PHYSICAL REVIEW D VOLUME 26, NUMBER 11 1 DECEMBER 1982

Forward production of baryons in quark and gluon jets

Gad Eilam* and M. Sajjad Zahir
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403
(Received 23 June 1981)

Inclusive production of protons in quark and gluon jets is calculated in perturbative QCD and the recombination model. We first calculate the inclusive distribution of a parton jet to decay into three quarks, and then recombine them to form a proton. The computation is performed in the framework of the same model that successfully describes the production of pions in parton jets. Our results, which are applicable to $x \gtrsim 0.5$, show a small production of baryons with $p/\pi \sim 1\%$. We point at other possible sources of baryons which must be operative if the abundant production of baryons shown in the low-x data will persist at higher x and high q^2.

I. INTRODUCTION

Perturbative quantum chromodynamics (QCD) has definite predictions for inclusive distributions of partons (quarks and gluons) originating from high-q^2 parton jets.[1] Once the partons degrade down from q^2, which sets the scale for the perturbative QCD calculation, to some typical hadronic scale (denoted hereafter by Q_0^2) they hadronize to form the particles observed in experiment. The hadronization process is still outside the realm of QCD calculations, and it therefore requires a specific model.

The recombination model[2] successfully describes the forward production of hadrons in hardon-hadron collisions.[3] That success prompted the application of the recombination model to another "soft" process: the above-mentioned hadronization of partons produced in QCD jets. Subsequently, very good agreement was achieved between the recombination model—used in conjunction with perturbative QCD—and data on e^+e^- annihilation into pions.[4] The ingredients used in this calculation were the jet calculus of QCD,[1] employed to yield the inclusive distribution of a parton jet to fragment into a quark-antiquark pair, and the recombination function describing the hadronization of these quarks to form a pion.[3] A similar calculation was performed for the fragmentation of parton jets into particles with significant coupling to gluon pairs, such as glueballs or η'.[5]

The recombination function for baryons is known too,[3] and has been derived from deep-inelastic scattering on protons.[6] Since no similar data are available for deep-inelastic scattering on pions, it is believed that the proton recombination function is even better known than the corresponding function for pions. Furthermore, the proton recombination function has been successfully applied to forward production of baryons in hadron-hadron collisions.[7] Therefore, it should be straightforward to generalize the calculation of Ref. 4 for pion production to proton production from quark and gluon jets. That is the subject of this paper.

The generalization from meson to baryon production is indeed straightforward but rather lengthy, and requires an assumption which restricts its applicability to large x. We first calculate from perturbative QCD the joint distribution of three quarks, and then recombine them to form a proton. Small production of protons is found in the forward direction, with $p/\pi \sim 1\%$. We compute the x and q^2 dependence of quark and gluon decay functions and parametrize the results in a simple form.

Recent experiments observed baryons which are presumably produced from parton jets. (1) The DASP II group[8] observed enhancement of protons in decays of the Υ meson, as compared to the neighboring e^+e^- continuum. (2) At PETRA (Ref. 9) there is a 10% to 15% fraction of baryons among quark fragments, with a p/π ratio that increases with x. (3) In deep-inelastic scattering the European Muon Collaboration (EMC) data[10] indicate a substantial ratio of p/π. It is clear that these data, being at low q^2, or integrated over the whole x region, are not directly of concern to our calculation here. However, if in future experiments, baryons with momenta of the order of 10 GeV and higher, and with $x \gtrsim 0.5$ are produced from parton jets with $p/\pi \gtrsim 10\%$ as observed in

Refs. 8–10, we would conclude that recombination of three partons in the manner described in this paper is not the major process for baryon production. We will comment in the Summary about other possible contributions to the decay functions of partons into baryons.

The paper is organized as follows. In Sec. II we summarize the successful computation of the fragmentation functions of partons into mesons; in Sec. III we present the formalism for the calculation of the fragmentation functions of partons into baryons. In Sec. IV we outline the method of the numerical computation and present our results for the decay functions of quarks and gluons into protons, and for $e^+e^-\to p\bar{p}+X$, and in Sec. V we summarize and discuss our results, add more comments about the data and mention other possible mechanisms for baryon production.

II. FRAGMENTATION FUNCTIONS FOR PARTON JETS INTO PIONS

The fragmentation function $D_i^\pi(x,Q^2)$ for the decay of a parton into a pion in the recombination model is[4]

$$xD_i^\pi(x,Q^2)=\int_0^1 dx_1\int_0^1 dx_2 G_{i\to kl}(x_1,x_2,Q^2,Q_0{}^2)R_{kl}^\pi(x_1,x_2,x)\,, \tag{1}$$

where $G_{i\to kl}$ stands for the probability to find a quark k at x_1 and antiquark l at x_2 (a summation is implicitly assumed to account for all the allowed pairs); it is derivable—once Q^2 is high—from the jet calculus of perturbative QCD. The pion recombination function R_{kl}^π specifies the probability for the formation of a pion at x from the recombination of a quark k at x_1 and an antiquark l at x_2. $G_{i\to kl}$ is represented in Fig. 1(a), where each of the three dark blobs represents a single-parton inclusive distribution. In the first step parton i with Q^2 (and momentum fraction of unity, a trivial fact which we should remember for the next section) decays inclusively into a parton j with $k^2<Q^2$ and with momentum fraction y. This step has the probability $G_{i\to j}(y,Q^2,k^2)$. The subsequent splitting $j\to j'j''$ is described by $P_{j\to j'}(z/y)$ where z is the momentum fraction for parton j. Partons j', j'' then decay inclusively to partons k, l each with $Q_0{}^2$ and carrying momentum fractions x_1,x_2, respectively. Therefore, from Fig. 1(a) we have,

$$\begin{aligned}x_1x_2G_{i\to kl}(x_1,x_2,s') &= \sum_{j,j',j''}\int_{Q_0{}^2}^{K^2(Q^2)}\frac{dk^2}{k^2}\frac{\alpha(k^2)}{2\pi}\int_{x_1+x_2}^{1}dy\, G_{i\to j}(y,Q^2,k^2)\int_{x_1}^{y-x_2}\frac{dz}{y}P_{j\to j'}\left[\frac{z}{y}\right]\frac{x_1}{z}G_{j'\to k}\left[\frac{x_1}{2},k^2,Q_0{}^2\right]\\ &\quad\times\frac{x_2}{y-z}G_{j''\to l}\left[\frac{x_2}{y-z},k^2,Q_0{}^2\right],\end{aligned} \tag{2}$$

where α is the QCD running coupling and

$$s'=\ln\left[\frac{\ln Q^2/\Lambda^2}{\ln Q_0{}^2/\Lambda^2}\right] \tag{3}$$

with $Q_0=0.8$ GeV as determined by structure-function analysis,[11] $\Lambda=0.65$ GeV,[6] and $K^2(Q^2)=\min(Q^2,Q_1{}^2)$ where $Q_1{}^2$ is some cutoff, taken for definiteness to be 30 GeV^2 to restrict the recombination of partons far away from each other in transverse momentum. The final answer is insensitive to the value of Q_1. The summation in Eq. (1) is over all parton types: quarks, antiquarks, and gluons.

To cross from the parton representation, which includes gluons as well as quarks, to the valence quark (valon) representation where $\pi^+=u\bar{d}$, etc., requires gluon conversion into $q\bar{q}$ pairs. In other words, three types of processes are summed in order to form a pion: (1) direct $q\bar{q}$ production; (2) qg or $g\bar{q}$ production followed by a $g\to q\bar{q}$ splitting; (3) production of gg followed by a splitting of each gluon into $q\bar{q}$. For each of the above processes the relevant quark pairs are recombined to form pions.

It is more convenient to work with moments of fragmentation functions D, evolution functions G and splitting function P. Define the moments

$$D_i^{\pi}(n,s')=\int_0^1 dx\, x^{n-1} D_i^{\pi}(x,s')\ , \tag{4}$$

$$G_{ij}^{n}(Q^2,k^2)=\int_0^1 dy\, y^{n-1} G_{i\to j}(y,Q^2,k^2)\ , \tag{5}$$

$$G_{i\to kl}^{mn}(s')=\int_0^1 dx_1 \int_0^1 dx_2 x_1{}^{m-1} x_2{}^{n-1} G_{i\to kl}(x_1,x_2,s')\ , \tag{6}$$

$$P_{jj'}^{mn}=\int_0^1 dz\, z^{m-1}(1-z)^{n-1} P_{j\to j'}(z)\ ; \tag{7}$$

then Eq. (2) is equivalent to the following equation for the moments

$$G_{i\to kl}^{mn}(s')=\sum_{j,j',j''}\int_{Q_0{}^2}^{K^2(Q^2)} \frac{dk^2}{k^2}\frac{\alpha(k^2)}{2\pi} G_{ij}^{m+n-1}(Q^2,k^2) P_{jj'}^{mn} G_{j'k}^{m}(k^2,Q_0{}^2) G_{j''l}^{n}(k^2,Q_0{}^2)\ . \tag{8}$$

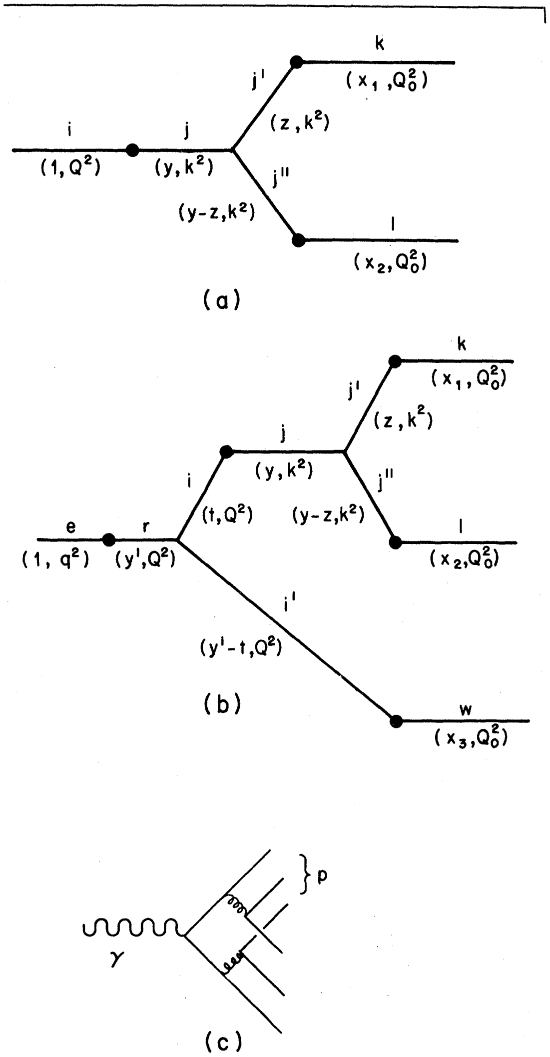

FIG. 1. (a) The inclusive decay of a parton i to two partons k,l. (y,k^2) are the longitudinal momentum fraction and momentum squared, respectively, of parton j, etc. (b) The inclusive decay of a parton e to three partons k,l,w. (c) A possible source of baryons in e^+e^- collisions: overlap of jets.

With the help of the pion recombination function[3,4]

$$R_{kl}(x_1,x_2,x)=\frac{x_1x_2}{x}\delta\left[\frac{x_1}{x}+\frac{x_2}{x}-1\right]$$

the moments defined in Eq. (4) are given, using Eq. (1), in terms of the moments of $G_{i\to kl}$ [see Eq. (8)] as follows:

$$D_i^{\pi}(n,s')=\sum_{m=0}^{n-3}\binom{n-3}{m} G_{i\to kl}^{m+2,n-m-1}(s')\ . \tag{9}$$

It is then clear that only $n\geq 3$ moments can be calculated using the present formalism.

The above equation for the moments holds for the direct production of $q\bar{q}$. When gluon conversion is needed the moments in Eq. (8) are multiplied by P_{gq}^{m1} [which is obtained from the more general definition in Eq. (7)] when parton k is a gluon, by P_{gq}^{n1} when parton l is a gluon, and by $P_{gq}^{m1}P_{gq}^{n1}$ when both k and l are gluons.

In Eq. (8) the moments of the evolution functions are given by G_f^n, G_u^n which represent the favored $(i\to i)$, unfavored $(i\to j\neq i)$ quark transitions, respectively, and by G_{gQ}^n, G_{Qg}^n, G_{gg}^n for gluon$\to$quark, quark$\to$gluon, and gluon$\to$gluon, respectively. For the splitting function we have the moments P_{QQ}^{mn}, P_{gQ}^{mn}, P_{Qg}^{mn}, P_{gg}^{mn} for quark $\to$quark + gluon, gluon$\to$quark + antiquark, quark$\to$gluon + quark, and gluon$\to$gluon + gluon, respectively. The formulas for all these moments in leading-order QCD are given in the Appendix.

Before moving on to the computation of proton production let us assume that the initial parton i carries a momentum fraction t, and not 1 as in Fig. 1(a). It is easy to see that the following changes are required in Eq. (2): (1) the upper limit of the y integration extends to t instead of 1; (2)

y/t replaces y in $G_{i\to j}(y,Q^2,k^2)$; (3) a factor of $1/t$ is inserted. Therefore Eq. (8) is multiplied by t^{m+n-2}, and we get

$$G^{mn}_{i\to kl}(t,s')=t^{m+n-2}G^{mn}_{i\to kl}(s') \tag{10}$$

for the moments of $i\to kl$ with i carrying a momentum fraction t.

III. FRAGMENTATION FUNCTION OF PARTON JETS INTO PROTONS

The inclusive production of three partons k,l,w each with $Q_0{}^2$ and carrying momentum fractions x_1,x_2,x_3, respectively, is described in Fig. 1(b). The decaying parton with q^2 is denoted by e. One immediately recognizes Fig. 1(a) as a subdiagram in Fig. 1(b), this time with i carrying a momentum fraction t. As compared with Fig. 1(a) we have in the baryon case two more evolutions and one more splitting. Since in the valon representation $p=uud$ then k,l,w are summed over all possible combinations including gluons which are converted into quarks, leading to a total of 24 configurations, compared with 8 for the pion case.

Using the notation of Fig. 1(b) we obtain the following equation describing the inclusive parton process $e\to klw$ as given in perturbative QCD:

$$\begin{aligned} x_1x_2x_3G_{e\to klw}(x_1,x_2,x_3,s) &= \sum_{r,i,i'}\int_{Q_0{}^2}^{K^2(q^2)}\frac{dQ^2}{Q^2}\frac{\alpha(Q^2)}{2\pi}\int_{y+x_3}^{1}dy'\,G_{e\to r}(y',q^2,Q^2) \\ &\quad\times\int_y^{y'-x_3}\frac{dt}{y'}P_{r\to i}\left[\frac{t}{y'}\right]\frac{x_3}{y'-t}G_{i'\to w}\left[\frac{x_3}{y'-t},s'\right]x_1x_2G_{i\to kl}(x_1,x_2,s') \end{aligned} \tag{11}$$

with s' given in Eq. (3) and

$$s=\ln\left[\frac{\ln\dfrac{q^2}{\Lambda^2}}{\ln\dfrac{Q_0{}^2}{\Lambda^2}}\right] \tag{12}$$

with the same parameters Q_0, Λ, and Q_1 (which determines the cutoff in the Q^2 integration) as for the pion case. Equation (11), the generalization of Eq. (2), is then translated into the following triple-moment equation, where we made use of Eqs. (5)–(7) and (10):

$$G^{mnv}_{e\to klw}(s)=\sum_{r,i,s}\int_{Q_0{}^2}^{K^2(q^2)}\frac{dQ^2}{Q^2}\frac{\alpha(Q^2)}{2\pi}G_{er}^{m+n+v-2}(q^2,Q^2)P_{ri}^{m+n-1,v}G^{v}_{i'w}(s')G^{mn}_{i\to kl}(s')\ . \tag{13}$$

Here we have defined

$$G^{mnv}_{e\to klw}(s)=\int_0^1 dx_1\int_0^1 dx_2\int_0^1 dx_3 x_1{}^{m-1}x_2{}^{n-1}x_3{}^{v-1}G_{e\to klw}(x_1,x_2,x_3,s)\ . \tag{14}$$

We then obtain from Eq. (8)

$$\begin{aligned} G^{mnv}_{e\to klw}(s)&=\sum_{r,i,i'}\sum_{j,j',j''}\int_{Q_0{}^2}^{K^2(q^2)}\frac{dQ^2}{Q^2}\frac{\alpha(Q^2)}{2\pi}\int_{Q_0{}^2}^{Q^2}\frac{dk^2}{k^2}\frac{\alpha(k^2)}{2\pi}G^{v}_{er}(q^2,Q^2)P_{ri}^{m+n-1,v} \\ &\quad\times G^{v}_{i'w}(s')G_{ij}^{m+n-1}(Q^2,k^2)P^{mn}_{jj''}\,G^{m}_{j'k}(k^2,Q_0{}^2)G^{n}_{j''l}(k^2,Q_0{}^2) \end{aligned} \tag{15}$$

which is a generalization of Eq. (8). The evolution and splitting function G and P, respectively, were discussed in the previous section and are explicitly given in the Appendix.

Once the inclusive distribution of three partons in the jet e is known we recombine them to form a proton. In analogy with Eq. (1) the decay function for $e\to$proton is

$$xD^p_e(x,q^2)=\int_0^1 dx_1\int_0^1 dx_2\int_0^1 dx_3G_{e\to klw}(x_1,x_2,x_3,s)R^p_{klw}(x_1,x_2,x_3,x)\ , \tag{16}$$

where the proton recombination function is given by[3,6]

$$R^{p}_{klw}(x_1,x_2,x_3,x)=19.9\frac{(x_1x_2)^{1.65}x_3{}^{1.35}}{x^{4.65}}\delta\left[\frac{x_1}{x}+\frac{x_2}{x}+\frac{x_3}{x}-1\right]. \tag{17}$$

This form is valid when k,l,w with x_1,x_2,x_3 are u,u,d quarks, respectively; for other assignments for k,l,w—which are always with x_1,x_2,x_3, respectively—a $u\,(d)$ quark goes with a power 1.65 (1.35). The factor of 19.9 is just the normalization factor[11] $1/B(1.65,3)B(1.65,1.35)$. There exists a higher normalization in the literature (see, e.g., Jones *et al.* in Ref. 4) but that calculation does not incorporate gluon conversion. Furthermore, the normalization in Eq. (17) can be convincingly justified in the framework of the valon model.[11] Defining the moments of the decay function as

$$D^{p}_{e}(n,s)=\int_0^1 dx\, x^{n-1}D^{p}_{e}(x,s)\,, \tag{18}$$

and substituting Eq. (17) into Eq. (15) we find that

$$D^{p}_{e}(n,s)=19.9\sum_{I=0}^{n'-1}\sum_{J=0}^{n'-1-I}\begin{bmatrix}n'-1\\ I\end{bmatrix}\begin{bmatrix}n'-1-I\\ J\end{bmatrix}G^{n'-I-J+\delta,J+\beta+1,I+\gamma+1}_{e\to klw}(s) \tag{19}$$

which is analogous to Eq. (9). In Eq. (19),

$$n'=n-4.65 \tag{20}$$

and δ,β,γ are equal to 1.65 (1.35) if quarks k,l,w, respectively, are u (d). Once gluon conversion is present, Eq. (19) will be multiplied by $P^{h,1}_{gQ}$ for each final gluon, with $h=n'-I-J+\delta$, $J+\beta+1$, $I+\gamma+1$ if k,l,w are gluons, respectively.

Moments in Eq. (19) are calculated for noninteger n [see Eq. (20)] and then inverted to yield an x distribution using the methods of Refs. 4–6. The anomalous dimensions and the moments of the splitting functions are well defined for all n, once factorials and $\psi(n)$ are analytically continued. While for the pion case [see Eq. (9)] all moments with $n\geq 3$ were calculable, here the restriction is more severe, i.e., $n\geq 5.65$ which limits our calculation to $x\gtrsim 0.25$. Let us comment here that this restriction is due to the method of calculation rather than to the model. If we could integrate in an intermediate stage instead of using the binomial expansion, lower x values will then be predictable. Unfortunately the calculations are already rather lengthy as outlined in the next section, and extra integrations would turn our computation into an unfeasible one.

Finally from Eqs. (15) and (19) we obtain the main result of this paper, in analogy with Eq. (9):

$$\begin{aligned}D^{p}_{e}(n,s)=19.9\sum_{I=0}^{n'-1}\sum_{J=0}^{n'-1-I}\begin{bmatrix}n'-1\\ I\end{bmatrix}\begin{bmatrix}n'-1-I\\ J\end{bmatrix}\sum_{r,i,i'}\sum_{j,j',j''}\int_{Q_0{}^2}^{K^2(q^2)}\frac{dQ^2}{Q^2}\frac{\alpha(Q^2)}{2\pi}\\ \times\int_{Q_0{}^2}^{Q^2}\frac{dk^2}{k^2}\frac{\alpha(k^2)}{2\pi}G^{n'+4.65}_{er}(q^2,Q^2)\\ \times P^{n'-I+\delta+\beta,I+\gamma+1}_{ri}G^{I+\gamma+1}_{i'w}(s')\\ \times G^{n'-I+\delta+\beta}_{ij}(Q^2,k^2)P^{n'-I-J+\delta,J+\beta+1}_{jj'}\\ \times G^{n'-I-J+\delta}_{j'k}(k^2,Q_0{}^2)G^{J+\beta+1}_{j''l}(k^2,Q_0{}^2)\,.\end{aligned} \tag{21}$$

Gluon conversion should be added if necessary, as explained above.

In the next section the method for computing $D_e^p(n,s)$ and its inverse $xD_e^p(x,s)$ is described, and our results are presented.

IV. COMPUTING $xD_e^p(x,s)$

It is now straightforward to compute the moments $D_e^p(n,s)$ given in Eq. (21) and then to invert them to obtain $xD_e^p(x,s)$, the fragmentation function describing the inclusive decay of a parton jet of type e into a proton.

Let us first discuss the problem of the double integration in Eq. (21). Since there are five moments of the evolution function G to integrate, the generic form of the integral is

$$\lambda_{ABCDE}(s)\equiv\int_{Q_0^2}^{K^2(q^2)}\frac{dQ^2}{Q^2}\frac{\alpha(Q^2)}{2\pi}\int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\frac{\alpha(k^2)}{2\pi}G_A^M(q^2,Q^2)G_B^N(Q^2,Q_0^2)G_C^U(Q^2,k^2)\times G_D^V(k^2,Q_0^2)G_E^W(k^2,Q_0^2)$$

$$=\int_{Q_0^2}^{K^2(q^2)}\frac{dQ^2}{Q^2}\frac{\alpha(Q^2)}{2\pi}\left[\frac{\alpha(q^2)}{\alpha(Q^2)}\right]^{d_A^M}\left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right]^{d_B^N}\Lambda_{CDE}(s')\ , \tag{22}$$

where the G's are written in the order in which they appear in Eq. (21), i.e.,

$$M=n'+4.65,\quad N=I+\gamma+1,\quad U=n'-I+\delta+\beta,\quad V=n'-I-J+\delta,\quad W=J+\beta+1\ . \tag{23}$$

A,B,C,D,E stand for the lower indices N, $+$, and $-$ (see Appendix). We express the functions G in terms of expressions with d_+, d_-, and d_N in the exponents. Thus, for instance, with $e=r=u$ quark, A will be N, $+$, or $-$, but when e=gluon, r=quark, A will only take the values $+$, $-$. The d's are the relevant anomalous dimensions, and

$$\Lambda_{CDE}(s')=\int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\frac{\alpha(k^2)}{2\pi}\left[\frac{\alpha(Q^2)}{\alpha(k^2)}\right]^{d_E^W}\left[\frac{\alpha(k^2)}{\alpha(Q_0^2)}\right]^{d_C^U+d_D^V}$$

$$=\frac{1}{2\pi b(d_E^W-d_C^U-d_D^V)}\left\{\left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right]^{d_C^U+d_D^V}-\left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right]^{d_E^V}\right\}. \tag{24}$$

Therefore Λ_{CDE} is a function of $d_C^U+d_D^V$ and d_E^W, a fact which can be symbolically written as

$$\Lambda_{C+D,E}=\Lambda_{CDE}\ . \tag{25}$$

The generic integral in Eq. (22) is then equal to (with its dependence on the d's explicitly shown through the indices of λ)

$$\lambda_{A,B,E,C+D}(s)=\frac{1}{2\pi b(d_E^W-d_C^U-d_D^V)}[\Lambda_{C+D+B,A}(s)-\Lambda_{B+E,A}(s)]\ . \tag{26}$$

Thus the two integrations in Eq. (21) have been fully performed, and there are no integrals in our numerical calculation.

The numerical evaluation of Eq. (21), with the appropriate λ's substituted for the double integral, will involve eight summations: two from the binomial expansion, and six summations over all possible intermediate partons in Fig. 1(b). There is also one summation yielding 24 terms to account for the possible labeling of the final partons. All of these are just for one value of n and one value of s, and for a fixed initial parton e. It turns out that there are too many summations for our computer to produce a result in a reasonable time. Fortunately, we found an approximation which made the calculation feasible. By looking at the anomalous dimensions which appear in the Appendix we find that for large enough n the unfavored $q \rightarrow q'$ transition is suppressed, i.e.,

$$G_u^N << G_f^n, G_{gQ}^n, G_{Qg}^n, G_{gg}^n , \tag{27}$$

and we therefore drop all terms in Eq. (21) in which G_u appears. Hence, if $e = u$ quark then r can be either a u quark or a gluon, etc. Of course d quarks will appear at the end of a chain originating from a u quark, but only as a result of gluon splitting. Numerically, Eq. (27) holds—down to a percent level—for all s values for $n \geq 3$, thus restricting the validity of our results to a slightly higher x than would be possible without the approximation $G_u^n = 0$. We have also repeated the calculation of Ref. 4 with $G_u^n = 0$, which greatly simplifies the computation, and found negligible changes for moments with $n > 5$. The physical implication of the $G_u = 0$ assumption is clear: It is equivalent to the neglect of the sea distribution with respect to valence distribution in hadrons, for high x, since the singlet and nonsinglet evolution function moments are equal [see Eq. (A3a)].

The computation is then performed using Eqs. (21) and (26) and the expressions for the moments in the Appendix with the assumption $G_u = 0$, and inserting gluon conversions when one or more of the final partons is a gluon. Our results for the moments $D_e^p(n,s)$ with $e = u,d,s,g$ are presented in Fig. 2 for $s = 2.92$, 3.30 corresponding to $q \simeq 30$, 180 GeV, respectively, using $Q_0 = 0.8$ GeV and $\Lambda = 0.65$ GeV. Scaling violations and the ordering with respect to parton type, are all as expected. The results for $D^p(n,s)$ are smaller by about 2 orders of magnitude than the corresponding results for $D^\pi(n,s)$. This is of course reflected in the fragmentation function $xD_e^p(x,s)$ presented as a function of x in Figs. 3 and 4 for two values of s, and various initial parton types e. The moments $D^p(n,s)$ were easily and accurately inverted assuming the following form for the fragmentation functions

$$xD_e^p(x,s) = \sum_i a_i(s)(1-x)^{b_i(s)} , \tag{28}$$

where $i = 1,2,3$ when e is a gluon, and $i = 1,2$ otherwise. The parameters are found to be well approximated by straight lines in s, i.e.,

$$\begin{aligned} a_i(s) &= a_i(0) + a_i' s , \\ b_i(s) &= b_i(0) + b_i' s . \end{aligned} \tag{29}$$

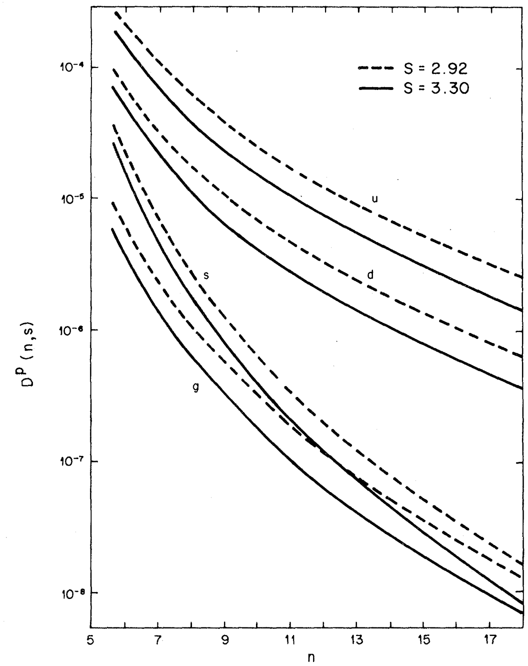

FIG. 2. Moments of the fragmentation functions of u,d,s,g to decay into protons as a function of n for two values of s [defined in Eq. (12)].

The values of $a_i(0)$, a_i', $b_i(0)$, b_i' are given in Table I for each of the four fragmentation functions.

If baryons are produced in e^+e^- from well separated quark jets, we can use our results to predict

$$N_{e^+e^-}^{p+\bar{p}} = \frac{1}{2\sigma} x \left[\frac{d\sigma^p}{dx} + \frac{d\sigma^{\bar{p}}}{dx} \right] . \tag{30}$$

Note the extra x in the above definition, which is not usually there. It is clear that

$$N_{e^+e^-}^{p+\bar{p}} = \tfrac{2}{3} xD_u^p + \tfrac{1}{6} xD_d^p + \tfrac{7}{6} xD_s^p \tag{31}$$

and using the computed values for the relevant fragmentation functions we find the results which are plotted as dashed lines in Figs. 3 and 4. Although the results are presented for small x, our calculation is reliable for forward production only because (1) only moments with $n \geq 5.65$ were used in the inversion and (2) the approximation $G_u = 0$ affects the low-x region of $xD^p(x,s)$.

V. SUMMARY AND CONCLUSIONS

We calculated the contribution of three-quark recombination to the forward production of pro-

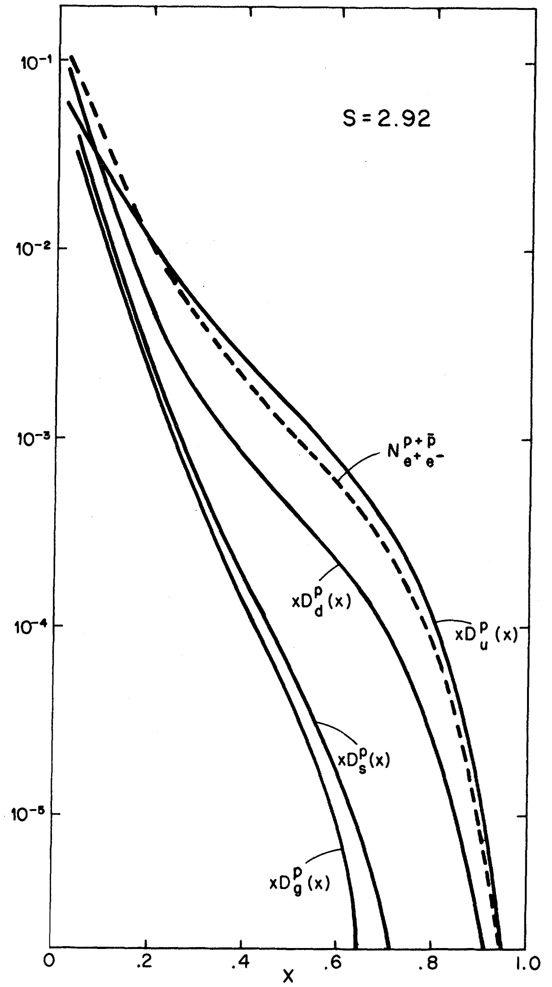

FIG. 3. Fragmentation functions for u,d,s,g to decay into protons as a function of x for $s=2.92$ ($q=30$ GeV). $N^{p+\bar p}_{e^+e^-}$ is defined in Eq. (30) (note the extra x).

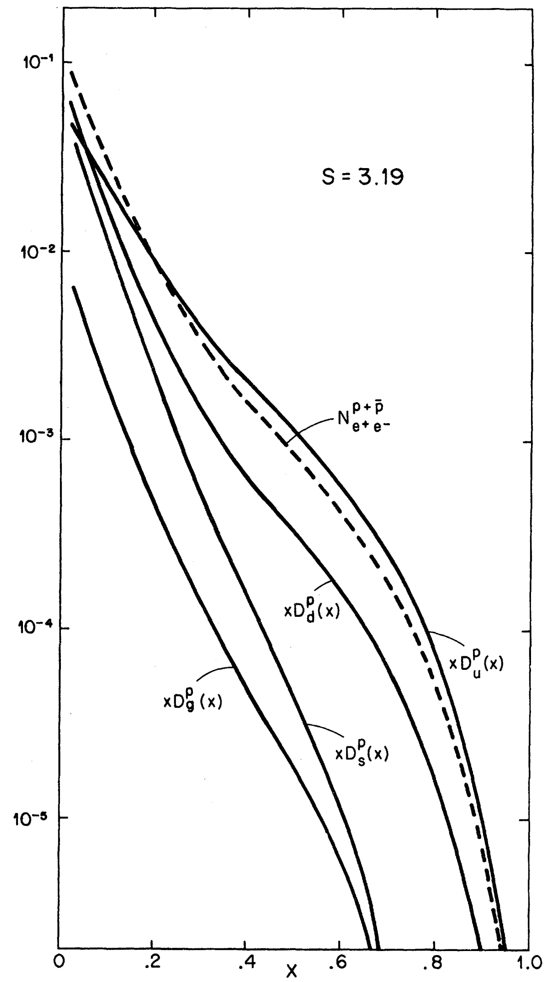

FIG. 4. The same as Fig. 3, for $s=3.19$ ($q=100$ GeV).

tons from high-q^2 ($q^2 \gtrsim 900$ GeV^2) partons. The joint distribution of three quarks in a jet was first obtained in perturbative QCD, followed by quark recombination as prescribed in the recombination model. The fragmentation functions of partons decaying into protons are about 2 orders of magnitude smaller than the corresponding functions for pions. We feel that it is too early to say whether the results are encouraging or discouraging as compared to present data which show $(p/\pi)_{\rm data} \sim 10(p/\pi)_{\rm theory}$.

The main reason for this feeling is that the present data are a little too far from the range of applicability of our results. First, the average multiplicity of baryons, and of pions for that matter, cannot be predicted reliably from perturbative QCD in leading-logarithmic approximation. Data for the Υ region[8] have q^2 values much below where we expect our results to be valid. In our results there is no indication for enhancement of the p/π ratio in gluon jets as compared with quark jets.[13] The EMC data[10] are below $q^2=900$ GeV^2, although they show enhancement for the p/π ratio at reasonably high x values. For the PETRA data[9] at $q=12$ and 30 GeV the maximum energy of the proton is around 2 GeV, which is certainly not high enough ($x \lesssim 0.15$ for $q=30$ GeV and $x \lesssim 0.30$ for $q=12$ GeV). Furthermore, the data at 30 GeV seem lower than the 12-GeV data, much more so than expected from scaling violations, which is encouraging. This may also indicate that the protons are not coming from well separated jets, a possibility depicted in Fig. 1(c).

If however, the trend showing high p/π ratios

TABLE I. Parameters for the fragmentation functions of various parton types decaying into protons. The parametrization, defined in Eqs. (28) and (29), is valid for $2.92 < s < 3.30$, covering $30 < q < 180$ GeV.

	xD_u^p	xD_d^p	xD_s^p	xD_g^p
$a_1(0)$	2.736×10^{-2}	1.186×10^{-2}	1.352×10^{-2}	5.238×10^{-4}
a_1'	-5.97×10^{-3}	-2.79×10^{-3}	-3.34×10^{-3}	-4.98×10^{-5}
$b_1(0)$	1.194	1.436	4.578	1.460
b_1'	0.55	0.56	0.48	1.00
$a_2(0)$	2.032×10^{-1}	5.914×10^{-1}	3.514×10^{-1}	1.676×10^{-2}
a_2'	-4.688×10^{-2}	-1.623×10^{-1}	-9.074×10^{-2}	-2.786×10^{-3}
$b_2(0)$	12.33	35.92	28.42	-0.49
b_2'	-0.45	-6.80	-3.80	4.60
$a_3(0)$				3.822×10^{-4}
a_3'				-6.49×10^{-5}
$b_3(0)$ b_3'				

persists at higher x and q^2 values and baryons are shown to emerge from well separated jets, then one is forced to conclude that the contribution of the process described here is about 10 times too small and that other sources of baryons are much more important. A major contender can be the process in which a diquark is produced with a probability which is not very different from the probability to produce a single quark. However, extended structure of diquarks may suppress this contribution. Consequently, the p/π ratio can be of order 1. Although a calculation of that source of baryons has not been carried out from first principles, the phenomenological models that incorporate it[14] are successful in explaining the present data.

Our particular recipe for recombination effectively forces us to use perturbative QCD, through the jet calculus, at low values of momenta down to Q_0/Λ where the strong coupling constant is big. We do this by resorting to the valon model which serves as a phenomenological substitute for our ignorance of the bound-state problem of QCD. We believe this part of our calculation to be the most vulnerable one, and we feel that alternatives to it are welcome.

ACKNOWLEDGMENTS

One of us (G.E). would like to thank the members of the ITS for their hospitality. We would like to thank Rudy Hwa for very helpful discussions. This work was supported in part by U.S. Department of Energy under Contract No. EY-76-S-06-2230, TA 4, Mod. A008.

APPENDIX: DEFINITIONS OF VARIOUS MOMENTS OF ANOMALOUS DIMENSIONS

We repeat here results for the leading-order QCD moments which appear in Ref. 12. m,n are in general nonintegers, and $b=(33-2f)/12\pi$ where f is the number of flavors ($f=3$ used in our computation). The moments of the splitting functions are

$$P_{QQ}^{mn}=P_{Qg}^{nm}=\frac{4}{3}\left[\frac{\Gamma(m)\Gamma(n-1)}{\Gamma(m+n-1)}+\frac{\Gamma(m+2)\Gamma(n-1)}{\Gamma(m+n+1)}\right], \tag{A1a}$$

$$P_{gQ}^{mn}=\frac{1}{2\Gamma(m+n+2)}[\Gamma(m+2)\Gamma(n)+\Gamma(m)\Gamma(n+2)], \tag{A1b}$$

$$P_{gg}^{mn}=G\left[\frac{\Gamma(m-1)\Gamma(n+1)+\Gamma(m+1)\Gamma(n-1)}{\Gamma(m+n)}+\frac{\Gamma(m+1)\Gamma(n+1)}{\Gamma(m+n+2)}\right]. \tag{A1c}$$

Define

$$d_N^n=\frac{1}{3\pi b}\left[-3-\frac{2}{n(n+1)}+4\gamma+4\psi(n+1)\right], \quad \text{(A2a)}$$

$$d_{Qg}^n=\frac{-2}{3\pi b}\frac{2+n+n^2}{n(n^2-1)}, \quad \text{(A2b)}$$

$$d_{gQ}^n=-\frac{f}{2\pi b}\frac{2+n+n^2}{n(n+1)(n+2)}, \quad \text{(A2c)}$$

$$d_{gg}^n=-\frac{3}{\pi b}\left[\frac{11}{12}+\frac{1}{n(n-1)}+\frac{1}{(n+1)(n+2)}-\frac{f}{18}-\gamma-\psi(n+1)\right], \quad \text{(A2d)}$$

$$\Delta^n=d_+^n-d_-^n=[(d_N^n-d_{gg}^n)^2+4d_{gQ}^n d_{Qg}^n]^{1/2}, \quad \text{(A2e)}$$

$$\rho^n=\frac{d_N^n-d_{gg}^n}{\Delta^n}, \quad \text{(A2f)}$$

$$d_\pm^n=\tfrac{1}{2}(d_N^n+d_{gg}^n\pm\Delta^n), \quad \text{(A2g)}$$

where

$$\psi(n+1)=\frac{\Gamma'(n+1)}{\Gamma(n+1)}.$$

Then the moments of the evolution functions are

$$G_u^n=\frac{1}{2f}G_S^n-\frac{1}{2f}G_N^n, \quad \text{(A3a)}$$

$$G_f^n=\left[1-\frac{1}{2f}\right]G_N^n+\frac{1}{2f}G_S^n, \quad \text{(A3b)}$$

$$G_N^n=e^{-d_N^n\xi}, \quad \text{(A3c)}$$

$$G_S^n=\tfrac{1}{2}(1+\rho^n)e^{-d_+^n\xi}+\tfrac{1}{2}(1-\rho^n)e^{-d_-^n\xi}, \quad \text{(A3d)}$$

$$G_{gQ}^n=\frac{-d_{gQ}^n}{2f\Delta^n}(e^{-d_-^n\xi}-e^{-d_+^n\xi}), \quad \text{(A3e)}$$

$$G_{Qg}^n=\frac{-d_{Qg}}{\Delta^n}(e^{-d_-^n\xi}-e^{-d_+^n\xi}), \quad \text{(A3f)}$$

$$G_{gg}^n=\tfrac{1}{2}(1-\rho^n)e^{-d_+^n\xi}+\tfrac{1}{2}(1+\rho^n)e^{-d_-^n\xi}, \quad \text{(A3g)}$$

where G_u^n, G_f^n, G_N^n, G_S^n, G_{gQ}^n, G_{Qg}^n, G_{gg}^n are moments of the evolution functions for favored $(i\to i)$ quark transitions, unfavored $(i\to j\neq i)$ quark transitions, nonsinglet, singlet, gluon→quark, quark→gluon, gluon→gluon, respectively; all moments can be expressed in terms of d_+, d_-, d_N only. ξ is defined as

$$\xi=\ln\left[\frac{\alpha(Q_0{}^2)}{\alpha(k^2)}\right] \quad \text{(A4)}$$

with

$$\alpha(k^2)=\frac{1}{b\ln\frac{k^2}{\Lambda^2}}. \quad \text{(A5)}$$

---

*Permanent address: Physics Department, Technion—Israel Institute of Technology, Haifa, Israel.

[1]K. Konishi, A. Ukawa, and G. Veneziano, Phys. Lett. 78B, 243 (1978); Nucl. Phys. B157, 45 (1979); Phys. Lett. 80B, 259 (1979); J. Kalinowski, K. Konishi, P. N. Scharbach, and T. R. Taylor, Nucl. Phys. B181, 221 (1981).

[2]K. P. Das and R. C. Hwa, Phys. Lett. 68B, 459 (1977).

[3]R. C. Hwa, in *Partons in Soft-Hadronic Processes,* proceedings of the Europhysics Study Conference, Erice, Italy, 1981, edited by R. T. Van de Walle (World Scientific, Singapore, 1981), and references therein.

[4]V. Chang and R. W. Hwa, Phys. Rev. Lett. 44, 439 (1980); 44, 1163(E) (1980); Phys. Rev. D 23, 728 (1981); see also L. M. Jones, K. E. Lassila, U. Sukhatme, and D. Willen, *ibid.* 23, 717 (1981).

[5]V. Chang, G. Eilam, and R. C. Hwa, Phys. Rev. D 24, 1818 (1981).

[6]R. C. Hwa and M. S. Zahir, Phys. Rev. D 23, 2539 (1981). We stick to their values of Q_0 and Λ in order to find out whether the same model with the same parameters can describe both meson and baryon production. Although Λ may seem a little too high, in any circumstances Q_0/Λ will be too high for a lowest-order QCD calculation. The justification for the procedure employed here in the hadronization stage can be found in Ref. 11.

[7]E. Takasugi and X. Tata, Phys. Rev. D 23, 2573 (1981); R. Migneron and J. L. Robinson, Report No. Fermilab-Pub-80/72 THY, 1980 (unpublished).

[8]DASP II Collaboration, H. Albrecht *et al.*, Phys. Lett. 102B, 291 (1981).

[9]TASSO Collaboration, R. Brandelik *et al.*, Phys. Lett. 89B, 418 (1980); 94B, 91 (1980); 94B, 444 (1980).

[10]J. J. Aubert *et al.*, Wuppertal report, 1981 (unpublished).

[11]R. C. Hwa, Phys. Rev. D 22, 1593 (1980).

[12]G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977).

[13]W. Hofmann [Z. Phys. C 10, 351 (1981)] claims to have calculated the inclusive production of baryons in

quark and gluon jets using the recombination model. His calculation of the function $G_{e \to klw}(x_1, x_2, x_3, x, s)$ describing the inclusive decay of a parton into three partons has no resemblance to our calculation. While we calculate it using perturbative QCD he emphasizes the phase-space restrictions on the parton spectrum. His criticism of the applications of the recombination model to inclusive production from parton jets is well taken, but it applies mainly to nonforward hadrons.

[14]G. Schierholz and M. Teper, DESY report, 1981 (unpublished); B. Andersson, G. Gustafson, and T. Sjöstrand, Nucl. Phys. B197, 45 (1982).