PHYSICAL REVIEW D VOLUME 26, NUMBER 10 15 NOVEMBER 1982

Interacting spin-two field on the light front

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(Received 19 July 1982)

The minimal (30-component) massive spin-two field is examined from the viewpoint of the Cauchy initial-value problem. The use of light-cone coordinates requires the existence of 25 independent constraints on the system, a number which is shown to obtain if one performs up to three successive differentiations of the primary constraint equations. These manipulations allow one to construct an algorithm for the evaluation of 25 dependent components in terms of a conveniently chosen set of five independent field variables. The introduction of an electromagnetic coupling is shown to result in a loss of some of the required constraint equations with two aspects of this breakdown being noteworthy. These are (a) the fact that (as in the spin-$\frac{3}{2}$ theory) the problem of obtaining the correct degrees of freedom can be solved if the condition $F_{-i}=0$ is imposed and (b) the loss of constraint occurs at the highest possible level, i.e., in the quaternary constraints rather than at the secondary or tertiary stages of the calculation.

I. INTRODUCTION

Among the problems encountered in the construction of a relativistic field theory of a massive particle with spin S is the requirement that a sufficient number of constraint equations be present in order to reduce the number of independent variables to $2(2S+1)$. If such conditions on the fields are present in the required number, then a suitable set of $2S+1$ independent variables (and their respective time derivatives) can be independently specified at each point on a given spacelike surface and the Cauchy initial-value problem can presumably be solved. One aspect of this problem which has generally come to be recognized is the fact that the occurrence of inconsistencies in the presence of interactions is most blatant in the case of theories which do not follow from a variational principle so that one hopes to find a solution in terms of a Lagrangian approach.

This prescription which is fairly straightforward for the low spin values (e.g., the scalar, spinor, and vector fields) even in the presence of interactions, encounters significant pitfalls with increasing spin. Although the Rarita-Schwinger spin-$\frac{3}{2}$ theory has a consistent formulation as a boundary-value problem (at least in the weak-coupling case for which the underlying differential equations remain hyperbolic), it is known that difficulties can arise in the spin-two case. In particular it has been observed by Fierz and Pauli[1] that the usual 50-component spin-two theory in terms of the symmetrical tensor $h_{\mu\nu}$ and the third-rank tensor $\Gamma_{\mu\nu,\lambda}=\Gamma_{\nu\mu,\lambda}$ suffers a loss of constraint in the case of a minimal electromagnetic coupling. They showed that the correct number of dynamical variables could, however, be restored by the inclusion of nonminimal terms, a result which was subsequently emphasized also by Federbush.[2] This somewhat surprising discovery was placed in perspective by the demonstration[3] that by using an alternative 30-component theory originally proposed by Chang[4] no modification of minimal coupling is required. Indeed, the two formulations (i.e., the 30-component theory with minimal coupling and the 50-component theory with the nonminimal Fierz-Pauli terms included) are completely equivalent inasmuch as each system has the same set of second-order differential equations for the ten-component $h_{\mu\nu}$.

The initial-value problem for spin S in Minkowski space is even more involved when examined in light-cone coordinates as defined by $x^\mu\equiv(x^1,x^2,x^-,x^+)$ with $x^\pm\equiv(1/\sqrt{2})(x^0\pm x^3)$ for which nonvanishing metric tensor components are seen to be

$$g_{11}=g_{22}=-g_{+-}=-g_{-+}=1 .$$

In this case one studies the field evolution along the

x^+ direction which thus plays a role analogous to the time. Since the D'Alembertian under this change of coordinates involves only a single derivative with respect to x^+, there occurs here a remarkable transformation of a set of hyperbolic differential equations into a set which is more nearly akin to a parabolic-type, inasmuch as it requires only half the number of independent variables usually present. The fact that nontrivial complications can result has been shown by us in an earlier work[5] for the case of a spin-$\frac{3}{2}$ field. It was found that although the free Rarita-Schwinger field yields the desired 12 constraints on the 16-component tensor spinor (leaving four independent variables as required by the $2S+1$ rule) the presence of an electromagnetic coupling causes a loss of constraint which can only be circumvented by the imposition of the condition $F_{-i}=0$ on the light-cone components of the electromagnetic field tensor. In the present work this is extended to the spin-two theory using Chang's minimal 30-component formalism.[6] The following section summarizes the free-field result by displaying the primary constraints as well as the various secondary, tertiary, and quaternary constraints which follow from successive differentiations. These are shown to be 25 in number so that the required five independent variables remain. Section III carries out the extension to the case of minimal coupling which (as stated above) is known to lead to a consistent Cauchy problem for the 30-component theory in conventional coordinates. The failure in light-cone coordinates is displayed and the role of the condition $F_{-i}=0$ is analyzed. A brief conclusion summarizes the principal results obtained and offers a few observations.

II. FREE FIELD

The 30-component description of a spin-two field follows from the Lagrangian

$$\mathscr{L}=-H_{\lambda\nu,\mu}\partial^\lambda h^{\mu\nu}+\tfrac{1}{4}(H_{\nu\lambda,\mu}H^{\nu\lambda,\mu}-H_\lambda H^\lambda) -\tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu}-h^2)\ ,$$

where the 20-component tensor $H_{\mu\nu,\lambda}=-H_{\nu\mu,\lambda}$ has the symmetry property

$$\epsilon^{\mu\nu\kappa\lambda}H_{\mu\nu,\lambda}=0$$

and the definitions $H_\lambda\equiv H^\mu{}_{\lambda,\mu}$ and $h\equiv h^\mu_\mu$ have been employed. The equations of motion are thus

$$H_{\nu\lambda,\mu}=\partial_\nu h_{\mu\lambda}-\partial_\lambda h_{\mu\nu}+g_{\mu\nu}(\partial_\lambda h-\partial h_\lambda) -g_{\mu\lambda}(\partial_\nu h-\partial h_\nu)\ ,$$

$$m^2(h_{\mu\nu}-g_{\mu\nu}h)=\tfrac{1}{2}\partial^\lambda(H_{\lambda\nu,\mu}+H_{\lambda\mu,\nu})\ ,$$

where $\partial h_\mu\equiv\partial^\nu h_{\mu\nu}$. By allowing each index to range over the four values[7] i, $+$, and $-$ one obtains a useful decomposition of the 30 equations into two sets according to whether the timelike derivative ∂_+ occurs or not. The former set includes

(i) $H_{+-,+}=\partial_+h_{ii}-\partial_jh_{ij}\ ,$

(ii) (iii) $H_{+-,i}=\partial_+h_{-i}-\partial_-h_{i+}\ ,$

(iv) (v) $H_{ij,j}=2\partial_ih_{+-}-\partial_+h_{-i}-\partial_-h_{+i}\ ,$

(vi) $H_{-i,i}=-2\partial_+h_{--}+\partial_ih_{-i}-\partial_-(h_{ij}-2h_{+-})\ ,$

(vii) (viii) $H^{\rm ST}_{+i,j}=\partial_+\bar{h}_{ij}-\tfrac{1}{2}(\partial_ih_{+j}+\partial_jh_{+i}-\delta_{ij}\partial_kh_{+k})\ ,$

(ix) (x) $H_{+i,+}=\partial_+h_{+i}-\partial_ih_{++}\ ,$

(xi) $H_{+i,i}=\partial_+(2h_{+-}-h_{ii})+\partial_ih_{+i}-2\partial_-h_{++}\ ,$

(xii) $m^2h_{++}=\partial_+H_{+-,+}+\partial_iH_{i+,+}\ ,$

(xiii) $m^2(h_{+-}+h)=\tfrac{1}{2}\partial_+H_{+-,-}-\tfrac{1}{2}\partial_-H_{+-,+}+\tfrac{1}{2}\partial_iH_{i+,-}+\tfrac{1}{2}\partial_iH_{i-,+}\ ,$

(xiv) (xv) $m^2h_{+i}=-\tfrac{1}{2}\partial_+(H_{-+,i}+H_{-i,+})-\tfrac{1}{2}\partial_-H_{+i,+}+\tfrac{1}{2}\partial_j(H_{j+,i}+H_{ji,+})\ ,$

(xvi) (xvii) $m^2h_{-i}=-\tfrac{1}{2}\partial_+H_{-i,-}-\tfrac{1}{2}\partial_-(H_{+-,i}+H_{+i,-})+\tfrac{1}{2}\partial_j(H_{j-,i}+H_{ji,-})\ ,$

(xviii) (xix) (xx) $m^2(h_{ij}-\delta_{ij}h)=-\tfrac{1}{2}\partial_+(H_{-i,j}+H_{-j,i})-\tfrac{1}{2}\partial_-(H_{+i,j}+H_{+j,i})+\tfrac{1}{2}\partial_k(H_{ki,j}+H_{kj,i})\ ,$

while the latter consists of the ten equations

$$H_{+-,-}=-\partial_- h_{kk}+\partial_i h_{-i}\ , \tag{1}$$

$$H_{+i,-}=-\partial_i(h+h_{+-})+\partial_j h_{ij}-\partial_- h_{+i}\ , \tag{2) (3}$$

$$H^{\mathrm{ST}}_{-i,j}=\partial_-\bar{h}_{ij}+\tfrac{1}{2}(\delta_{ij}\partial_k h_{-k}-\partial_i h_{-j}-\partial_j h_{-i})\ , \tag{4) (5}$$

$$H_{ij,\pm}=H_{\pm j,i}-H_{\pm i,j}=\partial_i h_{\pm j}-\partial_j h_{\pm i}\ , \tag{6) (7}$$

$$H_{-i,-}=\partial_- h_{-i}-\partial_i h_{--}\ , \tag{8) (9}$$

$$m^2 h_{--}=-\partial_- H_{+-,-}+\partial_i H_{i-,-}\ , \tag{10}$$

where $\bar{h}_{ij}\equiv h_{ij}-\frac{1}{2}\delta_{ij}h_{kk}$ and the notation $H^{\mathrm{ST}}_{+i,j}$ has been used to denote the symmetrical traceless part of a tensor. The labels of these two sets have been chosen to emphasize the number of independent conditions contained in each. Since the number of independent variables must be reduced to five, it is clear that the set of primary constraints [Eqs. (1)—(10)] is to be augmented by the derivation of 15 additional constraint equations.

One begins the search for these conditions by noting that Eqs. (ii)—(v) are redundant equations of motion for the two variables h_{-i}. Upon elimination of $\partial_+ h_{-i}$ there consequently follow the two additional conditions

$$\partial_i h_{+-}-\partial_- h_{+i}=\tfrac{1}{2}H_{+-,i}+\tfrac{1}{2}H_{ij,j}\ . \tag{11) (12}$$

In order to obtain more restraints upon the system the secondary constraints must be obtained. These are found by differentiation of Eqs. (1)—(10) with respect to x^+ and using the equations of motion (i)—(xx) to eliminate the x^+ derivative terms. By inspection one finds that only Eqs. (1), (4) and (5), (8) and (9), and (10) are suitable for the derivation of secondary constraints as the others contain terms which do not have their x^+ derivatives specified by the set (i)—(xx). From Eq. (10) there follows according to this prescription the result

$$m^2(\tfrac{3}{2}\partial_- h_{ii}-\tfrac{3}{2}\partial_i h_{-i}-\partial_- h_{+-})-\tfrac{1}{2}m^2 H_{-i,i}-\partial_i\partial_-(H_{i-,+}+H_{+-,i})+\partial_-^2 H_{+-,+}+\partial_i\partial_j H_{i-,j}=0\ . \tag{13}$$

Because of the fact that each term in (13) has its x^+ derivative given by the equations of motion a further or tertiary constraint follows by iteration. Thus one obtains

$$\partial_i\partial_j h_{ij}-\partial_i\partial_- h_{+i}-\partial_i^2(h_{kk}-h_{+-})+2m^2 h_{+-}-\tfrac{1}{2}m^2 h_{ii}+\partial_- H_{+-,+}-\partial_i H_{+-,i}=0\ , \tag{14}$$

where (11) and (12) have been used to eliminate the term $\partial_k H_{ki,i}$. Finally, upon rewriting (14) using the symmetry properties of $H_{\mu\nu,\lambda}$ one obtains

$$\partial_i\partial_j\bar{h}_{ij}-\partial_i\partial_- h_{+i}+(2m^2+\partial_i^2)(2h_{+-}-h_{ii}) +m^2 h_{ii}+2\partial_- H_{+-,+}+\partial_i(H_{-+,i}+H_{-i,+})=0 \tag{14'}$$

which allows yet another iteration and thus the quaternary constraint

$$6\partial_- h_{++}-3\partial_i h_{+i}+2H_{+i,i}+H_{+-,+}=0\ . \tag{15}$$

At this point the possibility of obtaining additional conditions must cease in view of the occurrence of the term $H_{+i,i}$ and the fact that it does not have a known time derivative.

In order to continue one turns instead to (1) and obtains from it the secondary constraint

$$m^2(h_{ii}-h_{+-})+\partial_- H_{+-,+}-\partial_i H_{i-,+}=0\ , \tag{16}$$

which by judicious use of (14′) and (2) and (3) to eliminate all $H_{\mu\nu,\lambda}$ terms acquires the covariant form

$$h=0$$

or, explicitly,

$$h_{ii}=2h_{+-}\ . \tag{16'}$$

From (16′) there follows the tertiary constraint

$$H_{+i,i}=\partial_i h_{+i}-2\partial_- h_{++}\ , \tag{17}$$

which in combination with (15) yields

$$H_{+i,i}=H_{+-,+}\ , \tag{17'}$$

at which point the iteration process must once again terminate.

The search continues with Eqs. (8) and (9) which imply the secondary conditions

$$2m^2h_{-i}-\tfrac{3}{2}\partial_i\partial_-h_{kk}+\partial_i\partial_jh_{-j}-\tfrac{3}{2}\partial_j{}^2h_{-i}+\partial_i\partial_-h_{+-}-\partial_-{}^2h_{+i}+2\partial_-\partial_jh_{ij}-\partial_-(H_{-i,+}+H_{-+,i})$$
$$+\partial_jH^{\mathrm{ST}}_{-i,j}+\partial_iH_{-k,k}=0\ . \qquad (18)\ (19)$$

An additional iteration is seen to be possible thereby leading to the tertiary set

$$(4m^2-\partial_j{}^2)\partial_-h_{+i}-\tfrac{1}{2}(m^2-\partial_j{}^2)\partial_ih_{kk}-m^2\partial_jh_{ij}+2\partial_-\partial_jH^{\mathrm{ST}}_{+i,j}+(2m^2-\partial_j{}^2)H_{+-,i}=0\ . \qquad (20)\ (21)$$

The last equations which can yield restraints are (4) and (5). They allow the derivation of

$$m^2\bar{h}_{ij}+\partial_i\partial_jh_{kk}-\tfrac{1}{2}\partial_l{}^2\delta_{ij}h_{kk}-\partial_i\partial_-h_{+j}-\partial_j\partial_-h_{+i}+\delta_{ij}\partial_-\partial_kh_{+k}+2\partial_-H^{\mathrm{ST}}_{+i,j}-2[\partial_iH_{+-,j}]^{\mathrm{ST}}=0\ . \qquad (22)\ (23)$$

If this is solved for $\partial_-H^{\mathrm{ST}}_{+i,j}$ and the result inserted into (20) and (21), one obtains

$$H_{+-,i}=\partial_jh_{ij}-2\partial_-h_{+i}\ . \qquad (20')\ (21')$$

The symmetry properties of $H_{\mu\nu,\lambda}$ together with (2) and (3) and (16′) allow this to be written as

$$H_{-+,i}+H_{-i,+}=-\partial_jh_{ij}+3\partial_-h_{+i}-\tfrac{1}{2}\partial_ih_{kk}\ ,$$

which form clearly allows (upon examination of the equations of motion) the iteration of the constraint procedure. The result is

$$(m^2-\partial_j{}^2)h_{+i}+\partial_i\partial_jh_{+j}+2\partial_-H_{+i,+}-\tfrac{1}{4}\partial_iH_{+k,k}-\tfrac{3}{4}\partial_iH_{+-,+}=0\ , \qquad (24)\ (25)$$

thereby completing the task of obtaining a full set of 25 constraint equations.

There is, however, an as-yet-unresolved question inherent in this procedure which has been outlined here, namely, the question of the independence of the constraint equations which have been derived. In fact these equations are independent and can be most easily demonstrated as such by means of an algorithm for the determination of the 25 dependent components. The prescription includes the following 16 steps:

1.	(2) (3)	yield $H_{+i,-}$	in terms of h's.
2.	(6) (7)	yield $H_{ij,\pm}$	in terms of h's.
3.	(4) (5)	yield $(H_{-i,j})^{\mathrm{ST}}$	in terms of h's.
4.	(1)	yields $H_{+-,-}$	in terms of h's.
5.	(8) (9)	yield $H_{-i,-}$	in terms of h's.
6.	(17)	yields $H_{+i,i}$	in terms of h's.
7.	(20′) (21′)	yield $H_{+-,i}$	in terms of h's.
8.	(16)	yields $H_{+-,+}$	in terms of h's.
9.	(11) (12)	yield $H_{ij,j}$	in terms of h's.
10.	(22) (23)	yield $H_{+i,j}{}^{\mathrm{ST}}$	in terms of h's.
11.	(24) (25)	yield $H_{+i,+}$	in terms of h's.
12.	(13)	yields $H_{-i,i}$	in terms of h's.

This completes the task of determining all H's.

13.	(10)	yields h_{--}	in terms of other h's.
14.	(15)	yields h_{++}	in terms of other h's (no h_{--}).
15.	(14)	yields h_{+-}	in terms of other h's (no $h_{\pm\pm}$).
16.	(18) (19)	yield h_{-i}	in terms of h_{ij},h_{+i}.

This yields all components in terms of a set of five (h_{ij} and h_{+i}). Thus one has a complete demonstration of the existence of the massive spin-two field as a consistent boundary-value problem on the light cone. It provides a useful framework for the discussion of the modifications which ensue when interactions are now introduced.

III. MINIMAL COUPLING

With increasing spin values there arises the possibility of ever larger numbers of couplings. Because of this it is unrealistic and of dubious value to attempt the most general interaction term. Instead, attention will be limited to the one coupling which is known to have a consistent set of constraints in conventional coordinates, namely, the case of minimal electromagnetic coupling. This choice has the additional advantage of having already been studied for spin $\frac{3}{2}$.

The coupling to the electromagnetic field is accomplished by means of the replacement

$$\partial_\mu \rightarrow i\pi_\mu = \partial_\mu - ieqA_\mu \ ,$$

where A_μ is the usual vector potential and q is a charge matrix. The latter implies, of course, that the spin-two field components have internal degrees of freedom and allows (if one so chooses) to retain them as Hermitian fields. No attempt will be made here to rewrite (i)—(xx) or (1)—(12) incorporating the above replacement for ∂_μ. The only nontrivial aspect is whether there exist analogs of Eqs. (13)—(25), thereby providing the full set of constraints required for a spin-two field.

As a first step toward the solution of this problem it should be observed that it is impossible for minimal coupling to eliminate any of the six secondary constraints. This is, of course, a consequence of the fact that the number of components of the spin-two field which appear in the corresponding six primary constraints does not increase by virtue of minimal coupling. Thus one obtains after some calculation the complete set of secondary constraints,

$$m^2[\tfrac{3}{2}\pi_- h_{ii} - \tfrac{3}{2}\pi_i h_{-i} - \pi_- h_{+-}] + \frac{i}{2}m^2 H_{-i,i} - i\pi_i\pi_-[H_{i-,+} + H_{+-,i}] + i\pi_-{}^2 H_{+-,+} + \frac{i}{2}\pi_i\pi_j(H_{j-,i} + H_{i-,j})$$
$$+ eq[F_{-i}(H_{i+,-} + H_{i-,+}) + F_{+i}H_{i-,-} - F_{+-}H_{+-,-} + \tfrac{3}{4}F_{ij}H_{ij,-}] = 0 \ , \qquad (13'')$$

$$m^2(h_{ii} - h_{+-}) + i\pi_- H_{+-,+} - i\pi_i H_{i-,+} - \frac{i}{2}eq[F_{+-}h_{kk} + F_{-i}h_{+i}] = 0 \ , \qquad (16'')$$

$$2im^2 h_{-i} + \tfrac{3}{2}i\pi_i\pi_- h_{jj} - i\pi_i\pi_j h_{-j} + \tfrac{3}{2}i\pi_j{}^2 h_{-i} - i\pi_i\pi_- h_{+-} + i\pi_-{}^2 h_{+i} - 2i\pi_-\pi_j h_{ij} + \pi_-(H_{-i,+} + H_{-+,i})$$
$$- \pi_j H^{\mathrm{ST}}_{-i,j} - \pi_i H_{-k,k} + eq(F_{+-}h_{-i} - F_{+i}h_{--} - 2F_{-i}h_{jj} - \tfrac{3}{2}F_{ij}h_{-j} + 2F_{-i}h_{+-}) = 0 \ , \qquad (18'')\,(19'')$$

$$im^2\bar{h}_{ij} - 2\pi_- H^{\mathrm{ST}}_{+i,j} + [\pi_i H_{+-,j}]^{\mathrm{ST}} + [\pi_k H_{ki,j}]^{\mathrm{ST}} + eq[F_{+-}\bar{h}_{ij} + \tfrac{1}{2}\delta_{ij}F_{+k}h_{-k} - \tfrac{1}{2}F_{+i}h_{-j} - \tfrac{1}{2}F_{+j}h_{-i}$$
$$+ (F_{-i}h_{+j})^{\mathrm{ST}}] = 0 \ , \qquad (22'')\,(23'')$$

where a double prime notation has been used to indicate a correspondence to the free-field limit of the preceding section.

The total number of constraints is now 18 so that 7 more are required. One now examines the above six equations for tertiary constraint possibilities. By using symmetry to write such things as

$$H_{i-,+} = -\tfrac{1}{2}H_{+i,-} - \tfrac{1}{2}(H_{-+,i} + H_{-i,+})$$

together with the analogs of (2) and (3) and (xiv) and (xv), one finds, for example, that the $H_{i-,+}$ term in (13″) does not preclude additional constraints. In fact, after some analysis (13″), (16″), and (18″) and (19″) are all seen to imply tertiary constraints, thereby bringing the total to 21. The tertiary constraints are exceedingly lengthy and as the concern here is with the existence of constraints rather than their form nothing is to be gained by displaying them. However, if there is to be an equation (15″) as a quaternary constraint, (14″) must be derived in order that its differentiability can be discussed. In addition to a great number of terms which present no complications, one encounters the combination

$$F_{-i}\left[-\frac{i}{2}\pi_i H_{+k,k} - i\pi_j H^{\mathrm{ST}}_{+i,j} + 2i\pi_- H_{+i,+} \right.$$
$$\left. - ieqF_{-i}h_{++} \right] ,$$

which does not allow the iteration of the constraint generating process. Although one might in principle hope that the coefficient of F_{-i} could vanish, it is easily verified that at least in one limit—namely, $e \rightarrow 0$ and ∂_i terms vanishingly small—there is no possibility of cancellation. The inescapable conclusion is that further progress is possible only if F_{-i} is taken to vanish. This condition is now incorporated and only the question as to the existence of the constraints (24″) and (25″) remains.

The search for (24″) and (25″) requires that the procedure leading up to the derivation of (24) and (25) be examined. It is seen that it entailed the elimination of $H^{\mathrm{ST}}_{+i,j}$ using (20) and (21) and (22) and (23). In the interacting case (22″) and (23″) have been derived and none of the coupling-dependent terms contains components which do not have equations of motion. If the same can be said of the tertiary constraints (20″) and (21″) which are known to exist but have not been written down, then the existence of (24″) and (25″) is assured.

Rather than calculate the lengthy equations (20″) and (21″) it is sufficient to ask what possible terms (other than $H^{ST}_{+i,j}$) could occur which do not have equations of motion. From (18″) and (19″) and the equations of motion for each of the terms in those two equations one finds that the troublesome terms in (20″) and (21″) involve $H_{+k,k}$; $H_{+i,+}$; and h_{++}. Considering each of these terms separately it is found without serious complication that the coefficient of each of these terms vanishes in the case $F_{-i}=0$. This completes the demonstration that the sequence of calculations which led to (24) and (25) in the free-field case can be carried out also for minimal coupling. The entire set of 25 constraints thereby is seen to exist if F_{-i} vanishes, but (somewhat surprisingly perhaps) no additional consistency condition on the electromagnetic field tensor is required.

IV. CONCLUDING REMARKS

This paper has examined the consistency of a spin-two field on the light cone at a very primitive level. It has been found that independent of whether that field is classical or quantized one cannot introduce minimal electromagnetic coupling without destroying the possibility of having a meaningful boundary-value problem. The fact that the $F_{-i}=0$ condition does restore the situation is of mathematical interest but, of course, is a requirement which can only be satisfied locally and in a limited set of Lorentz frames.

If one were to dare to extrapolate the result of experience with the spin-$\frac{3}{2}$ case[5] and the present work, it would suggest the hypothesis that all minimal spin-S theories suffer a loss of constraint on the light cone when minimally coupled to the electromagnetic field with consistency restored only in the peculiar case of $F_{-i}=0$. In view of the fact that a similar extrapolation suggests that each half-unit increase in the spin value will increase the order of the constraints by one, it is difficult to conceive of a general proof of this result. It appears all the more difficult when it is realized that the problem always seems to occur at the level of the highest-order constraint in the theory.

Finally it is to be mentioned that this study has demonstrated once again in yet another way that light-cone theories are not like other theories. They have their own special problems that lead to contradictions at a much more basic level than is the case with conventional fields. While it may not be impossible to repair them, there does not seem to exist any obvious clue as to how such modifications might be effected.

ACKNOWLEDGMENT

This research was supported in part by the U. S. Department of Energy.

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[6]The free spin-two field on the light front has been considered by R. G. Root [Phys. Rev. D 8, 3382 (1973)]. Since his approach does not allow an extension to the interacting case in any obvious way, it is necessary to obtain a solution of the constraint problem using the approach of Sec. II.

[7]Throughout this paper lower case Latin letters will be used to denote the 1 and 2 components of a tensor.