Incorporating pion effects into the naive quark model

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A hybrid of the naive nonrelativistic quark model and the Chew-Low model is proposed. The pion is treated as an elementary particle which interacts with the "bare baryon" or "baryon core" via the Chew-Low interaction. The baryon core, which is the source of the pion interaction, is described by the naive nonrelativistic quark model. It turns out that the baryon-core radius has to be as large as 0.8 fm, and consequently the cutoff momentum Λ for the pion interaction is $\lesssim 3m_\pi$, m_π being the pion mass. Because of this small Λ (as compared with $\Lambda\sim$ nucleon mass in the old Chew-Low model) the effects of the pion cloud are strongly suppressed. The baryon masses, baryon magnetic moments (except for Ξ^-), and the nucleon charge radii can be reproduced quite well. However, we found it singularly difficult to fit the axial-vector weak decay constant g_A.

I. INTRODUCTION

In the naive nonrelativistic quark model (NQM) a hadron is regarded as a bound system of quarks subject to the nonrelativistic Schrödinger equation. With an effective quark-quark (qq) potential containing the color-magnetic interaction, the NQM appears to be remarkably successful in reproducing the baryon masses, magnetic moments, etc. (see, e.g., Ref. 1 and references quoted therein). One can raise many basic questions as to the absence of relativistic effects in the model, etc., but its phenomenological success warrants its serious consideration. In the NQM the $\Delta(1232)$ resonance in the πN scattering is due to a $3q$ system which has appropriate quantum numbers and mass and decays into πN. The magnetic moments of the nucleons are given simply in terms of weighted sums of the (normal) magnetic moments of the constituent quarks.

Alternative, time-honored explanations of the Δ resonance and the nucleon magnetic moments are provided by the Chew-Low model[2,3] (CLM): the so-called driving term in the πN scattering, depicted in Fig. 1(a), gives an attractive interaction in the $I=J=\frac{3}{2}$ state, and its iteration, Fig. 1(b), builds up a resonance. This picture is in marked conflict with the way in which $\Delta(1232)$ appears in the NQM. The proton obtains a large anomalous magnetic moment due to the virtual dissociation $p\to\pi^+ n$ (Fig. 2).

The basic interaction in the CLM is the πNN Yukawa interaction, which can be written in standard notations as

$$H_{\rm int}=(4\pi)^{1/2}\frac{f_0}{m_\pi}\sum_\alpha \tau_\alpha \int d\vec{r}\rho(r)\vec{\sigma}\cdot\vec{\nabla}\phi_\alpha(\vec{r})=(4\pi)^{1/2}\frac{f_0}{m_\pi}\sum_\alpha \tau_\alpha\frac{i}{(2\pi)^{3/2}}\int d\vec{k}\,\frac{v(k)}{\sqrt{2\omega}}\vec{\sigma}\cdot\vec{k}a_{\alpha k}+\text{H.c.}\,, \qquad (1.1)$$

where $\rho(r)$ represents the interaction source due to the nucleon core fixed at $\vec{r}=0$, and $v(k)=\int \rho(r)e^{i\vec{k}\cdot\vec{r}}d\vec{r}$, $v(0)=1$. The radius of the interaction source $\rho(r)$ is usually assumed to be about the nucleon Compton wavelength, and hence the cutoff momentum Λ in $v(k)$ is $\Lambda\sim m_N$. Here we have in mind

$$v(k)=e^{-k^2/2\Lambda^2}\,. \qquad (1.2)$$

With the renormalized coupling constant f ($f^2=0.08$) and $\Lambda\sim m_N$, the energy and width of the $\Delta(1232)$ resonance can be well reproduced.[4] The process of Fig. 2 accounts for a substantial

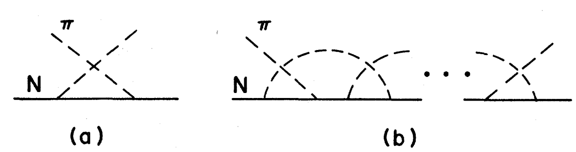

FIG. 1. πN scattering in the Chew-Low model. Diagram (a) is the driving term and diagram (b) is its repetition.

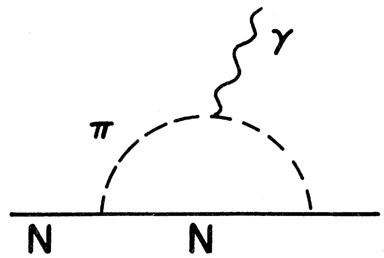

FIG. 2. The pion-current contribution to the electromagnetic structure of the nucleon.

part of the nucleon anomalous magnetic moments.[5,3]

We now face the well-known duality: The $\Delta(1232)$ and nucleon magnetic moments can be explained both by the NQM and CLM, but we cannot have both as such at the same time. The NQM is too nice to be dismissed simply because of the conflict with the CLM. On the other hand there is ample evidence for the Yukawa interaction (1.1) for the pion, e.g., the well established one-pion-exchange potential which results from the Yukawa interaction. In the hybrid model that we are going to propose below the pion effect is incorporated into the NQM such that the duality is resolved in a simple way. The basic assumptions in the hybrid model are as follows.

(1) All $3q$ systems in the NQM are regarded as "bare baryons" or "baryon cores." The pion, which we phenomenologically treat as an elementary particle, interacts with the baryon cores via a $\pi BB'$ Yukawa interaction, a generalization of Eq. (1.1).

(2) For the coupling constants $f_{BB'}$ we use $f^2=0.08$ and the SU(6) ratio for $f_{BB'}/f$. The source function $\rho(r)$ for the pion interaction is assumed to be common to all ground-state (octet and decuplet) baryons. The radius of this source is equal to or somewhat greater than the radius of the core which is determined as in the NQM.

The physical picture that underlies the above assumptions is that the pion interacts with the constituent u and d quarks in the core via a πqq interaction of the form of Eq. (1.1). Then the $\pi BB'$ interaction can be obtained by taking a matrix element of the πqq interaction between $3q$ wave functions for the baryon cores B and B'. The strange quark does not interact with the pion. If we use the SU(6) wave functions, which we believe to be a reasonable approximation, then the above assumption (2) follows. The source function $\rho(r)$ or $v(k)$ of Eq. (1.2) represents the quark distribution in the core. This however leads to embarrassing questions such as: what is the source function $\rho(r)$ in the πqq interaction? We will discuss (but not answer) these questions in the Appendix. Rather, let us proceed with the above assumptions as "working hypotheses" and see how the hybrid model works.

Our calculation goes as follows. First we assume some value for cutoff momentum Λ, and calculate all pion effects for the mass, magnetic moment, charge radius, etc. By stripping these pion corrections from observed quantities, we find the mass, magnetic moment, charge radius, etc., of the core. Next, we attempt to reproduce these core properties in terms of the NQM. Finally the adequacy of the cutoff Λ is examined in the light of assumption (2). For the form factor $v(k)$ of Eq. (1.2) we obtain

$$\left[\int r^2\rho(r)d\vec{r}\right]^{1/2}=\sqrt{3}/\Lambda\ . \qquad (1.3)$$

On the other hand we will find that the baryon-core radius determined by the $3q$ wave function is about 0.8 fm. It then follows from assumption (2) that $\Lambda \lesssim 3m_\pi$. This small value of Λ is crucial in resolving the duality between the NQM and CLM mentioned earlier. Because of the small Λ, effects of virtual pions are strongly suppressed, and can be incorporated into the NQM without destroying the successful features of the latter.

The "old" CLM is a "wrong" model in the sense that its basic interaction does not reflect the rich internal structure of the baryons. It is intriguing that such a defect was masked by using a "wrong" cutoff. Actually the old CLM has some difficulties which should have been taken more seriously. For example it is impossible to account for the $N^*(1470)$ resonance in the $(2I,2J)=(1,1)$ channel. The phase shift δ_{11} changes its sign from negative to positive around the c.m. energy 1220 MeV, and a natural interpretation of this would be that it is due to a Castillejo-Dalitz-Dyson (CDD) pole or something similar which would require the existence of a bare baryon corresponding to $N^*(1470)$. This is in contrast to the claim of Wei and Banerjee that the sign change of δ_{11} can be attributed to the strong inelasticity present in the 11 channel.[6]

Another example of the difficulty is as follows. If one extends the old CLM to $\pi\Sigma$ scattering, one finds that the π^+p and $\pi^+\Sigma^+$ scattering amplitudes have essentially the same structure; the rescattering process 1(b) leads to a $P_{3/2}$ resonance in the $I=2$ $\pi\Sigma$ scattering, which has not been observed. The driving term in the $(I,J)=(2,\frac{3}{2})$ $\pi\Sigma$ scattering is

$$\frac{2}{3}\left[\frac{f_{\Sigma}^{2}}{\omega}+\frac{f_{\Lambda\Sigma}^{2}}{\omega-m_{\Sigma}+m_{\Lambda}}\right] \tag{1.4}$$

which is compared with $\frac{4}{3}f^2/\omega$ for the $(\frac{3}{2},\frac{3}{2})$ πN scattering.[7] In the hybrid model the $\Delta(1232)$ resonance is caused by the bare Δ (Fig. 3), consisting of $3q$, and not by the process of Fig. 1(b). The absence of the $I=2$ $\pi\Sigma$ resonance is obvious in the hybrid model; there is no process like Fig. 3 in that channel because a $3q$ system cannot have $I=2$.

Before ending this section we should note that the pion effects have recently been examined in the context of the chiral bag model or cloudy bag model (CBM),[8–12] and our model resembles the CBM in many respects. In particular Miller *et al.*[10] advocated that the duality between the CLM and the quark model can be resolved in their model due to the large radius ($\sim$0.8 fm) of the bag. In the CBM the introduction of the pion field is essential in restoring the chiral symmetry, and furthermore the form factor for the pion interaction in the CBM is not arbitrary and is dictated by the chiral symmetry.[10] In our model, on the contrary, the introduction of the pion is quite *ad hoc*, and the chiral symmetry is not exploited. These are less satisfying sides of our model. However, it seems to us that the baryon-core structure can be much more easily analyzed in our model than in the CBM. For example it is easy in our model to examine the possibility and consequences of a strong admixture of the D state in the baryon core due to a strong tensor component in the qq interaction.

In Sec. II we give expressions for pion effects on various quantities, and show how the properties of the baryon core can be determined. In Sec. III we describe the NQM for the baryon core. The results are presented and discussed in Sec. IV.

II. PION EFFECTS

A. The $\pi BB'$ interaction

We first generalize the πNN interaction (1.1) to $\pi BB'$ where B and B' stand for any of the ground states of the octet and decuplet baryons. We do this by the following substitution in Eq. (1.1):

$$f_0\tau_\alpha\vec{\sigma}\cdot\vec{k}\rightarrow f_{BB'}\tau_\alpha^{BB'}\vec{\sigma}^{BB'}\cdot\vec{k}\ . \tag{2.1}$$

When $B=B'$, the suffix BB' is shortened to B. Also, the suffix N is suppressed whenever obvious, e.g., $f_N=f$, $\vec{\sigma}^N=\vec{\sigma}$. The spin and isospin matrices $\sigma_i^{BB'}$ and $\tau_\alpha^{BB'}$ are defined in the Appendix, and some useful relations involving σ's and/or τ's, together with the SU(6) ratios for $f_{BB'}/f$, are listed in Table I. We do not include excited-state baryons (like N^* and Δ^*). Their effects will be small because $\rho(r)$ between ground and excited B would be small and also higher excitation energies are involved. This is based on the speculation that $\rho_{NN^*}(r)$ contains the product of the quark wave functions for N and N^*, which are orthogonal to each other. For the cutoff momentum Λ that we eventually choose, i.e., $\Lambda=2\sim3m_\pi$, perturbation calculation turns out to be adequate, and hence we calculate mass, magnetic moment, etc., up to the order of f^2 (the lowest-order correction). We will not distinguish the bare and renormalized coupling constants. In fact we will see in Sec. IV that the renormalization factor $Z=f/f_0$ is not very different from unity: $1>Z\gtrsim0.95$.

B. Self-energy

The self-energy $\delta m_{B(B')}$ due to the diagram of Fig. 4 is given by

$$\delta m_{B(B')}=-f_{BB'}{}^2C_{B(B')}I\left[\frac{1}{\omega_{B(B')}}\right]. \tag{2.2}$$

Here

$$C_{B(B')}=(\vec{\sigma}^{BB'}\cdot\vec{\sigma}^{B'B})(\vec{\tau}^{BB'}\cdot\vec{\tau}^{B'B}) \tag{2.3}$$

which is listed in Table I, and

$$\omega_{B(B')}=\omega+m_{B'}^c-m_B^c\ , \tag{2.4}$$

where m_B^c is the mass of the baryon core of B, and

$$I[f(\omega)]=\frac{1}{3\pi m_\pi{}^2}\int_0^\infty dk\frac{k^4v^2(k)}{\omega}f(\omega)\ . \tag{2.5}$$

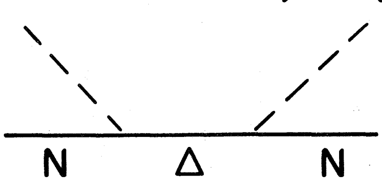

FIG. 3. πN scattering through the "bare" Δ.

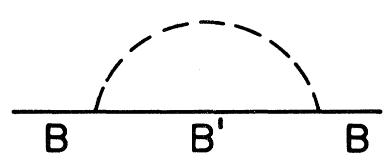

FIG. 4. The baryon self-energy $\delta m_{B(B')}$ due to the pion.

TABLE I. The pion coupling constants $f_{BB'}=f_{B'B}$ and spin and isospin factors. The ξ and η are defined by $\sum_i \sigma_i^{BB'}\sigma_j^{B'}\sigma_i^{B'B}=\xi\sigma_j^{B}$ and $\sum_\alpha \tau_\alpha^{BB'}\tau_\beta^{B'}\tau_\alpha^{B'B}=\eta\tau_\beta^{B}$. $C_{B(B')}$ is defined by Eq. (2.3).

BB'	$f_{BB'}/f$	$\vec{\sigma}^{BB'}\cdot\vec{\sigma}^{B'B}$	ξ	$\vec{\tau}^{BB'}\cdot\vec{\tau}^{B'B}$	η	$C_{B(B')}$
NN	1	3	-1	3	-1	9
$N\Delta$	$6\sqrt{2/5}$	2	$\frac{10}{3}$	2	$\frac{10}{3}$	4
ΔN		1	$\frac{1}{3}$	1	$\frac{1}{3}$	1
$\Delta\Delta$	$\frac{1}{5}$	15	11	15	11	225
$\Lambda\Lambda$	0	3	-1			
$\Lambda\Sigma$	$-2\sqrt{3/5}$	3	-1	3	1	9
$\Sigma\Lambda$		3	-1	1	1	3
$\Sigma\Sigma$	$\frac{4}{5}$	3	-1	2	1	6
$\Lambda\Sigma^*$	$\frac{6}{5}$	2	$\frac{10}{3}$	3	1	6
$\Sigma^*\Lambda$		1	$\frac{1}{3}$	1	1	1
$\Sigma\Sigma^*$	$-2\sqrt{3/5}$	2	$\frac{10}{3}$	2	1	4
$\Sigma^*\Sigma$		1	$\frac{1}{3}$	2	1	2
$\Sigma^*\Sigma^*$	$\frac{2}{5}$	15	11	2	1	30
$\Xi\Xi$	$-\frac{1}{5}$	3	-1	3	-1	9
$\Xi\Xi^*$	$-2\sqrt{3/5}$	2	$\frac{10}{3}$	3	-1	6
$\Xi^*\Xi$		1	$\frac{1}{3}$	3	-1	3
$\Xi^*\Xi^*$	$\frac{1}{5}$	15	11	3	-1	45

For the intermediate-state baryon B', we consider only the octet and decuplet ground states. The self-energy of B, $\delta m_B=\sum_{B'}\delta m_{B(B')}$, are given as follows:

$$\delta m_N=-I\left[\frac{9f^2}{\omega}+\frac{4f_{N\Delta}^2}{\omega_{N(\Delta)}}\right]=-9f^2I\left[\frac{1}{\omega}+\frac{32}{25\omega_{N(\Delta)}}\right], \tag{2.6}$$

$$\delta m_\Delta=-I\left[\frac{225f_\Delta^2}{\omega}+\frac{f_{N\Delta}^2}{\omega_{\Delta(N)}}\right]=-9f^2I\left[\frac{1}{\omega}+\frac{8}{25\omega_{\Delta(N)}}\right], \tag{2.7}$$

$$\delta m_\Lambda=-I\left[\frac{9f_{\Lambda\Sigma}^2}{\omega_{\Lambda(\Sigma)}}+\frac{6f_{\Lambda\Sigma^*}^2}{\omega_{\Lambda(\Sigma^*)}}\right], \tag{2.8}$$

$$\delta m_\Sigma=-I\left[\frac{6f_\Sigma^2}{\omega}+\frac{3f_{\Lambda\Sigma}^2}{\omega_{\Sigma(\Lambda)}}+\frac{4f_{\Sigma\Sigma^*}^2}{\omega_{\Sigma(\Sigma^*)}}\right], \tag{2.9}$$

$$\delta m_{\Sigma^*}=-I\left[\frac{30f_{\Sigma^*}^2}{\omega}+\frac{f_{\Lambda\Sigma^*}^2}{\omega_{\Sigma^*(\Lambda)}}+\frac{2f_{\Sigma\Sigma^*}^2}{\omega_{\Sigma^*(\Sigma)}}\right], \tag{2.10}$$

$$\delta m_\Xi=-I\left[\frac{9f_\Xi^2}{\omega}+\frac{6f_{\Xi\Xi^*}^2}{\omega_{\Xi(\Xi^*)}}\right], \tag{2.11}$$

$$\delta m_{\Xi^*}=-I\left[\frac{45f_{\Xi^*}^2}{\omega}+\frac{3f_{\Xi\Xi^*}^2}{\omega_{\Xi^*(\Xi)}}\right]. \tag{2.12}$$

We determine m_B^c by solving $m_B=m_B^c+\delta m_B$ for m_B^c. Here m_B is the observed mass of B. Since $\delta m \ll m$, however, it is in fact a very good approximation to replace m_B^c in Eq. (2.4) by m_B.

C. Magnetic moments; pion current

Unfortunately the notations are becoming rather tedious. We introduce four μ's: μ is the observed magnetic moment (*mm*); μ^c is the *mm* of the core, i.e., the *mm* is the absence of the pion; μ' is the *mm* due to the pion current; μ'' is the *mm* due to the baryon current; μ' and μ'' are manifestations of the pion effect. We will use similar notations also for the charge radius $\langle r^2\rangle^{1/2}$ later.

Now let us consider the *mm* of B due to Fig. 5 which we denote by $\mu'_{B(B')}$. For $B=B'=N$, we obtain[3]

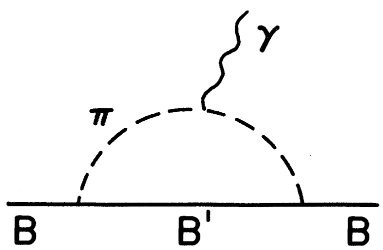

FIG. 5. The pion-current contribution to the electromagnetic structure of baryon B.

$$\mu'_{N(N)}=4ef^2 I\left[\frac{1}{\omega^3}\right] . \qquad (2.13)$$

In $\mu'_{N(N)}$ there is the following combination of vertex factors;

$$\tau_1\tau_2\vec{\sigma}\cdot\vec{k}\,\vec{\sigma}\cdot\vec{k}'+(1\leftrightarrow 2,\vec{k}\leftrightarrow\vec{k}')=-2\tau_3\vec{\sigma}\cdot(\vec{k}\times\vec{k}') . \qquad (2.14)$$

In the general $\mu'_{B(B')}$, the following will appear instead:

$$\tau_1^{BB'}\tau_2^{B'B}\vec{\sigma}^{BB'}\cdot\vec{k}\,\vec{\sigma}^{B'B}\cdot\vec{k}'+(1\leftrightarrow 2,\vec{k}\leftrightarrow\vec{k}') .$$

For $BB'=N\Delta$, using $\tau_\alpha^{N\Delta}\tau_\beta^{\Delta N}=\frac{1}{3}(2\delta_{\alpha\beta}-i\epsilon_{\alpha\beta\gamma}\tau_\gamma)$, this can be reduced to $-\frac{2}{9}\tau_3\cdot\vec{\sigma}\cdot(\vec{k}\times\vec{k}')$. The denominator has to be generalized as

$$\frac{1}{\omega^2}\rightarrow\frac{1}{2\omega_{B(B')}}\left[\frac{1}{\omega}+\frac{1}{\omega_{B(B')}}\right] . \qquad (2.15)$$

The results for the octet baryons are summarized as follows:

$$\mu'_N=eI\left[\frac{4f^2}{\omega^3}+\frac{2f_{N\Delta}{}^2}{9\omega\omega_{N(\Delta)}}\left[\frac{1}{\omega}+\frac{1}{\omega_{N(\Delta)}}\right]\right] , \qquad (2.16)$$

$$\mu'_\Lambda=0 , \qquad (2.17)$$

$$\mu'_\Sigma=eI\left[\frac{2f_\Sigma{}^2}{\omega^3}+\frac{f_{\Lambda\Sigma}{}^2}{\omega\omega_{\Sigma(\Lambda)}}\left[\frac{1}{\omega}+\frac{1}{\omega_{\Sigma(\Lambda)}}\right]\right.$$
$$\left.-\frac{f_{\Sigma\Sigma^*}{}^2}{3\omega\omega_{\Sigma(\Sigma^*)}}\left[\frac{1}{\omega}+\frac{1}{\omega_{\Sigma(\Sigma^*)}}\right]\right] , \qquad (2.18)$$

$$\mu'_\Xi=eI\left[\frac{4f_\Xi{}^2}{\omega^3}-\frac{2f_{\Xi\Xi^*}{}^2}{3\omega\omega_{\Xi(\Xi^*)}}\left[\frac{1}{\omega}+\frac{1}{\omega_{\Xi(\Xi^*)}}\right]\right] , \qquad (2.19)$$

$$\mu'_{\Lambda\Sigma}=eI\left[\frac{2f_\Sigma f_{\Lambda\Sigma}}{\omega^2\omega_{\Lambda(\Sigma)}}\left[1+\frac{\omega+\omega_{\Lambda(\Sigma)}}{2\omega}\right]\right.$$
$$\left.+\frac{2f_{\Sigma\Sigma^*}f_{\Lambda\Sigma^*}}{3\omega\omega_{\Lambda(\Sigma^*)}\omega_{\Sigma(\Sigma^*)}}\left[1+\frac{\omega_{\Lambda(\Sigma^*)}+\omega_{\Sigma(\Sigma^*)}}{2\omega}\right]\right] . \qquad (2.20)$$

We do not consider the pion correction to the *mm*'s of the decuplet baryons.

D. Magnetic moment; baryon current

Let a bare baryon state be $|B_0\rangle$ and the corresponding physical baryon state be $|B\rangle$. In first-order perturbation, we obtain

$$|B\rangle=\mathcal{N}_B{}^{1/2}[1-(H_0-m_B^c)^{-1}H_{\rm int}]\,|B_0\rangle , \qquad (2.21)$$

where H_0 is the unperturbed Hamiltonian. The normalization factor $\mathcal{N}_B$ is given by

$$\mathcal{N}_B{}^{-1}=1+\sum_{B'}f_{BB'}{}^2C_{B(B')}I\left[\frac{1}{\omega_{B(B')}{}^2}\right] . \qquad (2.22)$$

The second term on the right-hand side of Eq. (2.22) is the probability for finding a pion in the physical baryon state $|B\rangle$. Noticing the similarity between the probability term and δm_B of Eqs. (2.6)–(2.12), one can immediately write down $\mathcal{N}_B$ explicitly.

The baryon-current contribution μ'' to the *mm* is obtained from the expectation value of the z component of the baryon *mm* operator, M, defined by

$$M=\sum_{B,B'}M_{BB'}=\sum_{B,B'}(\mu_{BB'}^{cs}+\mu_{BB'}^{cv}\tau_3^{BB'})\sigma_3^{BB'} , \qquad (2.23)$$

where superscripts s and v refer to isoscalar and isovector parts, respectively. We obtain

$$\langle B\,|\,M\,|\,B\rangle=\mathcal{N}_B\langle B_0\,|\,M+\sum_{B'B''}D_{B(B'B'')}P_{B(B'B'')}\,|\,B_0\rangle , \qquad (2.24)$$

where

$$D_{B(B'B'')}=\sum_{i,\alpha}\sigma_i^{BB'}\tau_\alpha^{BB'}M_{B'B''}\sigma_i^{B''B}\tau_\alpha^{B''B} \qquad (2.25)$$

which can be calculated using ξ and η listed in

Table I, and

$$P_{B(B'B'')}=f_{BB'}f_{BB''}I\left[\frac{1}{\omega_{B(B')}\omega_{B(B'')}}\right] . \quad (2.26)$$

For example, $D_{N(B'B'')}$ are given by

$$D_{N(N)}=(-3\mu^{cs}+\mu^{cv}\tau_3)\sigma_3 , \quad (2.27)$$

$$D_{N(\Delta)}=[\tfrac{20}{3}\mu_\Delta^{cs}+(\tfrac{10}{3})^2\mu_\Delta^{cv}\tau_3]\sigma_3 , \quad (2.28)$$

$$D_{N(N\Delta)}=\tfrac{16}{9}\mu_{N\Delta}^{c}\tau_3\sigma_3 , \quad (2.29)$$

where $\mu^{cs,cv}\equiv\mu_N^{cs,cv}$. The terms containing $D_{N(N)}$, $D_{N(\Delta)}$, and $D_{N(N\Delta)}$ $[=D_{N(\Delta N)}]$ correspond to the diagrams of Fig. 6 with $(BB')=(NN)$, $(\Delta\Delta)$, and $(N\Delta)$. Using Eqs. (2.27)–(2.29) we obtain

$$\langle N\,|\,M\,|\,N\rangle=\mu''^{s}+\mu''^{v}\tau_3 , \quad (2.30)$$

$$\mu''^{s}=\mathcal{N}[\mu^{cs}-3Q_{N(N)}^{s}+\tfrac{20}{3}Q_{N(\Delta)}^{s}] , \quad (2.31)$$

$$\mu''^{v}=\mathcal{N}[\mu^{cv}+Q_{N(N)}^{v}+\tfrac{100}{9}Q_{N(\Delta)}^{v}+\tfrac{32}{9}Q_{N(N\Delta)}] , \quad (2.32)$$

where

$$Q_{B(B'B'')}^{s,v}=\mu_{B'B''}^{s,v}P_{B(B'B'')} . \quad (2.33)$$

The results for the hyperons are as follows:

$$\mu_\Lambda''=\mathcal{N}_\Lambda[\mu_\Lambda^{c}-3Q_{\Lambda(\Sigma)}^{s}+10Q_{\Lambda(\Sigma^*)}^{s}+8Q_{\Lambda(\Sigma\Sigma^*)}^{s}] , \quad (2.34)$$

$$\mu_\Sigma''^{s}=\mathcal{N}_\Sigma[\mu_\Sigma^{cs}-2Q_{\Sigma(\Sigma)}^{s}-Q_{\Sigma(\Lambda)}+\tfrac{20}{3}Q_{\Sigma(\Sigma^*)}^{s}+\tfrac{16}{3}Q_{\Sigma(\Sigma\Sigma^*)}^{s}] , \quad (2.35)$$

$$\mu_\Sigma''^{v}=\mathcal{N}_\Sigma[\mu_\Sigma^{cv}-Q_{\Sigma(\Sigma)}^{v}+\tfrac{10}{3}Q_{\Sigma(\Sigma^*)}^{v}+2Q_{\Sigma(\Lambda)}+\tfrac{8}{3}Q_{\Sigma(\Sigma\Sigma^*)}^{v}-\tfrac{8}{3}Q_{\Sigma(\Lambda\Sigma^*)}] , \quad (2.36)$$

$$\mu_{\Lambda\Sigma}''=(\mathcal{N}_\Lambda\mathcal{N}_\Sigma)^{1/2}\{\mu_{\Lambda\Sigma}-Q_{\Lambda\Sigma(\Sigma\Lambda)}+Q_{\Lambda\Sigma(\Sigma)}^{v}+\tfrac{4}{3}Q_{\Lambda\Sigma(\Sigma^*\Lambda)}-\tfrac{4}{3}[Q_{\Lambda\Sigma(\Sigma^*\Sigma)}^{v}+Q_{\Lambda\Sigma(\Sigma\Sigma^*)}^{v}]-\tfrac{10}{3}Q_{\Lambda\Sigma(\Sigma^*)}^{v}\} , \quad (2.37)$$

$$\mu_\Xi''^{s}=\mathcal{N}_\Xi[\mu_\Xi^{cs}-3Q_{\Xi(\Xi)}^{s}+10Q_{\Xi(\Xi^*)}^{s}+8Q_{\Xi(\Xi\Xi^*)}^{s}] , \quad (2.38)$$

$$\mu_\Xi''^{v}=\mathcal{N}_\Xi[\mu_\Xi^{cv}+Q_{\Xi(\Xi)}^{v}-\tfrac{10}{3}Q_{\Xi(\Xi^*)}^{v}-\tfrac{4}{3}Q_{\Xi(\Xi\Xi^*)}^{v}] , \quad (2.39)$$

where $Q_{\Lambda\Sigma(BB')}=\mu_{BB'}f_{\Lambda B}f_{\Sigma B'}I[1/\omega_{\Lambda(B)}\omega_{\Sigma(B')}]$. Combining the pion and baryon current contributions, we arrive at the total magnetic moment:

$$\mu_B=\mu_B^{s}+\mu_B^{v}\tau_3^{B}$$
$$=\mu_B''^{s}+(\mu_B''^{v}+\mu_B')\tau_3^{B} . \quad (2.40)$$

This is to be identified with the observed magnetic moment.

The renormalization factor for the axial-vector coupling constant g_A can be obtained in the same way as that for μ^v:

$$g_A=\mathcal{N}\{[1+P_{N(N)}]g_A^{c}+\tfrac{100}{9}P_{N(\Delta)}g_{A,\Delta}^{c}+\tfrac{32}{9}P_{N(N\Delta)}g_{A,N\Delta}^{c}\} , \quad (2.41)$$

where g_A^c is the axial-vector coupling constant for the nucleon core and is $\frac{5}{3}$ in SU(6), and more generally determined by Eq. (3.5) in the NQM. Since the pion interaction and the axial-vector weak interaction are mediated by the same operator ($\sigma\tau$ for the constituent quarks) we observe that $Z=f/f_0=g_A/g_A^c$. We assume that $g_{A,\Delta}^c=\frac{1}{5}g_A^c$ and $g_{A,N\Delta}^c=(6\sqrt{2}/5)g_A^c$.

In the "old" CLM one obtains[13,3]

$$Z=1-\tfrac{8}{9}P_{11}-P_{10} , \quad (2.42)$$

where $P_{2I,2J}$ is the probability of the pion cloud in the nucleon carrying isospin I and angular momentum J. In perturbation up to the order of f^2, $P_{10}=0$ and $P_{11}=9P_{N(N)}$, and hence Eq. (2.42) becomes equivalent to Eq. (2.41) with $g_{A,\Delta}^c=g_{A,N\Delta}^c=0$. In the old CLM, $P_{11}\approx0.6$, $P_{10}\approx0$, and hence $Z\approx0.5$, a tremendous reduction factor. In sharp contrast to this we will find in our hybrid model that $1>Z\gtrsim0.95$. There are two reasons for this difference: first, $P_{11}\lesssim0.2$ in our model, and second the terms with $g_{A,\Delta}$ and $g_{A,N\Delta}$ increase Z.

E. Charge radius; pion current

We evaluate the charge radius for the nucleon only. The contribution from the diagram of Fig. 5 with $B=B'=N$ is[14]

$$\langle r^2\rangle_{(N)}'=-3f^2I\left[\frac{1}{v}\left[\left[\frac{\partial}{\partial k'}+\frac{4}{k}\right]\frac{\partial}{\partial k'}\frac{v(k')}{\omega'(\omega+\omega')}\right]_{k=k'}\right]=6f^2I\left[\frac{5}{\omega^4}\left[\frac{3}{2}-\frac{k^2}{\omega^2}\right]+\left[\frac{1}{v}\frac{dv}{dk}\right]^2\frac{1}{\omega^2}\right] . \quad (2.43)$$

The prime indicates the pion-current contribution. Instead of Eq. (2.14) we have the combination

$$\tau_1\tau_2\vec{\sigma}\cdot\vec{k}\,\vec{\sigma}\cdot\vec{k}'-(1\leftrightarrow 2,\vec{k}\leftrightarrow\vec{k}')=2\tau_3\vec{k}\cdot\vec{k}' .$$

Its generalization to the diagram with $(BB')=(N\Delta)$ turns out to be $-\frac{4}{9}\tau_3\vec{k}\cdot\vec{k}'$. Furthermore we have to make the substitution $[\omega'(\omega+\omega')]^{-1}\rightarrow[\omega_{N(\Delta)}(\omega+\omega')]^{-1}$. We finally obtain

$$\langle r^2\rangle'_{(\Delta)}=-\tfrac{4}{3}f_{N\Delta}{}^2 I\left[\frac{5}{\omega\omega_{N(\Delta)}}\left[\frac{1}{2\omega}+\frac{1}{\omega_{N(\Delta)}}\right]-\frac{k^2}{\omega}\left[\frac{1}{2\omega^2}+\frac{1}{\omega\omega_{N(\Delta)}}+\frac{1}{\omega_{N(\Delta)}{}^2}\right]+\left[\frac{1}{v}\frac{dv}{dk}\right]^2\frac{1}{\omega_{N(\Delta)}{}^2}\right] . \tag{2.44}$$

F. Charge radius; baryon current

The baryon-current contribution

$$\langle r^2\rangle''=\langle r^2\rangle''^{s}+\langle r^2\rangle''^{v}\tau_3 \tag{2.45}$$

is given by

$$\langle r^2\rangle''^{s}=\mathcal{N}\{[1+9P_{N(N)}]\langle r^2\rangle^{cs}+4P_{N(\Delta)}\langle r^2\rangle^{cs}_{\Delta}\} , \tag{2.46}$$

$$\langle r^2\rangle''^{v}=\mathcal{N}\{[1-3P_{N(N)}]\langle r^2\rangle^{cv}+\tfrac{20}{3}P_{N(\Delta)}\langle r^2\rangle^{cv}_{\Delta}\} , \tag{2.47}$$

where $\langle r^2\rangle^{cs,v}$ are those of the core. Combining the above with $\langle r^2\rangle'$, we obtain

$$\begin{aligned}\langle r^2\rangle&=\langle r^2\rangle^{s}+\langle r^2\rangle^{v}\tau_3\\&=\langle r^2\rangle''^{s}+(\langle r^2\rangle''^{v}+\langle r^2\rangle')\tau_3\end{aligned} \tag{2.48}$$

which is to be compared with the experimental values for proton and neutron.

We have completed the derivation of the pion effects for the masses, magnetic moments, charge radii, and also g_A. As we said in Sec. I we first assume a momentum cutoff Λ and then calculate the pion effects. The calculation of δm^c_B which determines the bare mass m^c_B, is straightforward. The calculations for μ' and $\langle r^2\rangle'$ are also straightforward. However the baryon-current contributions μ'' and $\langle r^2\rangle''$ are somewhat complicated. In Eqs. (2.31) and (2.32), the P's are known. We assume the SU(6) ratios for μ^c_Δ/μ^c and $\mu^c_{N\Delta}/\mu^c$; then Eq. (2.40) can be solved for two unknowns $\mu^{cs,v}$. Equation (2.48) can be handled in the same manner using the SU(6) ratio for $\langle r^2\rangle^c_\Delta/\langle r^2\rangle^c$.

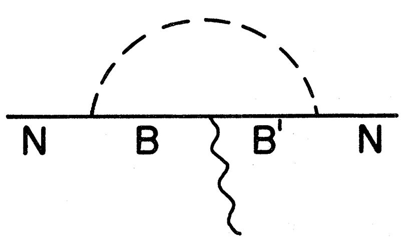

FIG. 6. The baryon-current contribution to the electromagnetic structure of nucleon.

III. STRUCTURE OF THE BARYON CORE

We have explained in the preceding section how the core quantities, m^c, μ^c, $\langle r^2\rangle^c$, and g^c_A can be determined by subtracting the pion effects from the observed quantities. So far we have treated the baryon core as an elementary particle. Now we look into the structure of the core by assuming that it consists of three quarks which interact with each other through a potential and obey the nonrelativistic Schrödinger equation. In this way we try to fit the core quantities m^c, μ^c, etc. In this section we consider the core quantities only, and hence the superscript c will be suppressed. For example μ^c is denoted by μ.

For the qq potential we consider

$$V_{ij}(\vec{r})=v_0(r)+\frac{1}{m_i m_j}[\,\vec{\sigma}_i\cdot\vec{\sigma}_j v_s(r)+S_{ij}v_t(r)] , \tag{3.1}$$

where $v_0(r)$ is a confining potential, $\vec{\sigma}$ is the spin operator for the quark, and S_{ij} is the standard tensor operator. The spin-dependent and tensor forces with the factor $1/m_i m_j$ is based on the analogy with the Fermi-Breit interaction, or the "hyperfine interaction."[15] We will specify the explicit form of the potential later, but let us mention here that the singular terms in the Fermi-Breit interaction, e.g., $\delta(\vec{r})$ of $v_s(r)$, will be replaced with a smooth finite range potential so that the three-body Schrödinger equation can be solved. For the quark masses, we understand that $m_u=m_d\neq m_s$. The wave function of the $3q$ system that we will use consists of S, S', P, and D components.

Before specifying the potential further, let us make some general observations. If we ignore components of the wave function other than the fully symmetric S, the magnetic moments can be easily calculated. To include S', P, and D becomes increasingly complex. Table II lists the isoscalar parts of the magnetic moments when only the S state is included.

Let us point out that

$$\frac{\mu^v_{BB'}}{\mu^v}=\frac{f_{BB'}}{f}=\frac{g_{A,BB'}}{g_A} \quad (3.2)$$

holds when only S and S' components are retained. Here $\mu_{BB}=\mu_B$. Actually $f_{BB'}/f=g_{A,BB'}/g_A$ is valid even if P, D,... are included (and even if pion effects are included).

We include the P- and D-state contributions for the following particularly interesting quantities,

$$\mu_p=(1-\tfrac{2}{3}S'-P-D)(e/2m_u)\ , \quad (3.3)$$

$$\mu_n=-\tfrac{2}{3}(1-S'-\tfrac{8}{9}P-D)(e/2m_u)\ , \quad (3.4)$$

$$\frac{g_A}{g_V}=\tfrac{1}{3}(5-4S'-4P-6D)\ , \quad (3.5)$$

where S', P, and D stand for the corresponding probabilities. If $S'=P=D=0$, then $\mu_p/\mu_n=-\frac{3}{2}$ in very good agreement with the experimental value of -1.46. But $g_A/g_V=\frac{5}{3}$ is much larger than the experimental value of 1.25. Glashow,[16] who did not consider S' and P, noted that if $D\approx 0.2$, the ratio g_A/g_V is reduced to the experimental value without changing the ratio μ_p/μ_n. However, $D\approx 0.2$ means that v_t in Eq. (3.1) is very strong. We will show that such a strong tensor force makes the fitting of other quantities extremely difficult.

For the qq potential we have tried with a variety of functions, and found that, as long as they are not very singular, i.e., if $rv_s(r)$ and $r^3v_t(r)$ vanish when $r\to 0$, the results are similar. Therefore we will present results only for one "standard" potential and its "modified" version. The standard potential, which will be referred to as potential I in the following, is defined by

$$v_0(r)=Ar^2+B\ , \quad (3.6)$$

$$v_s(r)=Cm_u{}^2e^{-(r/r_0)^2}\ , \quad (3.7)$$

$$v_t(r)=-\left[1+\frac{1}{2}\left[\frac{r_0}{r}\right]^2\right]v_s(r)+\frac{3\sqrt{\pi}}{8}Cm_u{}^2\left[\frac{r_0}{r}\right]^3\operatorname{erf}\left[\frac{r}{r_0}\right]\ , \quad (3.8)$$

where A, B, C, and r_0 are parameters. This $v_t(r)$ may look peculiar, but it is determined from the assumption that v_s and v_t both derive from a vector exchange (see, e.g.,Ref. 17),

$$\vec{\sigma}_1\cdot\vec{\sigma}_2v_s(r)+S_{12}v_t(r)=\frac{1}{(2\pi)^3}\int d\vec{q}\,e^{i\vec{q}\cdot\vec{r}}\frac{F(q)}{q^2}(\vec{\sigma}_1\times\vec{q})(\vec{\sigma}_2\times\vec{q})\ . \quad (3.9)$$

TABLE II. The isoscalar parts of the magnetic moments of baryon cores (in units of $e/2m_u$) when only the S state is retained; $x=m_u/m_s$. The isovector parts are obtained from $f_{BB'}/f$ listed in Table I, together with Eq. (3.2) and $\mu^v=\frac{1}{6}$.

N	$\frac{1}{6}$
Δ	$\frac{1}{2}$
Λ	$-\frac{x}{3}$
Σ	$\frac{2+x}{3}$
Σ^*	$\frac{1-x}{3}$
Ξ	$-\frac{1+8x}{6}$
Ξ^*	$\frac{1-4x}{6}$

Because of the above relationship between v_s and v_t, potential I is not very flexible with respect to the strength of the tensor force. In order to examine the possibility of large D-state admixture in relation to g_A we also consider the following potential II:

$$v_0(r)=Ar^2+B\ , \quad (3.10)$$

$$v_s(r)=m_u{}^2[Ce^{-(r/r_0)^2}+C']\ , \quad (3.11)$$

$$v_t(r)=\zeta[v_t(r)\text{ of Eq. (3.7)}]\ . \quad (3.12)$$

Potential II has two additional parameters, C' and ζ. The ζ controls the tensor force and hence the D state, while the S' state is sensitive to v_S. The parameters A, B, C, and r_0 will take different values between potentials I and II.

In treating the $3q$ wave function we expand it into harmonic-oscillator bases with respect to the standard Jacobi coordinates, and truncate the expansion beyond two-quantum excitation. The S, S', P, and D components are included. The Hamiltonian is diagonalized and the ground-state energy minimized by varying the oscillator parameter in the wave function. Because the potentials that we consider are very smooth (we will find that $r_0 \approx 1$ fm), our approximation is very accurate. We believe that the error in the energy of the $3q$ system is typically 5 MeV or less. We do not give any details of the three-body calculation in this paper because it will be explained elsewhere.[18]

IV. RESULTS AND DISCUSSION

We determine the parameters in the potential and the quark mass $m_u = m_d$ by fitting the quantities in the *ud* sector. Then the only remaining parameter m_s is determined by fitting the mass of Ω. Therefore *the results for hyperons (other than the mass of* Ω*) are all predictions.* Note that Ω does not interact with the pion. The parameters thus determined are listed in Table III, and the results in Tables IV and V.

Before examining the results of the hybrid model, let us consider the "old" NQM ($\Lambda = 0$), with potential I. The fit is good except for g_A, $N^*(1470)$, and $\Delta^*(1690)$. Nobody has been able to fit g_A satisfactorily in the NQM, and as we discuss later this difficulty persists in our hybrid model. Potential II improves the fit for g_A, but it is still far from being satisfactory.

Let us comment on the difficulty in fitting $N^*(1470)$ and $\Delta^*(1690)$. Isgur and Karl[21] did obtain a remarkably good fit for baryon-excited states in their NQM calculations, but we have the following reservations. First, since their qq interaction is singular, the oscillator constant (they also used the same harmonic-oscillator bases as ours) in their wave function was not determined by minimizing the energy; that is to say, their oscillator constant was chosen in an *ad hoc* way. Moreover they took very different values of the oscillator constant for the ground and excited states. The quark masses were also different between the ground and excited states. This is not very satisfying.

There is a strong correlation between $\langle r^2 \rangle_p$ and the NN^* excitation energy. If one tries to fit $\langle r^2 \rangle_p$, then NN^* excitation energy tends to be too small. Harvey[22] also examined the baryon spectrum versus the qq interaction. He fitted $\langle r^2 \rangle_p{}^{1/2}$ to 0.8 fm, and obtained $N^* \sim 1340$ MeV. His results in this respect are rather similar to ours. However, Harvey also used a singular qq potential and hence his oscillator constant was not determined dynamically. It is rather strange that practically no dynamical calculation of the ground *and* excited states of the $3q$ systems have been done. To our knowledge our NQM is the first such calculation except for those using very simple schematic potentials (the constant spin-dependent potential[23] and the harmonic-oscillator potential[24]).

Concerning the range r_0 of the "hyperfine interaction", it has been advocated[1,21] that the qq hyperfine interaction is very short-ranged. In the typical NQM calculations like those of Refs. 15 and 21, only the matrix elements of the hyperfine interaction with respect to prescribed wave functions are used. In this sense, the details of the r dependence of $v_s(r)$ and $v_t(r)$ have not been fully tested. In our calculation the optimum value of r_0 turned out to be ~ 1 fm, not a very short range. Let us point out that as r_0 becomes smaller $dv_s(r)/dr$ and hence S' increases. This results in an excessive increase of the magnitude of $\langle r^2 \rangle_n$.[25]

TABLE III. The parameters in the model. Potentials I and II are defined in Eqs. (3.5)–(3.7) and Eqs. (3.9)–(3.11), respectively.

Potential	I	I	I	II
Λ/m_π	0	2	2.5	2.5
m_u (MeV)	323.8	341.3	352.4	342.6
m_s (MeV)	529.8	517.9	501.3	469.7
r_0 (fm)	1.0	0.966	0.983	0.80
A (MeV fm^{-2})	49.5	42.7	40.7	15.64
B (MeV)	−140.3	−121.1	−105.5	−18.89
C (MeV)	158.5	173.4	171.2	341.7
C' (MeV)	0	0	0	0.03
ζ	1	1	1	0.66

TABLE IV. The "bare" quantities m in MeV, μ in nuclear magnetons, $\langle r^2 \rangle$ in fm^2. For example, for potential I and cutoff $\Lambda/m_\pi=2.5$, the bare mass of Σ, 1242 MeV (in parentheses), was determined from $m_\Sigma = m_\Sigma^c + \delta m_\Sigma$. The value obtained from the NQM calculation of Sec. II is 1235 MeV. Therefore the physical mass of Σ predicted by our model is 7 MeV less than the experimental value of 1193 MeV. The bare quantities are determined once the cutoff Λ is given. They are independent of parameters introduced in Sec. III, i.e., quark masses and those in the potential. For experimental values of the charge radii, we used $\langle r^2 \rangle_p = 0.74\ \mathrm{fm}^2$ (Ref. 19) and $\langle r^2 \rangle_n = -0.12\ \mathrm{fm}^2$ (Ref. 20).

Potential		I		I		I		II
Λ/m_π		0		2		2.5		2.5
m^c	N	(939)	939	(989)	989	(1047)	1047	1047
	Δ	(1232)	1231	(1280)	1280	(1340)	1340	1354
μ^c	p	(2.79)	2.81	(2.64)	2.64	(2.55)	2.55	2.47
	n	(−1.91)	−1.84	(−1.68)	−1.72	(−1.53)	−1.66	−1.56
$\langle r^2 \rangle^c$	p	(0.74)	0.61	(0.65)	0.64	(0.64)	0.63	0.72
	n	(−0.12)	−0.09	(−0.02)	−0.10	(−0.01)	−0.10	−0.15
g_A^c		(1.25)	1.60	(1.30)	1.58	(1.32)	1.58	1.47
S'			0.05		0.06		0.06	0.146
P			0.000		0.000		0.000	0.000
D			0.002		0.002		0.003	0.002
m^c	$N^*(1470)$		1389		1417		1461	1481
	$\Delta^*(1690)$		1445		1472		1518	1483
	Λ	(1115)	1110	(1142)	1140	(1175)	1175	1167
	Σ	(1193)	1187	(1217)	1208	(1242)	1235	1239
	Ξ	(1318)	1327	(1327)	1331	(1334)	1338	1332
	Σ^*	(1385)	1376	(1419)	1410	(1457)	1450	1459
	Ξ^*	(1530)	1522	(1554)	1540	(1571)	1560	1564
	Ω	(1670)	1670	(1670)	1670	(1670)	1670	1670

On the other hand, as was correctly emphasized by Isgur and Karl,[21] the p-wave baryon spectra (which we have not examined) seem to favor a short range for $v_s(r)$.

We now examine the hybrid model. First let us see how various pion effects depend on the momentum cutoff Λ. Table VI shows $\delta m_{N(N)}$, $P_{N(N)}$, $\mu'_{N(N)}$, and $\langle r^2 \rangle'_{N(N)}$ versus Λ. Note that δm is drastically reduced as Λ varies from $\sim m_N$ to 2 $\sim 3m_\pi$. For Λ we pointed out in Sec. I that $\Lambda \lesssim 3m_\pi$ follows from assumption 2; otherwise the radius of the pion interaction source $\rho(r)$ would be-

TABLE V. Magnetic moments in units of nuclear magnetons and $\langle r^2 \rangle_{p,n}$ in fm^2 calculated with potential I; μ^c is the magnetic moment of the baryon core, while $\mu=\mu'+\mu''$ includes the pion correction, and is to be compared with the experimental value. For the sources of the experimental data, see Table I of Ref. 11 and references quoted therein.

		$\Lambda/m_\pi=2$		$\Lambda/m_\pi=2.5$	
	Expt.	μ^c	μ	μ^c	μ
p	2.793	2.64	2.79	2.55	2.80
n	−1.913	−1.72	−1.96	−1.66	−2.03
Λ	-0.61 ± 0.01	−0.61	−0.61	−0.63	−0.64
Σ^+	2.33 ± 0.13	2.65	2.60	2.58	2.47
Σ^0		0.81	0.74	0.80	0.67
Σ^-	-1.41 ± 0.27	−1.03	−1.12	−0.98	−1.12
Ξ^0	-1.25 ± 0.02	−1.42	−1.41	−1.43	−1.41
Ξ^-	-0.75 ± 0.07	−0.49	−0.48	−0.54	−0.50
$\Lambda\Sigma$	-1.82 ± 0.22	−1.59	−1.67	−1.54	−1.68
$\langle r^2 \rangle_p$	0.74	0.64	0.73	0.63	0.72
$\langle r^2 \rangle_n$	−0.12	−0.10	−0.20	−0.10	−0.21

TABLE VI. The dependence of $\delta m_{N(N)}$ in MeV, P_{11}, $\mu_{N(N)}$ in nuclear magnetons and $\langle r^2\rangle'_{N(N)}$ in fm^2 on the cutoff momentum Λ.

Λ/m_π	$-\delta m_{N(N)}$	P_{11}	$\mu'_{N(N)}$	$\langle r^2\rangle'_{N(N)}$
2	29	0.09	0.24	0.19
2.5	61	0.16	0.37	0.26
3	110	0.25	0.53	0.33
4	274	0.49	0.86	0.50
5	550	0.81	1.21	0.69
6	967	1.21	1.58	0.90

come smaller than the radius of the core. We tried with $\Lambda \gtrsim 3m_\pi$ and found that this was indeed the case. We also examined the πN scattering dynamics and found that $\Lambda = 2 \sim 3m_\pi$ is the most appropriate range of Λ. Furthermore we will point out later that the fit for μ_p^c/μ_n^c deteriorates rapidly when Λ exceeds $3m_\pi$. We present the results for $\Lambda/m_\pi = 2$ and 2.5. We think that the overall results are about the best for $\Lambda = 2.5m_\pi$.[26]

Since the pion effects are rather small, we expect that the NQM part of the calculation goes in a way similar to that in the old NQM. This is indeed the case. As long as we do not attempt to fit g_A, potential I of Eqs. (3.6)–(3.8) works well except that the difficulty concerning N^* and Δ^* persists. In the hybrid model, however, this difficulty with N^* and Δ^* is somewhat lessened because the pion cloud tends to increase $\langle r^2\rangle_p$.

It is interesting that the bare masses, listed in Table IV, satisfy the Gell-Mann–Okubo mass formula well. Myhrer *et al.*[9] also found that the masses of their bags satisfy the same mass formula well. In detail, however, there are some differences between our results and those of Myhrer *et al.* Generally our δm is smaller than their corresponding δm. The ΔN mass difference is affected very little by the pion effect in our case. In Ref. 9 about 25 MeV of the ΔN difference is due to the pion contribution. For $N^*(1470)$ and $\Delta^*(1690)$, we have not estimated their bare masses, but we expect that the changes in their masses due to the pion are much smaller than those of N and Δ.

For g_A^c, the value determined from Eq. (2.41) together with the experimental value $g_A = 1.25$, is 1.32 for $\Lambda = 2.5m_\pi$. This should be compared with the NQM result for the case, $g_A^c = 1.58$. This means that our hybrid model with $\Lambda = 2.5m_\pi$ predicts $g_A = 1.51$ which is much too large. Note that the suppression of g_A due to the pion is very small; $Z = 1.25/1.32 = 0.95$. Since the pion effect is unimportant in suppressing g_A, we have to seek another mechanism. Could it be ascribed to a very large admixture of the D state? We tried with potential I to improve the fit for g_A, but as soon as we change the parameters in the potential to increase D, everything starts going haywire. In particular, the Δ mass turned out to be much more sensitive to D than N, and as D increases the ΔN mass splitting decreases. It is quite impossible to increase D to about 5% without seriously affecting the fit for the ΔN splitting.

Potential II is more flexible than potential I. As shown in Table IV the fit for g_A^c is somewhat improved with potential II. Contrary to our expectation, however, the improvement is due to the increase in the S' probability rather than D. Note that the spin-dependent term in potential II is much stronger than that in potential I (see Table III). The price for the improved g_A^c is that the ΔN mass splitting tends to be too large and the fit for $\langle r^2\rangle^c$ deteriorates. (See the note added in proof.)

Thomas *et al.*[10] claim that g_A can be well reproduced in their CBM calculation. What they did was to calculate g_A by using the Goldberger-Treiman relation $g_A = 2\sqrt{4\pi} f_\pi f_{\pi NN}/m_\pi$ where f_π (f in their notation) is the pion-decay constant, 93 MeV, and $f_{\pi NN}$ is f in our notation. By substituting $f_{\pi NN}{}^2 = 0.078$ in the above they obtained $g_A = 1.33$. Note however that the value of f_π has not been derived from the model. One should be able to calculate g_A directly based on the structure of the bag plus pion corrections.

Vento *et al.*[27] examined the possibility of a very strong D-state admixture in relation to g_A in their CBM approach. In their case the D state is caused mainly by the pion exchange between quarks, and hence the mechanism is different from what we have considered. They did not examine other consequences of such a strong D-state admixture. We suspect that the mass spectrum, particularly the octet-decuplet splitting, would be substantially affected.

The results for the magnetic moments with potential I are given in Table V. For μ_p and μ_n, S, S', P, D are all included, but for hyperons we used the formulas listed in Table II in which only the S contribution was considered. (In determining the wave function and energy we always include S, S', P, and D for all baryons.) If we include $S', \ldots$, the magnetic moments of the hyperons will be slightly reduced from those shown in Table V. We do not present results with potential II, which after all did not succeed in fitting g_A.

The agreement between the experimental and calculated values is overall reasonable except for Ξ^-. The results for $\Sigma^\pm$ are not very satisfactory but are still almost within the experimental errors. We do not know what is wrong with the magnetic moment of Ξ^-. Concerning the dependence on the cutoff momentum Λ, the ratio μ_p^c/μ_n^c increases as Λ increases. The increase is very slow up to $\Lambda \approx 2.5m_\pi$. But for $\Lambda \gtrsim 3m_\pi$ the ratio becomes appreciably greater than the canonical value $\frac{3}{2}$, and it becomes difficult to fit it by means of the NQM. This is another reason why the cutoff momentum Λ should be $\lesssim 3m_\pi$.

Our results for the magnetic moments are very similar to those of Théberge *et al.*[11] For the pion effects, essentially the same $\pi BB'$ interaction is used in the two calculations, although Théberge *et al.* used $v(k)=3j_1(kR)/kR$ with $R=0.82$ fm. The baryon core is treated differently; we use the NQM while they use the bag model. Nevertheless the results are very similar. Unlike Brown *et al.*[8] we do not have any serious discrepancy with the Σ^- magnetic moment.

Are there any quantities in which the difference between the NQM and the bag-type model shows up in a significant way? One such quantity is probably the charge form factor of the nucleon. If one takes the sharp boundary of the bag literally, the charge density of the nucleon exhibits large discontinuity at $r=R$ (see, e.g., Figs. 10 and 11 of Thomas *et al.*[10]). This is in contrast to a smooth charge density which follows from the NQM. This difference will be reflected in the behavior of the form factor $F(q^2)$ for large values of q^2.

Before ending, let us note that the effect of the bare Δ on the nucleon structure and πN scattering were examined in the 1950's[28] before the advent of the CLM. At that time, however, there were no clues as to $f_{N\Delta}$, μ_Δ, etc., and also the bare N and Δ were thought to be almost point particles.

Note added in proof. By making the tensor force in potential II very strong, it is actually possible to fit $g_A^c=1.32$ without appreciably jeopardizing other quantities. But a close scrutiny reveals that Δ and other decouplet baryons in that case are mainly in the D' state. Although this is an interesting possibility, we excluded it by requiring that the S-state probability in Δ be larger than 80%.

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APPENDIX A: SPIN AND ISOSPIN MATRICES

Let us discuss the isospin matrices $\vec{\tau}^{BB'}$ rather than $\vec{\sigma}^{BB'}$ because there are more isospin combinations for BB' than spin combinations. Of course in all relations τ can be replaced with σ.

(1) $I_B=I_{B'}=\frac{1}{2}$, I_B being the isospin of B; $(BB')=(NN)$, $(\Xi\Xi)$, $(\Xi\Xi^*)$, $(\Xi^*\Xi)$, $(\Xi^*\Xi^*)$. In this case, $\vec{\tau}^{BB'}=\vec{\tau}$, the conventional Pauli matrices.

(2) $I_B=\frac{3}{2}$, $I_{B'}=\frac{1}{2}$; $(BB')=(\Delta N)$. In this case,

$$(\tau_\alpha^{\Delta N})_{\beta\beta'}=(\tfrac{1}{2}\beta' 1\alpha \mid \tfrac{3}{2}\beta)\ ,$$

$$\tau_{\pm 1}^{\Delta N}=(\mp 1/\sqrt{2})(\tau_1^{\Delta N}\pm i\tau_2^{\Delta N}),\ \tau_0^{\Delta N}=\tau_3^{\Delta N}\ .$$

Here β and β' are the z components of the isospins of B and B', respectively.

(3) $I_B=\frac{1}{2}$, $I_{B'}=\frac{3}{2}$; $(BB')=(N\Delta)$. In this case,

$$\tau_\alpha^{N\Delta}=(\tau_\alpha^{\Delta N})^\dagger\ .$$

(4) $I_B=I_{B'}=\frac{3}{2}$; $(BB')=(\Delta\Delta)$. In this case,

$$(\tau_\alpha^{\Delta})_{\beta\beta'}=\sqrt{15}(\tfrac{3}{2}\beta' 1\alpha \mid \tfrac{3}{2}\beta)\ .$$

The diagonal elements of τ_α^Δ are (3, 1, -1, -3).

(5) $I_B=1$, $I_{B'}=0$; $(BB')=(\Sigma\Lambda)$, $(\Sigma^*\Lambda)$. In this case,

$$(\tau_\alpha^{\Sigma\Lambda})_\beta=\delta_{\alpha\beta},\ \tau_\alpha^{\Lambda\Sigma}=(\tau_\alpha^{\Sigma\Lambda})^\dagger\ .$$

(6) $I_B=I_{B'}=1$; $(BB')=(\Sigma\Sigma)$, $(\Sigma\Sigma^*)$, $(\Sigma^*\Sigma)$, $(\Sigma^*\Sigma^*)$. In this case,

$$(\tau_\alpha^{BB'})_{\beta\beta'}=\sqrt{2}(1\beta' 1\alpha \mid 1\beta)\ .$$

The diagonal elements of τ_3^Σ are (1, 0, -1).

APPENDIX B: THE PION-QUARK INTERACTION

An inevitable question that we have to face is if the baryon core is made up from quarks, how does the pion interact with the quarks? As we mentioned in Sec. I, the picture that we have in mind is that the pion interacts with the quarks in the core via a πqq interaction of the form of Eq. (1.1). The $\pi BB'$ can then be obtained in the same manner as the pion interaction with ^{3}H and ^{3}He can be derived from the interaction (1.1):

$$f_{BB'}\tau_\alpha^{BB'}\sigma_i^{BB'} = f_q \left\langle B \left| \sum \tau_\alpha \sigma_i \right| B' \right\rangle . \tag{B1}$$

Here f_q is the πqq coupling constant, $\sum$ stands for the summation over all u and d quarks, τ's and σ's are for the operators for the quarks, and $|B\rangle$ is an appropriate $3q$ wave function. If we use SU(6) wave functions for the quarks one obtains

$$f = \tfrac{5}{3} f_q \tag{B2}$$

and also the SU(6) ratios for $f_{BB'}/f$. If one uses more sophisticated wave functions, the factor $\frac{5}{3}$ in Eq. (A2) may be replaced by the right-hand side of Eq. (3.5).

In the calculation of the self-energies, if we ignore $m_{B'}^c - m_B^c$ in Eq. (2.4), i.e., if $\omega_{B(B')} = \omega$, then δm_B is reduced to

$$\delta m_B \to -f_Q{}^2 \Sigma_B I \left[\frac{1}{\omega} \right] , \tag{B3}$$

where Σ_B is the same quantity that appears in the self-energy calculation in the CBM [see Eq. (2.18c) of Myhrer *et al.*[9]], and is defined by

$$\Sigma_B = \left\langle \sum_{i,j} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \right\rangle . \tag{B4}$$

Here $\sum_{i,j}$ is the summation over all constituent ud quarks, including $i=j$. Let us also point out that

$$\sum_{B'} f_{BB'} C_{B(B')} \Big/ \sum_{B'} f_{NB'} C_{N(B')} = \Sigma_B / \Sigma_N , \tag{B5}$$

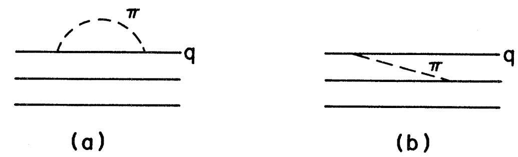

FIG. 7. The self-energy of the $3q$ system due to pion interaction.

where f's and C's are those given in Table I. Now the meaning of Eq. (B3) is quite clear. The self-energy δm_B contains both the quark self-energy, Fig. 7 (a), and the one-pion-exchange contribution, Fig. 7(b). The pion-current contributions to the magnetic moment μ' and the charge radius $\langle r^2 \rangle'$ also arise from both of the diagrams of Fig. 7, of course an external photon line being attached to the pion line this time.

There is one strange aspect of Eq. (B3). If one examines the quark self-energy which is contained in Eq. (B3), i.e., the terms with $i=j$ in Eq. (B4), it looks as if the πqq interaction has the same form factor $v(k)$ as that for the $\pi BB'$. The latter form factor represents the quark distribution in the baryon core. The πqq form factor would become essentially the same as that of $\pi BB'$ if the quarks somehow remain frozen in the "ground state" as far as the spatial part of the wave function is concerned.[29] We do not know how to justify this, and this should be regarded as part of our "working hypotheses."

Throughout this work we do not consider the size of the pion except that we allow the possibility that the core radius for the $\pi BB'$ interaction could be larger than the radius of the baryon core.[26] Actually, the size of the pion is comparable with that of the baryon core: $\langle r^2 \rangle_\pi{}^{1/2} \approx 0.66$ fm. If the πqq vertex has a form factor which corresponds to this pion size, the excitation of the quarks due to the pion interaction would be strongly suppressed. This could be a justification of the above working hypotheses.

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