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Search for composite models of quarks and leptons with preons having strong and electroweak quantum numbers

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The impossibility of composite models of two or more generations of quarks and leptons is shown using preons with strong $SU_C(3)$ quantum numbers and either the electroweak $SU_L(2)\times U(1)$ or $SU_L(2)\times SU_R(2)\times U_{B-L}(1)$ quantum numbers. The preon models considered are the simplest: *FFF* and *FB* (*F* is fermion and *B* is boson). The constraints imposed on the models are the absence of exotics and anomalies in the strong and electroweak sectors.

I. INTRODUCTION

There are at least two reasons for giving serious consideration to a common origin of quarks and leptons[1]: (1) the proliferation of an equal number of families of quark doublets and lepton doublets (quark-lepton symmetry) and (2) the pointlike behavior of both quarks and leptons down to distances below 10^{-16} cm.[2] If we had just one family, a composite model would be less attractive. Thus, a composite model of quarks and leptons must explain why Nature repeats itself at least three times with rapid changes in mass[3] and can only give structure to quarks and leptons below 10^{-16} cm. Most of the composite models proposed so far are inadequate from this point of view: preons[4] (or subquarks and subleptons or whatever) must repeat themselves to produce generations (or they have radial excitations, which is very unlikely.[5]) We do not gain any insight about the generations of quarks and leptons.

The postulated compositeness of quarks and leptons has led some authors to also hypothesize the compositeness of gauge bosons[6] (at least, weak bosons). However, one then loses the beauty of the gauge theory of electroweak interactions and the weak interaction becomes a van der Waals-type force like the nuclear interaction is supposed to be.[7] We recall the old flavor group SU(3) where the symmetry of the composite particles, the hadrons, was the same as that of the three quarks *u,d,s*. Moreover, the SU(3) color force, which is supposed to bind quarks together, does not disturb the flavor SU(3). Following this analogy, we propose to investigate composite models of quarks and leptons where the preons carry strong and electroweak quantum numbers. We hope to reproduce the generation (or family) structure nontrivially. From our vantage point, no gauge boson is composite.

Can we invent such a model? We try one example. Suppose we have two kinds of preons (Weyl spinors) as follows:

$$\begin{aligned} &T\colon (\bar{3},2,1)_L\ ,\\ &V\colon (3,1,2)_L\ , \end{aligned} \tag{1.1}$$

where the bracket indicates the quantum numbers of $SU_C(3)\times SU_L(2)\times SU_R(2)$. Then, we obtain the following group decomposition of particles constructed out of three preons, without attempting to answer the question of how they bind together:

$$\begin{aligned} TVV&=(3+3+\bar{6}+15,2,1+3)_L\ ,\\ TTV&=(\bar{3}+\bar{3}+6+\overline{15},1+3,2)_L\ ,\\ TTT&=(1+8+8+\overline{10},2+2+4,1)_L\ ,\\ VVV&=(1+8+8+10,1,2+2+4)_L\ . \end{aligned} \tag{1.2}$$

The apparent similarity to the Harari-Shupe model[8] is illusory. In our model, *TVV* represents $(u,d)_L$ and *TTV* represents $(u^c,d^c)_L$, in contrast to the Harari-Shupe model where *TTV* is (u,u^c) and *TVV* is (d^c,d) (since *T* and *V* are Dirac—not Weyl—spinors). We then note that there exist *no* $(\bar{3},2,1)_L$ and *no* $(3,1,2)_L$ in our model while we have two $(3,2,1)_L$ and two $(\bar{3},1,2)_L$. In addition, we have two $(1,2,1)_L$ and two $(1,1,2)_L$. Hence, we have two families of quarks and leptons. Other composite states are either color exotics or weak exotics with high-dimension representations.

Note that in our model the origin of the two families is different for quarks and leptons: doubling comes from the color group for quarks and from the weak group for leptons. This may indicate that the mass differences among quark families are larger than those among lepton families. These mass differences seem to be a type of hyperfine splitting.[9] So far so good. Unfortunately, this model has only two families, not three or four families, and, more seriously, it is not anomaly-free in the weak sector. The reason is as follows. Using the fact that the weak hypercharges Y $[=\frac{1}{2}(B-L)]$ are $Y(T)=-\frac{1}{6}$ and $Y(V)=\frac{1}{6}$, we get $\mathrm{Tr} I_{3L}{}^2 Y \neq 0$. Therefore, our model is not renormalizable. We are in trouble. However, because our model is simple and it will not give any "normal" exotics such as $(\bar{3},2,1)_L$ nor $(3,1,2)_L$, it is worth pursuing further.

In the search for composite models of quarks and leptons, we can take two different paths: The Georgi-Glashow way[10] and the Pati-Salam way.[11] In the former, we use the $\mathrm{SU}_L(2)\times \mathrm{U}(1)$ electroweak group and we construct $(3,2)_L$, $2(\bar{3},1)_L$, $(1,2)_L$, and $2(1,1)_L$. In the latter, we use the left-right-symmetric $\mathrm{SU}_L(2)\times \mathrm{SU}_R(2)\times \mathrm{U}_{B-L}(1)$ electroweak group and we construct $(3,2,1)_L$, $(1,2,1)_L$, $(3,1,2)_R$, and $(1,1,2)_R$ [or $(3,2,1)_L$, $(\bar{3},1,2)_L$, $(1,2,1)_L$, and $(1,1,2)_L$], where the second and the third indices denote the $\mathrm{SU}_L(2)\times \mathrm{SU}_R(2)$ quantum numbers. The basic assumptions of composite models with strong and weak quantum numbers are presented in Sec. II.

Although we can construct quarks and leptons out of preons in many ways, we restrict ourselves to the simplest: *FFF* and *FB* where *F* (*B*) denotes fermion (boson). In each case, we discuss both the Georgi-Glashow and the Pati-Salam approaches. In Sec. III, we discuss the pure-fermion composite model *FFF*.[12] In Sec. IV, we discuss the fermion-boson composite model *FB*.[13] One example has already been proposed[14]: preons are fermions $F=(1,2)$, and bosons $\psi_+=(3,1)$ and $\psi_0=(1,1)$. However, in this model, SU(2) is a global symmetry and is not gauged.[15] This model does satisfy the 't Hooft anomaly condition[16] for the case of the $\mathrm{SU}(4N_f)$ binding force, where N_f denotes the number of families. Section V is devoted to our conclusions.

II. BASIC ASSUMPTIONS OF COMPOSITE MODELS WITH STRONG AND WEAK QUANTUM NUMBERS

Since we assign strong and electroweak quantum numbers to the basic preon fields, we naturally limit the number of such fields. For the pure-fermion models, we choose two sets of Weyl spinors appropriate to the Georgi-Glashow (GG) and Pati-Salam (PS) approaches.

Assumption Ia. Preons are of the following types in the GG approach:

$$T_+:(3,2)_L\ ,\quad V_+:(3,1)_L\ ,$$
$$T_0:(1,2)_L\ ,\quad V_0:(1,1)_L\ ,$$
$$T_-:(\bar{3},2)_L\ ,\quad V_-:(\bar{3},1)_L\ ,$$

where the numbers in parentheses indicate the $\mathrm{SU}_C(3)\times \mathrm{SU}_L(2)$ quantum numbers and the $+$, $-$, 0 denote the triality of $\mathrm{SU}_C(3)$.

Assumption Ib. Preons are of the following types in the PS approach:

$$T_+:(3,2,1)_L\ ,\quad V_+:(3,1,2)_L\ ,$$
$$T_0:(1,2,1)_L\ ,\quad V_0:(1,1,2)_L\ ,$$
$$T_-:(\bar{3},2,1)_L\ ,\quad V_-:(\bar{3},1,2)_L\ ,$$

where the numbers in parentheses indicate the $\mathrm{SU}_C(3)\times \mathrm{SU}_L(2)\times \mathrm{SU}_R(2)$ quantum numbers and the $+$, $-$, 0 denote the triality of $\mathrm{SU}_C(3)$ as before.

For the fermion-boson models, we use the following.

Assumption I'a. Preons are of the following types in the GG approach:

$$T_+:(3,2)_L\ ,\quad V_+:(3,1)_L\ ,\quad \phi_+:(3,2)\ ,\quad \psi_+:(3,1)\ ,$$
$$T_0:(1,2)_L\ ,\quad V_0:(1,1)_L\ ,\quad \phi_0:(1,2)\ ,\quad \psi_0:(1,1)\ ,$$
$$T_-:(\bar{3},2)_L\ ,\quad V_-:(\bar{3},1)_L\ ,\quad \phi_-:(\bar{3},2)\ ,\quad \psi_-:(\bar{3},1)\ ,$$

where T,V (ϕ,ψ) denote fermions (bosons).

Assumption I'b. Preons are of the following type in the PS approach:

$$T_+: (3,2,1)_L\ ,\quad V_+:(3,1,2)_L\ ,$$
$$T_0: (1,2,1)_L\ ,\quad V_0: (1,1,2)_L\ ,$$
$$T_-: (\bar{3},2,1)_L\ ,\quad V_-: (\bar{3},1,2)_L\ ,$$
$$\phi_+: (3,2,1)\ ,\quad \psi_+: (3,1,2)\ ,\quad \Omega_+: (3,2,2)\ ,\quad \Lambda_+: (3,1,1)\ ,$$
$$\phi_0: (1,2,1)\ ,\quad \psi_0:(1,1,2)\ ,\quad \Omega_0:(1,2,2)\ ,\quad \Lambda_0: (1,1,1)\ ,$$
$$\phi_-: (\bar{3},2,1)\ ,\quad \psi_-: (\bar{3},1,2)\ ,\quad \Omega_-: (\bar{3},2,2)\ ,\quad \Lambda_-: (\bar{3},1,1)\ ,$$

where T,V $(\phi,\psi,\Omega,\Lambda)$ denote fermions (bosons).

The compositeness of quarks and leptons produce many exotics. There exist arguments why we should throw away some of these exotics.[17] However, we believe that it is preferable for exotics not to exist from the beginning. So, we make the following assumption.

Assumption II. No spin-$\frac{1}{2}$ bound states exist which have quantum numbers $(\bar{3},2)_L$ or $(3,1)_L$.

The possibility of such a model was demonstrated in the Introduction, although it is not anomaly free. Assumption II is motivated by the demonstration[18] that grand unification groups are limited to either SU(5) or SO(10) if the "observed" quarks and leptons are only in the $(3,2)_L$, $(3,1)_L$, $(1,2)_L$, $(1,1)_L$ representations but not $(\bar{3},2)_L$ or $(3,1)_L$.

Our insistence on the $\mathrm{SU}_C(3)\times\mathrm{SU}_L(2)\times\mathrm{U}(1)$ quantum numbers comes also from taking note of the fact that these are the only quantum numbers that are known at present except masses. If the grand unified theory is confirmed—e.g., by observing proton decay at the predicted level of $10^{31\pm1}$ yr—one could characterize quarks and leptons by some new quantum numbers or whatever. As long as we do not have any clear-cut evidence for SU(5) or SO(10) GUT, it seems reasonable to attempt to understand the postulated composite structure of quarks and leptons by using the $\mathrm{SU}_C(3)\times\mathrm{SU}_L(2)\times\mathrm{U}(1)$ quantum numbers as the tool to distinguish between quarks and leptons and both from nonexisting exotics. Of course, if we had some method to tell us which composites should be superheavy, then it would not be necessary to invoke assumption II.

We do not impose the Pauli principle, since its use requires the complete specification of preon dynamics, although this may eliminate some exotics. That is the lesson we have learned from the old SU(3) where the application of the Pauli principle to the flavor group SU(3) led to inconsistencies.

Finally, we add the following assumption.

Assumption III. No anomalies exist, at least in the strong and weak sectors of preons.

In the GG approach to model building, one normally accepts assumption III. In the PS approach, one usually hypothesizes *mirror fermions to* cancel the weak anomalies; we do not accept this hypothesis because it introduces too much arbitrariness into the PS approach.

III. THREE-FERMION COMPOSITE MODELS

We prove that it is impossible to have composite models of quarks and leptons, based on three fermions, which satisfy all three assumptions above. We first discuss composite models within the $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\times\mathrm{U}_{B-L}(1)$ framework. The assignments of quantum numbers are as follows:

$$\begin{aligned} &T_+: (3,2,1)_L\ ,\quad V_+: (3,1,2)_L\ ,\\ &T_0: (1,2,1)_L\ ,\quad V_0: (1,1,2)_L\ ,\\ &T_-: (\bar{3},2,1)_L\ ,\quad V_-: (\bar{3},1,2)_L\ .\end{aligned} \tag{3.1}$$

The $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$ quantum numbers for composite states of three preons are

$$\begin{aligned} &TTT: 2(2_L,1_R)+(4_L,1_R)\ ,\\ &TVV: (2_L,1_R)+(2_L,3_R)\ ,\\ &TTV: (1_L,2_R)+(3_L,2_R)\ ,\\ &VVV: 2(1_L,2_R)+(1_L,4_R)\ .\end{aligned} \tag{3.2}$$

The color states, 3, $\bar{3}$, and 1, can be constructed as

$$\begin{array}{lll} 3:\ ++- & \bar{3}:\ +-- & 1:\ +++ \\ \quad +\,0\,0 & \quad -\,0\,0 & \quad +-0 \\ \quad --0 & \quad ++0 & \quad --- \\ & & \quad 0\,0\,0 \end{array} \tag{3.3}$$

where the use of triality helps in the enumeration of states. Hence, the relevant $SU_C(3)\times SU_L(2)\times SU_R(2)$ states are

$$(3,2,1)_L=\begin{bmatrix} + & + & - \\ + & 0 & 0 \\ - & - & 0 \end{bmatrix}\otimes\begin{bmatrix} T & T & T \\ T & V & V \end{bmatrix}, \quad (3.4)$$

$$(\bar{3},1,2)_L=\begin{bmatrix} + & - & - \\ - & 0 & 0 \\ + & + & 0 \end{bmatrix}\otimes\begin{bmatrix} T & T & V \\ V & V & V \end{bmatrix}, \quad (3.5)$$

$$(1,2,1)_L=\begin{bmatrix} + & + & + \\ + & - & 0 \\ - & - & - \\ 0 & 0 & 0 \end{bmatrix}\otimes\begin{bmatrix} T & T & T \\ T & V & V \end{bmatrix}, \quad (3.6)$$

$$(1,1,2)_L=\begin{bmatrix} + & + & + \\ + & - & 0 \\ - & - & - \\ 0 & 0 & 0 \end{bmatrix}\otimes\begin{bmatrix} T & T & V \\ V & V & V \end{bmatrix}. \quad (3.7)$$

Forbidden states are constructed by

$$(\bar{3},2,1)_L=\begin{bmatrix} + & - & - \\ - & 0 & 0 \\ + & + & 0 \end{bmatrix}\otimes\begin{bmatrix} T & T & T \\ T & V & V \end{bmatrix}, \quad (3.8)$$

$$(3,1,2)_L=\begin{bmatrix} + & + & - \\ + & 0 & 0 \\ - & - & 0 \end{bmatrix}\otimes\begin{bmatrix} T & T & V \\ V & V & V \end{bmatrix}. \quad (3.9)$$

We obtain three possibilities:

$$(1)\ T_+\neq 0\,,\ T_0=T_-=V_-=0\,,\ V_+V_0=0\,, \quad (3.10)$$

$$(2)\ T_0\neq 0\,,\ T_+=T_-=V_+=0\,,\ V_0V_-=0\,, \quad (3.11)$$

$$(3)\ T_-\neq 0\,,\ T_+=T_0=V_0=0\,,\ V_+V_-=0\,, \quad (3.12)$$

where $T_i=0$ means that the number of T_i type is zero and $V_iV_j=0$ means that either the number of V_i type or that of V_j type is zero. The demand that we should get particles with quantum numbers $(3,2,1)_L$ and $(\bar{3},1,2)_L$ yields finally:

Case 1: $T_+V_0V_0=(3,2,1)_L$, $T_+T_+V_0=(\bar{3},1,2)_L$, $T_+T_+T_+=(1,2,1)_L$, $V_0V_0V_0=(1,1,2)_L$;

Case 2: $T_0V_-V_-=(3,2,1)_L$, $T_0T_0V_-=(\bar{3},1,2)_L$, $T_0T_0T_0=(1,2,1)_L$, $V_-V_-V_-=(1,1,2)_L$;

Case 3: $T_-V_+V_+=(3,2,1)_L$, $T_-T_-V_+=(\bar{3},1,2)_L$, $T_-T_-T_-=(1,2,1)_L$, $V_+V_+V_+=(1,1,2)_L$.

Now we investigate each case.

Case 1. We have only $T_+=(3,2,1)_L$ and $V_0=(1,1,2)_L$. The numbers of quarks and leptons are

$$N_q=n_T\frac{n_V(n_V+1)}{2!}\,,\quad N_q=\frac{n_T(n_T+1)n_V}{2!}\quad \text{so that } n_T=n_V\,,$$
$$N_l=2\frac{n_T(n_T+1)(n_T+2)}{3!}\,,\quad N_l=2\frac{n_V(n_V+1)(n_V+2)}{3!}\,, \quad (3.13)$$

where n_i denotes the number of i-type preons and N_j denotes the number of j-type families. If we insist that $N_q=N_l$ (equal number of quarks and lepton families), the solution is

$$N_q=N_l=40 \quad (3.14)$$

which is highly unlikely. If we allow $N_q\neq N_l$ (different numbers of quark and lepton families), we find

$$(N_q,N_l)=(1,2),(6,8),\ldots\,. \quad (3.15)$$

Case 2 where we have only $T_0=(1,2,1)_L$ and $V_-=(3,1,2)_L$ yields the same solution as case 1. Thus, we do not have three or four quark families for cases 1 and 2.

Case 3 where we have $T_-=(\bar{3},2,1)_L$ and $V_+=(3,1,2)_L$ yields

$$N_q=2n_T\frac{n_V(n_V+1)}{2!}\,,\quad N_q=2n_V\frac{n_T(n_T+1)}{2!}\quad \text{so that } n_T=n_V\,,$$
$$N_l=2\frac{n_T(n_T+1)(n_T+2)}{3!}\,,$$
$$N_l=2\frac{n_V(n_V+1)(n_V+2)}{3!}\,. \quad (3.16)$$

The solution for $N_q=N_l$ is

$$N_q = N_l = 2 \ ,$$

and for $N_q \neq N_l$,

$$(N_q, N_l) = (12,8), \ldots \ . \tag{3.17}$$

The solution $N_q = N_l = 2$ is the example in the Introduction. In all cases, we have shown that composite models of three preons cannot provide three or four families of quarks and leptons. Moreover, in all cases, they are not anomaly-free in the weak sector. The reason is the same as that given in the Introduction. Thus, the PS approach does not work.

One may wonder whether the use of $SU_L(2) \times U(1)$ preons (GG approach) works or not. Since the quantum numbers of $SU_C(3) \times SU_L(2)$ are the same as in the case of $SU_C(3) \times SU_L(2) \times SU_R(2)$, we obtain three possibilities as before. However, in the GG approach, we do not have two distinct $(\bar{3},1)_L$. We have just TTV. We cannot have both u_L^c and d_L^c, otherwise we can not assign the U(1) quantum numbers to T and V. Thus, we have shown that three-fermion composite models in the GG approach do not work.

IV. FERMION-BOSON PREON MODELS

Since the purely fermion composite models are "no go", we try the fermion-boson composite model FB. We note that we forbid composites of type $F\bar{B}$. This is motivated by the following observation: If the superstrong force which binds preons is a gauged one with a suitable gauge group G, the situation may arise where FB is singlet under this superstrong force; but then $F\bar{B}$ can not be a singlet with the assignment of representation r to fermions F and $\bar{r}$ to bosons B, except SU(2).

To reduce the numerical calculations, we make an additional assumption which is consistent with all observations thus far.

Assumption IV. The number of quark doublets is equal to the number of lepton doublets, i.e., $N_q = N_l$ (quark-lepton symmetry).

A. GG solutions for FB

First, we explain how $SU_C(3) \times SU_L(2)$ quantum numbers are constructed out of F and B:

$$\begin{aligned} &SU_L(2) \text{ doublet: } T\psi \text{ or } V\phi \ , \\ &SU_L(2) \text{ singlet: } T\phi \text{ or } V\psi \ , \end{aligned} \tag{4.1}$$

$$\begin{aligned} &\text{the color } 3\text{: } +0 \text{ or } -- \ , \\ &\bar{3}\text{: } -0 \text{ or } ++ \ , \\ &1\text{: } +- \text{ or } 0\,0 \ . \end{aligned} \tag{4.2}$$

Forbidden quantum numbers arise from

$$\begin{bmatrix} + & + \\ - & 0 \end{bmatrix} \otimes \begin{bmatrix} T & \psi \\ V & \phi \end{bmatrix}$$

or

$$\begin{bmatrix} + & 0 \\ - & - \end{bmatrix} \otimes \begin{bmatrix} T & \phi \\ V & \psi \end{bmatrix} \ . \tag{4.3}$$

It is easy to see that at least one of T_+, T_0, T_- *and* one of V_+, V_0, V_- should not exist, since otherwise scalars are not allowed. Moreover, at least one of T_+, T_-, V_+, V_- should not be present by the same reasoning. Also, if both T_0 and V_0 are present and the rest of the fermions are not, we have $\phi_+ = \phi_- = \psi_+ = \psi_- = 0$, i.e., we cannot construct colored states. Combined with assumption III, which requires an anomaly-free strong sector, we obtain the following possibilities.

Case 1: $T_+ T_- \neq 0$, $T_0 = V_+ = V_- = 0$.

Case 2: $V_+ V_- \neq 0$, $V_0 = T_+ = T_- = 0$.

Case 3: $T_+ V_- \neq 0$, $T_- = V_+ = 0$.

Case 4: $T_- V_+ \neq 0$, $T_+ = V_- = 0$.

Case 5: $T_+ T_- V_+ \neq 0$, $T_0 = V_- = 0$.

Case 6: $T_+ V_+ V_- \neq 0$, $T_- = V_0 = 0$.

Case 7: $T_+ T_- V_- \neq 0$, $T_0 = V_+ = 0$.

Case 8: $T_- V_+ V_- \neq 0$, $T_+ = V_0 = 0$.

Case 9: $T_0 \neq 0$, the rest of fermions vanish .

Case 10: $V_0 \neq 0$, the rest of fermions vanish .

Cases of even number are related to cases of odd number (among cases 1–8) by $T \leftrightarrow V$.

Since we could not find a simple method to solve the above ten cases, we resort to brute force. For each case, we construct the representations $(3,2)_L$, $(\bar{3},1)_L$, $(1,2)_L$, $(1,1)_L$ which are listed in Table I. It is straightforward to see that cases 5–10 should be dismissed since we must have two distinct kinds of $(\bar{3},1)_L$ (u_L^c and d_L^c). In other words, we cannot assign the U(1) quantum numbers of $SU_L(2) \times$ U(1) to preons. Hence, cases 5–10 must be discussed in the left-right-symmetric electroweak model.

TABLE I. Contents of *FB* composite states in the GG approach.

Case	$(3,2)_L$	$(\bar{3},1)_L$	$(1,2)_L$	$(1,1)_L$
1	$T_-\psi_-, V_0\phi_+$	$T_+\phi_+, V_0\psi_-$	$T_+\psi_-$	$T_-\phi_+$
2	$T_0\psi_+, V_-\phi_-$	$T_0\phi_-, V_+\psi_+$	$V_+\phi_-$	$V_-\psi_+$
3	$T_+\psi_0, V_-\phi_-, V_0\phi_+$	$T_+\phi_+, V_-\psi_0, T_0\phi_-$	$T_0\psi_0, V_-\phi_+$	$T_+\phi_-, V_0\psi_0$
4	$T_0\psi_+, V_+\phi_0, T_-\psi_-$	$T_-\phi_0, V_+\psi_+, V_0\psi_-$	$T_-\psi_+, V_0\phi_0$	$T_0\phi_0, V_+\psi_-$
5	$T_-\psi_-$	$V_0\psi_-$	$T_+\psi_-$	$V_+\psi_-$
6	$V_-\phi_-$	$T_0\phi_-$	$V_+\phi_-$	$T_+\phi_-$
7	$V_0\phi_+$	$T_+\phi_+$	$V_-\phi_+$	$T_-\phi_+$
8	$T_0\psi_+$	$V_+\psi_+$	$T_-\psi_+$	$V_-\psi_+$
9	$T_0\psi_+$	$T_0\phi_-$	$T_0\psi_0$	$T_0\phi_0$
10	$V_0\phi_+$	$V_0\psi_-$	$V_0\phi_0$	$V_0\psi_0$

Next we discuss cases 1–4.

Case 1. It is easy to show that $T_+\phi_+$ behaves as d_L^c, while $V_0\psi_-$ as u_L^c, by examining the U(1) quantum number, Y, of $\mathrm{SU}_L(2)\times\mathrm{U}(1)$. [The relations, $Y(V_0)+Y(\psi_-)=\frac{2}{3}$ and $Y(V_0)+Y(\phi_+)=\frac{1}{6}$ must be satisfied.] Then we have

$$N_q=n(T_-)n(\psi_-)+n(V_0)n(\phi_+)\ , \tag{4.4}$$

$$N_q=n(T_+)n(\phi_+)\ , \tag{4.5}$$

$$N_q=n(V_0)n(\psi_-)\ , \tag{4.6}$$

$$N_l=n(T_+)n(\psi_-)\ , \tag{4.7}$$

$$N_l=n(T_-)n(\phi_+) \tag{4.8}$$

while assumption III yields

$$n(T_+)=n(T_-)\ . \tag{4.9}$$

Equations (4.7), (4.8), and (4.9) yield

$$n(\psi_-)=n(\phi_+)$$

which leads to

$$n(V_0)=0$$

by Eq. (4.4). This implies the nonexistence of u_L^c.

Case 2. We make the transformations $V\leftrightarrow T$ and $\phi\leftrightarrow\psi$. Then we get $n(T_0)=0$, which leads to the absence of u_L^c.

Case 3. The U(1) quantum numbers require

$$Y(T_+)+Y(\psi_0)=\tfrac{1}{6}\ , \tag{4.10}$$

$$Y(V_-)+Y(\phi_-)=\tfrac{1}{6}\ , \tag{4.11}$$

$$Y(V_0)+Y(\phi_+)=\tfrac{1}{6}\ , \tag{4.12}$$

$$Y(T_0)+Y(\psi_0)=-\tfrac{1}{2}\ , \tag{4.13}$$

$$Y(V_-)+Y(\phi_+)=-\tfrac{1}{2}\ . \tag{4.14}$$

If $Y(T_+\phi_+)=Y(T_+)+Y(\phi_+)=\frac{1}{3}$, that is $T_+\phi_+$ is d_L^c type, then we get $Y(V_-\psi_0)=-\frac{2}{3}$ ($V_-\psi_0$ is u_L^c type), using Eqs. (4.10) and (4.14). Consequently, we get $Y(V_0\psi_0)=0$ ($V_0\psi_0$ is ν_L^c type). Since we must have e_L^c type, $T_+\phi_-$ must be the solution. Then $Y(T_0\phi_-)=\frac{1}{3}$ ($T_0\phi_-$ is d_L^c type), using Eqs. (4.10) and (4.13). Summarizing, if $T_+\phi_+$ is d_L^c type, then $V_-\psi_0$, $V_0\psi_0$, $T_+\phi_-$, $T_0\phi_-$ are u_L^c, ν_L^c, e_L^c, d_L^c types, respectively. Hence, we have

$$N_q=n(T_+)n(\psi_0)+n(V_-)n(\phi_-)+n(V_0)n(\phi_+)\ , \tag{4.15}$$

$$N_q=n(T_+)n(\phi_+)+n(T_0)n(\phi_-)\ , \tag{4.16}$$

$$N_q=n(V_-)n(\psi_0)\ , \tag{4.17}$$

$$N_l=n(T_0)n(\psi_0)+n(V_-)n(\phi_+)\ , \tag{4.18}$$

$$N_l=n(T_+)n(\phi_-)\ , \tag{4.19}$$

$$N_0=n(V_0)n(\psi_0)\ . \tag{4.20}$$

The anomaly-free condition yields

$$2n(T_+)=n(V_-)\ . \tag{4.21}$$

Using assumption IV, namely $N_q=N_l$, and Eqs. (4.15), (4.17), (4.19), and (4.21), we have

$$\begin{aligned} n(V_0)n(\phi_+)&=N_q-n(T_+)n(\psi_0)-n(V_-)n(\phi_-)\\ &=N_q-\tfrac{1}{2}n(V_-)n(\psi_0)-2n(T_+)n(\phi_-)\\ &=N_q-\tfrac{1}{2}N_q-2N_l<0\ . \end{aligned}$$

In the same way, we can show that if $T_+\phi_+$ is u_L^c type, then $V_-\psi_0$, $V_0\psi_0$, $T_+\phi_-$, $T_0\phi_-$ are d_L^c, e_L^c,

ν_L^c, u_L^c types, respectively. Hence,

$$N_q = n(T_+)n(\psi_0) + n(V_-)n(\phi_-) + n(V_0)n(\phi_+)\ , \tag{4.22}$$

$$N_q = n(T_+)n(\phi_+) + n(T_0)n(\phi_-)\ , \tag{4.23}$$

$$N_q = n(V_-)n(\psi_0)\ , \tag{4.24}$$

$$N_l = n(T_0)n(\psi_0) + n(V_-)n(\phi_+)\ , \tag{4.25}$$

$$N_l = n(V_0)n(\psi_0)\ , \tag{4.26}$$

$$N_0 = n(T_0)n(\phi_-)\ , \tag{4.27}$$

and

$$2n(T_+) = n(V_-)\ . \tag{4.28}$$

Equations (4.24) and (4.26) yield

$$n(V_-) = n(V_0) \equiv n_V\ . \tag{4.29}$$

Then, for $N_q = N_l = N$, we have

$$N = n(T_+)n(\psi_0) + n_V n(\phi_+) + n_V n(\phi_-)\ , \tag{4.30}$$

$$N = n(T_+)n(\phi_+) + n(T_0)n(\phi_-)\ , \tag{4.31}$$

$$N = n_V n(\psi_0)\ , \tag{4.32}$$

$$N = n(T_0)n(\psi_0) + n_V n(\phi_+)\ . \tag{4.33}$$

Using Eqs. (4.31) and (4.33), we have

$$n(\phi_+)[n(T_+) - n_V] = n(T_0)[n(\psi_0) - n(\phi_-)]$$

while Eqs. (4.30) and (4.32) yield

$$n(\psi_0)[n_V - n(T_+)] = n_V[n(\phi_+) + n(\phi_-)]\ .$$

Thus, we have

$$n(\psi_0) < n(\phi_-) \tag{4.34}$$

or

$$N = n_V n(\psi_0) < n_V n(\phi_-) < n(T_+)n(\psi_0) + n_V n(\phi_+) + n_V n(\phi_-) = N\ .$$

Therefore, there is no solution for $N_q = N_l$.

Case 4. Making use of the transformations $T \leftrightarrow V$ and $\phi \leftrightarrow \psi$ we have no solutions.

We conclude that there are no solutions for the FB preon model in the GG approach.

B. PS solutions for FB preon model

Now, we look for the left-right-symmetric composite models involving one fermion (spin-$\frac{1}{2}$) and one boson (spin-0). We construct $SU_L(2)\times SU_R(2)$ quantum numbers for composite states as

$$(*,2,1)\colon\ T\Lambda, V\Omega,\quad (*,1,2)\colon\ T\Omega, V\Lambda,$$
$$(*,2,2)\colon\ T\psi, V\phi,\quad (*,1,1)\colon\ T\phi, V\psi\ .$$

Since we do not want quarks and leptons with (*,2,2) or (*,1,1), we conclude:

$$\phi = 0 \text{ and } \psi = 0\ . \tag{4.35}$$

Actually, this equation holds for FB^n (n=odd), since $\phi^n \sim (*,2,1)$ and $\psi^n \sim (*,1,2)$ for n=odd.

The forbidden quantum numbers are $(\bar{3},2,1)_L$ and $(3,1,2)_L$, to wit,

$$\begin{bmatrix} + & + \\ - & 0 \end{bmatrix} \otimes \begin{bmatrix} T & \Lambda \\ V & \Omega \end{bmatrix}$$

and (4.36)

$$\begin{bmatrix} - & - \\ + & 0 \end{bmatrix} \otimes \begin{bmatrix} T & \Omega \\ V & \Lambda \end{bmatrix}$$

or

$$\begin{aligned} &T_+\Lambda_+ = 0,\quad T_0\Lambda_- = 0,\quad T_-\Lambda_0 = 0,\\ &T_+\Omega_0 = 0,\quad T_0\Omega_+ = 0,\quad T_-\Omega_- = 0,\\ &V_+\Omega_+ = 0,\quad V_0\Omega_- = 0,\quad V_-\Omega_0 = 0,\\ &V_+\Lambda_0 = 0,\quad V_0\Lambda_+ = 0,\quad V_-\Lambda_- = 0\ . \end{aligned} \tag{4.37}$$

If $V_+V_-T_+T_- \neq 0$, then $\Lambda_+ = \Lambda_- = \Lambda_0 = \Omega_+ = \Omega_- = \Omega_0 = 0$, i.e., no bosons. Hence, at least one of T_+, T_-, V_+, V_- must not exist. Note that the simultaneous transformation of $T \leftrightarrow V$ and $\Omega \leftrightarrow \Lambda$ leaves Eq. (4.37) invariant. The candidates for solutions are as follows:

(1)	$T_+T_0V_+V_-$	Ω_-,
(2)	$T_+T_-V_0V_-$	Ω_+,
(3)	$T_0T_-V_+V_0$	Ω_0,
(4)	$T_+T_-V_0$	$\Omega_+\Lambda_-$,
(5)	$T_+T_0V_-$	$\Omega_-\Lambda_0$,
(6)	$T_0T_-V_+$	$\Omega_0\Lambda_+$,
(7)	T_+V_-	$\Omega_+\Omega_-$,
(8)	T_+V_-	$\Omega_+\Omega_-\Lambda_0$

and their ($T \leftrightarrow V$) and ($\Omega \leftrightarrow \Lambda$) conjugates. The requirements of the anomaly-free condition in color and $N_q = N_l$ limit candidates to only cases 3 and 7 (and their conjugates). Then, the anomaly-free condition in the electroweak sector uniquely picks case 3 and its conjugate as solutions:

$$(T_0, T_-, V_+, V_0, \Omega_0)$$

and

$$(T_+, T_0, V_0, V_-, \Lambda_0)\ .$$

TABLE II. Contents of *FB* composite states in the PS approach.

(a)

Case	$(3,2,1)_L$	$(\bar{3},1,2)_L$	$(1,2,1)_L$	$(1,1,2)_L$
1	$T_+\Lambda_0$	$V_-\Lambda_0$	$T_0\Lambda_0$	$V_0\Lambda_0$
2	$V_+\Omega_0$	$T_-\Omega_0$	$V_0\Omega_0$	$T_0\Omega_0$

(b)

Case	$(3,2,1)_L$	$(3,1,2)_R$	$(1,2,1)_L$	$(1,1,2)_R$
1	$T_+\Lambda_0$	$V_+\Lambda_0$	$T_0\Lambda_0$	$V_0\Lambda_0$
2	$T_0\Lambda_+$	$V_0\Lambda_+$	$T_-\Lambda_+$	$V_-\Lambda_+$
3	$T_-\Lambda_-$	$V_-\Lambda_-$	$T_+\Lambda_-$	$V_+\Lambda_-$

Their contents are listed in Table II(a).

The change of the assignment of V as $V_L \rightarrow V_R$ yields different solutions. They are in Table II(b). In either case, solutions are *trivial* and lead to only one family.

V. CONCLUSIONS

Under the major assumption that preons carry strong and electroweak quantum numbers, we have investigated composite models of quarks and leptons of the types *FFF* and *FB* in the GG and PS approaches.[12,13] Interestingly, there exist no solutions in the GG approach, using the $SU_L(2)\times U(1)$ electroweak group. In the PS approach, the left-right-symmetric electroweak model $SU_L(2) \times SU_R(2)\times U_{B-L}(1)$ does provide solutions of the *FB* type that are anomaly-free.

If we were to accept "mirror" fermions[11] in the PS approach in order to cancel the electroweak anomaly of preons ($\mathrm{Tr} I_{3L}{}^2 Y \neq 0$), there would be additional solutions that become anomaly-free. However,the allowance for mirror preons introduces too much arbitrariness, in our view, into the PS approach. In this regard, we note that if we allow $(\bar{3},2)_L$ or $(3,1)_L$ in the pure-fermion composite models, we can also find solutions in the GG approach. For example, giving up assumption II leads to the Casalbuoni-Gatto model[19] where equal numbers of right- and left-handed composite states appear. However, it is completely unclear why right-handed particles should become superheavy. Incidently, 't Hooft[16] also ended up with no solution in his search for composite models of quarks and leptons constructed out of fermions, although our motivation is quite different from that of 't Hooft.

Since the pure-fermion preon model with assigned strong and electroweak quantum numbers appeared to be incapable of providing a viable composite model of quarks and leptons (with at least three generations), we turned to a composite model of fermion and boson preons, *FB*, which was also unsuccessful. Thus, we can summarize our finding: *no composite model with only strong and electroweak quantum numbers exists which produces just ordinary quarks and leptons.* This may imply that we should give up assuming both strong and electroweak quantum numbers for preons. Or, it may imply the existence of a rich spectrum of right-handed particles at higher energies. It is too early to say which implication will prevail or whether some new approach will be required at the intermediate mass scale.

Note added in proof. We have found a three-preon composite model in $SU_C(4) \otimes SU_L(2) \otimes SU_R(2)$. See Y. Tosa and R. E. Marshak, VPI Report No. HEP-81/10 (unpublished); Y. Tosa, P. Xve, and R. E. Marshak, VPI Report No. HEP-82/3 (unpublished); Y. Tosa, VPI Report No. HEP-82/4 (unpublished).

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[1]Reviews on composite models are, e.g., H. Terazawa, Phys. Rev. D 22, 184 (1980); Report No. INS-416, 1981 (unpublished); M. Peskin, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn,* edited by W. Pfeil (Universität Bonn, Bonn, 1981), and references therein.

[2]J. G. Branson, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn* (Ref. 1).

[3]A striking feature of the quark and lepton mass spectra is their relative inversion for the second and third generations (the first generation is ambiguous because of the small absolute values of the quark and lepton masses). See. R. E. Marshak and R. N. Mohapatra, Phys. Lett. 91B, 222 (1980). See also H. Terazawa, Prog. Theor. Phys. 64, 1763 (1980).

[4]We shall always use the term "preons" to designate the proposed constituents of quarks and leptons. Preons may be fermions or a combination of fermions and bosons.

[5]The mass of the radially excited states increases as $\sim n^{\alpha}$ where $\alpha = 2\nu/(\nu+2) \lesssim 2$, $V(r) = ar^{\nu}$ and n is the radial quantum number [*Problems in Quantum Mechanics,* edited by D. ter Haar (Pion Limited, London, 1975)]. The empirical variation of quark and lepton masses from generation to generation is much more rapid than that suggested by the above formula. See Ref. 3.

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[7]It has been suggested (see Ref. 8 and R. Barbieri *et al.* in Ref. 14) that the weak interaction, like the strong nuclear interaction, has a "van der Waals" origin. We believe the analogy is misleading because the mass differences between $I_{3w} = \pm 1/2$ (I_{3w} is the third component of the weak isospin) quarks and leptons is so much larger than the $I_{3s} = \pm 1/2$ (I_{3s} is the third component of the strong isospin) mass difference between p and n, etc.

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[10]Cf. H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); H. Georgi, H. R. Quinn, and S. Weinberg, *ibid.* 33, 451 (1974).

[11]Cf. J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* 11, 566, 2559 (1975); R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. 44, 1316 (1980).

[12]The *FFFFF* preon model has also been studied with the same negative results as the *FFF* preon model.

[13]The fermion-boson preon models FB^n ($n > 1$) have also been studied. Although the number of generations increases as n increases, solutions are trivial and unattractive.

[14]This is a variation of the old Nagoya model, where baryons were postulated to be composed of leptons and scalar objects called B matter [Z. Maki, M. Nakagawa, Y. Ohnuki, and S. Sakata, Prog. Theor. Phys. 23, 1174 (1970)]; K. Matumoto, *ibid.* 52, 1973 (1974); M. Yasue, Phys. Lett. 91B, 85 (1980) and references therein; O. W. Greenberg and J. Sucher, *ibid.* 99B, 339 (1981); L. F. Abbott and E. Fahri, *ibid.* 101B, 69 (1981); Nucl. Phys. B189, 547 (1981); R. Barbieri, R. N. Mohapatra, and A. Masiero, Phys. Lett. 105B, 369 (1981).

[15]See R. Barbieri *et al.*, in Ref. 14.

[16]G. 't Hooft, in *Recent Developments in Gauge Theories,* edited by G. 't Hooft *et al.* (Plenum, New York, 1980).

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