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Primordial nucleosynthesis including radiative, Coulomb, and finite-temperature corrections to weak rates

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We calculate the changes in the yields of primordial nucleosynthesis which result from small corrections to rates for weak processes that connect neutrons and protons. We correct the weak rates by improved treatment of Coulomb and radiative corrections, and by inclusion of plasma effects. Our calculations lead to a systematic decrease in the predicted ^{4}He abundance of about $\Delta Y = 0.0025$. The relative changes in other primordial abundances are also $1-2\,\%$.

I. INTRODUCTION

The standard hot big-bang model of the Universe seems to provide a reliable framework for understanding the origin and evolution of our universe.[1] One of the features of our present universe which is naturally explained in this model is the large abundance of ^{4}He. The success of primordial nucleosynthesis in predicting the large primoridal abundance of ^{4}He, and the relatively large abundance of D, is usually considered to be the strongest evidence that the Universe can be described by a Friedmann-Robertson-Walker cosmology at very early times. Because of this concordance, it is attractive to assume that the Friedmann-Robertson-Walker cosmology was applicable at the time of nucleosynthesis, and then demand that the resulting primordial abundances of the light elements be within bounds extrapolated from present observations. This approach results in a limit on the contribution to the energy density from additional particles present in the universe at $T \leq 1$ MeV, such as additional neutrino species.[2]

Because primordial nucleosynthesis provides such a powerful probe of the conditions in the Universe at early times, it is important to have precise predictions of the primordial light-element abundances, particularly that of ^{4}He. In this paper we consider modifications to the calculation of the ^{4}He abundance due to (1) use of an explicit numerical integration of the rates for n-p transitions rather than fits to the numerical rates, (2) correct treatment of Coulomb corrections, (3) inclusion of radiative corrections—both the usual radiative corrections and the finite-temperature and finite-density radiative corrections that depend on the presence of a plasma, (4) inclusion of the effect of the plasma on the mass of the electron, and (5) heating of electron neutrinos in e^+e^- annihilation. We find that the above five effects result in a systematic *decrease* in the ^{4}He abundance of about 0.003, or about a one-percent relative decrease. (Addition of a light neutrino species leads to a ΔY of $\Delta Y \simeq 0.01$.) There are similar $1-2\,\%$ changes in the abundances of the other light elements which are produced (D, ^{3}He,^{7}Li, etc.).

Although we find that the corrections to the weak rates are in general temperature dependent, we can get a rough estimate of the sensitivity of the primordial ^{4}He abundance, , on the weak rates by considering the change in Y resulting from a change in the neutron half-life.[3] For instance, a decrease in the neutron half-life from 10.6 min to 10.4 min ($\Delta\lambda/\lambda \cong +0.02$ where $\lambda \equiv \tau_{1/2}{}^{-1}$) results in a decrease in the ^{4}He abundance of about 0.004. This decrease in the neutron half-life is equivalent to an increase in G_F, the Fermi constant, hence a temperature-independent increase in all the weak rates. This suggests that the dependence of Y on $\Delta\lambda$ may be approximated as

$$\Delta Y \cong 0.2 \frac{\Delta\tau_{1/2}}{\tau_{1/2}} \cong -0.2 \frac{\Delta\lambda}{\lambda} . \tag{1.1}$$

Our corrections to λ are in the $1-2\,\%$ range at the relevant temperature, so we expect a decrease in Y of order $0.002-0.004$. Some corrections will increase Y, while others will decrease Y.

It should be emphasized that the corrections $(1)-(5)$ above are universal in the sense that they must be applied regardless of the values of the neutron lifetime, the number of neutrinos, or the ratio of baryons to photons.

II. CORRECTIONS TO THE WEAK RATES

Before discussing the corrections to the weak rates, it will be useful for an understanding of the sensitivity of the ^{4}He abundance on them to give a brief review of primordial nucleosynthesis. For a detailed discussion, the reader is referred to the original literature.[4–6] Primordial nucleosynthesis can be divided into two stages. In the first, the weak interactions freeze out fixing the neutron-proton ratio. In the second, practically all available neutrons are processed into ^{2}H, ^{3}He, and mostly into ^{4}He. The reactions relevant in the first stage are the weak reactions, while the strong and electromagnetic interactions are important in the second stage. We will be most interested in the first stage, as the corrections we treat are corrections to the weak rates.

At temperatures above 1 MeV, the rates for the weak reactions $n \leftrightarrow ep\nu$, $ne^+ \leftrightarrow p\bar{\nu}$, and $n\nu \leftrightarrow pe^-$, given by $\lambda_W \sim G_F{}^2 T^5$, are much larger than the expansion rate of the Universe[1]

$$\lambda_e = (8\pi\rho/3m_P{}^2)^{1/2} ,$$

where G_F is the Fermi constant, m_P is the Planck mass, and ρ is the total energy density. When λ_W is much greater than λ_e, the neutron-proton ratio is given by its equilibrium value,

$$\left[\frac{n}{p} \right]_{\text{eq}} \cong e^{-\Delta m/T} , \tag{2.1}$$

where Δm is the neutron-proton mass difference (1.293 MeV). When λ_W is less than λ_e, the neutron-proton ratio is no longer able to track its equilibrium ratio, Eq. (2.1), and except for free neutron decay and the effect of the strong (n,p) reactions, it can be approximated by

$$\frac{n}{p} \cong e^{-\Delta m/T_f} , \tag{2.2}$$

where T_f is the freeze-out temperature, defined by the condition $\lambda_W = \lambda_e$. Freeze out occurs at about $T_f \cong 0.7$ MeV. An increase in λ_W leads to a decrease in T_f, hence to a smaller neutron-photon ratio. In the second stage of primordial nucleosynthesis almost all available neutrons end up in ^{4}He. Thus an increase in the weak rates leads to a decrease in the neutron proton ratio which results in less ^{4}He.

A. The uncorrected rates

The six weak reactions which interconvert neutrons and protons are

$$n \to p + e^- + \bar{\nu}_e , \tag{2.3a}$$

$$n + e^+ \to p + \bar{\nu}_e , \tag{2.3b}$$

$$n + \nu_e \to p + e^- , \tag{2.3c}$$

$$p + e^- + \bar{\nu}_e \to n , \tag{2.3d}$$

$$p + \bar{\nu}_e \to n + e^+ , \tag{2.3e}$$

$$p + e^- \to n + \nu_e . \tag{2.3f}$$

The rates for these processes were first calculated in the context of the early Universe by Alpher, Follin, and Herman,[4] and we will adopt their notation (see also Ref. 1). The rates depend on $Q = m_n - m_p$, the photon temperature T, the neutrino temperature T_ν, and the electron energy E_e. The rates depend on these quantities through the dimensionless variables

$$z \equiv m_e/T_\gamma, \quad z_\nu \equiv m_e/T_\nu ,$$
$$q \equiv Q/m_e, \quad \epsilon \equiv E_e/m_e . \tag{2.4}$$

In terms of these variables, the rates for the processes given in (2.3) are

$$\lambda_{n\to pe\nu} = (\tau\lambda_0)^{-1} \int_1^q d\epsilon \frac{\epsilon(\epsilon-q)^2(\epsilon^2-1)^{1/2}}{[1+\exp(-\epsilon z)]\{1+\exp[(\epsilon-q)z_\nu]\}} , \tag{2.5a}$$

$$\lambda_{\nu+n\to p+e}=(\tau\lambda_0)^{-1}\int_q^\infty d\epsilon\frac{\epsilon(\epsilon-q)^2(\epsilon^2-1)^{1/2}}{[1+\exp(-\epsilon z)]\{1+\exp[(\epsilon-q)z_\nu]\}}\ , \tag{2.5b}$$

$$\lambda_{e+n\to p+\nu}=(\tau\lambda_0)^{-1}\int_1^\infty d\epsilon\frac{\epsilon(\epsilon+q)^2(\epsilon^2-1)^{1/2}}{[1+\exp(\epsilon z)]\{1+\exp[-(\epsilon+q)z_\nu]\}}\ , \tag{2.5c}$$

$$\lambda_{p+e+\nu\to n}=(\tau\lambda_0)^{-1}\int_1^q d\epsilon\frac{\epsilon(\epsilon-q)^2(\epsilon^2-1)^{1/2}}{[1+\exp(\epsilon z)]\{1+\exp[(q-\epsilon)z_\nu]\}}\ , \tag{2.5d}$$

$$\lambda_{p+e\to n+\nu}=(\tau\lambda_0)^{-1}\int_q^\infty d\epsilon\frac{\epsilon(\epsilon-q)^2(\epsilon^2-1)^{1/2}}{1+\exp(\epsilon z)]\{1+\exp[(q-\epsilon)z_\nu]\}}\ , \tag{2.5e}$$

$$\lambda_{p+\nu\to e+n}=(\tau\lambda_0)^{-1}\int_1^\infty d\epsilon\frac{\epsilon(\epsilon+q)^2(\epsilon^2-1)^{1/2}}{1+\exp(-\epsilon z)]\{1+\exp[(q+\epsilon)z_\nu]\}}\ , \tag{2.5f}$$

where τ is the neutron lifetime [$\tau=(\ln 2)\tau_{1/2}$], and λ_0 is defined to be

$$\lambda_0\equiv\int_1^q d\epsilon\,\epsilon(\epsilon-q)^2(\epsilon^2-1)^{1/2}$$
$$=1.63615\ . \tag{2.6}$$

The units of the rates in (2.5) are $\sec^{-1}$. The factor $(\tau\lambda_0)^{-1}$ fixes the nuclear matrix element to be used in the calculation of the rates. Note that, in the limit $T\to 0$, we have $T_\nu\to 0$, $\lambda_{n\to pe\nu}\to\tau^{-1}$, while the rest of the rates in (2.5) go to zero. This, of course, is just the result that at low temperature and density the only change in the neutron-proton ratio from weak reactions is due to free decay of neutrons.

It will be convenient to combine the rates in (2.5) into two groups:

$$\lambda_n\equiv\lambda_{n\to p+e+\nu}+\lambda_{\nu+n\to p+e}+\lambda_{e+n\to p+\nu}\ , \tag{2.7a}$$

$$\lambda_p\equiv\lambda_{p+e+\nu\to n}+\lambda_{p+e\to\nu+n}+\lambda_{p+\nu\to e+n}\ . \tag{2.7b}$$

At high temperature, when the weak reactions are much faster than the expansion rate, the neutrino temperature is equal to the photon temperature and $\lambda_p=e^{-qz}\lambda_n$.

In the remainder of this section we will discuss corrections to the weak rates, and in the next section we will discuss how these changes affect the outcome of primordial nucleosynthesis.

B. Numerical evaluation of the rates

In the computer code developed by Wagoner,[7] the rates (2.7) are fit by an analytic function of the photon temperature. The fit to λ_n is good to an accuracy of about 1.4% over the temperature range $T_9=100$ to $T_9=0.3$ (T_9 is the temperature in units of 10^9 K).[7] The fit to λ_p is good to about 1.4% over a range $T_9=100$ to $T_9=3$. This fit is rather remarkable considering that λ_n and λ_p change by more than seven orders of magnitude over the above temperature range. However, based on our approximate formula for ΔY given in Eq. (1.1), a 1.4% error in the weak rates leads to a potential ΔY of $|\Delta Y|=(0.2)(1.4\%)\cong 0.003$.

We have modified Wagoner's code to evaluate the rates numerically every time step. We evaluate the integrals to an accuracy of better than 0.005%, which should introduce an error in ΔY of $|\Delta Y|\lesssim 0.0001$. The ratios of the numerical evaluation of λ_n and λ_p to Wagoner's fit to λ_n and λ_p at different temperatures are given in Fig. 1. The resultant change in Y from the usual result is discussed in Sec. III.

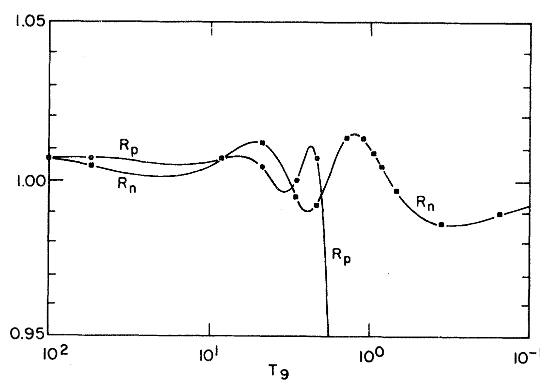

FIG. 1. The ratio of the numerical evaluation of λ_n and λ_p to the analytic fit used by Wagoner. R is the ratio of the numerical rate to the analytic fit. For $T_9\leq 2$, where the error in the fit for λ_p becomes greater than 5%, λ_p is very small and the rate of $p\to n$ conversion is negligible.

C. Coulomb and radiative corrections at zero temperature and density

Here we evaluate the Coulomb and radiative corrections to the weak rates ignoring effects of the plasma on these corrections. To order α, in neutron decay, the zero-temperature radiative and Coulomb corrections are given in terms of the diagrams of Fig. 2 by the interference of the Born term 2(a) with the diagrams 2(b), 2(c), and 2(d), plus the square of the sum of diagrams 2(e) and 2(f). We first address the zero-temperature corrections with all propagators in their usual vacuum form. Later we consider the changes in the radiative and Coulomb corrections due to plasma effects; these necessitate modification of the propagators and inclusion of photons in the initial state, [diagrams 2(g) and 2(h)].

It is standard to separate the electromagnetic corrections into a Coulomb part proportional to $Z\alpha$, and a radiative part proportional to α. The separation of the diagram 2(b) into a Coulomb part and a radiative part is somewhat arbitrary because the nuclear charge $Z=1$. We adopt the standard convention that the zero-temperature *radiative* corrections for all rates be the same and proportional to[8–12]

$$I^R \propto -i\alpha \int \frac{d^4k}{k^2-\lambda^2+i\epsilon}\frac{1}{k^2+2l\cdot k+i\epsilon}\frac{1}{k^2+2p\cdot k+i\epsilon}, \tag{2.8}$$

where l and p are the lepton and nucleon momenta and λ is a photon mass inserted to control the infrared divergences. For instance, in neutron decay, diagram 2(b) leads to a correction of the form

$$I_{2(b)} \propto -i(q_p q_{e^-}) \int \frac{d^4k}{k^2-\lambda^2+i\epsilon}\frac{1}{k^2+2l\cdot k+i\epsilon}\frac{1}{k^2-2p\cdot k+i\epsilon} \tag{2.9}$$

where q_p and q_{e^-} are the charges of the proton and electron. $I_{2(b)}$ may be rewritten by adding and subtracting I^R:

$$I_{2(b)} \propto I^R + i\alpha \int \frac{d^4k}{k^2-\lambda^2+i\epsilon}\frac{1}{k^2+2l\cdot k+i\epsilon}\left[\frac{1}{k^2-2p\cdot k+i\epsilon}+\frac{1}{k^2+2p\cdot k+i\epsilon}\right]. \tag{2.10}$$

The I^R part is absorbed into the radiative correction terms, and with the approximation $k^2 \ll 2p\cdot k$, we have

$$\frac{1}{k^2-2p\cdot k+i\epsilon}+\frac{1}{k^2+2p\cdot k+i\epsilon} \cong -2\pi i\delta(2p\cdot k). \tag{2.11}$$

The real part of the first term in Eq. (2.10) in the approximation (2.11) contributes to $I_{2(b)}$ a term proportional to $\alpha\pi/\beta$, where β is the velocity of the electron in the proton rest frame. The $\alpha\pi/\beta$ term is the first-order approximation that results from including the Fermi function $F_+(\beta)$ in the matrix element. The Fermi function is usually approximated as

$$F_+(\beta) \cong \frac{2\pi\alpha/\beta}{1-e^{-2\pi\alpha/\beta}}. \tag{2.12}$$

Therefore for neutron decay, diagram 2(b) contributes a term to the radiative corrections, I^R, and results in the insertion of $F_+(\beta)$ in the matrix element. By detailed balance the same corrections ap-

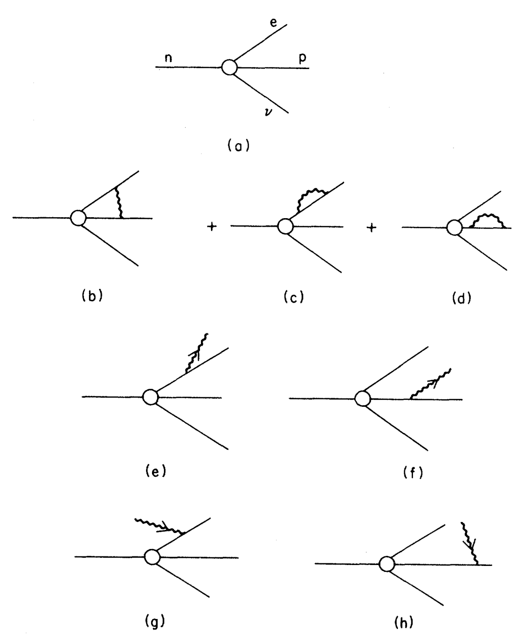

FIG. 2. Born term (a) and diagrams contributing to $O(\alpha)$ corrections to neutron decay.

ply to $e^-p\bar{\nu}\to n$. Since the direction of the neutrino momentum did not enter the above considerations, the same corrections apply to $\nu n\to pe$ and $\pi\to\nu n$.

With our convention that every reaction has a radiative correction proportional to I^R, it is easy to show that the correction due to the diagram in Fig. 3 is just given by I^R, i.e., there is no *Coulomb* correction to $ne^+\leftrightarrow p\bar{\nu}$.

For the vector part of the *V-A* nucleon current, the radiative corrections are known to be essentially independent of the structure of the nucleons for the small momentum transfers considered here.[8–12] For the axial-vector current there is no proof that the radiative corrections are independent of nucleon structure, but there is no evidence that the structure dependence is large. In any case, we cannot calculate the corrections due to the structure of the nucleons, so we will take the radiative corrections to be those for a point nucleon. Therefore we will multiply all the integrands of Eq. (2.5) by

$$1+\frac{\alpha}{2\pi}C(\beta,y)\ , \qquad (2.13)$$

where β is the electron's velocity and y is the neutrino energy divided by m_e [y is different for each of the rates in Eq. (2.5)]. C is given by[8–12]

$$C(\beta,y)\cong 40+4(R-1)(y/3\epsilon-\tfrac{3}{2}+\ln 2y)+R\,[2(1+\beta^2)+y^2/6\epsilon^2-4\beta R]$$
$$-4[2+11\beta+25\beta+25\beta^2+30\beta^3+20\beta^4+8\beta^5]/(1+\beta)^6\ , \qquad (2.14)$$

where the last term in the square brackets is from expansions of Spence functions, and R is defined to be

$$R\equiv\beta^{-1}\tanh^{-1}\beta\ . \qquad (2.15)$$

In conclusion, the zero-temperature Coulomb and radiative corrections result in the following: including $F_+(\beta)$ in the integrands of (2.5a), (2.5b), (2.5d), and (2.5e); multiplying the integrands of all of the rates (2.5) by $[1+(\alpha/2\pi)C(\beta,y)]$; and, finally, correcting λ_0 by multiplying the integrand by $F_+(\beta)[1+(\alpha/2\pi)C(\beta,y)]$. This last change increases λ_0 more than 7% from 1.63615 to 1.75321. The purpose of this last correction is to undo the radiative and Coulomb corrections to free neutron decay in order to obtain the bare matrix element.

It should be noted that the largest part of these corrections is due to the *radiative* corrections. The radiative corrections are large because of the constant 40 in Eq. (2.14), which arises mostly from a term $3\ln(m_W/m_e)$ in the radiative corrections.[8–12] For instance, of the 7% correction to λ_0, 3.4% comes from the Coulomb correction, while 3.7% arises from radiative corrections.

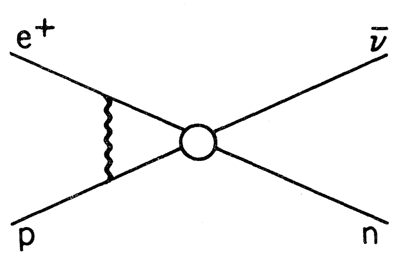

FIG. 3. One of the $O(\alpha)$ corrections to $e^-p\to\bar{\nu}n$. This term does not lead to a Coulomb correction.

Changes in the matrix elements of the rates in (2.5), which are compensated by an identical change in λ_0, will have no effect. Therefore the largest part of the radiative corrections, the constant $40\alpha/2\pi$ is irrelevant and could have been eliminated by redefining the coupling constant in the standard way. However, since the Coulomb corrections are β dependent, and the integrals in (2.5) sample β in a different manner than the integral for λ_0, the Coulomb corrections potentially lead to a 2 to 3% correction in λ_n, hence a 0.004 to 0.006 correction in Y. It should also be noted that $\lambda_{ne\leftrightarrow p\nu}$ feels the full 3.4% change in λ_0 due to Coulomb corrections, as there are no corresponding Coulomb corrections in the numerator.

For the purpose of comparison with previous work we compare our treatment of radiative and Coulomb corrections with that used by Wagoner. Wagoner ignores radiative corrections, and accounts for Coulomb corrections by simply effectively increasing λ_0 by 2%. This approximation has the undesirable feature that at low temperature $\lambda_{n\to pe\nu}$ approaches 0.98 rather than unity. Figure 4 gives the ratio of the numerical integration of the ratio corrected for zero-temperature Coulomb and radiative corrections to the rates used by Wagoner.

D. Finite-temperature and -density corrections to the electron mass

At finite temperature or density, the propagation of an electron or a photon is modified by its interac-

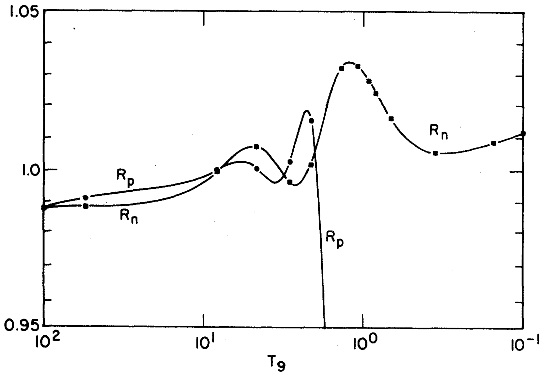

FIG. 4. The same as Fig. 1, but with the correct treatment of the zero-temperature Coulomb and radiative corrections.

tion with the ambient background gas. In the early Universe the background gas is quite hot, $\mu/T \lesssim 10^{-9}$, where μ is the electron chemical potential. Therefore, in the early Universe "density" effects which are parametrized by μ are much less important than "temperature" effects which are parametrized by T. We will work in the approximation $\mu = 0$.

In the real-time formalism[13] at nonzero temperature, the photon propagator has an additional term proportional to the photon phase-space density:

$$D_T^{\mu\nu}(k) = -i[(k^2+i\epsilon)^{-1} - 2\pi i f_B(k)\delta(k^2)] \times \left[g^{\mu\nu} - (1-\alpha)\frac{k^\mu k^\nu}{k^2+i\epsilon} \right], \quad (2.16)$$

where the phase-space density $f_B(k)$ is given by the Bose-Einstein distribution

$$f_B(k) = [\exp(\omega/T) - 1]^{-1} \quad (\omega = |\vec{k}|). \quad (2.17)$$

In Eq. (2.16), the term proportional to α is the result of adding a gauge-fixing term $-(2\alpha)^{-1}(\partial^\mu A_\mu)^2$. For the electron, the propagator at finite temperature is

$$S_T(k) = i[(\not{k} - m + i\epsilon)^{-1} + 2\pi i f_D(k)(\not{k}+m)\delta(k^2-m^2)], \quad (2.18)$$

where $f_D(k)$ is the phase-space density for a particle obeying Fermi-Dirac statistics:

$$f_D(k) = [\exp(\omega/T) + 1]^{-1} \quad [\omega = (|\vec{k}|^2 + m^2)^{1/2}]. \quad (2.19)$$

The electron mass at finite temperature is found by calculating the electron self-energy diagram with the propagators given above. If one defines mass renormalization at zero temperature by the addition of a counterterm $\delta m = -\mathrm{Re}\Sigma(p^2 = m_e^2)$ with $\Sigma(p)$ the electron self-energy diagram, then finite-temperature effects add to the electron mass a term δm_T defined by[14]

$$\delta m_T = \delta m + \mathrm{Re}\Sigma_T(p^2 = m_e^2), \quad (2.20)$$

where Σ_T is the electron self-energy evaluated with the finite-temperature photon and electron propagators. This results in a temperature-dependent effective electron mass m_T of the form[15]

$$m_T = m_e + \delta m_T = m_e + B\alpha T^2/m_e \quad (\alpha T^2/m_e^2 \le 1), \quad (2.21)$$

where m_e is the zero-temperature electron mass (0.511 MeV) and B is a slowly varying function of the temperature with a value between 1 and 2.[16]

It is easy to include this correction in the rates (2.5) by the substitution of m_T for m_e in the definitions (2.4), and by multiplying all the rates by $(m_T/m_e)^5$, since the cancellation of the m_e^5 from the numerator with the m_e^5 from λ_0 is true only at zero temperature. (We did not explicitly include the factor of m_e^5 in the definitions of the rates in (2.5) and in the definition of λ_0.)

The modification of the electron mass also has the effect of changing the relationship between the neutrino and photon temperatures. The neutrino temperature differs from the photon temperature because, to a good approximation, the neutrinos are decoupled when the entropy in the e^+e^- gas is released as e^+e^- annihilate. Therefore, the entropy released heats the photons, but not the neutrinos. (In Appendix B, we discuss the validity of the approximation that the neutrinos are not heated by e^+e^- annihilations, and the effect on Y of relaxing this assumption.) Assuming conservation of entropy during e^+e^- annihilation, the neutrino and photon temperatures are related by

$$T_\nu/T_\gamma = (\tfrac{4}{11})^{1/3}[\zeta(m_e/T_\gamma)]^{1/3}, \quad (2.22)$$

where $\zeta(x)$ is defined by the integral

$$\zeta(x) = 1 + \frac{45}{2\pi^4}\int_0^\infty dy \frac{y^2[(x^2+y^2)^{1/2} + y^2(x^2+y^2)^{-1/2}/3]}{\exp[(x^2+y^2)^{1/2}]+1}. \quad (2.23)$$

We evaluated Eq. (2.23) numerically and found, for temperatures in the range $0.1 \le T_9 \le 8$, $\Delta T_\nu / T_\mu \cong -(m/5T)^2 \Delta m/m$. We have also studied the dependence of the rates on changes in the electron mass, and changes in the neutrino temperature. In the same temperature range we found $\Delta\lambda/\lambda \cong -0.2(m/T)^2 \Delta m/m$ and $\Delta\lambda/\lambda \cong 2\Delta T_\nu / T_\nu$. Using $\Delta m = \alpha T^2/m_e$, then, as a rough estimate, we expect a change in the weak rates of about $\Delta\lambda/\lambda \cong -0.0015$, which would result in a ΔY of about $\Delta Y \cong +0.0003$.

An additional effect of the change in the electron mass that leads to an even smaller effect on ΔY, is that as the electron mass changes, the electron's contribution to the total energy density, and hence to the expansion rate, changes. The total energy density ρ_T changes by an amount $\Delta\rho_T$ due to a change in the electron mass, Δm, given approximately by $\Delta\rho_T/\rho_T \cong -(m/4T)^2 \Delta m/m$ in the range $0.3 \le T_9 \le 8$. For Δm given by $\Delta m = \alpha T^2/m$, $\Delta\rho_T/\rho_T$ is about -6×10^{-4}. The change in Y due to a change in $\Delta\rho_T$ was considered in studies of the sensitivity of Y on the number of massless neutrinos.[3] In those studies it was found that $\Delta Y \cong 0.1 \Delta\rho_T/\rho_T$. Therefore, the change in ρ_T due to the change in the electron mass should result in a ΔY of order $\Delta Y \cong -6\times10^{-5}$. Therefore, as a rough estimate, combining the effect of the change in the electron mass on T_ν, λ, and ρ_T, we might expect a ΔY of order $+2.4\times10^{-4}$. These effects are included exactly in the numerical calculations of Sec. III.

E. Finite-temperature radiative corrections

The finite-temperature parts of the propagators (2.16) and (2.18) give additional radiative corrections to Feynman graphs involving virtual photons and electrons. To these must be added the additional processes of spontaneous absorption, induced emission, and induced absorption. [Spontaneous emission was, of course, already included in (2.11).] The virtual-photon contributions have infrared divergences which go as

$$\int_0 \frac{d\omega}{\omega} \frac{1}{e^{\omega/T}-1} , \qquad (2.24)$$

which are canceled by divergences in real-photon processes. An infrared divergence of the form

$$\int_0 d\omega \frac{1}{e^{\omega/T}-1} \qquad (2.25)$$

in the emission of real photons is canceled by a similar term from the absorption of real photons.

The finite-temperature radiative corrections involve double integrals which must be evaluated numerically. The integrals are too tedious to write here; they are given in Appendix A. The finite-temperature radiative corrections cause the rates in (2.5) to be multiplied by an additional factor of the form

$$1 + \frac{\alpha}{\pi} C_i'(T) , \qquad (2.26)$$

where the C_i' differ for different processes. The magnitude of the finite-temperature corrections to the weak rates are shown in Fig. 5. As there are no corresponding corrections to λ_0, the weak rates absorb the full change due to the finite-temperature radiative corrections. Because the temperature-dependent parts of the propagators are more convergent in the ultraviolet, there is no term that corresponds to the factor $\ln(m_W/m_e)$ present in the zero-temperature radiative corrections.

It is seen from Fig. 5 that the finite-temperature corrections depend strongly on the temperature and on the individual reaction considered. It is also obvious that the rate for neutron decay receives the

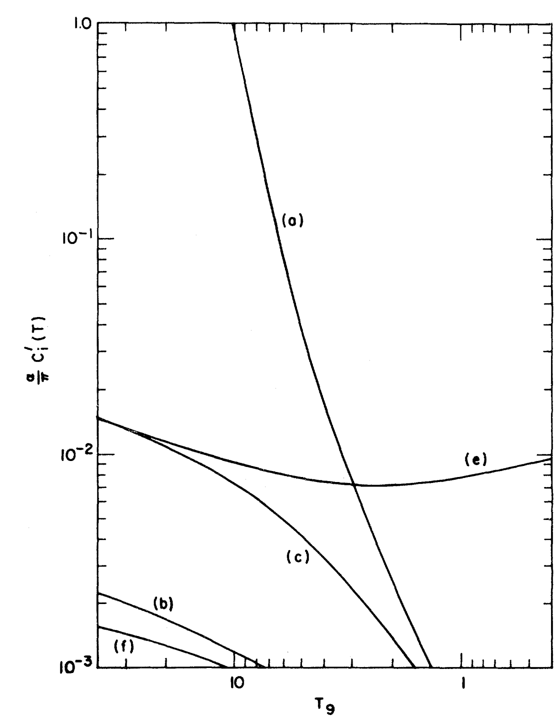

FIG. 5. Temperature-dependent parts of the radiative corrections to some of the weak rates. The curves labeled a through f refer to the corrections to rates (2.5a) through (2.5f).

largest correction. This large correction is mostly due to the effect of a photon in the initial state: $\gamma n \rightarrow pe\nu$. The effect of the photon can be thought of as an effective increase in the Q value for $n \rightarrow pe\nu$, $Q \rightarrow Q+\omega$, where ω is the energy of the photon, $\langle \omega \rangle \cong 2.7T$. Since the neutron decay rate is a more sensitive function of Q then the $2 \rightarrow 2$ weak processes, this particular consequence of finite temperature is larger for $\lambda_{n \rightarrow pe\nu}$. However, at the time of the freeze out of the neutron-proton ratio, $\lambda_{n \rightarrow pe\nu}$ makes only a small contribution to the total λ_n. It is also seen that the contributions to λ_p start to increase at low temperatures, again this can be understood as an increase in the effective Q value. Since λ_p at low temperatures decrease as $\exp(-Q/T)$, the rate is sensitive to the small increase in the Q value caused by having the additional photon energy in the initial state.

The final ratio of the numerical evaluation of λ_n and λ_p to the fit employed by Wagoner is given in Fig. 6. The rates in Fig. 6 include both zero-temperature and finite-temperature Coulomb and radiative corrections, and all corrections in the rates due to the temperature dependence of the electron mass. In the next section we calculate the result of primordial ^{4}He production with the new, improved rates.

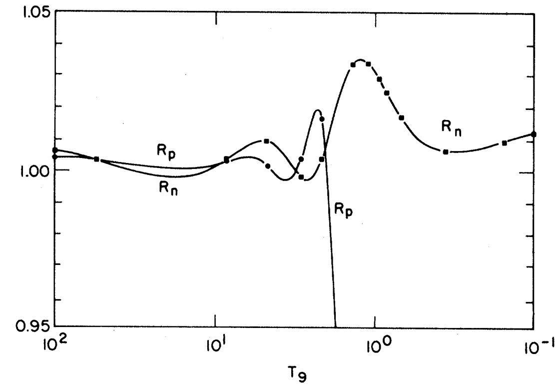

FIG. 6. The same as Fig. 4, but with finite-temperature radiative corrections and finite-temperature effects on the electron mass.

III. RESULTS AND CONCLUSIONS

The changes in the predicted ^{4}He mass fraction due to all the effects discussed in Sec. II, and due to

TABLE I. Sensitivity of the primordial ^{4}He abundance to the corrections discussed in the text. (The value quoted are for $n_B/n_\gamma = 3\times10^{-10}$, three light neutrinos, and $\tau_{1/2}=10.6$ min.)

Wagoner 1973	$Y_0=0.2456$		
Wagoner 1973 with numerical evaluation of λ_p and λ_n	$Y_1=0.2443$	$Y_0-Y_1=0.0013$	
Above corrections plus Coulomb corrections	$Y_2=0.2434$	$Y_0-Y_2=0.0022$	$Y_1-Y_2=0.0009$
Above corrections plus zero-temperature radiative corrections	$Y_3=0.2439$	$Y_0-Y_3=0.0017$	$Y_2-Y_3=-0.0005$
Above corrections plus finite-temperature radiative corrections	$Y_4=0.2435$	$Y_0-Y_4=0.0021$	$Y_3-Y_4=0.0004$
Above corrections plus corrections to electron mass	$Y_5=0.2436$	$Y_0-Y_5=0.0020$	$Y_4-Y_5=-0.0001$
Above corrections plus e^+e^- heating of ν_e	$Y_6=0.2434$	$Y_0-Y_6=0.0022$	$Y_5-Y_6=0.0002$

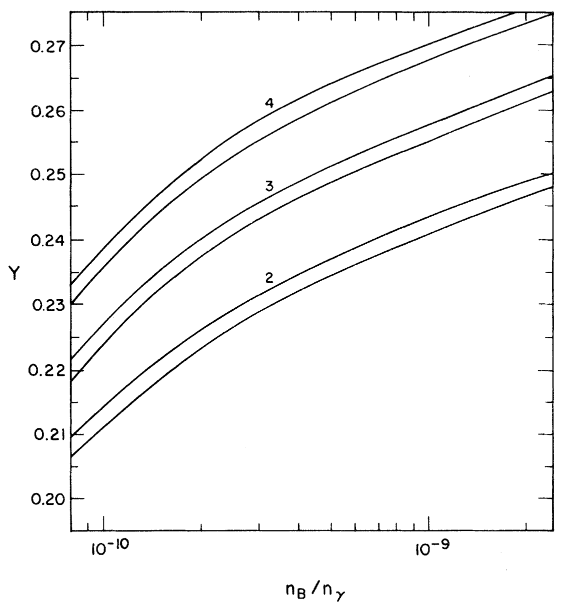

FIG. 7. The primordial ^{4}He mass fraction as a function of the baryon-to-photon ratio with two, three, and four light neutrino families assuming a neutron half-life of 10.61 min. The upper curve in each set shows the result of Wagoner's calculation, and the lower curve shows the results of including all the corrections to the weak rates discussed in Sec. II and summarized in Table I.

the slight heating of the electron neutrinos by $e^{\pm}$ annihilations (see Appendix B) are summarized in Table I. For a wide range of input parameters ($\tau_{1/2}=10.1-11.1$ min, $N_\nu=2-10$, and $\eta=3\times10^{-11}-3\times10^{-9}$), the sum of all these corrections results in an approximately constant, systematic decrease in Y, $\Delta Y \cong -0.0025$ ($\sim 1\%$ relative change). The changes in the predicted abundances of the other light elements (D, ^{3}He, and ^{7}Li) are also in the range of a few percent. However, their present abundances are known to much less precision than the present ^{4}He abundance, typically to only within a factor of 2. In Fig. 7 we show the predicted primordial ^{4}He abundance with and without the corrections we have discussed in this paper, as a function of η for $\tau_{1/2}=10.6$ min and $N_\nu=2$, 3, and 4.

The net ΔY we find is about the size of Wagoner's estimated uncertainty, and just slightly less than the uncertainty due to a 1 σ change (±0.16 min) in $\tau_{1/2}$. Nevertheless, it is a *systematic decrease* in the predicted primordial abundance of ^{4}He. Within a few years experiments with confined neutrons should significantly improve the determination of $\tau_{1/2}$. Recent studies of extragalactic, very metal-poor objects (in which the stellar contribution to the ^{4}He abundance should be small) have led to more accurate and reliable determination of the *primordial* mass fraction of ^{4}He.[17] A recent result typical of these studies of $Y_p=0.24\pm0.01$.[18] As more objects are studied the uncertainty in Y_p should continue to decrease. It has been argued on the basis of the abundances of D and ^{3}He that η must be greater than $(2-3)\times10^{-10}$.[19] For $N_\nu=3$ the standard model did predict $Y_p \gtrsim 0.240-0.246$—leaving little room for concordance. It now predicts $Y_p \gtrsim 0.237-0.243$— easing the situation a bit. In any case, since primordial nucleosynthesis is the most vigorous test of the standard, hot big-bang model, and is also our most powerful probe of the early Universe, it is important to continue to sharpen and reexamine its predictions as the uncertainties in the input parameters decrease.

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APPENDIX A: FINITE TEMPERATURE RADIATIVE CORRECTIONS

In this appendix we give the corrections to the weak rates (2.5) due to finite-temperature radiative corrections. The corrections naturally divide into a part arising from finite-temperature modifications to the photon propagator, and a part arising from finite-temperature modifications to the electron propagator. We first give the photon propagator corrections, then discuss the electron propagator corrections.

From photon corrections, the rate $\lambda_{n\to pe\nu}$ receives an additional term:

$$(\tau\lambda_0)^{-1}(\alpha/\pi)\int_1^q d\epsilon\frac{2\epsilon(\epsilon^2-1)^{1/2}}{e^{-\epsilon z}+1}$$

$$\times\Bigg\{(1-\beta^{-1}\tanh^{-1}\beta)\int_0^{q-\epsilon}\frac{d\eta}{\eta}\frac{1}{e^{\eta z}-1}[2F_\nu(\epsilon-q)-F_\nu(\epsilon-q+\eta)-F_\nu(\epsilon-q-\eta)]$$

$$+(1-\beta^{-1}\tanh^{-1}\beta)\int_{q-\epsilon}^{\infty}\frac{d\eta}{\eta}\frac{1}{e^{\eta z}-1}[2F_\nu(\epsilon-q)-F_\nu(\epsilon-q-\eta)]$$

$$+\tfrac{1}{2}(\epsilon^{-2}\beta^{-1}\tanh^{-1}\beta)\int_0^{q-\epsilon}d\eta\frac{\eta}{e^{\eta z}-1}F_\nu(\epsilon-q+\eta)$$

$$+\tfrac{1}{2}(\epsilon^{-2}\beta^{-1}\tanh^{-1}\beta)\int_0^{\infty}d\eta\frac{\eta}{e^{\eta z}-1}F_\nu(\epsilon-q-\eta)$$

$$+\epsilon^{-1}(\beta^{-1}\tanh^{-1}\beta-1)\int_0^{q-\epsilon}d\eta\frac{1}{e^{\eta z}-1}[F_\nu(\epsilon-q+\eta)-F_\nu(\epsilon-q-\eta)]$$

$$-\epsilon^{-1}(\beta^{-1}\tanh^{-1}\beta-1)\int_{q-\epsilon}^{\infty}d\eta\frac{1}{e^{\eta z}-1}F_\nu(\epsilon-q-\eta)\Bigg\}$$

$$-(\tau\lambda_0)^{-1}(\alpha/\pi)\int_q^{\infty}d\epsilon\frac{2\epsilon(\epsilon^2-1)^{1/2}}{\epsilon^{-\epsilon z}+1}\int_{\epsilon-q}^{\infty}d\eta\frac{1}{e^{\eta-1}}[(1-\beta^{-1}\tanh^{-1}\beta)\eta^{-1}F_\nu(\epsilon-q-\eta)$$

$$+\epsilon^{-1}(\beta^{-1}\tanh^{-1}\beta-\tfrac{3}{2})F_\nu(\epsilon-q-\eta)$$

$$-\epsilon^{-2}\beta^{-1}(\tanh^{-1}\beta)\eta F_\nu(\epsilon-q-\eta)]\;, \qquad \text{(A1)}$$

where $F_\nu(x)$ is defined as

$$F_\nu(x)=x^2[\exp(xz_\nu)+1]^{-1}\;. \qquad \text{(A2)}$$

From photon corrections, the rate $\lambda_{\nu n\to pe}$ receives a correction

$$(\tau\lambda_0)^{-1}(\alpha/\pi)\int_q^{\infty}d\epsilon\frac{2\epsilon(\epsilon^2-1)^{1/2}}{\epsilon^{-\epsilon z}+1}$$

$$\times\Bigg\{(1-\beta^{-1}\tanh^{-1}\beta)\int_0^{\epsilon-q}\frac{d\eta}{\eta}\frac{1}{e^{\eta z}-1}[2F_\nu(\epsilon-q)-F_\nu(\epsilon-q+\eta)-F_\nu(\epsilon-q-\eta)]$$

$$+(1-\beta^{-1}\tanh^{-1}\beta)\int_{\epsilon-q}^{\infty}\frac{d\eta}{\eta}\frac{1}{e^{\eta z}-1}[2F_\nu(\epsilon-q)-F_\nu(\epsilon-q+\eta)]$$

$$+\tfrac{1}{2}(\epsilon^{-2}\beta^{-1}\tanh^{-1}\beta)\int_0^{\epsilon-q}d\eta\frac{\eta}{e^{\eta z}-1}F_\nu(\epsilon-q-\eta)$$

$$+\tfrac{1}{2}(\epsilon^{-2}\beta^{-1}\tanh^{-1}\beta)\int_0^{\infty}d\eta\frac{\eta}{e^{\eta z}-1}F_\nu(\epsilon-q+\eta)$$

$$+\epsilon^{-1}(\beta^{-1}\tanh^{-2}\beta-1)\int_0^{\epsilon-q}d\eta\frac{1}{e^{\eta z}-1}[F_\nu(\epsilon-q+\eta)-F_\nu(\epsilon-q-\eta)]$$

$$+\epsilon^{-1}(\beta^{-1}\tanh^{-1}\beta-1)\int_{\epsilon-q}^{\infty}d\eta\frac{1}{e^{\eta z}-1}F_\nu(\epsilon-q+\eta)\Bigg\}$$

$$-(\tau\lambda_0)^{-1}(\alpha/\pi)\int_1^{q}d\epsilon\frac{2\epsilon(\epsilon^2-1)^{1/2}}{e^{-\epsilon z}+1}\int_{q-\epsilon}^{\infty}d\eta\frac{1}{e^{\eta z}-1}[(1-\beta^{-1}\tanh^{-1}\beta)\eta^{-1}F_\nu(\epsilon-q+\eta)$$

$$-\epsilon^{-1}(\beta^{-1}\tanh^{-1}\beta-\tfrac{3}{2})F_\nu(\epsilon-q+\eta)$$

$$-\epsilon^{-2}\beta^{-1}(\tanh^{-1}\beta)\eta F_\nu(\epsilon-q+\eta)] \qquad \text{(A3)}$$

and the rate $\lambda_{ne^+\to p\bar\nu}$ receives a correction

$$(\tau\lambda_0)^{-1}(\alpha/\pi)\int_1^\infty d\epsilon \frac{2\epsilon(\epsilon^2-1)^{1/2}}{e^{\epsilon z}+1}$$
$$\times\left\{(1-\beta^{-1}\tanh^{-1}\beta)\int_0^{\epsilon+q}\frac{d\eta}{\eta}\frac{1}{e^{\eta z}-1}[2F_\nu(-\epsilon-q)-F_\nu(-\epsilon-q-\eta)-F_\nu(\eta-\epsilon-q)]\right.$$
$$+(1-\beta^{-1}\tanh^{-1}\beta)\int_{\epsilon+q}^\infty\frac{d\eta}{\eta}\frac{1}{e^{\eta z}-1}[2F_\nu(-\epsilon-q)-F_\nu(-\epsilon-q-\eta)]$$
$$+\tfrac{1}{2}(\epsilon^{-2}\beta^{-1}\tanh^{-1}\beta)\int_0^\infty d\eta\frac{\eta}{e^{\eta z}-1}F_\nu(-\epsilon-q-\eta)$$
$$+\tfrac{1}{2}(\epsilon^{-2}\beta^{-1}\tanh^{-1}\beta)\int_0^{\epsilon+q} d\eta\frac{\eta}{e^{\eta x}-1}F_\nu(\eta-q-\epsilon)$$
$$+\epsilon^{-1}\beta^{-1}\tanh^{-1}\beta\int_0^{\epsilon+q} d\eta\frac{1}{e^{\eta z}-1}[F_\nu(-\epsilon-q-\eta)-F_\nu(\eta-q-\epsilon)]$$
$$\left.+\epsilon^{-1}\beta^{-1}\tanh^{-1}\beta\int_{\epsilon+q}^\infty d\eta\frac{1}{e^{\eta z}-1}F_\nu(-\epsilon-q-\eta)\right\}. \quad \text{(A4)}$$

The corrections for the reverse reactions may be obtained from the above corrections with the substitution rule

$$\lambda_{i\to j}=\lambda_{j\to i}(z_\nu\leftrightarrow -z_\nu; e^{\pm\epsilon z}\leftrightarrow e^{\mp\epsilon z}) . \quad \text{(A5)}$$

Finally, we have included corrections to the electron propagators due to finite-temperature effects. Since they result in smaller corrections than the corrections to the photon propagators, we do not list them here.

These radiative corrections have also been calculated by Cambier, Primack, and Sher.[20] Their results for the corrections are identical to ours.

APPENDIX B: NEUTRINO HEATING DUE TO e^+e^- ANNIHILATION

Part of the "early Universe" lore is that neutrinos decouple at a temperature $T_d\cong 1$ MeV and therefore do not share in the entropy release from $e^\pm$ annihilations which occur at temperatures $\lesssim 0.5$ MeV. As a consequence, neutrinos are predicted to have a lower temperature today than the photons, $T_\nu=(\frac{4}{11})^{1/3}T_\gamma$. The assumption of complete neutrino decoupling is explicitly incorporated into Wagoner's code. However, as we mentioned earlier, the weak rates are extremely sensitive to the temperature of the electron neutrinos $\Delta\lambda/\lambda \cong 2\Delta T_{\nu_e}/T_{\nu_e}$ (for $T\cong 0.3-10$ MeV). This results in a Y dependence upon T_{ν_e} of[21]

$$\Delta Y\simeq -0.15(\Delta T_{\nu_e}/T_{\nu_e}) . \quad \text{(B1)}$$

Taking into account both the dependence of Y upon the energy density contributed by all the neutrino species and the T_{ν_e} dependence of the weak rates we estimate that

$$\Delta Y\cong -0.1\Delta T_{\nu_e}/T_{\nu_e}+0.04\Delta T_{\nu_\mu}/T_{\nu_\mu} +0.04\Delta T_{\nu_\tau}/T_{\nu_\tau} , \quad \text{(B2)}$$

where all the changes are with respect to the standard calculation in which the neutrinos do not share in any of the energy from $e^\pm$ annihilations. At the crucial epoch ($T_f\cong 0.7$ MeV), when the n/p ratio "freezes out," $T_\nu\cong 0.985\ T_\gamma$. Thus if the electron neutrino species remains in good thermal contact ($T_\nu=T_\gamma$) until this epoch, $\Delta T_{\nu_e}/T_{\nu_e}\cong 0.015$, and Eq. (B2) predicts a change in Y of $\Delta Y\cong -0.0015$—comparable to the changes due to the other effects we have discussed. (The electron neutrino interacts via both the charged and neutral currents and decouples at a lower temperature than ν_μ and ν_τ, which have only neutral current interactions with electrons.)

This estimate and the fact that $T_d\cong 1$ MeV is not too different from $T_f\cong 0.7$ MeV motivated the following more precise calculation of the evolution of the T_{ν_i}.

The number density of a neutrino species is governed by the Boltzmann equation

$$\dot{n}_\nu + (3\dot{R}/R)n_\nu = \langle \sigma v \rangle [n_e{}^2 - f(T_\nu) n_\nu{}^2] \ , \quad \text{(B3)}$$

where n_e and n_ν are the number density of e^- (or e^+) and a given neutrino species (ν_e, ν_μ, or ν_τ), respectively, $\langle \sigma v \rangle$ is the thermally averaged cross section times relative velocity for $e^+ + e^- \to \nu_i + \bar{\nu}_i$, and f will be evaluated below. We have assumed that the momentum distribution of each neutrino species is that of a Fermi-Dirac particle with temperature T_{ν_i}. The $(3\dot{R}/R)n_\nu$ term represents the dilution due to the expansion of the Universe. Neglecting the expansion for a moment, in thermal equilibrium when $T_\nu = T_\gamma$, $\dot{n}_\nu = 0$, so that $f(T) = [n_e(T)/n_\nu(T)]_{\text{eq}}{}^2$. This is usually referred to as "detailed balance." Clearly $f(T_\nu)$ only depends upon the neutrino distribution—which by assumption only depends upon T_ν; thus

$$f(T_\nu) = \left[\frac{n_e(T_\nu)}{n_\nu(T_\nu)} \right]_{\text{eq}}^2 , \quad \text{(B4)}$$

regardless of whether or not $T_\nu = T_\gamma$. We have calculated that for electron neutrinos $\langle \sigma v \rangle \cong 0.9\, G_F{}^2 T^2$, and for μ and τ neutrinos $\langle \sigma v \rangle \cong 0.2\, G_F{}^2 T^2$; these forms are valid for $T_\nu \gtrsim 0.3$ MeV ($\sin^2\theta_W \simeq 0.23$ has been assumed). For reference, these values for $\langle \sigma v \rangle$ imply that ν_e "decouples" at $T_d \cong 2$ MeV, and ν_μ, ν_τ decouple at $T_d \cong 3.5$ MeV, where T_d is defined by $\lambda_W(T_d)/\lambda_e(T_d) = 1$.

By inventing a fictitious fiducial neutrino species x which does not participate at all in $e^\pm$ annihilations, so that $\dot{n}_x + (3\dot{R}/R)n_x = 0$, Eq. (B3) can be recast in a more useful form:

$$\frac{d}{dt}(n_\nu/n_x) = \langle \sigma v \rangle n_x \left[\left[\frac{n_e}{n_x} \right]^2 - f \left[\frac{n_\nu}{n_x} \right]^2 \right] , \quad \text{(B5)}$$

where the temperature of x is T_x, of e is T_γ, and of ν is T_ν. For $\delta = (T_\nu - T_x)/T_x \ll 1$, $n_\nu/n_x \cong 1 + 3\delta$. Since δ is the temperature difference between ν and a species x which is truly "decoupled" during the annihilation epoch, it measures the slight heating of the neutrinos due to $e^\pm$ annihilations.

After introducing $y \equiv (1 \text{ MeV}/T)$ and $\epsilon = (T_\gamma - T_x)/T_x$, and employing the numerical fits $n_e(y)/n_x(y) \cong 2(1 - 4.6 \times 10^{-2} y^2)$ and $\epsilon \cong 5.9 \times 10^{-3} y^2$, Eq. (B5) can be written in the more suggestive form

$$\delta' \cong a y^{-4}(\epsilon - \delta) \begin{cases} e: a \cong 0.2 \ . \\ \mu,\tau: a \cong 0.02 \ , \end{cases} \quad \text{(B6)}$$

$$\epsilon \simeq 5.9 \times 10^{-3} y^2 \ , \quad \text{(B7)}$$

where a prime denotes d/dy. From (B6) it is clear that the rise in the γ and $e^\pm$ temperature drives the slight heating of the neutrinos. If a ($\propto n_x \langle \sigma v \rangle / \lambda_e$) were large, then ϵ would track δ, and the neutrinos would maintain "good thermal contact."

Equation (B5) is easily integrated:

$$\delta(y) = 5.9 \times 10^{-3} \left[\frac{a}{3} \right]^{2/3} \exp(a y^{-3}/3) \times \int_{a y^{-3}/3}^{\infty} v^{-2/3} e^{-v} dv \ . \quad \text{(B8)}$$

We find that for ν_e,

$$\delta = (T_{\nu_e} - T_x)/T_x = \begin{cases} 2 \times 10^{-3}, & T_\gamma \cong 0.7 \text{ MeV} \ , \\ 3 \times 10^{-3}, & T_\gamma \cong 0 \ , \end{cases} \quad \text{(B9)}$$

while for ν_μ and ν_τ,

$$\delta = (T_{\mu_\nu} - T_x)/T_x = \begin{cases} 9 \times 10^{-4}, & T_\gamma = 0.7 \text{ MeV} \ , \\ 1 \times 10^{-3}, & T_\gamma \cong 0 \ . \end{cases} \quad \text{(B10)}$$

This means that at n/p "freeze out," $T_f \cong 0.7$ MeV, electron neutrinos should be slightly warmer, $\Delta T_\nu / T_\nu \cong 2 \times 10^{-3}$, then they are when it is assumed that they do not share in the $e^\pm$ entropy release. The corresponding change for ν_μ and ν_τ should be $\Delta T_\nu / T_\nu \cong 9 \times 10^{-4}$. The estimated change in the ^{4}He production due to this slight heating is [cf. Eq. (B2)]

$$\Delta Y \cong -0.00015 \ . \quad \text{(B11)}$$

Although it is the reaction $e^- e^+ \leftrightarrow \nu_i \bar{\nu}_i$ which keeps the neutrinos in chemical equilibrium ($\mu_\nu = \mu_{\bar{\nu}} = 0$), elastic scatterings like $e^- \nu_i \leftrightarrow e^- \nu_i$ and $e^+ \nu_i \leftrightarrow e^+ \nu_i$ can keep the neutrinos in the kinetic equilibrium ($T_\nu = T_\gamma$). The weak rates depend both upon the number density of neutrinos and their average energy. To estimate the effect of kinetic equilibrium being partially maintained by the elastic processes, we have integrated (B6) including in $\langle \sigma v \rangle$ all the reactions mentioned above (for e, $a = 0.8$; for μ,τ, $a = 0.2$). In this case δ_e ($T = 0.7$ MeV) $= 4 \times 10^{-3}$ and $\delta_{\mu,\tau}$ ($T = 0.7$ MeV) $= 2 \times 10^{-3}$, resulting in a predicted change in Y of $\Delta Y = -0.0003$. These two estimates, $\Delta Y = 0.000\,15$ and $\Delta Y = 0.0003$, should serve to bracket the size of the actual effect.

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[21]Note that Eq. (1.1) is not directly applicable here because $\Delta\lambda_n/\lambda_n$ is not equal to $\Delta\lambda_p\lambda_p$, so that the changes in λ_n and λ_p are *not* equivalent to a change in the neutron half-life.