Are the scaling violations observed in deep-inelastic lepton-hadron scattering purely $O(1/Q^2)$ effects?

B. P. Mahapatra
Institute of Physics, Bhubaneswar 751001, India
*and Post-Graduate Department of Physics, Sambalpur University, Burla 768017, India**

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A very simple quark-parton model, incorporating power-law scaling violation, quark-counting rules, Regge behavior, positivity, and sum rules, is shown to provide a superior description of the world data on deep-inelastic lepton-hadron scattering with only two adjustable parameters.

With the availability of abundant and precise data over a wide kinematic range, deep-inelastic lepton-hadron scattering (DIS) continues to provide new insights into hadronic structure at short distance and the underlying dynamics. It has been widely believed[1] that the pattern of scaling violations observed in DIS is in quantitative agreement with the prediction of perturbative quantum chromodynamics (QCD) characterized by logarithmic departure from scaling at asymptotic values of Q^2. The subasymptotic correction to scaling associated, e.g., with higher-twist (HT) contributions are expected to be fast suppressed[2] as $O(1/Q^2)$ compared to the dominant leading-twist (LT) contributions.

However, there has been, of late, growing evidence of substantial $O(1/Q^2)$ violations of scaling persisting to quite large values of Q^2 contrary to theoretical expectations.[2] Recently, Abbot and Barnett[3] have observed that the CERN-Dortmünd-Heidelberg-Saclay (CDHS) and Big European Bubble Chamber (BEBC) combined data on xF_3^ν and the corresponding moments could be explained by HT contributions alone. Abbott, Atwood, and Barnett[4] reinforced this conclusion from an analysis of the Stanford Accelerator Center—Massachusetts Institute of Technology (SLAC-MIT) data on F_2^p and F_2^{p-n}. Donnachie and Landshoff[5] showed that the electromagnetic (EM) structure functions at large $x \gtrsim 0.2$ are associated with mainly $O(1/Q^2)$ scaling violations. The most dramatic and model-independent demonstration of $O(1/Q^2)$ effects in scaling violations is the observation[6] that the precise data on the Nachtmann moments of the EM structure functions F_2^p, F_2^d obtained by the Chicago-Harvard-Illinois-Oxford (CHIO) group are straight lines in the variable $1/Q^2$ over the range $3 \lesssim Q^2 \lesssim 40$ GeV^2. More recently, high-statistics data at much larger values of Q^2 from the European Muon Collaboration (EMC),[7] Bologna-CERN-Dubna-Münich-Saclay (BCDMS),[8] Michigan State University—Fermilab (MSU-Fermilab)[9] groups show the absence of any appreciable scaling violations for $Q^2 \gtrsim 50$ GeV^2. Such behavior would be natural if all scaling violations were purely low-Q^2 phenomena as would be expected from $O(1/Q^2)$ effects alone.

Theoretically, such $O(1/Q^2)$ scaling violations could arise from a variety of reasons which are of both kinematic and dynamical origin and as diverse as target-mass effects,[10] gauge invariance,[11] canonical dimension of nonleading light-cone sing-

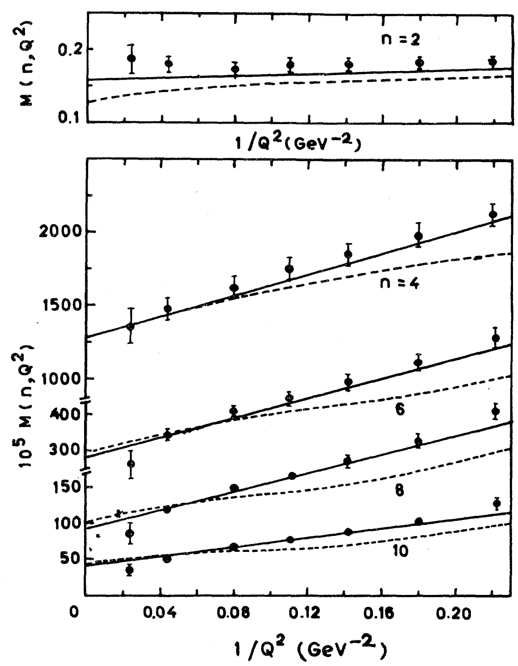

FIG. 1. Fits to the proton Nachtmann moments (data from Ref. 17). The dashed curves are the prediction of the QCD parton model (Ref. 19).

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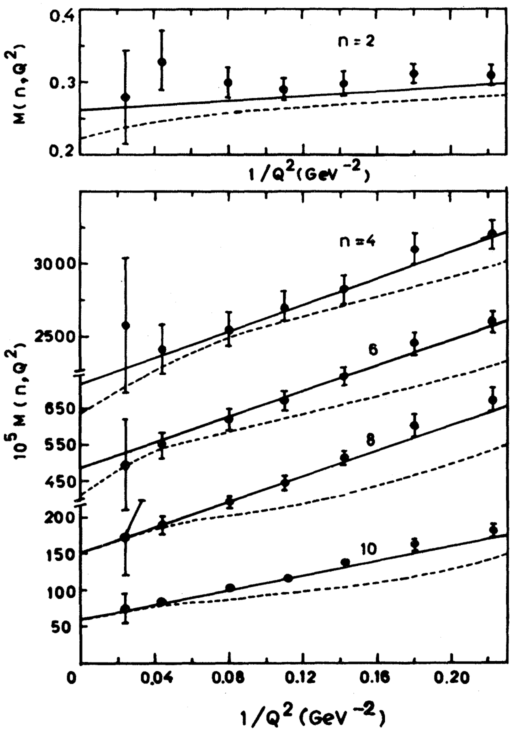

FIG. 2. Fits to the deuteron Nachtmann moments (data from Ref. 17). The dashed curves are the prediction of the QCD parton model (Ref. 19).

ularity,[12] higher twists,[13] diquarks,[14] etc.

The above accumulating evidence, therefore, make it imperative for finding an answer to the question posed in the title of this note, i.e., "Are *all* scaling violations observed in DIS purely $O(1/Q^2)$ effects?." It is obvious that an answer in the affirmative would have far-reaching repercussions on the large number of current investigations based on asymptotic freedom.[15]

This note constitutes an effort in finding an answer to the above question. Obviously, this requires a simultaneous analysis of the totality of the DIS data in a general framework incorporating only power-law departure from scaling. To this end, we propose a very simple quark-parton model (QPM) in which all scaling violations are purely $O(1/Q^2)$ and obtain excellent *simultaneous* description of the SLAC-MIT,[16] CHIO,[17] MSU-Fermilab,[9] and BCDMS[8] data on F_2^p, F_2^d, and the corresponding Nachtmann moments, and the CDHS neutrino data[18] on F_2^ν and xF_3^ν. The most important feature of the model is that it provides, as yet, the most economical description of the DIS data with *only two adjustable parameters.* Besides, compared to the description of these data in QCD parton models[19,20] (which involve sixteen[19] or

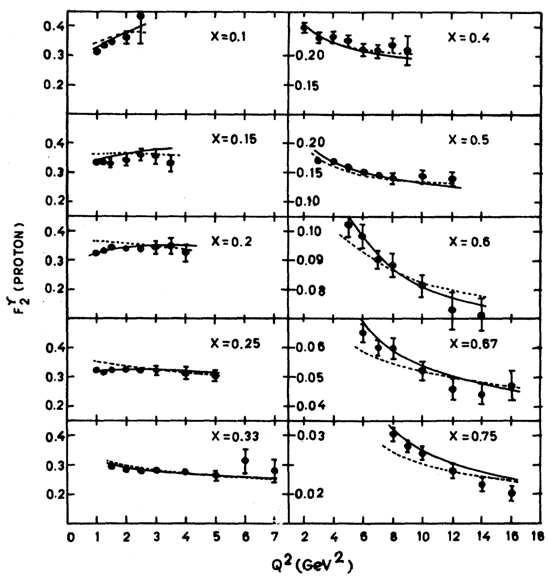

FIG. 3. Fits (solid curves) to the SLAC-MIT data (Ref. 16) on $F_2^{\gamma p}$ compared with those (dashed curves) of the Buras-Gaemers model (Ref. 19).

more[20] number of parameters) the present model yields a much superior description of the same.

The deep-inelastic structure functions are given by the standard expressions:

$$F_2^p = \tfrac{4}{9} x u_v + \tfrac{1}{9} x d_v + \tfrac{20}{9} x S \ , \tag{1}$$

$$F_2^n = \tfrac{1}{9} x u_v + \tfrac{4}{9} x d_v + \tfrac{20}{9} x S \ , \tag{2}$$

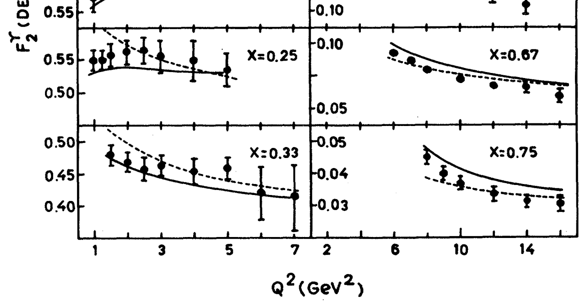

FIG. 4. Fits (solid curves) to the SLAC-MIT data (Ref. 16) on $F_2^{\gamma d}$ compared with those (dashed curves) of the Buras-Gaemers model (Ref. 19).

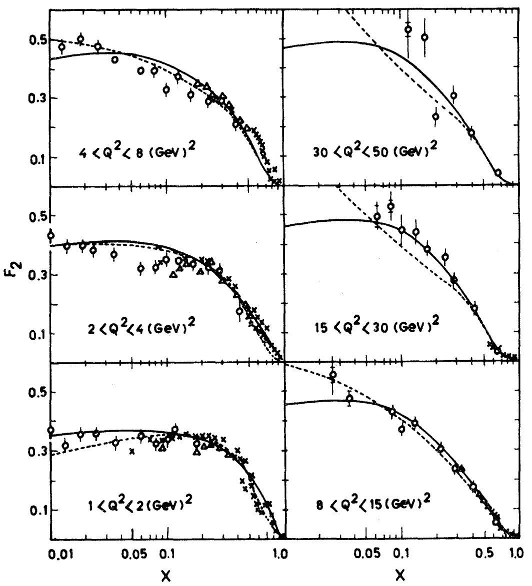

FIG. 5. Fits (solid curves) to the CHIO data (Ref. 17) on $F_2^{\gamma p}$ compared with the prediction (dashed curves) of the QCD parton model, Ref. 19.

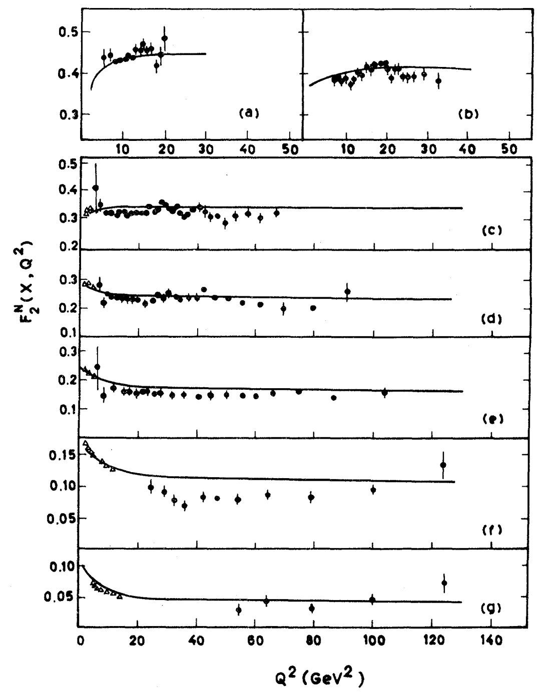

FIG. 6. Predictions of the model compared with the MSU-Fermilab data (Ref. 9) for several x bins (a) $0.03<x<0.06$; (b) $0.06<x<0.1$; (c) $0.1<x<0.2$; (d) $0.2<x<0.3$; (e) $0.3<x<0.4$; (f) $0.4<x<0.5$; (g) $0.5<x<0.7$. Errors shown are statistical only.

$$F_2^{\nu N}=x(u_v+d_v+8S)\ , \tag{3}$$

$$xF_3^{\nu N}=x(u_v+d_v)\ , \tag{4}$$

where we have assumed four-effective flavors, and SU(4)-symmetric sea and followed standard notation for the parton densities. The latter are parametrized, in the scaling limit, as

$$xq_i(x)=A_i x^{\alpha_i}(1-x)^{\beta_i}\ ,\quad i=1,2,3,4\ , \tag{5}$$

where q_i, for $i=1,\ldots,4$ represents u_v, d_v, S, and the gluon distribution G, respectively. The parametrization (5), which was first derived by Kuti and Weisskopf[21] in their relativistic QPM, is the simplest one consistent with positivity,[22], Regge[23] and threshold behavior[24] for appropriately chosen parameters α_i and β_i. It has, therefore, become standard in current literature. *Regge-behavior*[23] in the $x\to 0$ limit demands

$$\alpha_1=\alpha_2=\tfrac{1}{2}\ ,\quad \alpha_3=\alpha_4=0\ . \tag{6}$$

Quark-counting rules[24] suggest

$$\beta_1=\beta_2=3\ ,\quad \beta_3=7\ ,\quad \beta_4=5\ . \tag{7}$$

The normalizations $A_{1,2}$ are then fixed by the *valence sum rule*[25]

$$\int_0^1 u_v(x)dx=2\int_0^1 d_v(x)dx=2 \tag{8}$$

and given by

$$A_1=2A_2=2/B(\alpha_1,1+\beta_1)\ , \tag{9}$$

where $B(x,y)$ is the Euler beta function. The *Callan-Gross sum rule*[26] given by

$$\int_0^1 F_2^p(x)dx=C_p\ ,\quad \int_0^1 F_2^n(x)dx=C_n\ , \tag{10}$$

where C_p, C_n are constants measuring weighted mean-square charge per parton in the target and given by[21,27]

$$C_p=13/81\simeq 0.16,\quad C_n=10/81\simeq 0.123 \tag{11}$$

determine the sea normalization,

$$A_3=8/45\simeq 0.177\ . \tag{12}$$

Finally, the *energy-momentum sum rule*[28]

$$\int_0^1 dx\,(F_2^{\nu N}+xG)=1 \tag{13}$$

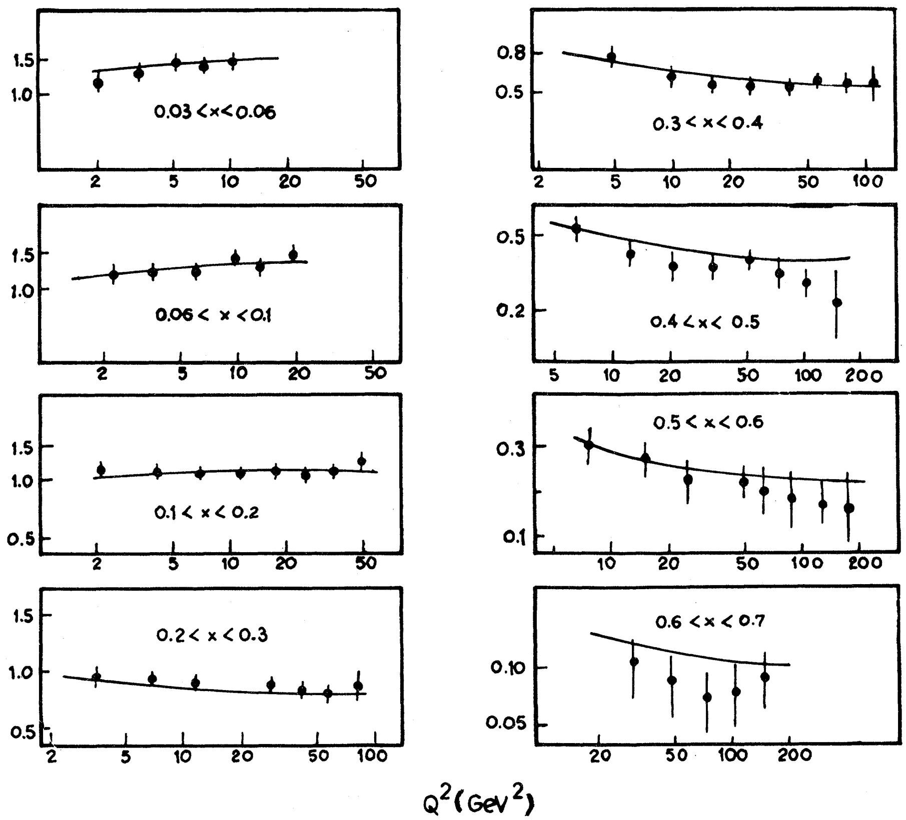

FIG. 7. Predictions of the model compared with CDHS data (Ref. 18) on F_2^ν.

fixes A_4, the gluon normalization,

$$A_4 = 44/15 \simeq 2.93 . \tag{14}$$

Thus, the parton distributions are *completely* specified in the scaling limit ($Q^2 \to \infty$, x fixed).

At subasymptotic values of Q^2, we assume the validity of the quark-parton description and, therefore, of Eqs. (1)—(4), (8), (11), and (13) with Q^2-dependent parton distributions. Besides, it is also proposed that scaling violations are purely $O(1/Q^2)$ effects consistent with known analyticity properties[29] in Q^2 and the kinematic constraint[30] in the $Q^2 \to 0$ limit. These considerations lead to the following simple ansatz for the Q^2-dependent parton densities,

$$xu_v(x,Q^2) = \frac{2x^{\alpha_1}(1-x)^{\tilde{\beta}_1(Q^2)}}{B(\alpha_1, 1+\tilde{\beta}_1(Q^2))} , \tag{15}$$

$$xd_v(x,Q^2) = \frac{x^{\alpha_2}(1-x)^{\tilde{\beta}_2(Q^2)}}{B(\alpha_2, 1+\tilde{\beta}_2(Q^2))} , \tag{16}$$

$$xS(x,Q^2) = \tilde{A}_3(Q^2) x^{\alpha_3}(1-x)^{\beta_3} , \tag{17}$$

$$xG(x,Q^2) = \tilde{A}_4(Q^2) x^{\alpha_4}(1-x)^{\beta_4} , \tag{18}$$

with

$$\tilde{A}_3(Q^2) = A_3 Q^2/(Q^2 + m_\rho{}^2) ,$$

$$m_\rho{}^2 = (\rho \text{ mass})^2 \simeq 0.6 \text{ GeV}^2 , \tag{19}$$

$$\tilde{\beta}_i(Q^2) = \beta_i Q^2/(Q^2 + Q_i^2) , \quad i = 1,2 , \tag{20}$$

where other parameters (except $\tilde{A}_4$) are as given above. In (15) and (16) α_1, and α_2 retain their Q^2-independent values since these are related (linearly) to Q^2-independent intercepts (at $t=0$) of the contributing Regge trajectories. The threshold exponents β_1 and β_2 acquire Q^2-dependence[31] through Eq. (20). A similar Q^2 dependence of A_3 given by (19) accounts for the vanishing of the structure functions[30] in the $Q^2 \to 0$ limit and the contribution from the nearby vector-meson pole[32]

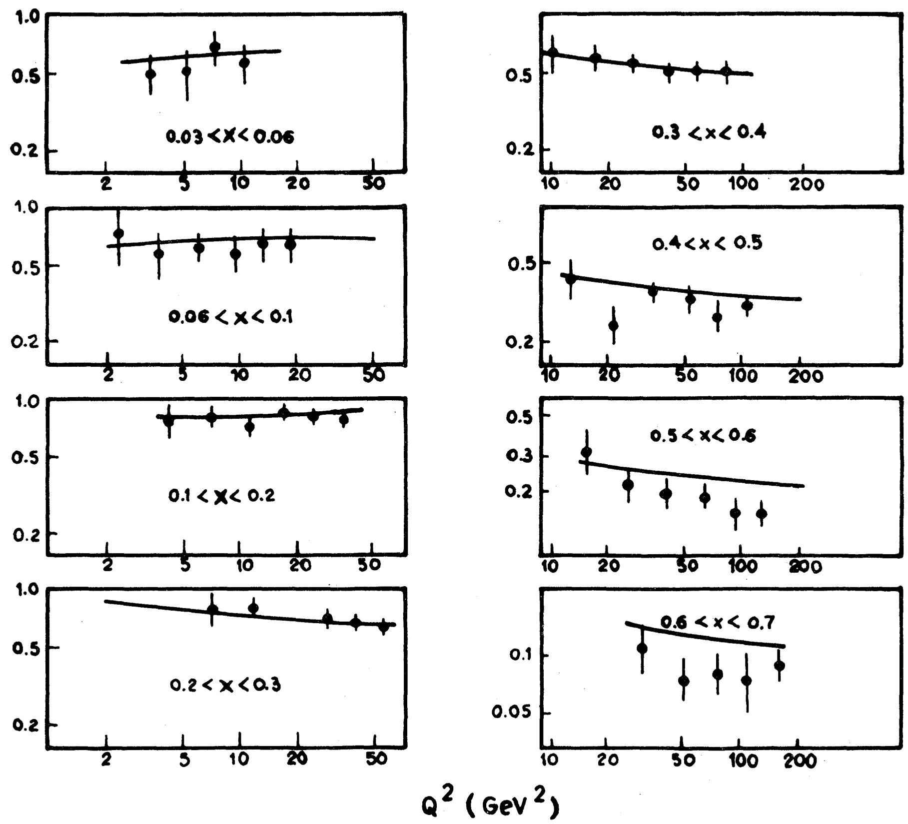

FIG. 8. Predictions of the model compared with CDHS data (Ref. 18) on xF_3^{ν}.

$(\rho,\omega,\ldots)$. The gluon normalization $\widetilde{A}_4(Q^2)$ is then determined from Eq. (13). Finally, therefore, there are only two free parameters ${Q_1}^2$ and ${Q_2}^2$.

These two parameters are determined from a simultaneous fit to the SLAC-MIT[16] and CHIO[17] data on F_2^p, F_2^d, and the corresponding Nachtmann moments.[17] The fits are displayed in Figs. 1–5. The values obtained are[33]

$$Q_1^2=2.1\pm0.23 \text{ GeV}^2 ,$$
$$Q_2^2=0.75\pm0.26 \text{ GeV}^2 . \tag{21}$$

For comparison, we have also indicated, in Figs. 1–5, the prediction of the QCD parton model of Buras and Gaemers[19] (BG). The $O(1/Q^2)$ effects are particularly transparent in Figs. 1–2 displaying the Nachtmann moments of proton and deuteron structure functions as straight lines in $1/Q^2$.

A χ^2 test establishes the supremacy of the present description over that of the QCD parton models.[19,20] We obtain, a total $\chi^2\simeq632$ for 446 data points with only two adjusted parameters. The corresponding figures for the BG model[19] are $\chi^2\simeq1300$ with sixteen parameters[34] while the Glück-Reya[20] model gives $\chi^2\simeq2830$ for the above data.

The predictions of the model are confronted with the MSU-Fermilab[9] data (Fig. 6) and the CDHS[18] neutrino data (Figs. 7–8). Good agreements result, noting that systematic errors of about 5% are not shown in these data. We also obtain good agreement with the BCDMS[8] data which are generally consistent with those of the MSU-Fermilab[9] group.

The Q^2-dependent parton distribution functions resulting in the model are shown in Fig. (9) for two values of Q^2 ($Q^2=10$ and 1000 GeV^2). The gluon distribution is much flatter compared to the predictions of the BG model[19] (shown by dashed curves, Fig. 9).

In conclusion, we emphasize that the analysis presented here constitutes, perhaps, the strongest evidence of dominant $O(1/Q^2)$ contributions in

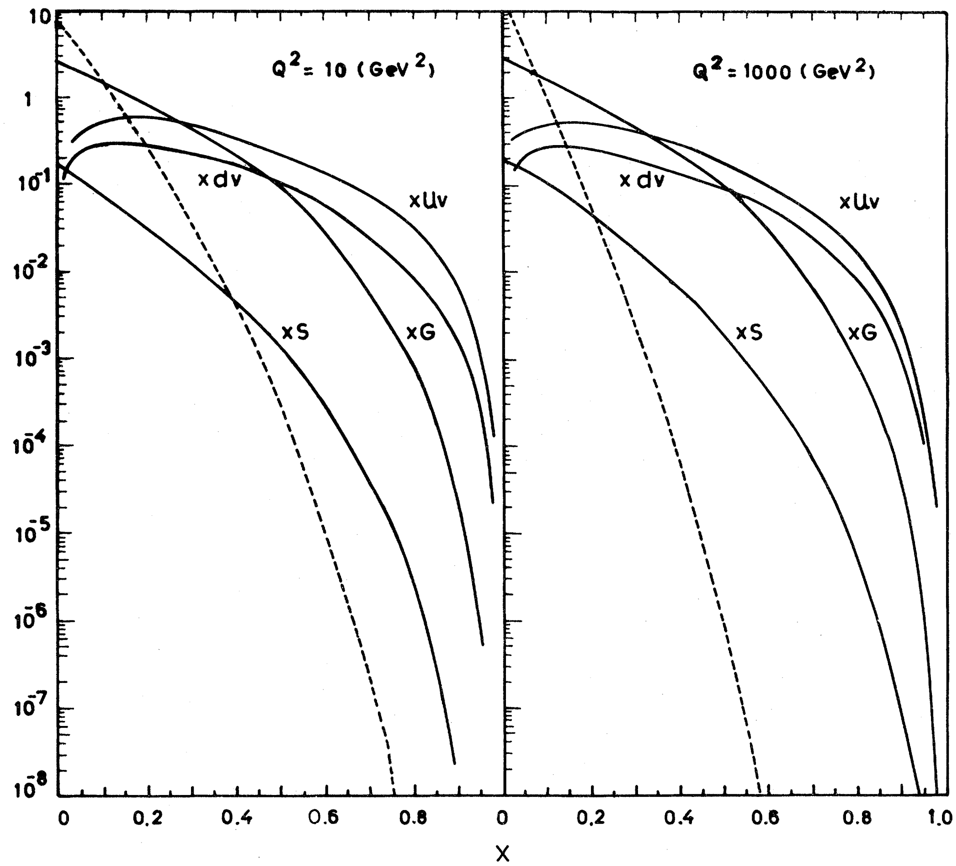

FIG. 9. Prediction of the present model for the parton densities at $Q^2 = 10$ and 1000 GeV^2. The dashed curves are the gluon distributions in the BG model (Ref. 19).

scaling violation, by demonstrating that the latter *alone* suffice to provide for a description of the DIS data which is much superior and economical over that of any other model hitherto proposed. Simultaneously, it also demonstrates the power and versatility of the quark-parton description when supplemented by the general requirements of positivity, Regge behavior, quark-counting rules, and quark-parton sum rules. It would be interesting to confront this model with other deep inelastic processes admitting of a quark-parton treatment.

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*Permanent address.

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