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Supersymmetry at ordinary energies. Masses and conservation laws

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An assessment is made of the general problems encountered in formulating a realistic supersymmetric theory in which the spontaneous breakdown of supersymmetry occurs at ordinary energies accessible to accelerators. As a starting point, three problems are identified in $SU(3)\times SU(2)\times U(1)$ supersymmetric models with only quark and lepton chiral superfields: the up quarks get no masses, baryon and lepton (B and L) conservation are violated by renormalizable and hence unsuppressed interactions, and the scalar counterparts of the quarks and leptons are too light. An interesting $SU(3)\times SU(2)\times U(1)$ model of Dimopoulos and Georgi that avoids these problems is considered; it is found that this model contains B- and L-nonconserving effective interactions of dimensionality 5 that lead to proton decay at too rapid a rate. To guarantee natural B and L conservation in effective interactions of dimensionality 4 and 5, it is suggested that the gauge group that describes physics at ordinary energies contains a factor, such as another U(1), in addition to $SU(3)\times SU(2)\times U(1)$. Such theories do not contain dimension-5 L-nonconserving interactions which could produce an observable neutrino mass, but they do allow dimension-6 B- and L-nonconserving interactions that would lead to proton decay at an observable rate. Supersymmetry is found to constrain the matrix elements for proton decay in a phenomenologically interesting way. A general explanation is given of how such theories naturally avoid the problem of light scalars, as found by Fayet. The formalism is used to derive general approximate mass relations for the scalar superpartners of the quarks and leptons. The problem of anomalies in the new U(1) current is considered, and one attractive scheme for avoiding them is offered, in which the anomalies cancel for precisely three generations of quarks and leptons.

I. INTRODUCTION

We know that if nature at a fundamental level really obeys supersymmetry,[1] then the supersymmetry must be spontaneously broken. However, we do not know whether the vacuum expectation values involved in this breakdown are of an "ordinary" scale, say of order 300 GeV, like those involved in the breakdown of the electroweak gauge symmetry, or whether they are much larger, perhaps as high as the Planck mass. One reason to suspect that supersymmetry is broken only at ordinary energies arises from the hierarchy problem[2]: if supersymmetry is unbroken at higher energies, then it can protect some scalar fields from getting enormous masses in the spontaneous breakdown of whatever symmetry connects strong and electroweak interactions; these scalars would then survive to provide a second stage of symmetry breaking, in which the electroweak gauge symmetry and supersymmetry are both spontaneously broken at ordinary energies. At any rate, the hypothesis that supersymmetry is unbroken above some ordinary energy scale of order 300 GeV is worth careful attention, because it has direct experimental implications at the energies that will soon be accessible to accelerators. Models which are supersymmetric down to ordinary energies have already been developed and their consequences studied, most notably by Fayet.[3]

The purpose of this paper is to take a fresh look at the implications of supersymmetry at ordinary energies, and especially to apply to supersymmetric theories certain developments that have occurred in the last few years, particularly in our point of view regarding baryon and lepton conservation.

One of the successes of the $SU(3)\times SU(2)\times U(1)$ gauge theory of strong and electroweak interactions was that it explained the experimentally observed conservation laws for baryon and lepton number (B and L) without needing to invoke separate global conservation laws. The most general renormalizable interaction that one can write down involving just ordinary quarks, leptons, Higgs doublets, and

$SU(3)\times SU(2)\times U(1)$ gauge fields is forced by $SU(3)\times SU(2)\times U(1)$ gauge invariance to conserve B and L. However, supersymmetric theories necessarily contain additional particles, including scalar superpartners of the quarks and leptons, and in general these could participate in renormalizable B- or L-nonconserving interactions. Of course, one could simply impose B or L conservation laws as global symmetries of supersymmetric models. This does not seem to me to be the most fruitful approach. Global symmetries, like strangeness, isospin, etc., increasingly appear to us as incidental consequences of gauge symmetries and renormalizability, with no status as *a priori* constraints on a fundamental level. It would be preferable to hold on to the feature of $SU(3)\times SU(2)\times U(1)$ gauge theories, that B and L conservation follow automatically from gauge symmetries and renormalizability, when the particle content of the theories is extended to incorporate supersymmetry. As we shall see, this requirement proves to be a useful guide in constructing a satisfactory supersymmetric model of physics at ordinary energies.

The approach advocated here will turn out to be indispensable if B and L are found to be not exactly conserved. If the renormalizable interactions of particles of ordinary masses ($\lesssim 300$ GeV) automatically conserve B and L, then any observable B- and L-nonconserving interactions among these particles would have to be due to the effects of virtual superheavy (10^{15} GeV?) particles with different gauge quantum numbers. Such effects would show up in the effective Lagrangian which describes physics at ordinary energies as nonrenormalizable effective interactions of dimensionality $d>4$, with coupling constants suppressed by $d-4$ powers of the superheavy mass. Now, just as $SU(2)\times U(1)$ can be used[4] directly to study the structure of the B- and L-nonconserving nonrenormalizable effective interactions, because even though $SU(2)\times U(1)$ is spontaneously broken at ordinary energies it is not broken at the superheavy masses of the particles whose exchange violates B and L conservation, in the same way if supersymmetry is unbroken at these superhigh energies, then it can be used to constrain the effective interactions with $d>4$ that are responsible for B- and L-nonconserving interactions of ordinary particles. Supersymmetry, like $SU(2)\times U(1)$, may become manifest in the selection rules for proton decay.

The general rules for constructing renormalizable and nonrenormalizable supersymmetric effective interactions are very simple. Suppose a theory involves a set of left-handed chiral scalar superfields, generically called S, together with their right-handed adjoints S^*. Leaving aside the possibility of extra derivatives and gauge couplings, the most general supersymmetric Lagrangian will be of the form

$$\mathscr{L}=f(S)_F+f(S)_F^*+d(S^*,S)_D\ , \tag{1}$$

where f and d are arbitrary functions, and as usual F and D denote the terms in these functions proportional to $\theta_L\theta_L$ or $\theta_L\theta_L\theta_R\theta_R$, respectively, with θ_L and θ_R the left- and right-handed parts of the fermionic superfield coordinate θ. Note that we do not include F terms of functions of both S^* and S, because these would not be supersymmetric, and we do not include D terms of functions of S or S^* alone, because these would be total derivatives. It is easy to extend this Lagrangian so that it has a local gauge symmetry: in the same way as is familiar for renormalizable theories,[1] we add a Yang-Mills-type term for the real vector superfield $V\equiv\sum_\alpha g_\alpha t_\alpha V_\alpha$ and replace S^* in the D term with S^*e^{2V}. However, the inclusion of gauge couplings (or extra derivatives) in an effective interaction would generally increase its dimensionality without introducing new possibilities for B or L nonconservation. The expression for the Lagrangian (1) in terms of ordinary component fields is given in Appendix A.

Now, the dimensionality of a scalar superfield is $+1$ (in powers of mass), and the dimensionality of the F term or D term of any function is equal to the dimensionality of the function plus 1 or 2, respectively. Thus the terms in the effective Lagrangian (1) of various dimensionalities have the structure

$$\begin{aligned}
&d=2:\ (S)_F\ ,\\
&d=3:\ (SS)_F\ ,\\
&d=4:\ (S^*S)_D\ ,\ \ (SSS)_F\ ,\\
&d=5:\ (S^*SS)_D\ ,\ \ (SSSS)_F\ ,\\
&d=6:\ (S^*SSS)_D\ ,\ \ (S^*S^*SS)_D\ ,\ \ (SSSSS)_F
\end{aligned} \tag{2}$$

and so on, plus the Hermitian conjugates. Of course, the S superfields have a number of indices which must be suitably contracted in order to maintain invariance under all gauge symmetries. In this way, we can construct all possible terms in the effective Lagrangian which can occur up to some definite degree of suppression by superheavy masses.

In order to implement this program, it is necessary at least to know what are the gauge symmetries and particle spectrum which appear in a supersymmetric model at ordinary energies, and this opens up the whole range of phenomenological problems faced by supersymmetric theories. For orientation, in Sec. II, we inspect the properties of a minimal supersymmetric model, with only a $SU(3)\times SU(2)\times U(1)$ gauge symmetry and only chiral scalar superfields corresponding to the known quarks and leptons. This turns out to have severe problems: no up-quark masses, renormalizable (and hence unsuppressed) B- and L-nonconserving interactions, and unobserved light scalars. Then in Sec. III, we explore a model proposed by Dimopoulos and Georgi,[5] in which renormalizable B- and L-nonconserving interactions are ruled out by a discrete reflection symmetry; Higgs superfields are introduced to give masses to all quarks and leptons; and light scalars are avoided by supposing that supersymmetry is explicitly but softly broken by terms in the Lagrangian with $d=2$ and $d=3$. We find that although proton decay is suppressed in this model, it is not suppressed enough: there are B- and L-nonconserving terms with $d=5$ in the effective Lagrangian, and these lead to processes like $p\to\mu^+K^0$ with a proton lifetime of order 10^{28} yr. To avoid such catastrophes, we consider in Sec. IV the addition of an extra gauge symmetry to the invariance group of the model. It is found that all $d=5$ terms are forbidden, including not only the B- and L-nonconserving terms which gave trouble in the Dimopoulos-Georgi model, but also any L-nonconserving terms which could give an observable neutrino mass. However, there remain allowed $d=6$ terms, which could produce a "normal" $(B-L)$-conserving proton decay at an observable rate. Supersymmetry and gauge symmetries constrain the matrix elements for this decay, with interesting phenomenological consequences.

The extra gauge symmetry which is introduced in order automatically to suppress B and L nonconservation also has important implications for particle masses. These are described in Sec. V, and as an illustration a special case of the models described by Fayet is analyzed in detail in Appendix B. It is shown that supersymmetric theories can quite naturally account for the fact that we observe quarks and leptons with masses much less than those of their scalar superpartners, without having to break supersymmetry explicitly in the Lagrangian. Supersymmetry is broken only spontaneously here, the scale of this breaking and of $SU(2)\times U(1)$ breaking being set by the coefficients of Fayet-Iliopoulos terms[6] $(V)_D$. As shown by Witten,[7] it is natural for these coefficients not to get large values from the spontaneous breakdown of a semisimple group at superlarge energies, or from perturbative corrections, so this part of the hierarchy problem is solved in such theories. There remains the vexing question of why the $(V)_D$ terms are present at all, and with such small coefficients relative to the superheavy masses. One can hope that this will be explained by nonperturbative effects[7] yielding $(V)_D$ terms proportional to $\exp(-\text{constant}/g^2)$, but such questions will not be addressed in this paper.

Section VI describes one way of canceling the Adler-Bell-Jackiw (ABJ) anomalies[8] in a model with precisely three generations of quarks and leptons. A future paper[9] will apply to "R invariance" the same sort of analysis used here for B and L, but with special attention to the effects of ABJ anomalies and instantons, and will attempt to summarize the problems that still face the formulation of a realistic supersymmetric theory.

II. VARIETIES OF TROUBLE: A MINIMAL MODEL

To appreciate the difficulties encountered in formulating a realistic model that is supersymmetric down to ordinary energies, consider first an $SU(3)\times SU(2)\times U(1)$ gauge theory containing just those chiral scalar superfields whose spin-$\frac{1}{2}$ components are the ordinary leptons and quarks. We will denote these superfields by capital letters: the spin-$\frac{1}{2}$ component of the left chiral scalar superfield $Q_L=(U_L,D_L)$ is the left-handed quark doublet $q_L=(u_L,d_L)$; the spin-$\frac{1}{2}$ components of the left chiral superfields U_R^*,D_R^* are the antiparticles u_R^*,d_R^* of the right-handed singlet quarks; and similarly the superfields L_L and E_R^* have as spin-$\frac{1}{2}$ components the lepton doublet $l_L^-=(\nu_L,e_L^-)$ and singlet e_R^*. The scalar components of these superfields are labeled by capital script letters:

$$\mathcal{Q}_L=(\mathcal{U}_L,\mathcal{D}_L)\ ,\ \mathcal{U}_R^*\ ,\ \mathcal{D}_R^*\ ,\ \mathcal{L}_L=(\mathcal{N}_L,\mathcal{E}_L)\ ,\ \mathcal{E}_R^*\ .$$

Here and wherever not explicitly otherwise indicated, the symbols for particles in the lowest generation are used to represent any particle with the same quantum numbers; thus e stands for e, μ, or

TABLE I. Summary of notation for superfields, with their spin-$\frac{1}{2}$ and spin-0 components and SU(3)×SU(2)×$\tilde{\text{U}}$(1) quantum numbers. Note that where not otherwise indicated, the letters u,d,ν,e refer to quarks and leptons of any generation with the indicated quantum numbers.

Left-chiral scalar superfield	Spin-$\frac{1}{2}$ component	Spin-0 component	SU(3)	SU(2)	Y
$Q_L=(U_L,D_L)$	$q_L=(u_L,d_L)$	$(\mathscr{Q}=(\mathscr{U}_L,\mathscr{D}_L)$	3	2	$-\frac{1}{6}$
U_R^*	u_R^*	$\mathscr{U}_R^*$	$\bar{3}$	1	$\frac{2}{3}$
D_R^*	d_R^*	$\mathscr{D}_R^*$	$\bar{3}$	1	$-\frac{1}{3}$
$L_L=(N_L,E_L)$	$l_L=(\nu_L,e_L)$	$\mathscr{L}_L=(\mathscr{N}_L,\mathscr{E}_L)$	1	2	$\frac{1}{2}$
E_R^*	e_R^*	$\mathscr{E}_R^*$	1	1	-1
H_L	h_L	$\mathscr{H}_L=(\mathscr{Y}_L^0,\mathscr{U}_L^-)$	1	2	$\frac{1}{2}$
H_L'	h_L'	$\mathscr{H}_L'=(\mathscr{H}_L^{+\prime},\mathscr{U}_L^{0\prime})$	1	2	$-\frac{1}{2}$

τ, and so on. For convenience this notation is summarized in Table I.

Apart from kinematic terms and gauge couplings, the most general renormalizable supersymmetric SU(3)×SU(2)×U(1)-invariant interaction among the quark and lepton superfields is a linear combination of the trilinear F terms

$$(L_L L_L E_R^*)_F\ ,\quad (L_L Q_L D_R^*)_F\ ,\quad (D_R^* D_R^* U_R^*)_F\ , \tag{3}$$

with SU(3) and SU(2) indices contracted in an obvious way. There are three conspicuous things wrong with such a theory.

(1) Although a vacuum expectation value of the neutral scalar field $\mathscr{N}_L$ of L_L will break SU(2)×U(1) in the usual way and give mass to the charged leptons and charge $-\frac{1}{3}$ quarks, there is no neutral scalar field here whose vacuum expectation value can give mass to the charge $+\frac{2}{3}$ quarks.

(2) There is no way of extending the definition of baryon and lepton number (B and L) to the scalar fields so that B and L are conserved. In particular, exchange of the $\mathscr{D}_R$ scalar between the last two interactions in (3) can produce the proton decay process $q_L d_R u_R \rightarrow \bar{l}_L$ at a catastrophic rate. (However, $B-L$ can be still defined as a conserved R symmetry.[10])

(3) Such theories contain unobserved light scalars. This is shown most clearly by a theorem of Dimopoulos and Georgi[5] which states that in any supersymmetric theory with gauge group SU(3)×SU(2)×U(1) and in which quark and gluon superfields are the only colored fields, the spontaneous breakdown of supersymmetry and SU(2)×U(1) must leave at least one scalar lighter than the lightest d or u quark.

Different attempts at realistic supersymmetry models can be conveniently characterized by the features that are put in to avoid these three problems.

III. SU(3)×SU(2)×U(1) SUPERSYMMETRIC MODELS

Let us first consider what must be done to surmount the problems discussed in the last section, if we do not wish to expand the gauge group at ordinary energies beyond SU(3)×SU(2)×U(1). In this case the B- and L-nonconserving renormalizable interaction $(D_R^* D_R^* U_R^*)_F$ must be prohibited by some sort of global symmetry. This could of course be B or L conservation itself, but it is also possible to forbid B- and L-nonconserving interactions of dimensionality 4 with weaker symmetries, which would not require complete conservation of B and L. It is also necessary to add at least one Higgs superfield to give mass to the charge $\frac{2}{3}$ quarks, and if the global symmetry which rules out the B- and L-nonconserving term $(D_R^* D_R^* U_R^*)_F$ also rules out the other terms in (3), we have to add a second Higgs superfield to give mass to the charged leptons and charge $-\frac{1}{3}$ quarks as well. [The second Higgs superfield also serves to cancel an SU(2)×U(1) ABJ anomaly introduced by the first spin-$\frac{1}{2}$—Higgs—fermion doublet.] Finally, there is still the problem of the light scalars. The theorem of Dimopoulos and Georgi[5] makes it clear that in this class of models, the unacceptably low values of some scalar masses can be avoided only by supposing that supersymmetry is explicitly broken, but perhaps only "softly," by terms in the Lagrangian of dimensionality $d<4$.

Such a model has been developed by Dimopoulos and Georgi[5] (DG). In their work, the effective Lagrangian that governs physics at ordinary energies arises from an underlying grand unified model, but it can be described here in terms of the effective Lagrangian itself.

In the DG model, the problem of B and L nonconservation is dealt with by imposing a discrete symmetry: invariance under a change of sign of all quark and lepton superfields. Such a symmetry immediately rules out all the interactions (3), B and L conserving as well as nonconserving ones.

To restore the possibility of quark and lepton masses, DG add a pair of color-singlet electroweak-doublet left-chiral Higgs superfields H_L and H'_L with weak hypercharge ($Y\equiv T_3-Q$) equal to $+\frac{1}{2}$ and $-\frac{1}{2}$, which are even under the above discrete symmetry. Apart from kinematic terms and gauge couplings, the complete set of renormalizable supersymmetric interactions which are invariant under $SU(3)\times SU(2)\times U(1)$ and the discrete symmetry consists of just the F terms

$$\begin{aligned}&(H_L H'_L)_F\ ,\ \ (H_L L_L E_R^*)_F\ ,\\&(H_L Q_L D_R^*)_F\ ,\ \ (H'_L Q_L U_R^*)_F\end{aligned}\tag{4}$$

with obvious index contractions. Up as well as down quarks can now get masses, and B and L are automatically conserved, with H_L and H'_L assigned vanishing B and L values.

The remaining problem found in Sec. II was (3), the problem of light scalars, To obviate this, DG suppose that only interactions of dimension 4 are supersymmetric, and that supersymmetry is explicitly (though softly) broken by terms of dimension 2 and 3. With this assumption, particle masses can be given values that do not conflict with observations.

Now, what about the suppressed nonrenormalizable terms in the effective Lagrangian? The least suppressed terms are those of dimensionality 5: either trilinear D terms or quadrilinear F terms. It is easy to see that the most general $SU(3)\times SU(2)\times U(1)$-invariant $d=5$ terms, which are also invariant under the DG discrete symmetry, have the form

$$\begin{aligned}&(L_L E_R^* H_L'^*)_D\ ,\ \ (Q_L D_R^* H_L'^*)_D\ ,\ \ (Q_L U_R^* H_L^*)_D\ ,\\&(L_L L_L H'_L H'_L)_F\ ,\ (Q_L Q_L U_R^* D_R^*)_F\ ,\ (Q_L U_R^* L_L E_R^*)_F\ ,\\&(Q_L Q_L Q_L L_L)_F\ ,\ \ (U_R^* U_R^* D_R^* E_R^*)_F\ ,\end{aligned}\tag{5}$$

again with SU(3) and SU(2) indices contracted in the obvious way. (For notation, see Sec. II or Table I.) The coupling constants of these effective interactions are in general expected to be of order

$$G_5\approx f^2/M\ ,\tag{6}$$

where f is a typical superheavy-particle coupling constant (Higgs or gauge), and M is a typical superheavy mass.

Most of the interactions (5) are innocuous, conserving both B and L. The $(L_L L_L H'_L H'_L)_F$ term provides the sort of $d=5$ L-nonconserving interaction[11] $\nu_L\nu_L\mathscr{H}'^0_L\mathscr{H}'^0_L$ which would give the neutrino a small but possibly observable neutrino mass of order $f^2G_F{}^{-1}/M$. The dangerous terms are the last two, which violate both B and L. These are $d=5$ interactions, so the B- and L-violating matrix elements here are suppressed by only one power of the superheavy mass M, in contrast with the usual case[4] of theories without scalar superpartners of quarks and lepton, where the B- and L-violating effective interactions had $d=6$ and were suppressed by two powers of M. The DG model thus runs the risk of predicting much too fast a rate of proton decay.

We still must ask whether the B- and L-nonconserving interactions $(Q_L Q_L Q_L L_L)_F$ and $(U_R^* D_R^* D_R^* E_R^*)_F$ actually appear in the effective Lagrangian with couplings of order (6), and if so, whether these interactions really produce proton decay at an unacceptable rate.

The first question can of course only be answered in the context of a specific theory of the superheavy degrees of freedom. In the grand unified form of the DG model, there are superheavy color-triplet superfields, where exchange actually does produce the effective interactions $(Q_L Q_L Q_L L_L)_F$ and $(U_R^* D_R^* D_R^* E_R^*)_F$ in tree approximation, with coupling of order (6), where f is a typical Higgs-particle coupling.[12]

As to the effect of these interactions, note that they include the two-fermion—two-scalar terms (see Appendix A)

$$\begin{aligned}&q_L q_L \mathscr{D}_L \mathscr{L}_L\ ,\ \ q_L l_L \mathscr{D}_L \mathscr{D}_L\ ,\\&u_R^* u_R^* \mathscr{D}_R^* \mathscr{E}_R^*\ ,\ \ u_R^* e_R^* \mathscr{U}_R^* \mathscr{D}_R^*\ ,\\&e_R^* d_R^* \mathscr{U}_R^* \mathscr{U}_R^*\ ,\ \ d_R^* u_R^* \mathscr{U}_R^* \mathscr{E}_R^*\ .\end{aligned}\tag{7}$$

(Recall that capital script letters denote scalars.) Each of the heavy-scalar pairs $\mathscr{Q}_L\mathscr{L}_L$, $\mathscr{Q}_L\mathscr{Q}_L$, $\mathscr{U}_R\mathscr{D}_R$, and $\mathscr{U}_R\mathscr{E}_R$ can be created from a fermion pair u_Re_R, u_Rd_R, q_Lq_L, and q_Ll_L, respectively, by emitting a pair of fermion partners h_L, h_L' of the Higgs bosons, which then annihilate through the Majorana mass of these Higgs fermions. This yields the proton decay effective interactions $q_Lq_Lu_Re_R$ and $q_Ll_Lu_Rd_R$. However, the proton decay rate produced in this way is enormously suppressed by the four powers of small Higgs-particle couplings in the matrix element, and is probably too small to be observed. On the other hand, the heavy boson pairs in (7) can also be produced from the corresponding light fermion pairs q_Ll_L, q_Lq_L, u_Rd_R, u_Re_R by emitting a pair of the fermion superpartners of the gauge bosons Z^0 or $W^\pm$, which then annihilate through their Majorana mass term (which DG explicitly included in their model as a soft supersymmetry breaking). This yields the proton-decay effective interactions $q_Lq_Lq_Ll_L$ and $u_R^*u_R^*d_R^*e_R^*$, with coefficients of order

$$\frac{1}{8\pi^2}\frac{f^2}{M}\frac{e^2}{m_W} . \tag{8}$$

[The factor $1/8\pi^2$ is put in because this is a one-loop graph; the factor of f^2/M is the coupling constant (6) of the $d=5$ effective interaction; the factor e^2 arises from the emission and absorption of the fermion superpartners of the $W^\pm$ and Z; and the factor $1/m_W$ arises from the integral on the assumption that m_W is the characteristic mass of the scalar superpartners of the quarks and leptons and the fermion superpartners of the $W^\pm$ and Z^0.] The proton decay rate is then roughly of order

$$\Gamma \approx m_p{}^5 \left| \frac{1}{8\pi^2}\frac{f^2}{M}\frac{e^2}{m_W} \right|^2 . \tag{9}$$

For the process $p \to \mu^+ K^0$, the coupling f^2 is a product of Higgs-particle couplings,

$$f^2 \approx G_F m_d m_s \simeq 1.3\times 10^{-8} . \tag{10}$$

For M of order 10^{15} GeV, the proton lifetime would be of order 10^{24} yr. Even for $M=10^{17}$ GeV (the appropriate value for the grand unified version of the DG model), the proton lifetime is only 10^{28} yr, too short by $2-3$ orders of magnitude.

This difficulty could be avoided, if we do not insist on embedding the DG model in a grand unified theory, by adding additional global symmetries. This is not very attractive—even the modest discrete symmetry of the DG model was not entirely appealing. It seems better to attempt a different solution.

IV. SU(3)×SU(2)×U(1)×$\tilde{G}$ THEORIES: B AND L CONSERVATION

We will now consider a different way of avoiding the problem of insufficient suppression of B and L violations encountered in Secs. II and III. Instead of imposing global symmetries, we will suppose that the gauge group which survives down to ordinary energies is not just SU(3)×SU(2)×U(1), but contains an additional factor $\tilde{G}$. Depending on the transformation properties of the quark and lepton superfields under $\tilde{G}$, it is then quite plausible that B and L nonconservation could be ruled out in $d=4$ and $d=5$ interactions, and yet be not altogether forbidden.

To take one example, suppose that $\tilde{G}=\tilde{\mathrm{U}}(1)$, where $\tilde{\mathrm{U}}(1)$ is another U(1) group, commuting with the original SU(3)×SU(2)×U(1). Suppose also that all of the left-chiral quark and lepton superfields $Q_L, U_R^*, D_R^*, L_L, E_R^*$ have values of the $\tilde{\mathrm{U}}(1)$ quantum number $\tilde{Y}$ with the same sign. Then clearly *all* F terms that involve only quarks and lepton superfields are forbidden, including the B- and L-nonconserving effective interactions with $d=4$ and $d=5$ in (3) and (5), respectively.

To allow us to catalog the varieties of possible B and L nonconservation which remain, let us suppose for definiteness that the $\tilde{\mathrm{U}}(1)$ quantum number $\tilde{Y}$ has equal values, say $+1$, for all left-chiral quark and lepton superfields. In order to provide for quark and lepton masses, we must suppose, as in Sec. III, that in addition there are SU(3)-singlet SU(2)-doublet Higgs superfields H_L and H_L', with ordinary weak hypercharges $Y=+\frac{1}{2}$ and $Y=-\frac{1}{2}$, respectively, and now also with the new $\tilde{\mathrm{U}}(1)$ quantum number $\tilde{Y}=-2$. (As discussed in Sec. VI and Appendix B, other superfields will also have to be added in order to cancel anomalies and to obtain suitable symmetry-breaking solutions.) A complete list (apart, as always, from kinematic terms, gauge couplings, and extra derivatives) of SU(3)×SU(2)×U(1)×$\tilde{\mathrm{U}}(1)$-invariant supersymmetric interactions involving quark, lepton, and Higgs superfields up to dimensionality $d=6$ is

$$d=4:\ (L_LE_R^*H_L)_F\ ,\ (Q_LD_R^*H_L)_F\ , \\ (Q_LU_R^*H_L')_F\ , \tag{11}$$

$$d=5\text{: None ,}$$

$$d=6\text{: } (Q_L Q_L U_R E_R)_D \text{ , } (Q_L U_R D_R L_L)_D \text{ ,}$$

$$\text{etc. ,} \tag{12}$$

where etc. denotes a large number of allowed terms of form $(S^*SS^*S)_D$ that conserve both B and L.

Some remarks are now in order about the implications of this list.

(1) The baryon-number-violating effective interactions of lowest dimensionality have $d=6$, and are therefore adequately suppressed, by two powers of superheavy masses. According to the rules given in Appendix A, these superfield interactions contain the usual four-fermion operators[4]

$$q_L q_L u_R e_R \text{ , } q_L u_R d_R l_L \text{ ,} \tag{13}$$

which produce proton decay directly.

(2) Not all of the four-fermion proton-decay interactions which would be allowed by SU(3) $\times$SU(2)$\times$U(1) are produced in this way; we do not obtain the other two interactions[4]

$$q_L q_L q_L l_L \text{ , } u_R u_R d_R e_R \text{ .} \tag{14}$$

Thus whatever the underlying mechanism of proton decay, it would be expected to have matrix elements of the *LLRR* form which would be expected from the exchange of vector bosons, and not of the *LLLL* or *RRRR* forms which could only be produced by scalar-boson exchange. This has well-known phenomenological consequences,[4] including model-independent (but strong-interaction-dependent) predictions for ratios of all decay rates for $\Delta S=0$ modes, and universal lepton polarizations. Although this result (that *LLRR* terms may occur while *LLLL* and *RRRR* terms are forbidden) can be derived here from the $\widetilde{\mathrm{U}}(1)$ symmetry, it is actually quite general, and follows directly from the supersymmetric nature of the effective interaction, whatever the structure of $\widetilde{G}$, as shown by the last term of Eq. (A7).

(3) There are no $d=5$ effective interactions which could produce an observable neutrino mass. To generate such interactions, one would need Higgs superfields with $\widetilde{Y}=-1$ as well as $\widetilde{Y}=-2$.

(4) The $\widetilde{\mathrm{U}}(1)$ symmetry rules out Higgs mass terms $(H_L H_L')_F$ which would otherwise be allowed by SU(3)$\times$SU(2) $\times$U(1) and supersymmetry. Thus supersymmetry together with SU(3)$\times$SU(2)$\times$U(1)$\times\widetilde{\mathrm{U}}(1)$ is serving its hoped for role of prohibiting scalar mass terms.

V. SU(3)$\times$SU(2)$\times$U(1)$\times G$ THEORIES: MASSES

The extra gauge symmetry $\widetilde{G}$, which was introduced in the previous section in order to preserve the natural suppression of B and L nonconservation, has another attractive feature: it helps in understanding how a reasonable spectrum of masses can arise in a supersymmetric theory. An extra gauge symmetry [with $\widetilde{G}=\mathrm{U}(1)$] was introduced for this purpose by Fayet.[3] I will come back at the end of this section to the relation between his approach and that taken here.

First, what is the problem? It is often said that the chief difficulty faced in constructing models which are supersymmetric down to ordinary energies is to understand why the scalar superpartners of the quarks and leptons do not occur at masses low enough for them to have been seen. In my opinion this somewhat misstates the real problem. After all, there is a natural scale of masses to be expected in a gauge theory, the scale of the masses of the gauge bosons associated with broken symmetries, such as the $W^{\pm}$ at 80 GeV. Clearly any number of scalar counterparts of quarks and leptons could be lurking at energies of order 80–100 GeV, and we would not yet know it.[13] The real problem is one that has bedeviled gauge theories of electroweak intereactions from the start—not why are the scalars so heavy, but why are at least some quarks and leptons so light?

I have no new answer to this problem, but at least we can try to apply a not very satisfying old answer in a supersymmetric context: The observed quarks and leptons are much lighter than the W or Z because they have very weak Yukawa couplings to the Higgs bosons. In a supersymmetric model, this would mean that the $d=4$ interactions of the quark, lepton, and Higgs superfields in (11) all have very small coupling constants. We do not know why this should be the case, but if by assuming these couplings to be small we can understand qualitatively why the quarks and leptons are much lighter than the $W^{\pm}$ while the scalars are not, then we will at least be no worse off than without supersymmetry.

To simplify matters, let us go all the way, and imagine that the coupling constants of renormalizable $(S^3)_F$-type interactions involving quark and lepton superfields [like those in (11)] all vanish. We will however leave it an open possibility that the Higgs superfields may have $(S^3)_F$-type interactions with other superfields, denoted X_L. [It is implicit in these assumptions that the quark and lepton superfields form representations of the $SU(3)\times SU(2)\times U(1)\times \tilde{U}(1)$ gauge group which are separate from those of the Higgs and X superfields.] Our aim then is to see if the broken-symmetry solutions of such a theory with $\langle \mathscr{H}^0\rangle \neq 0$, $\langle \mathscr{H}^{0\prime}\rangle \neq 0$ will have massless quarks and leptons and massive scalars. If so, then we will not have to worry about whether the scalars are heavy enough; they would have to have masses of the order of the only remaining mass scale in the theory, that of the $W^{\pm}$ and Z^0.

The potential of the scalar fields in this sort of theory will have the general form[14]

$$V(\phi,\sigma)=\sum_n \left| \frac{\partial f(\phi)}{\partial \phi_n} \right|^2 + \tfrac{1}{2}\sum_\alpha (g_\alpha \phi^\dagger t_\alpha^\phi \phi + g_\alpha \sigma^\dagger t_\alpha^\sigma \sigma + \xi_\alpha)^2 \;, \tag{15}$$

where σ stands for all the scalar superpartners of the quarks and leptons, ϕ stands for the scalar components of all other superfields, including the scalar Higgs fields $\mathscr{H}_L, \mathscr{H}'_L$ and the scalar components of whatever other superfields X_L may have $(S^3)_F$-type interactions with them, $f(H_L,H'_L,\dots)$ is the trilinear function whose F component describes any such interactions, t_α^ϕ and t_α^σ are the matrices representing the αth gauge generator on the ϕ and σ fields, g_α is the corresponding coupling constant, and ξ_α is the coefficient of the term[6] $(V_\alpha)_D$ which may appear in the Lagrangian for U(1) gauge superfields.

This potential can usefully be rewritten in the form

$$V(\phi,\sigma)=V(\phi,0)+\sum_\alpha g_\alpha D_\alpha(\phi)\sigma^\dagger t_\alpha^\sigma \sigma + \tfrac{1}{2}\sum_\alpha g_\alpha{}^2(\sigma^\dagger t_\alpha^\sigma \sigma)^2 \;, \tag{16}$$

where

$$D_\alpha(\phi)\equiv g_\alpha \phi^\dagger t_\alpha^\phi \phi + \xi_\alpha \;. \tag{17}$$

It is immediately apparent that if we find a value ϕ_0 of ϕ which minimizes the potential $V(\phi,0)$ with $\sigma=0$, then the point $\phi=0$, $\sigma=0$ will be a local minimum of the full potential if and only if the matrix

$$\mathscr{M}^2 \equiv \sum_\alpha g_\alpha D_\alpha(\phi_0) t_\alpha^\sigma \tag{18}$$

is positive definite. In practice if $\mathscr{M}^2$ is positive definite, this is usually (but not always; see the Added Notes) also an absolute minimum, because the only way to get a lower value of $V(\phi,\sigma)$ would be to go to ϕ's for which some of the eigenvalues of $\sum_\alpha g_\alpha D_\alpha(\phi) t_\alpha^\sigma$ have changed sign, and these will be far from the point $\phi=\phi_0$ which minimizes $V(\phi,0)$. On the other hand, if $\mathscr{M}^2$ is not positive, then the true minimum of $V(\phi,\sigma)$ definitely has $\sigma\neq 0$.

The importance here of finding a minimum with $\sigma=0$ is that this is a sufficient condition to have vanishing quark and lepton masses. (Of course, we want the scalar counterparts of the quarks and charged leptons to have vanishing vacuum expectation values also in order to preserve color and charge conservation.) In general, the mass matrix of the left-handed fermions in a renormalizable supersymmetric theory is given in tree approximation by[15]

$$m_{nm}=\left[\frac{\partial^2 f(\mathscr{S})}{\partial \mathscr{S}_n \partial \mathscr{S}_m} \right]_0 , \tag{19}$$

$$m_{\alpha n}=m_{n\alpha}=-\sqrt{2}g_\alpha(\mathscr{S}_0^* t_\alpha)_n \;, \tag{20}$$

$$m_{\alpha\beta}=0 \;, \tag{21}$$

where n and α label the left-handed fermion fields in left-chiral scalar superfields and real gauge vector superfields, respectively, $\mathscr{S}_n$ is the scalar component (σ or ϕ) of the left-chiral scalar superfield S_n, $(t_\alpha)_{nm}$ is the matrix representing the αth generator of the gauge group on the S_n, $f(S)$ is the function whose F term appears in the renormalizable part of the interaction (1), and the subscript zero means that these quantities are evaluated at the minimum of the potential $V(\mathscr{S})$. For the scalar superpartners σ of the quarks and leptons, (19) vanishes under the assumption that σ does not appear in $f(\mathscr{S})$, and (20) vanishes because we are considering a minimum of $V(\mathscr{S})$ with $\sigma_0=0$. Hence all quarks and leptons have zero mass. Of course in the real world $f(\phi,\sigma)$ would be assumed to depend weakly on σ, and the quarks and leptons would not be massless but only relatively light.

We see that the positivity of $\mathscr{M}^2$ will lead to the result we would like, that quarks and leptons are very light, so let us suppose $\mathscr{M}^2$ *is* positive definite. What about the scalars? Equation (16) shows that their mass-squared matrix is just $\mathscr{M}^2$, which we have just now assumed is positive definite. There is no reason to expect that any of the positive-definite eigenvalues of (18) would be much smaller than the natural scale $g\phi_0 \approx m_W$, so all scalar counterparts of the quarks and leptons are expected to be too heavy to have been observed yet.

We can now see why it was necessary to introduce the extra gauge group $\tilde{G}$. The matrices t_α^σ which represent the generators of $SU(3)\times SU(2)\times U(1)$ on the known quarks and leptons all have zero trace, so if there were no extra gauge generator $\tilde{Y}$, then (18) would give $\mathrm{Tr}\mathscr{M}^2=0$, and hence $\mathscr{M}^2$ could not be positive definite. (This argument is closely related to that of Dimopoulos and Georgi.[5]) Of course, to be of use the generators $\tilde{t}_\alpha^\sigma$ which represents $\tilde{G}$ on the quarks and leptons must not themselves be traceless. [However, if the $\mathrm{SU(3)}\times\mathrm{SU(2)}\times\mathrm{U(1)}\times\tilde{G}$ theory comes from a semisimple grand unified theory, $\mathrm{Tr}\tilde{t}_\alpha^\phi+\mathrm{Tr}\tilde{t}_\alpha^\sigma$ would have to vanish.] We see that $\tilde{G}$ must contain at least one $\tilde{\mathrm{U}}(1)$ factor, whose generator $\tilde{Y}$ has nonvanishing trace on the known quarks and leptons. We are almost inevitably led again to the conclusion that $\tilde{G}$ is just $\tilde{\mathrm{U}}(1)$, with quarks and leptons all having $\tilde{Y}$ values of the same sign, as assumed in Sec. IV to enforce a natural suppression of B and L nonconservation.

The general picture outlined here has interesting immediate consequences for the scalar-particle masses, which became exact in the limit of vanishing quark and lepton masses and Yukawa couplings. We note that all members of a given SU(2) multiplet have the same behavior under SU(3), U(1), and $\tilde{G}$, as (18) tells us that the mass splitting in any doublet has a common value

$$M^2(\mathscr{U}_L)-M^2(\mathscr{D}_L)=M^2(\mathscr{N}_L)-M^2(\mathscr{E}_L)\equiv\Delta M^2\ , \tag{22}$$

independent of the generations of the corresponding quarks and leptons. We can also set an upper bound on this splitting. Since SU(2) has $\xi=0$, Eqs. (17) and (18) give

$$\Delta M^2=g^2(\phi^\dagger t_3\phi)_0 \tag{23}$$

with g and t_3 now specifically denoting the SU(2) coupling and generator. This may be compared with the usual formula for the $W^\pm$ mass

$$m_W{}^2=g^2[\phi^\dagger(t_1{}^2+t_2{}^2)\phi]\ . \tag{24}$$

We see that

$$\Delta M^2/m_W{}^2=\langle t_3\rangle/\langle t_1{}^2+t_2{}^2\rangle\ , \tag{25}$$

the averages being weighted with the SU(2)-nonsinglet ϕ-field vacuum expectation values. If only electroweak singlets and doublets have vacuum expectation values (so that m_Z and m_W have the usual ratio), then $|\langle t_3\rangle|\le\frac{1}{2}$, while $\langle t_1{}^2+t_2{}^2\rangle=\frac{1}{2}$, so (25) gives

$$|\Delta M^2|\le m_W{}^2\ . \tag{26}$$

Many more mass relations arise if we make simple assumptions about the extra gauge group $\tilde{G}$. For instance, suppose that $\tilde{G}$ is a U(1) group, and that its generator $\tilde{Y}$ has equal values for all left-handed quark and lepton superfields, say $\tilde{Y}=1$. Then (18) gives the masses of all scalar superpartners of the quarks and leptons in terms of just three unknowns, the values of $g_\alpha D_\alpha(\phi_0)$ for the generators T_3, Y, and $\tilde{Y}$. The scalars corresponding to a quark or lepton of a given $\mathrm{SU(3)}\times\mathrm{SU(2)}\times\mathrm{U(1)}$ type will thus have a mass which is equal for all generations. Also, these generation-independent masses will be subject to the relations

$$M^2(\mathscr{U}_L)+M^2(\mathscr{D}_L)=\tfrac{1}{3}M^2(\mathscr{U}_R^*)+\tfrac{5}{3}M^2(\mathscr{D}_R^*)\ , \tag{27}$$

$$M^2(\mathscr{N}_L)+M^2(\mathscr{E}_L)=\tfrac{5}{3}M^2(\mathscr{U}_R^*)+\tfrac{1}{3}M^2(\mathscr{D}_R^*)\ , \tag{28}$$

$$M^2(\mathscr{E}_R^*)=-\tfrac{2}{3}M^2(\mathscr{U}_R^*)+\tfrac{5}{3}M^2(\mathscr{D}_R^*)\ . \tag{29}$$

If $\tilde{Y}$ varies from generation to generation but is the same within each generation, then the scalar masses will no longer be generation-independent, but all mass splittings within each generation will be the same for all generations, and subject to (27)–(29). Of course, all these results are only approximate, with corrections of order $(m_{\text{quark}}/m_W)^2$.

Fayet[3] has derived Eq. (18) as a formula for the difference between scalar and fermion squared masses, without using the approximation of weak Yukawa couplings. From this, it is straightforward to derive an improved version of Eqs. (22)–(29), and also the Dimopoulos-Georgi theorem.[5] However, as recognized by Fayet, although his derivation applies when there is an unbroken R symmetry, and in some other cases, in general there may appear additional terms in (18). The assumption of weak F-term couplings of the quark and lepton superfields was made here in order to avoid having to invoke R invariance or the details of specific models in deriving (18), and also because we need some such assumption to explain why the quarks and leptons are so light. By using color conservation, Dimopoulos and Georgi were able to obtain their result on the existence of light scalar quarks, without having to make any of the above assumptions.

Our conclusion is that there is no difficulty in understanding why quarks and leptons are so much lighter than scalar and vector bosons, provided the model without quark and lepton fields has a minimum of the potential at which the matrix (18) is positive definite. Whether or not this is the case is a question that must be checked in specific models. As an "existence proof," a semirealistic model which satisfies this positivity condition is given in Appendix B.

VI. ANOMALIES

The problems raised by the minimal model of Sec. II have been satisfactorily avoided in the $SU(3)\times SU(2)\times U(1)\times\widetilde{G}$ models discussed in Secs. IV and V and Appendix B. These models have a variety of other phenomenological problems, including new neutral currents and massless fermionic superpartners of gluons. However, before tinkering with the models to avoid these other problems, it will be more useful first to address the outstanding problem of mathematical consistency raised by the introduction of the extra gauge group, the problem of Adler-Bell-Jackiw (ABJ) anomalies.[8] After seeing what new superfields need to be added to cancel these anomalies, we will be better able (in a future paper[9]) to consider what phenomenological problems may remain.

Let us assume for definiteness that the gauge group is $SU(3)\times SU(2)\times U(1)\times\widetilde{U}(1)$, and that among the left-chiral scalar superfields there are N_g "generations" $(Q_L,U_R^*,D_R^*,L_L,E_R^*)$ of quark and lepton superfields with $\widetilde{Y}=+1$ and N_h pairs of Higgs doublets H_L,H_L' with $Y=\pm\frac{1}{2}$ and $\widetilde{Y}=-2$. Then the introduction of the extra $\widetilde{U}(1)$ gauge group produces the ABJ anomalies

$$SU(3)^2\widetilde{U}(1):\ \ \mathrm{Tr}({T_{SU(3)}}^2\widetilde{Y})=2N_g\ , \tag{30}$$

$$SU(2)^2\widetilde{U}(1):\ \ \mathrm{Tr}({T_{SU(2)}}^2\widetilde{Y})=2N_g-2N_h\ , \tag{31}$$

$$U(1)^2\widetilde{U}(1):\ \ \mathrm{Tr}(Y^2\widetilde{Y})=\tfrac{10}{3}N_g-2N_h\ , \tag{32}$$

$$\widetilde{U}(1)^3:\ \ \mathrm{Tr}(\widetilde{Y}^3)=15N_g-32N_h\ . \tag{33}$$

We note in particular that colored fields with $\widetilde{Y}$ negative must be added to cancel the $SU(3)^2\widetilde{U}(1)$ anomaly. This raises the possibility that the problem of B and L nonconservation which was solved by the introduction of $\widetilde{U}(1)$ may reappear.

As an example of the sort of trouble we can get into in adding new colored superfields, suppose we try to cancel anomalies by embedding $SU(3)\times SU(2)\times U(1)\times\widetilde{U}(1)$ in a larger group whose representations are known to be anomaly free, E_6.[16] This group is chosen because E_6 has an $SO(10)\times\widetilde{U}(1)$ subgroup, and therefore leads to generations of quarks and leptons all with the same value of $\widetilde{Y}$. Specifically, the $\underline{27}$ of E_6 consists of one generation of quarks and leptons with $\widetilde{Y}=+1$, including an extra $SU(3)\times SU(2)\ \times U(1)$-neutral neutrino N_R^* with $\widetilde{Y}=+1$, plus a pair of Higgs doublets H_L,H_L' with $\widetilde{Y}=-2$, plus a pair of color-triplet (antitriplet) SU(2)-singlets with $Y=+\frac{1}{3}$ $(-\frac{1}{3})$ and $\widetilde{Y}=-2$, plus an $SU(3)\times SU(2)\ \times U(1)$-neutral singlet with $\widetilde{Y}=+4$. All anomalies automatically cancel. It is striking that we find here all the superfields of the models we have been considering, including the $\widetilde{Y}=+4$ singlet discussed in Appendix B. Unfortunately, we find other superfields as well. The extra singlet neutrino ν_R^* cannot be superheavy [since $\widetilde{U}(1)$ has to survive down to ordinary energies], so the neutrinos would be expected to get large Dirac masses from their Higgs couplings. Even worse, the color-antitriplet superfield with $Y=-\frac{1}{3}$ and $\widetilde{Y}=-2$ would be expected to have renormalizable F-term interactions with both L_LQ_L and $D_R^*U_R^*$, leading to unsuppressed proton decay at a disastrous rate, just as in the minimal model of Sec. II. One other unattractive feature of this set of superfields is that to maintain the cancellations of anomalies, we must add new Higgs etc. fields for each new generation of quarks and leptons.

Inspection of (30)–(33) suggests a different approach to the cancellation of anomalies. Note that for $N_g=3$ generations, the trace (30) has the value $+6$. This is neatly canceled by a *single* color octet O_L of chiral superfields with $\widetilde{Y}=-2$. To avoid reintroducing an $SU(3)^2U(1)$ anomaly, we must take O_L to be neutral under U(1) as well as SU(2); in other words, O_L is a member of the adjoint representation of $SU(3)\times SU(2)\times U(1)$. Suppose we try adding another "adjoint superfield" with $\widetilde{Y}=-2$, a *single* SU(2) triplet T_L which is neutral under SU(3) and U(1). No new anomalies are produced, and T_L contributes -4 to the trace (31), which cancels the $SU(2)^2\widetilde{U}(1)$ anomaly for a single Higgs pair and three generations of quarks and leptons. Doubtless these cancellations could be understood by embedding $SU(3)\times SU(2)\times U(1)\times\widetilde{U}(1)$ in some large group with anomaly-free representations containing just three generations of nonsuperheavy quarks and leptons.

Not all anomalies have been canceled. With $N_g=3$ and $N_h=1$, the trace (32) has the value $+8$, so new charged $SU(3)\times SU(2)$-singlet superfields must be added with $\widetilde{Y}$ negative to cancel the $U(1)^2\widetilde{U}(1)$ anomaly. One possibility which would not reintroduce a $U(1)\widetilde{U}(1)^2$ or $U(1)^3$ anomaly is to add two pairs of additional singlet superfields J_L,J_L' with charges ±1 and $\widetilde{Y}=-2$. [At any rate

TABLE II. An anomaly-free set of left-chiral scalar superfields for an SU(3)×SU(2) ×U(1)×$\tilde{\mathrm{U}}$(1)-invariant supersymmetric theory.

	SU(3)	SU(2)	Y	$\tilde{Y}$	
Q_L	3	2	$-\frac{1}{6}$	1	three generations
U_R^*	$\bar{3}$	1	$\frac{2}{3}$	1	
D_R^*	$\bar{3}$	1	$-\frac{1}{3}$	1	
L_L	1	2	$\frac{1}{2}$	1	
E_R^*	1	1	-1	1	
H_L	1	2	$\frac{1}{2}$	-2	
H_L'	1	2	$-\frac{1}{2}$	-2	
O_L	8	1	0	-2	
T_L	1	3	0	-2	
J_L	1	1	1	-2	two each
J_L'	1	1	-1	-2	
X_L	1	1	0	?	several

it is encouraging that we do not have to add color-neutral particles with noninteger charges, which as seen from (32) is another special feature of three generations.] The only remaining anomaly is $\tilde{\mathrm{U}}(1)^3$, but this can be canceled in any number of ways by adding SU(3)×SU(2)×U(1)-neutral fields with various values of $\tilde{Y}$, one of which could be the X_L of Appendix B.

We now must check that we have not reintroduced the possibility of renormalizable B- and L-violating interactions. To be specific, suppose that the only left-chiral scalar superfields are those listed in Table II. Then the only trilinear F terms which involve a pair of the left-chiral quark and/or lepton superfields are those listed in Eq. (11), plus the new interaction $(L_L L_L J_L')_F$. Also, as long as we do not introduce new X_L superfields with $\tilde{Y}_L=+1$, there are no trilinear F terms which involve 1 or 3 of the quark and lepton superfields. Hence B and L are automatically conserved by all renormalizable SU(3)×SU(2)×U(1)×$\tilde{\mathrm{U}}$(1)-invariant supersymmetric interactions, with B and L assigned conventional values for quark and lepton superfields; zero values for H_L, H_L', O_L, T_L, and X_L; and $B=0$ and $L=+1$ (-1) for J_L (J_L'). Also, there is no trouble with $d=5$ effective interactions; as long as the X_L are limited to certain $\tilde{Y}$ values (including 1, -2, 4, 5, etc.) no $d=5$ terms are allowed, B and L conserving or not.

The phenomenological viability of a theory with this set of superfields now depends on the pattern of scalar field vacuum expectation values. In particular, $\mathscr{H}_L^0$ and $\mathscr{H}_L'^0$ must both get nonvanishing vacuum expectation values to give masses to all quarks and leptons, the charged scalar components of H_L, H_L', T_L, J_L, and J_L' must have vanishing vacuum expectation values to preserve charge conservation, $\mathscr{O}_L$ must have vanishing vacuum expectation value to preserve color invariance, and the neutral scalar component of T_L must have zero or small vacuum expectation value to preserve the usual result for the Z/W mass ratio. Finally, some of the $\mathscr{X}_L$ must have nonvanishing vacuum expectation values. Otherwise the neutral U(1) gauge boson $\tilde{Z}$ would get its mass solely from the same $\mathscr{H}_L^0$ and $\mathscr{H}_L'^0$ vacuum expectation values as Z^0, and the effective low-energy neutral-current couplings would be related by

$$\frac{\tilde{g}^2}{m^2(\tilde{Z}^0)}=\frac{1}{16}\frac{g^2+g'^2}{m^2(Z^0)}\ . \tag{34}$$

(This is for $\langle\mathscr{H}^0\rangle=\langle\mathscr{H}'^0\rangle$, for $\langle\mathscr{H}^0\rangle\neq\langle\mathscr{H}'^0\rangle$ the Z^0 and $\tilde{Z}^0$ are mixed.) Since quarks and leptons have larger values of $\tilde{Y}$ than of the SU(2)×U(1) quantum numbers, the neutral-current coupling (34) is probably inconsistent with experiment,[17] and so we must suppose that $\tilde{Z}^0$ gets part of its mass from other neutral scalar fields $\mathscr{X}_L^0$ which carry nonvanishing values of $\tilde{Y}$. Unfortunately, the question of whether or not the vacuum expectation values have all these necessary properties cannot be settled until we decide what sorts of X_L superfields to include in the theory.

The particular set of superfields that has been introduced in this section of course represents just one solution to the problem of canceling anomalies without reintroducing an unsuppressed violation of B and L conservation. It has the attractive feature that it works only for three generations of quarks and leptons. In addition, this solution has one other significant feature, of canceling anomalies in R symmetries,[9] which will be discussed in a future paper.

Added notes

(1) After this article was submitted for publication, I received a paper by N. Sakai and T. Yanagida [Munich Report No. MPI-PAE/PThSS/81 (unpublished)] which deals with the problem of baryon and lepton nonconservation in supersymmetric theories, using an approach similar to that presented here.

(2) M. Claudson and M. Wise have pointed out to me that the supersymmetry-breaking solution of the model presented in Appendix B is actually destabilized by the introduction of leptons: With two generations of leptons it is possible to find a deeper potential minimum at which charge is broken and supersymmetry is unbroken. They also note that this new supersymmetry-preserving solution could be avoided if there were an SU(2) triplet as well as an SU(2)-singlet field with $\widetilde{Y}=+4$. One can go further, and show that if all left-chiral superfields have $\widetilde{Y}=+1$, -2, and $+4$, and if there are enough superfields with $\widetilde{Y}=+4$ to allow all pairs of $\widetilde{Y}=-2$ superfields to have separate couplings with them, and if $\widetilde{\xi}/\widetilde{g}>0$, then supersymmetry must be spontaneously broken. (This is because a supersymmetry-preserving solution would have to have $[\partial f(\mathscr{S})/\partial\mathscr{S}_a]_0$ for all scalar fields, and in particular for all $\widetilde{Y}=4$ fields, which would imply that all $\widetilde{Y}=-2$ fields vanish; and it would also have to have $D_\alpha=0$ for all gauge fields, and in particular for the $\widetilde{\mathrm{U}}(1)$ field, which would only be possible if some fields with $\widetilde{Y}$ negative do not vanish.) Once one eliminates the possibility of a supersymmetry-preserving solution, the heights of the potential at the various local minima will generally be functions of the parameters of the theory, so that the sort of solution found in Appendix B will be the deepest minimum for at least a finite range of parameters. Unfortunately, as Claudson and Wise point out, it is very difficult to construct a theory with enough $\widetilde{Y}=+4$ fields to avoid a supersymmetry-conserving solution without also introducing a $\widetilde{Y}^3$ ABJ anomaly.

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APPENDIX A: SUPERSYMMETRIC LAGRANGIANS

This appendix will present expressions for the Lagrangian in terms of ordinary fields, corresponding to the general Lagrangian for left-chiral scalar superfields $S_n(x,\theta)$ discussed in Sec. I:

$$\mathscr{L}=[f(S)]_F+[f(S)]_F^*+[d(S,S^*)]_D\ . \tag{A1}$$

First, let us establish our notation. Since we want our results to appear at the end in a conventional Dirac formalism, the superfield coordinate θ in $S(x,\theta)$ will be taken as a Majorana four-component spinor, and all spinors and associated matrices will be four-dimensional. We define the component fields $\mathscr{S}_n$ (scalar), s_n (spinor), and $\mathscr{M}_n$ (auxiliary scalar) of a left-chiral scalar superfield S_n by the expansion

$$S_n=\mathscr{S}_n-i\sqrt{2}(\theta_L^T\epsilon s_{nL})-i(\theta_L^T\epsilon\theta_L)\mathscr{M}_n+\tfrac{1}{2}(\theta^T\epsilon\gamma^\mu\theta)\partial_\mu\mathscr{S}_n+\frac{i}{\sqrt{2}}(\theta_L^T\epsilon\theta_L)(\theta_R^T\epsilon\gamma^\mu\partial_\mu s_{nL})-\tfrac{1}{8}(\theta^T\epsilon\theta)^2\Box\mathscr{S}_n\ . \tag{A2}$$

Our metric is $++\,+\,-$, and our Dirac matrices can be taken (in supermatrix notation) as

$$\vec{\gamma}=\begin{bmatrix}0 & -i\vec{\sigma}\\ i\vec{\sigma} & 0\end{bmatrix},\quad \gamma^0=\begin{bmatrix}0 & -i\\ -i & 0\end{bmatrix}$$

with $\vec{\sigma}$ the 2×2 Pauli matrices. Also, γ_5 and ϵ are two diagonal supermatrices

$$\gamma_5=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\quad \epsilon=\begin{bmatrix}i\sigma_2 & 0\\ 0 & i\sigma_2\end{bmatrix}.$$

A subscript L or R denotes multiplication with $\frac{1}{2}(1+\gamma_5)$ or $\frac{1}{2}(1-\gamma_5)$, respectively. Finally, θ and s are Majorana spinors, in the sense that

$$s_n=\epsilon\gamma_5\beta s_n^*,\quad \theta=\epsilon\gamma_5\beta\theta^*\ ,$$

where $\beta=i\gamma^0$. It is useful to note that $\bar{s}\equiv s^\dagger\beta=s^T\epsilon\gamma_5$.

The F term of any function of left-chiral superfields $S_n(x,\theta)$ is defined as the coefficient of $\theta_L^T\epsilon\theta_L$ in the expansion of the function in powers of θ_L and θ_R. Using the expansion (A2), it is straightforward to show that the first term in the general Lagrangian (A1) has the form

$$[f(S)]_F=\frac{1}{2}(s_{Ln}^T\epsilon s_{Lm})\left[\frac{\partial^2 f(\mathscr{S})}{\partial\mathscr{S}_n\partial\mathscr{S}_m}\right]-i\mathscr{M}_n\frac{\partial f(\mathscr{S})}{\partial\mathscr{S}_n}\ . \tag{A3}$$

The D term of any real function of left-handed chiral superfields $S_n(x,\theta)$ and their adjoints $S_n^*(x,\theta)$ is defined as the coefficient of $-\frac{1}{2}(\theta^T\epsilon\theta)^2$ in the expansion of the function in powers of θ_L and θ_R. By a still straightforward, though somewhat tedious, calculation, this gives the last term in (A1):

$$\begin{aligned}[d(S,S^*)]_D=&\frac{1}{4}\frac{\partial d}{\partial\mathscr{S}_n}\Box\mathscr{S}_n+\frac{1}{4}\frac{\partial d}{\partial\mathscr{S}_n^*}\Box\mathscr{S}_n^*-\frac{1}{2}\partial_\mu\mathscr{S}_n\partial^\mu\mathscr{S}_m^*\frac{\partial^2 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m^*}\\
&+\frac{1}{4}\partial_\mu\mathscr{S}_n\partial^\mu\mathscr{S}_m\frac{\partial^2 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m}+\frac{1}{4}\partial_\mu\mathscr{S}_n^*\partial^\mu\mathscr{S}_m^*\frac{\partial^2 d}{\partial\mathscr{S}_n^*\partial\mathscr{S}_n^*}\\
&-\frac{1}{2}(s_n^T\epsilon\gamma_5\gamma^\mu\partial_\mu s_m)\frac{\partial^2 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m^*}+\frac{1}{2}(s_{Ln}^T\epsilon\gamma^\mu s_{Rm})\left[\frac{\partial^3 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m^*\partial\mathscr{S}_l}\partial_\mu\mathscr{S}_l-\frac{\partial^3 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m^*\partial\mathscr{S}_l^*}\partial_\mu\mathscr{S}_l^*\right]\\
&-\frac{1}{4}(s_{Ln}^T\epsilon s_{Lm})(s_{Rl}^T\epsilon s_{Rk})\frac{\partial^4 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m\partial\mathscr{S}_l^*\partial\mathscr{S}_k^*}+\mathscr{M}_n\mathscr{M}_m^*\frac{\partial^2 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m^*}\\
&-\frac{i}{2}\mathscr{M}_n^*(s_{Lm}^T\epsilon s_{Ll})\frac{\partial^3 d}{\partial\mathscr{S}_n^*\partial\mathscr{S}_m\partial\mathscr{S}_l}+\frac{i}{2}\mathscr{M}_n(s_{Rm}^T\epsilon s_{Rl})\frac{\partial^3 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m^*\partial\mathscr{S}_l^*}\ .\end{aligned} \tag{A4}$$

Note that although d and f are here not constrained to be polynomials, the Lagrangian (A1) is still quadratic in the auxiliary fields $\mathscr{M}_n$ and $\mathscr{M}_n^*$. Hence by requiring that the action be stationary with respect to these fields, we can obtain the closed-form result for $\mathscr{M}$:

$$\mathscr{M}_n=J_{nm}{}^{-1}\left[-\frac{i}{2}(s_{Lk}^T\epsilon s_{Ll})\frac{\partial^3 d}{\partial\mathscr{S}_m^*\partial\mathscr{S}_k\partial\mathscr{S}_l}-i\left[\frac{\partial f}{\partial\mathscr{S}_m}\right]^*\right], \tag{A5}$$

where J is the ubiquitous matrix

$$J_{nm}=\frac{\partial^2 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m^*}\ . \tag{A6}$$

Inserting this in (A3) and (A4) and discarding terms that vanish on integrations, we find the Lagrangian

$$\mathscr{L}=-J_{nm}\partial_\mu\mathscr{S}_n\partial^\mu\mathscr{S}_m^*-J_{nm}{}^{-1}\left[\frac{\partial f}{\partial\mathscr{S}_n}\right]\left[\frac{\partial f}{\partial\mathscr{S}_m}\right]^*-\tfrac{1}{2}J_{nm}(s_n^T\epsilon\gamma_5\gamma^\mu\partial_\mu s_m)$$
$$+\tfrac{1}{2}(s_{Ln}^T\epsilon s_{Lm})\left[\frac{\partial^2 f}{\partial\mathscr{S}_n\partial\mathscr{S}_m}-J_{kl}{}^{-1}\frac{\partial f}{\partial\mathscr{S}_k}\frac{\partial^3 d}{\partial\mathscr{S}_l^*\partial\mathscr{S}_n\partial\mathscr{S}_m}\right]$$
$$+\tfrac{1}{2}(s_{Rn}^T\epsilon s_{Rm})\left[\left[\frac{\partial^2 f}{\partial\mathscr{S}_n\partial\mathscr{S}_m}\right]^*-J_{lk}{}^{-1}\left[\frac{\partial f}{\partial\mathscr{S}_k}\right]^*\left[\frac{\partial^3 d}{\partial\mathscr{S}_l\partial\mathscr{S}_n^*\partial\mathscr{S}_m^*}\right]\right]$$
$$+\tfrac{1}{2}(s_{nL}^T\epsilon\gamma^\mu s_{mR})\left[\frac{\partial^3 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m^*\partial\mathscr{S}_l}\partial_\mu\mathscr{S}_l-\frac{\partial^3 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m^*\partial\mathscr{S}_l^*}\partial_\mu\mathscr{S}_l^*\right]$$
$$-\tfrac{1}{4}(s_{nL}^T\epsilon s_{mL})(s_{lR}^T\epsilon s_{kR})\left[\frac{\partial^4 d}{\partial\mathscr{S}_n\partial\mathscr{S}_m\partial\mathscr{S}_l^*\partial\mathscr{S}_k^*}+J_{qp}{}^{-1}\frac{\partial^3 d}{\partial\mathscr{S}_p^*\partial\mathscr{S}_n\partial\mathscr{S}_m}\frac{\partial^3 d}{\partial\mathscr{S}_q\partial\mathscr{S}_l^*\partial\mathscr{S}_k^*}\right]. \tag{A7}$$

(This can be easily written in a more familiar notation by using the relation $s^T\epsilon=\bar{s}\gamma_5$.) This result is already well known[14] for the special case of a quadratic D term and arbitrary F term, where J is a constant matrix.

It is interesting that even though we have allowed $f(S)$ and $d(S,S^*)$ to contain terms of arbitrarily high order in the superfields, the Lagrangian has turned out to involve only terms quadratic and quartic in the fermion fields. Furthermore, the quartic fermion interactions are only of the *LLRR* type.

It is not difficult to extend these results to gauge theories, or to Lagrangians with extra derivatives. It may be that these results will prove useful not only for the analysis of effective B- and L-nonconserving interactions, but also for the study of radiative corrections in renormalizable supersymmetric theories, because any term that does not appear in the most general supersymmetric Lagrangian cannot be produced by radiative corrections.

APPENDIX B: AN ILLUSTRATIVE MODEL

One necessary condition that must be met in order to apply the results of Sec. V to a specific model is that when the quark and lepton superfields are omitted the model must have an absolute potential minimum at which the matrix $\Sigma_\alpha g_\alpha D_\alpha t_\alpha$ is positive definite. For a proof that this is possible we may refer to a specific version of a class of models developed by Fayet.[3] To the best of my knowledge, Fayet has not yet presented a complete analysis of the symmetry-breaking solutions of his models, so I will go into some detail here.

The superfield content of this model is limited to just the left-chiral scalar quark, lepton, and Higgs superfields and $SU(3)\times SU(2)\times U(1)\times\widetilde{U}(1)$ gauge vector superfields described in Secs. II–V, plus one additional $SU(3)\times SU(2)\times U(1)$-neutral left-chiral superfield X_L. We assume that all quark and lepton superfields have $\widetilde{U}(1)$ quantum number $\widetilde{Y}=+1$, the Higgs superfields H_L, H_L' have $\widetilde{Y}=-2$, and the neutral superfield X_L has $\widetilde{Y}=+4$. With these quantum numbers, the most general renormalization interaction among the chiral scalar superfields has the form

$$g_U(Q_L U_R^* H_L')_F+g_D(Q_L D_R^* H_L)_F+g_E(L_L E_R^* H_L)+g_H(H_L H_L' X_L)_F+\text{H.c.}$$

In accordance with the discussion in Sec. V, we are interested in the case where g_U, g_D, and g_E are much smaller than the gauge couplings, and as an approximation we can begin by dropping these terms altogether. With all quark and lepton scalars set equal to zero, the potential takes the form

$$V=g_H{}^2\left[\,|\mathscr{H}_L^T\epsilon\mathscr{H}'_L\,|^2+(\mathscr{H}_L^\dagger\mathscr{H}_L)\,|\,\mathscr{L}_L\,|^2+(\mathscr{H}_L'^\dagger\mathscr{H}'_L)\,|\,\mathscr{L}_L\,|^2\right]+\tfrac{1}{2}\left[g(\mathscr{H}_L^\dagger\vec{t}\mathscr{H}_L)+g(\mathscr{H}_L'^\dagger\vec{t}\mathscr{H}'_L)\right]^2$$
$$+\tfrac{1}{2}\left[\frac{g'}{2}(\mathscr{H}_L^\dagger\mathscr{H}_L)-\frac{g'}{2}(\mathscr{H}_L'^\dagger\mathscr{H}'_L)+\xi\right]^2+\tfrac{1}{2}\left[-2\tilde{g}(\mathscr{H}_L^\dagger\mathscr{H}_L)-2\tilde{g}(\mathscr{H}_L'^\dagger\mathscr{H}'_L)+4\tilde{g}\,|\,\mathscr{L}_L\,|^2+\tilde{\xi}\right]^2, \tag{B1}$$

where ξ and $\tilde{\xi}$ are the coefficients of the term $[V_a]_D$ in the Lagrangian for U(1) and $\tilde{\mathrm{U}}(1)$, respectively. [We are now using a somewhat less schematic notation for SU(2) indices: $\mathscr{H}_L$ and $\mathscr{H}'_L$ are two-component columns, ϵ is the totally antisymmetric matrix with $\epsilon_{12}=+1$, and $\vec{t}$ is the matrix of electroweak isospin.]

It is now straightforward to show the following.

(a) At the minimum of the potential, $\mathscr{H}_L^\dagger\vec{t}\mathscr{H}_L$ and $\mathscr{H}_L'^\dagger\vec{t}\mathscr{H}'_L$ will be parallel or antiparallel according as $2g_H{}^2$ is greater or less than g^2. We will adopt the assumption that

$$2g_H{}^2<g^2\,, \tag{B2}$$

because then with these vectors antiparallel we can [by an SU(2) transformation] bring the Higgs doublets at the potential minimum to the form

$$\mathscr{H}_L=\begin{bmatrix}\mathscr{H}_L^0\\ 0\end{bmatrix},\quad \mathscr{H}'_L=\begin{bmatrix}0\\ \mathscr{H}_L'^0\end{bmatrix}, \tag{B3}$$

corresponding to conservation of electric charge.

(b) There is no stationary point of the potential with $\mathscr{H}_L^0$, $\mathscr{H}_L'^0$, and $\mathscr{L}_L$ all nonzero.

(c) The minimum of the potential lies at a point with $\mathscr{H}_L^0\neq 0$, $\mathscr{H}_L'^0\neq 0$, and $\mathscr{L}_L=0$ if and only if

$$\frac{|\,\xi g'\,|}{g^2+g'^2-2g_H{}^2}<\frac{2\tilde{\xi}\tilde{g}}{8\tilde{g}^2+g_H{}^2}\,. \tag{B4}$$

Equation (B2) ensures that the denominator on the left-hand side is positive so (B4) requires in particular that $\tilde{\xi}$ should have the same sign as $\tilde{g}$. We assume that (B4) is satisfied because we want $\langle\mathscr{H}_L^0\rangle$ and $\langle\mathscr{H}_L'^0\rangle$ to have nonvanishing values so that all quarks and leptons can get small masses when the Yukawa couplings are turned on.

(d) Under the conditions (B2) and (B4), the Higgs fields at the minimum of the potential have the values

$$\left.\begin{matrix}|\,\mathscr{H}_L^0\,|^2\\ |\,\mathscr{H}_L'^0\,|^2\end{matrix}\right\}=\frac{2\tilde{\xi}\tilde{g}}{8\tilde{g}^2+g_H{}^2}\pm\frac{\xi g'}{g^2+g'^2-2g_H{}^2}\,. \tag{B5}$$

With this information, we can now compute the mass-squared matrix (18) of the scalar superpartners of the quarks and leptons. Our result is

$$\mathscr{M}^2=\frac{-\xi g'}{g^2+g'^2-2g_H{}^2}\left[g^2t_3-(g^2-2g_H{}^2)y\right]+\frac{g_H{}^2\tilde{\xi}\tilde{g}}{g_H{}^2+8\tilde{g}^2}\tilde{y}\,, \tag{B6}$$

where t_3, y, and $\tilde{y}$ are the SU(2), U(1), and $\tilde{\mathrm{U}}(1)$ generators for the corresponding quarks and leptons. The matrix $\tilde{y}$ is positive definite (in the special case considered here, it is the unit matrix) and the conditions (B2) and (B4) set no upper bound on $\tilde{\xi}$, so as long as g_H and $\tilde{g}$ are nonzero we can clearly make $\mathscr{M}^2$ positive by making $\tilde{\xi}$ sufficiently large.

Incidentally, (B6) shows why the field $\mathscr{L}_L$ has to be added to the model, even though its value vanishes at the minimum of the potential. Without $\mathscr{L}_L$ we would in effect have $g_H=0$, and although the conditions (B2) and (B4) could still be satisfied, the matrix $\mathscr{M}^2$ could not be positive definite.

For comparison, we also note the vector-boson mass-matrix elements

$$m_W{}^2=\tfrac{1}{2}g^2(\,|\,\mathscr{H}_L^0\,|^2+|\,\mathscr{H}_L^{0\prime}\,|^2)=\frac{2\tilde{\xi}\tilde{g}g^2}{8\tilde{g}^2+g_H{}^2}\,,$$
$$m_Z{}^2=(g^2+g'^2)m_W{}^2/g^2\,,$$
$$m_{\tilde{Z}}{}^2=16\tilde{g}^2m_W{}^2/g^2\,,$$
$$m_{Z\tilde{Z}}{}^2=g\tilde{g}(\,|\,\mathscr{H}_L^0\,|^2-|\,\mathscr{H}_L^{0\prime}\,|^2)=\frac{2\xi g'g\tilde{g}}{g^2+g'^2-2g_H{}^2}\,.$$

It is interesting that it is $\tilde{\xi}$ alone that sets the scale of the W mass and the overall scale of the Z and $\tilde{Z}$ masses, while ξ only determines the Z-$\tilde{Z}$ mixing.

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