac{1}{2})$	$-\frac{2}{9}$ $(\frac{3}{2})$
$\Xi^-\to\Lambda$	$\frac{1}{2}$	$\frac{1}{2\sqrt{5}}$	0	$\frac{1}{\sqrt{5}}$	$-(\frac{2}{15})^{1/2}$ $(\frac{1}{2})$	0 $(\frac{3}{2})$
$\Xi^-\to\Sigma^0$	$\frac{1}{2\sqrt{3}}$	$-\frac{3}{2\sqrt{5}}$	$\frac{2}{\sqrt{15}}$	$-\frac{1}{\sqrt{15}}$	$-\frac{1}{9}(\frac{2}{5})^{1/2}(\frac{1}{2})$	$\frac{2}{9}\sqrt{2}$ $(\frac{3}{2})$

TABLE IV. Comparison of the experimental data for HSD with the Cabibbo model with higher representations of SU(3) in the axial-vector current. The parameters F, D, F_{10}, $F_{\overline{10}}$, and F_{27} are the SU(3) reduced form factors of the axial-vector current.

Process	Predicted value	Contribution to χ^2
$n \to pe\nu$ (rate)	1.083	0.22
$\Sigma^+ \to \Lambda e\nu$ (rate)	0.229	0.17
$\Sigma^- \to \Lambda e\nu$ (rate)	0.380	0.01
$\Lambda \to pe\nu$ (rate)	3.164	0.00
$\Sigma^- \to ne\nu$ (rate)	7.079	0.00
$\Xi^- \to \Lambda e\nu$ (rate)	3.206	0.03
$\Xi^- \to \Sigma^0 e\nu$ (rate)	0.609	0.48
$\Lambda \to p\mu\nu$ (rate)	0.601	0.00
$\Sigma^- \to n\mu\nu$ (rate)	3.248	0.36
$\Xi^- \to \Lambda\mu\nu$ (rate)	0.975	0.15
$n \to pe\nu$ ($\alpha_{e\nu}$)	−0.076	0.21
$n \to pe\nu$ (α_e)	−0.084	0.01
$n \to pe\nu$ (α_ν)	0.988	0.11
$\Sigma^+ \to \Lambda e\nu$ ($\alpha_{e\nu}$)	−0.404	0.00
$\Sigma^- \to \Lambda e\nu$ ($\alpha_{e\nu}$)	−0.412	0.00
$\Sigma^- \to \Lambda e\nu$ (A)	0.054	0.02
$\Sigma^- \to \Lambda e\nu$ (B)	0.898	0.42
$\Lambda \to pe\nu$ ($\alpha_{e\nu}$)	−0.004	0.08
$\Lambda \to pe\nu$ (α_e)	0.018	2.64
$\Lambda \to pe\nu$ (α_ν)	0.973	6.41
$\Lambda \to pe\nu$ (α_p)	−0.580	1.24
$\Sigma^- \to ne\nu$ ($\alpha_{e\nu}$)	0.306	0.69
$\Sigma^- \to ne\nu$ (α_e)	−0.647	22.77
$\Xi^- \to \Lambda e\nu$ (A)	0.599	0.00
Total value of χ^2		36.02

$\sin\theta=0.230$ $F=1.123$ $D=-1.389$ $F_{10}=-0.122$ $F_{\overline{10}}=0.034$ $F_{27}=-0.080$

Table I. This scheme cannot be discriminated from the first-order symmetry-breaking schemes of Table II. We thus see that the presence of higher representations in A_μ cannot explain the new world average for α_e in $\Sigma^- \to ne\nu$ either.

V. DISCUSSION

We have compared the most recent data on HSD with various versions of the CT. The standard CT is not able to explain the data well. The strongest discrepancies show up for the rate in $\Sigma^- \to \Lambda e\nu$ and α_e in $\Sigma^- \to ne\nu$. We have considered two generalizations of the Cabibbo model to explain the data. The first modification—a natural one, incorporating first-order SB—improves the agreement significantly and the rate in $\Sigma^- \to \Lambda e\nu$ can be explained very well, while the prediction for α_e in $\Sigma^- \to ne\nu$ still deviates very much from its experimental value. This situation repeats itself in the second modification when admixtures of higher representations in the axial-vector current are considered.

It is a very remarkable fact that none of these two approaches can explain the value of α_e in $\Sigma^- \to ne\nu$. It has probably got to do with the fact that for $\Sigma^- \to ne\nu$ decay there is a discrepancy in the sign and not in the absolute value of the g_1 form factor. The difference in the sign cannot be easily explained by the perturbative schemes nor by small admixtures of higher representations and this is exactly why the discrepancy for α_e in $\Sigma^- \to ne\nu$ always remains.[18] It is, therefore, a very serious discrepancy that requires both experimental and theoretical attention. From the experimental point of view it is most important to obtain an independent determination of the value of α_e in $\Sigma^- \to ne\nu$. From the theoretical point of view, if the value of

α_e in $\Sigma^- \to ne\nu$ is not changed, it can mean that the Cabibbo model may need important modifications. One might suspect that a large second-class-current contribution would be able to explain the value of α_e in $\Sigma^- \to ne\nu$. However in a recent paper it has been shown[19] that the CT with a second-class octet g_2 term (which brings two additional parameters) the prediction for α_e in $\Sigma^- \to ne\nu$ is moved only insignificantly from the standard CT prediction $\alpha_e \approx -0.6$.

From the analysis of this paper and that of Ref. 19, we are faced with the unique conclusion that, if the current value of α_e in $\Sigma^- \to ne\nu$ is confirmed by future experiments, then strong SB must be present in HSD. This would be a major change to the CT. Such strong SB might be very difficult to be computed perturbatively and, therefore, a different approach that redefines the concept of SU(3) symmetry might be required altogether. The only approach of this kind known to us that is in agreement with the present value of α_e in $\Sigma^- \to ne\nu$ is the approach that treats SU(3) as a spectrum-generating group. In fact, it was already noted when the data on HSD were much poorer that the value of α_e is $\Sigma^- \to ne\nu$ will be the crucial test that discriminates between the CT and the spectrum-generating SU(3) approach.[20]

Summarizing, our main conclusion can be stated in terms of two mutually exclusive statements, depending on the future experimental value of α_e in $\Sigma^- \to ne\nu$:

(1) If the value $\alpha_e = 0.26 \pm 0.19$ in $\Sigma^- \to ne\nu$ persists, then the CT will need some essential modification.

(2) If the value of α_e in $\Sigma^- \to ne\nu$ will move in the direction closer to the CT-favored value then the only modification of CT that is required is first-order symmetry breaking. Future more detailed experimental results on HSD can decide on the particular SB scheme.

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