Effective fermion masses of order gT in high-temperature gauge theories with exact chiral invariance

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It is shown that, at finite temperature, chiral invariance does not imply that fermion propagators have poles at $K^2=0$. Instead, a zero-momentum fermion has energy $K^0=M$, where $M^2=g^2C(R)T^2/8$ and $C(R)$ is the quadratic Casimir of the fermion representation. The dispersion relation for $\vec{K}\neq 0$ is computed and can be crudely approximated (to within 10%) by $K^0\approx(M^2+\vec{K}^2)^{1/2}$. Applications to high-temperature QCD, SU(2)×U(1), and grand unified theories are discussed.

INTRODUCTION

A theory containing only fermions and gauge bosons in which there are no bare masses for the fermions is chirally invariant to all orders. For definiteness, suppose that the theory is also parity invariant, such as massless QED. At zero temperature, chiral invariance has two consequences.

(i) There are no $\bar{\psi}\psi$ couplings induced in any finite order of perturbation theory.

(ii) The fermion self-energy is of the form $\Sigma=-a\not{K}$ for a particle of momentum K^α, where a is some function of K^2. Consequently the fermion propagator is $S=\not{K}/(1+a)K^2$ but the pole remains at $K^2=0$. The function $1+a$ only modifies the residue of the pole.

For the same chirally invariant theory at nonzero temperature, (i) still holds but (ii) does not. At finite temperature the plasma of particles and antiparticles that constitutes the heat bath introduces a special Lorentz frame: viz., the rest frame (center-of-mass frame) of the plasma. In a general frame the heat bath has four-velocity u^α with $u^\alpha u_\alpha=1$. The presence of this four-vector means that the fermion self-energy will be of the form

$$\Sigma(K)=-a\not{K}-b\not{u}\ , \tag{1.1}$$

where a and b are Lorentz-invariant functions.[1] These functions can depend on two Lorentz scalars

$$\begin{aligned}\omega &\equiv K^\alpha u_\alpha\ ,\\ k&\equiv[(K^\alpha u_\alpha)^2-K^2]^{1/2}\ .\end{aligned} \tag{1.2}$$

Since $K^2=\omega^2-k^2$, one may interpret ω and k as Lorentz-invariant energy and three-momentum. The full fermion propagator is

$$S(K)=[(1+a)\not{K}+b\not{u}]^{-1}\ . \tag{1.3}$$

Inverting the matrices gives

$$S(K)=[(1+a)\not{K}+b\not{u}]/D\ , \tag{1.4}$$

where D is the Lorentz-invariant function

$$D(k,\omega)=(1+a)^2K^2+2(1+a)bK\cdot u+b^2\ . \tag{1.5}$$

Although the propagator (1.4) manifests the chiral symmetry, it is quite possible that the zeros of D do not occur at $\omega^2=k^2$.[2]

In Sec. II, a computation of the functions a and b is presented for the large-T regime and the zeros of D are determined both for parity-conserving and for parity-violating theories. Section III discusses possible extensions. Appendix A contains the details of the one-loop calculation for gauge bosons and Appendix B incorporates Yukawa couplings.

II. CHIRALLY INVARIANT GAUGE THEORIES

A. With parity conservation

Consider a non-Abelian gauge theory with Lagrange density

$$\mathscr{L}=-\tfrac{1}{4}F^A_{\mu\nu}F^{\mu\nu}_A+\bar{\psi}_m\gamma^\mu(\delta_{mn}i\partial_\mu-gL^A_{mn}A^A_\mu)\psi_n\ ,$$

where A runs over the generators of the group and m,n over the states of the fermion representation. The representation matrices L^A_{mn} are normalized by

$$\mathrm{Tr}(L^AL^B)=T(R)\delta^{AB}\ , \tag{2.1}$$

where $T(R)$ is the index of the representation.[3] For example, in SU(N) the fundamental (N-dimensional) representation has $T(R)=\frac{1}{2}$.

The one-loop self-energy in Fig. 1(a) between $\bar{\psi}_m$

and ψ_n is proportional to $(L^A L^A)_{mn}$, where A is summed over all the gauge bosons. This gives a diagonal self-energy proportional to

$$(L^A L^A)_{mn} = C(R)\delta_{mn} , \tag{2.2}$$

where $C(R)$ is the quadratic Casimir invariant of the representation. This may be computed from the index by using

$$T(R)\times \text{Dim}G = C(R)\times \text{Dim}R . \tag{2.3}$$

The contribution of Fig. 1(a) is thus

$$\Sigma(K) = ig^2 C(R) \int \frac{d^4p}{(2\pi)^4} D_{\mu\nu}(p)\gamma^\mu S(p+K)\gamma^\nu , \tag{2.4}$$

where $D_{\mu\nu}$ and S are the finite-temperature propagators of massless gauge bosons and fermions, respectively.

The computation of (2.4) in the high-temperature regime ($T \gg k, T \gg \omega$) is performed in Appendix A. Because the maximum divergence of Σ at zero temperature is linear, one might expect Σ to be linear in T at high temperature. However, explicit calculation shows that the functions a and b of (1.1) are actually proportional to T^2, viz.,

$$a(k,\omega) = \frac{M^2}{k^2}\left[1 - \frac{\omega}{2k}\ln\left[\frac{\omega+k}{\omega-k}\right]\right] ,$$
$$b(k,\omega) = \frac{M^2}{k}\left[-\frac{\omega}{k} + \left[\frac{\omega^2}{k^2} - 1\right]\frac{1}{2}\ln\left[\frac{\omega+k}{\omega-k}\right]\right] , \tag{2.5}$$

where for convenience we have defined

$$M^2 \equiv g^2 T^2 C(R)/8 . \tag{2.6}$$

The results (2.5) are both gauge invariant and, despite appearances, finite at $k=0$.[4]

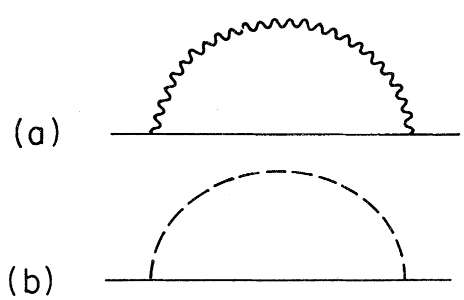

FIG. 1. The one-loop fermion self-energy. The gauge-boson contribution (a) and scalar-boson contribution (b) are calculated in Appendices A and B, respectively.

(1) Fermion dispersion relation. The poles in the propagator (1.4) occur when ω and k are such as to produce a zero in the denominator (1.5),

$$D(k,\omega) = [\omega(1+a)+b]^2 - [k(1+a)]^2 .$$

The positive-energy root of $D(k,\omega)=0$ occurs at

$$\omega(1+a)+b = k(1+a) , \tag{2.7}$$

where $k = |\vec{k}| > 0$.[5] Substituting (2.5) gives the dispersion relation

$$\omega - k = \frac{M^2}{k}\left[1 + \left[1 - \frac{\omega}{k}\right]\frac{1}{2}\ln\left[\frac{\omega+k}{\omega-k}\right]\right] . \tag{2.8}$$

This has the property that at $k=0$ the solution is not $\omega=0$ but rather $\omega=M$. More generally, for small k the analytic form of the solution is

$$\omega = M + \frac{k}{3} + \frac{k^2}{3M} + \cdots \quad (k \ll M) .$$

Note that $k = |\vec{k}| > 0$ here. The minimum energy for a fermion with real momentum is M. Fermions with $\omega < M$ necessarily have k complex and cannot propagate. This cutoff energy M plays the role for fermions that the plasma frequency does for gauge bosons.[6]

The numerical solution to (2.8) is plotted in Fig. 2 and is seen to be a monotonic function of real k.

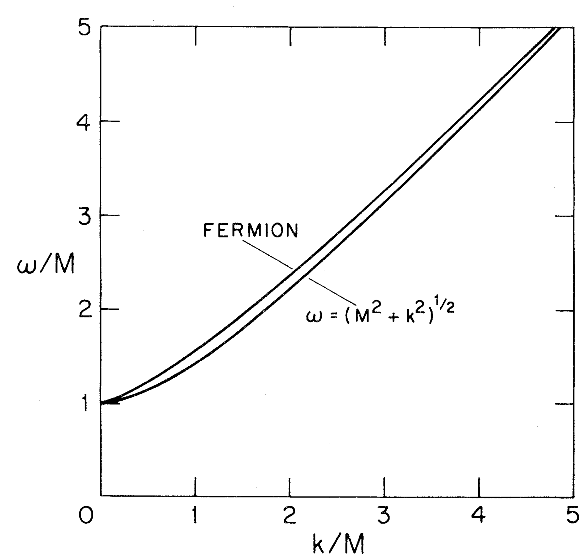

FIG. 2. A plot of the position of the poles (2.8) in the chirally invariant fermion propagator. For comparison, the free-particle dispersion relation $\omega = (M^2+k^2)^{1/2}$ is also plotted.

For very large k it goes over to

$$\omega=k+\frac{M^2}{k}-\frac{M^4}{2k^3}\ln\left[\frac{2k^2}{M^2}\right]+\cdots \quad (k\gg M)\ .$$

By way of comparison, the free-particle dispersion relation $\omega=(M^2+k^2)^{1/2}$ is plotted in Fig. 2. The free-particle curve is always an underestimate of the actual dispersion relation but the error is never more than 10%. In this sense the parameter M really serves as a mass.

(2) The modified Dirac equation. Corresponding to the propagator (1.4) is a new Dirac equation

$$[(1+a)\not{K}+b\not{u}]U=0 \qquad (2.9)$$

that determines the spinor wave function U. In the rest frame of the heat bath $[u^\mu=(1,0,0,0)]$ this becomes[7]

$$\{[(1+a)\omega+b]\gamma^0-(1+a)\vec{\gamma}\cdot\vec{k}\}=0\ .$$

A solution exists only when ω and k are such that the matrix in curly brackets has zero determinant. This condition is the same as (2.7). Consequently one must solve

$$\left[k\gamma^0-\vec{\gamma}\cdot\vec{k}\right]U=0\ .$$

This coincides with the usual Dirac equation for a free particle with no mass and has solution

$$U_s=\begin{bmatrix}\chi_s\\ \vec{\sigma}\cdot\hat{k}\chi_s\end{bmatrix}, \qquad (2.10)$$

where χ_s $(s=\pm\frac{1}{2})$ is a two-component spinor. This is the wave function to be used for external fermions in Feynman diagram calculations at finite temperature. The on-shell mass condition, however, is not $\omega=k$ but relation (2.8). Near its pole, the fermion propagator (1.4) behaves as

$$S(K)\underset{\text{pole}}{\longrightarrow}\frac{\sum_{\pm s}U_s\bar{U}_s}{2[\omega(1+a)+b-k(1+a)]}\ ,$$

with the residue determined by the wave functions (2.10) as expected.

3. SU(3) color theory. The simplest example of this type of fermion mass generation is provided by the SU(3) color theory of strong interactions. There is a critical temperature $T_c\sim$GeV above which quarks are unconfined.[8] For any $T>T_c$ chiral invariance is a good approximation for the light quarks (u,d,s) and the previous discussion applies. For $T>300$ GeV the restoration of SU(2)$\times$U(1) symmetry removes all current-algebra masses from the Lagrangian, chiral invariance is exact, and the previous discussion applies to all quarks. From Table I the quadratic Casimir invariant for SU(3) triplets is $\frac{4}{3}$ and consequently the effective quark mass (2.6) is $M^2=g^2T^2/6$, where g^2 is the temperature-dependent running coupling constant. For typical values $1>g^2/4\pi>0.25$ this gives $1.45T>M>0.72T$.

B. With parity violation

It is very simple to extend the discussion to chirally invariant theories in which the left-handed and right-handed fermions are in different representations of the gauge group. Let[7]

$$\eta\equiv\tfrac{1}{2}(1-\gamma_5),\quad \bar{\eta}\equiv\tfrac{1}{2}(1+\gamma_5)\ . \qquad (2.11)$$

The general fermion self-energy is of the form

$$-\Sigma(K)=\bar{\eta}(a_L\not{K}+b_L\not{u})\eta+\eta(a_R\not{K}+b_R\not{u})\bar{\eta}\ ,$$

where a_L,b_L,a_R,b_R are all Lorentz-invariant functions of ω and k. It is convenient to define two four-vectors

$$L^\mu\equiv(1+a_L)K^\mu+b_Lu^\mu\ ,$$
$$R^\mu\equiv(1+a_R)K^\mu+b_Ru^\mu\ , \qquad (2.12)$$

with the property that

$$L^2=[\omega(1+a_L)+b_L]^2-[k(1+a_L)]^2\ ,$$
$$R^2=[\omega(1+a_R)=b_R]^2-[k(1+a_R]^2\ .$$

TABLE I. The index $T(R)$ and the quadratic Casimir invariant $C(R)$, as defined in (2.1) and (2.2), for various representations R.

Group	Dim (R)	$T(R)$	$C(R)$
U(1)	1	1	1
SU(2)	2	$\frac{1}{2}$	$\frac{3}{4}$
	3	2	2
SU(3)	3	$\frac{1}{2}$	$\frac{4}{3}$
	8	3	3
SU(5)	5	$\frac{1}{2}$	$\frac{12}{5}$
	10	$\frac{3}{2}$	$\frac{18}{5}$
	24	5	5

Since the inverse propagator is

$$S^{-1}(K)=\bar{\eta}\not{L}\eta+\eta\not{R}\bar{\eta}\ ,$$

the propagator itself becomes

$$S(K)=\frac{\eta\not{L}\bar{\eta}}{L^2}+\frac{\bar{\eta}\not{R}\eta}{R^2}\ . \tag{2.13}$$

The high-temperature limit of a_L,b_L,a_R,b_R to one-loop order is just (2.5) but with a different value of the parameter M^2 for the left-handed and the right-handed fermions. It may be useful to consider a few examples.

(1) SU(2) × U(1) electroweak theory. For $T \gg M_Z/e$ the SU(2)×U(1) symmetry[9] is unbroken; the gauge bosons and fermions are all massless at the tree level. Because the principal radiative contribution to the quark mass comes from the strong interactions (see Sec. II A), we now concentrate on the leptons. The right-handed leptons (e_R,μ_R,τ_R) couple only to the U(1) gauge field (with strength g') and from (2.8) they have an effective mass $M_R{}^2=g'^2T^2/8$. The left-handed leptons $(e_L,\mu_L,\tau_L,\nu_e,\nu_\mu,\nu_\tau)$ couple to the U(1) gauge field (with strength $g'/2$) and to the SU(2) gauge field (with strength g). From (2.8) and Table I their effective mass is $M_L{}^2=(g'^2+3g^2)T^2/32$. For measured values of g' and g (Ref. 10) this gives $M_R \approx 0.12T$ and $M_L \approx 0.22T$.

(2) SU(5) grand unified theory. For $T \gg 10^{14}$ GeV the electroweak and strong interactions may be unified into an unbroken SU(5) theory[11] which would be chirally invariant at such a temperature. Although there is only one coupling constant g, the Casimir invariant for the ten-dimensional representation $(\bar{e}_R,u_L,d_L,\bar{u}_R$ per generation) differs from that for the five-dimensional representation $(e_L,\nu_L,\bar{d}_R)$. From Table I, $M_{10}{}^2=g^2T^2(\frac{9}{20})$ and $M_5{}^2=g^2T^2(\frac{3}{10})$. For $g^2/4\pi \approx \frac{1}{45}$ this gives $M_{10} \approx 0.35T$ and $M_5 \approx 0.29T$. It is curious that in the SU(5) theory ν_L has a smaller effective mass than e_R but in the SU(2)×U(1) theory it has a larger effective mass than e_R.

III. DISCUSSION

It is well known that in the low-temperature, broken-symmetric phase of SU(2)×U(1), chiral symmetry is also broken. Quarks and lepton masses are proportional to the temperature-dependent vacuum expectation value of a scalar field. The one-loop corrections to these masses are, of course, temperature dependent as well.[12]

In the high temperature, symmetric phase of SU(2)×U(1) there are exact chiral symmetries. Consequently, a process such as $\nu_L e_L \to \nu_L e_R$ has zero cross section. However, despite the chiral symmetry of the finite-temperature fermion propagators (1.4), the actual dispersion relation for the fermion poles is that plotted in Fig. 2 and not $\omega^2=k^2$. The fermions all have (chirally invariant) effective masses of order T. Of course, scalar bosons are all known to have masses of order T (Ref. 13) as do vector bosons from the plasmon effect.[14] These effective masses all tend to suppress reaction rates at high temperature by reducing the available phase space. They also modify the thermodynamics of the early universe by reducing the pressure, energy, entropy, and number density. This reduces the expansion rate of the universe and modifies the magnitude of the baryon excess that is generated. These effects will be investigated in a separate publication.

It should be noted that although Yukawa couplings of fermions to scalar fields were omitted from the discussion in Sec. II, they also contribute to the fermion self-energy. The one-loop diagram shown in Fig. 1(b) is computed in Appendix B for massless scalars. The resulting dispersion relation is exactly the same as for the gauge bosons (2.8); only the effective mass M^2 changes to the value quoted in (B3).[15]

The restriction to chirally invariant theories has simplified our results considerably. However, it should be clear that in any finite-temperature calculation the fluid velocity u_μ will generally modify the structure of the propagators, vertices, and Green's functions. For example, when chiral invariance is broken at low temperature the fermion self-energy will be a linear combination of the matrices 1, $\not{K}$, $\not{u}$, and $[\not{K},\not{u}]$. The form of the resulting propagator is discussed in Appendix C.

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APPENDIX A: COMPUTATION OF Σ

It is convenient to use the real-time formulation of finite-temperature field theory[13] in order to maintain manifest Lorentz covariance. The free-particle propagators for massless fermions and gauge bosons are

$$S(p)=\not{p}\left[\frac{1}{p^2+i\eta}+i\Gamma_f(p)\right],$$
$$D_{\mu\nu}(p)=-\eta_{\mu\nu}\left[\frac{1}{p^2+i\eta}-i\Gamma_b(p)\right], \tag{A1}$$

where all temperature dependence occurs via

$$\Gamma(p)\equiv 2\pi\delta(p^2)n(p),$$
$$n(p)=(e^{|p\cdot u|/T}\pm 1)^{-1}, \tag{A2}$$

and u^α is the four-velocity of the heat bath. The calculation will first be performed in Feynman gauge; it will then be shown that the order-T^2 result is the same in any covariant gauge. The one-loop contribution to the fermion self-energy is

$$\Sigma(K)=ig^2C(R)\int\frac{d^4p}{(2\pi)^4}D_{\mu\nu}(p)\gamma^\mu S(p+K)\gamma^\nu .$$

Because the T dependence of the propagators (A1) is additive, it is easy to separate off the $T=0$ self-energy by

$$\Sigma=\Sigma(T=0)+\Sigma',$$

where the prime denotes the finite-T corrections. This correction Σ' is complex. In computing $\mathrm{Im}\Sigma'$ in the region $\omega^2>k^2$, the p integration is limited by two-body phase space and thus $\mathrm{Im}\Sigma'$ is of order ω or k even for large T. We will ignore $\mathrm{Im}\Sigma'$ and compute only the real part.

Substituting the propagators (A1) gives

$$\mathrm{Re}\Sigma'=2g^2C(R)\int\frac{d^4p}{(2\pi)^4}(\not{p}+\not{K})\left[\frac{\Gamma_b(p)}{(p+K)^2}-\frac{\Gamma_f(p+K)}{p^2}\right], \tag{A3}$$

with the denominators defined by their principal value. Changing p to $-p-K$ in the second term yields

$$\mathrm{Re}\Sigma'=2g^2C(R)\int\frac{d^4p}{(2\pi)^4}[(\not{p}+\not{K})\Gamma_b(p)+\not{p}\Gamma_f(p)]\frac{1}{(p+K)^2}. \tag{A4}$$

[Note that in the absence of the thermal-distribution function n, the δ functions in Γ would allow (A4) to diverge quadratically. This is the ultimate source of the T^2 behavior of Σ'.] Since (A4) is manifestly covariant, it must be a linear combination of $\not{K}$ and $\not{u}$ as in (1.1). Consequently, if we multiply (A4) by either $\not{K}$ or $\not{u}$ and then take the trace of the product, the result will be a pair of Lorentz-invariant integrals. In each of these, the p^0 integration and the two angular integrations are elementary and leave an integration over $|\vec{p}|\equiv p$ to perform:

$$\tfrac{1}{4}\mathrm{Tr}(\not{K}\mathrm{Re}\Sigma')=g^2C(R)\int_0^\infty\frac{dp}{8\pi^2}\left\{\left[4p+\frac{K^2}{2k}L_1(p)\right]n_f(p)+\left[4p-\frac{K^2}{2k}L_1(p)\right]n_b(p)\right\},$$

$$\tfrac{1}{4}\mathrm{Tr}(\not{u}\mathrm{Re}\Sigma')=\frac{g^2C(R)}{k}\int_0^\infty\frac{dp}{8\pi^2}\left\{\left[2p\ln\left[\frac{\omega_+}{\omega_-}\right]-pL_2(p)\right]n_f(p)+\left[2p\ln\left[\frac{\omega_+}{\omega_-}\right]-pL_2(p)-\omega L_1(p)\right]n_b(p)\right\},$$

with ω and k as defined in (1.2) and

$$L_1(p)\equiv\ln\left[\frac{p+\omega_+}{p+\omega_-}\right]-\ln\left[\frac{p-\omega_+}{p-\omega_-}\right],$$
$$L_2(p)\equiv\ln\left[\frac{p+\omega_+}{p+\omega_-}\right]+\ln\left[\frac{p-\omega_+}{p-\omega_-}\right],$$
$$\omega_\pm\equiv\tfrac{1}{2}(\omega\pm k).$$

All logarithms are to be understood in the principal-value sense.

Only the order-T^2 contributions will be kept. These arise from the terms which would diverge quadratically if there were no thermal-distribution functions to cut off the integration at $p\sim 0(T)$. In particular,

$$\int_0^\infty dp\, p n_f(p)=\pi^2T^2/12,$$
$$\int_0^\infty dp\, p n_b(p)=\pi^2T^2/6. \tag{A5}$$

For very large p, $L_1(p)\to k/p$ and $L_2(p)\to -\omega k/p^2$. Consequently all the integrations over L_1 and L_2 would diverge at most logarithmically if there were no thermal-distribution functions. These integrations only give $\ln T$ and will be dropped.[16] From (A5) the large-T behavior is thus

$$\tfrac{1}{4}\mathrm{Tr}(\not{K}\mathrm{Re}\Sigma')=g^2C(R)T^2/8,$$
$$\tfrac{1}{4}\mathrm{Tr}(\not{u}\mathrm{Re}\Sigma')=g^2C(R)\frac{T^2}{16k}\ln\left[\frac{\omega_+}{\omega_-}\right]. \tag{A6}$$

Since Σ' is of the form (1.1) the coefficient functions a and b follow from (A6),

$$a(k,\omega)=g^2C(R)\frac{T^2}{8k^2}\left[1-\frac{\omega}{2k}\ln\left[\frac{\omega+k}{\omega-k}\right]\right],$$

$$b(k,\omega)=g^2C(R)\frac{T^2}{8k}\left[-\frac{\omega}{k}+\left[\frac{\omega^2}{k^2}-1\right]\times\frac{1}{2}\ln\left[\frac{\omega+k}{\omega-k}\right]\right]. \quad \text{(A7)}$$

These are the results used in (2.5).

Note on gauge invariance. It is straightforward to show that the order-T^2 part of the fermion self-energy is gauge invariant. In an arbitrary covariant gauge the vector propagator (A1) becomes

$$D_{\mu\nu}(p)=\left[-\eta_{\mu\nu}+\alpha\frac{p_\mu P_\nu}{p^2}\right]\left[\frac{1}{p^2+in}-i\Gamma_b(p)\right], \quad \text{(A8)}$$

where α is the gauge parameter. Because the $1/p^2$ in the longitudinal propagator is a principal-value singularity, its product with $\delta(p^2)$ is defined by

$$\frac{-1}{p^2}\delta(p^2)\rightarrow\delta'(p^2), \quad \text{(A9)}$$

where the prime denotes the derivative with respect to $p_0{}^2$. The gauge-dependent part of the propagator is

$$\delta D_{\mu\nu}(p)=\alpha p_\mu p_\nu\left[\frac{1}{p^2(p^2+in)}+i2\pi\delta'(p^2)n_b(p)\right], \quad \text{(A10)}$$

so that the gauge-dependent part of Σ is

$$\delta\Sigma=ig^2C(R)\int\frac{d^4p}{(2\pi)^4}\delta D_{\mu\nu}(p)\gamma^\mu S(p+K)\gamma^\nu .$$

After separating off the $T=0$ contribution as before, the real part of the finite-T correction becomes

$$\begin{aligned}\mathrm{Re}\delta\Sigma'&=\mathrm{Re}\delta\Sigma_f'+\mathrm{Re}\delta\Sigma_b' ,\\ \mathrm{Re}\delta\Sigma_f'&\equiv-\alpha g^2C(R)\int\frac{d^4p}{(2\pi)^4}\frac{\not p(\not p+\not K)\not p}{p^2p^2}2\pi\delta[(p+K)^2]n_f(p+K) ,\\ \mathrm{Re}\delta\Sigma_b'&\equiv-\alpha g^2C(R)\int\frac{d^4p}{(2\pi)^4}\frac{\not p(\not p+\not K)\not p}{(p+K)^2}2\pi\delta'(p^2)n_b(p) .\end{aligned} \quad \text{(A11)}$$

Neither of these integrals will turn out to have a T^2 contribution. In the fermionic integral it is convenient to change p to $-p-K$ and then use $\delta(p^2)$ to set $p^2=0$:

$$\mathrm{Re}\delta\Sigma_f'=\alpha g^2C(R)\int\frac{d^4p}{(2\pi)^4}\frac{-K^2\not p+2p\cdot K\not K}{(2p\cdot K+K^2)^2}2\pi\delta(p^2)n_f(p) .$$

In the absence of n_f this integral would diverge linearly with $|\vec{p}|$. With the thermal-distribution function it thus behaves at most like T for large temperature and can be neglected. For the bosonic integral (A11) one must evaluate

$$\mathrm{Re}\delta\Sigma_b'=\alpha g^2C(R)\int\frac{d^4p}{(2\pi)^4}\left[-\not p+\frac{K^2\not p+p^2\not K}{(p+K)^2}\right]n_b(p)\frac{\pi}{p_0}\frac{d}{dp_0}\delta(p^2) .$$

The isolated $\not p$ term is odd and integrates to zero. Integrating by parts on p_0 and then setting $p^2=0$ gives

$$\mathrm{Re}\delta\Sigma_b'=\alpha g^2C(R)\int\frac{d^4p}{(2\pi)^4}\left[-K^2\not p\frac{dn_b(p^0)}{dp^0}+\left[\frac{2(p^0+K^0)K^2\not p}{2p\cdot K+K^2}-2p_0\not K-\frac{K^2\vec{\gamma}\cdot\vec{p}}{p_0}\right]n_b(p)\right]\frac{\pi\delta(p^2)}{p^0(2p\cdot K+K^2)} .$$

In the absence of the thermal-distribution functions this integral also diverges linearly with $|\vec{p}|$. Consequently it also behaves like T for large temperature. This demonstrates the gauge invariance of the T^2 results (A6).

APPENDIX B: INCLUSION OF YUKAWA COUPLINGS

If scalar fields are included in the gauge theory, there will generally be Yukawa couplings to the fermions of the form

$$\mathcal{L}_Y=f\bar{\psi}_m^L\Gamma^i_{mn}\psi_n^R\varphi_i+\text{H.C.} , \quad \text{(B1)}$$

where f is the coupling constant and the Γ^i are matrices of Clebsch-Gordan coefficients. These matrices will not be square if the left- and right-handed fermions are in different dimensional representations. The one-loop fermion self-energy in Fig. 1(b) will be proportional to

$$(\Gamma^i\Gamma^{i+})_{mm'}\equiv C'_L\delta_{mm'}\ ,$$
$$(\Gamma^{i+}\Gamma^i)_{nn'}\equiv C'_R\delta_{nn'}\ , \tag{B2}$$

for left and right chiralities, respectively. There is no standard convention for normalizing these constants.

The order $|f|^2$ contribution to the self-energy of either chirality fermion is

$$\Sigma(K)=i|f|^2C'\int\frac{d^4p}{(2\pi)^4}D(p)S(p+K)\ .$$

The bare propagator for the fermion is given in (A1) and for the scalar is

$$D(p)=\frac{1}{p^2+in}-i\Gamma_b(p)\ .$$

The temperature-dependent part of the self-energy Σ', thus has a real part

$$\mathrm{Re}\Sigma'=|f|^2C'\int\frac{d^4p}{(2\pi)^4}(\not p+\not K)\times\left[\frac{\Gamma_b(p)}{(p+K)^2}-\frac{\Gamma_f(p+K)}{p^2}\right]$$

with the denominators defined by their principal value. This integral is exactly the same as the gauge-boson contribution (A3) except that $2g^2C(R)$ is replaced by $|f|^2C'$. Consequently the effective mass becomes

$$M^2=g^2T^2C(R)/8+|f|^2T^2C'/16\ , \tag{B3}$$

but the form of the dispersion relation (2.8) is unchanged.

APPENDIX C: WITHOUT CHIRAL INVARIANCE

At low temperature, chiral invariance will often be broken. The fermion self-energy then no longer anticommutes with γ_5 as before but will be a combination of the matrices $\not K,\not u$,1, and $[\not K,\not u]$. However, one-loop diagrams do not generate a $[\not K,\not u]$ term and it will be omitted for simplicity. With this restriction, the general form of the inverse propagator for a parity-conserving theory is

$$S^{-1}=(1+a)\not K+b\not u-c\ , \tag{C1}$$

where c is a real function of ω and k that parametrizes the chirality violation. The propagator itself is

$$S=[(1+a)\not K+b\not u+c]/D\ , \tag{C2}$$

where the poles are determined by the vanishing of the function

$$D(k,\omega)\equiv(1+a)^2K^2+2(1+a)bK\cdot u+b^2-c^2\ .$$

This is applicable to pure QED, for example, without weak-interaction corrections.

Including parity violation leads to an inverse propagator of the form

$$S^{-1}=\bar\eta\not L\eta+\eta\not R\bar\eta-c\eta-c^*\bar\eta\ , \tag{C3}$$

where c is a complex function of ω and k that parametrizes the chirality violation; η and $\bar\eta$ the projection matrices (2.11); L^μ and R^μ are the vectors (2.12). (Note that $[\not K,\not u]$ terms from multiloop graphs are again omitted.) The poles of the propagator define the mass and occur when the determinant of (C3) vanishes. This determinant is

$$D(k,\omega)\equiv L^2R^2-2|c|^2(L\cdot R)+|c|^4\ .$$

By inverting (C3), one obtains for the propagator

$$S=\frac{1}{D}[\bar\eta(L^2\not R-|c|^2\not L)\eta+\eta(R^2\not L-|c|^2\not R)\bar\eta+c^*\eta(L\cdot R-|c|^2+\tfrac{1}{2}[\not L,\not R])\eta+c\bar\eta(L\cdot R-|c|^2-\tfrac{1}{2}[\not L,\not R])\bar\eta]\ . \tag{C4}$$

This reduces to (1.4) when both chirality and parity are good ($c=0$ and $L^\mu=R^\mu$), to (2.13) when chirality holds but not parity ($c=0$ and $L^\mu\neq R^\mu$), and to (C2) when chirality is broken but parity holds (c=real and $L^\mu=R^\mu$). The complicated spin structure of (C4) can have observable effects as, for example, in the birefringence of polarized photons propagating through a hot electron gas.

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[1]In the noncovariant formulation of finite-temperature field theory (i.e., with imaginary time and discrete energies) the only constraint on Σ is rotational invariance. Consequently, Σ is a linear combination of γ^0 and $\vec\gamma\cdot\vec K$ with two coefficient functions.

[2]This would require the function b to vanish at $\omega^2=k^2$.

[3]These are tabulated in J. Patera and D. Sankoff, *Tables of Branching Rules for Representations of Simple Lie*

Algebras (Les Presses de l'Universite de Montreal, Montreal, 1973). Note that these authors normalize the index so that $T(R)=1$ for the fundamental of SU(N).

[4]At $k=0$, $a=-\frac{1}{3}$ and $b=-2M/3$. Along the dispersion relation plotted in Fig. 2 both a and b are monotonically increasing functions of k within the limits $-\frac{1}{3}\leq a\leq 0$ and $-2M/3\leq b\leq 0$.

[5]If ω solves (2.7) and ω' solves $\omega'(1+a)+b=-k(1+a)$, then $\omega'=-\omega$ because a and b are even and odd functions, respectively, of ω.

[6]In the more familiar gauge-boson example, when the boson energy is less than the plasma frequency the momentum is pure imaginary. However, for a fermion with $\omega<M$ the corresponding k is actually complex.

[7]The γ-matrix convention is that of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[8]A. Polyakov, Phys. Lett. 72B, 477 (1978); L. Susskind, Phys. Rev. D 20, 2610 (1979); T. Banks and E. Rabinovici, Nucl. Phys. B160, 349 (1979).

[9]S. Weinberg, Phys. Rev. Lett. 13, 168 (1964); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8),* edited by N. S. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[10]$g^2\approx e^2/(0.2)$ and $g'^2\approx e^2/(0.8)$.

[11]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[12]G. Peressutti and B. S. Skagerstam [Phys. Lett. 110B, 406 (1982)] have calculated the T dependence of the one-photon correction to the electron mass in the low temperature, broken chirality phase.

[13]See L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320 (1974).

[14]See, for example, H. A. Weldon, Phys. Rev. D 26, 1394 (1982).

[15]The temperature-dependent fermion mass for the Wess-Zumino model computed by L. Girardello, M. T. Grisaru, and P. Salomonson, Nucl. Phys. B178, 331 (1981), is a chiral-breaking mass proportional to the vacuum expectation value of a scalar field. Although their conclusion that supersymmetry is broken at all T is correct, their argument should be modified for $T>T_c$. At such temperatures the chiral-breaking fermion mass certainly vanishes, but it should not be compared with the scalar mass because it does not describe the location of the fermion pole. Instead the dispersion relation (2.8) should be compared with the relation $\omega^2-k^2=(\text{mass})^2$ for scalars. Since the momentum dependence of the two dispersion relations is different, supersymmetry is broken in this temperature regime. The argument does not even depend on a comparison of the scalar mass with the effective fermion mass (B3) generated by Yukawa couplings.

[16]This was checked by explicit computation.